Quantitative Aptitude

1. Find the value of (a+b) (b-a) if 0.976767676..........= $\frac{a}{b}$

Answer: 45011

Solution: $0.976767676 = \frac{9}{10} + \frac{76}{1000} + \frac{76}{10^5} + \frac{76}{10^7} + \ldots\ldots.$

$= \frac{9}{10} + \frac{76}{1000}\left(1 + \frac{1}{100} + \frac{1}{1000} + \ldots\ldots\right)$

$= \frac{9}{10} + \frac{76}{1000}\left(\frac{100}{99}\right)$

$= \frac{9}{10} + \frac{76}{990}$

$= \frac{967}{990}$

So, a = 967 and b = 990

⇨ (a+b) (b-a) = (967+990)(990-967)

= 45011

2. There are M employees working in a factory. The n^{th} employee does n units of work each day, working n hours for n number of days, where n varies from 1 to M. The total amount of work completed by them is 2025 units in M days. Find the number of employees working in the factory?

Answer: 9

Solution: It is given that nth employee do n units of work, working n hours for n days.

⇨ 1^{st} employee does 1*1 unit of work for 1 day. Total work done by 1^{st} employee is 1*1*1= 1 unit

- ⇨ 2^{nd} employee works for 2 days. Total work done by 2^{nd} employee
  2*2*2 = 8 units
- ⇨ Total work done by M^{th} employee is M*M*M = M^3 units
- ⇨ Work done is $1^3+2^3+3^3+4^3+ \ldots\ldots.+M^3 = 2025$
- ⇨ $\Sigma n^3 = [n(n+1)/2]^2$
- ⇨ On solving this we get m=9

3. For how many positive integral values of a does (an – a) is completely divisible by 10 for all positive values of n, where a and nare natural numbers between 1 and 1000.

Answer: 40

Solution: Any positive integral power of 0, 1, 5 and 6 always ends in 0, 1, 5 and 6 respectively.

Number of numbers ending in 0 = 10

Number of numbers ending in 1 = 10

Number of numbers ending in 5 = 10

Number of numbers ending in 6 = 10

Total integral values of a = 40

4. A, B and C divides a work amongst themselves in the ratio 2:3:5. Their efficiencies of doing the work are in the ratio 1:2:3. It takes A 12 days to complete his part. What is the amount of work completed by them in 8 days from the start?
   a. 4/5
   b. 3/5
   c. 31/40
   d. 31/45

Solution: Proportion of work done by A, B and C is 2/10, 3/10 and 5/10 respectively.

- ⇨ Rate of doing work are in the ratio = 1:2:3
  ∴ Time required to complete the work are in the ratio = 1: 1/2: 1/3
  Let A complete the entire work in ‘x’ days
  ∴ Work done by A in 1 day = 1/x
  According to question,

(1/x)*12 = 2/10

⇨ X= 60 days

Thus, B and C will complete the work in 30 and 20 days respectively.

∴ Total work done by three of them in 8 days = (1/60) + (2/60) + (3/60)*8 = 4/5

5. For the given equation $x^9+5x^8-x^3+7x+3=0$, how many maximum real roots are possible?
   a. 3
   b. 5
   c. 6
   d. 7

Solution: let $f(x) = x^9+5x^8-x^3+7x+3=0$

We can solve this question easily using Descartes' Rule

According to Descartes' rule maximum number of positive real roots = number of sign Changes in f(x) = 2

Similarly, Maximum number of negative real roots = number of sign changes in f (-x)

Note: To find f (-x) replace "x" by "-x" in each instance

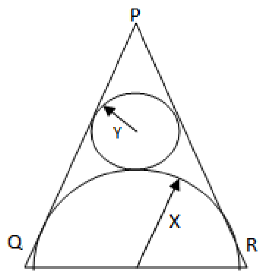
∴f (-x) = $- x^9+5x^8+x^3-7x+3$

Maximum number of negative real roots = number of sign changes in f (-x) = 3

Zero cannot be a root because constant part is also involved in equation.

So maximum number of real roots = 2 + 3 = 5.

6. In the given equilateral triangle, the smaller circle and the bigger semi-circle are inscribed inside. What is the value of $\frac{X}{Y}$?

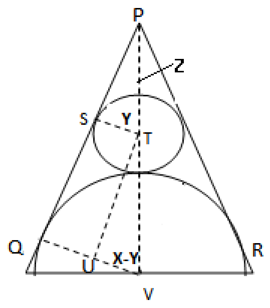


   a. 3
   b. 2
   c. 4
   d. 5

Solution : Line PV bisects line QR. ∆PST and ∆TUV are similar.

Hence, $\frac{Z}{Y+X} = \frac{Y}{X-Y}$; => $\frac{Y}{Z} = \frac{X-Y}{X+Y}$;

But in ∆PST, $\frac{Y}{Z} = \sin 30 = \frac{1}{2}$.



Hence, $\left(\frac{1}{2}\right) = \frac{X-Y}{X+Y}$; $=> \frac{X}{Y} = 3$.

7.If 2a + 5b = 103. How many pairs of positive integer values can a, b take such that a > b?

Answer: 7

Solution:

Let us find the one pair of values for a, b.

a = 4, b = 19 satisfies this equation.

2×4 + 5×19 = 103.

Now, if we increase ‘a’ by 5 and decrease ‘b’ by 2 we should get the next set of numbers. We can keep repeating this to get all values.

a = 4, b = 19

a = 9, b = 17

a = 14, b = 15

..

and so on till

a = 49, b = 1

Number of pairs that satisfy the condition a > b are 7.

8. If x, y, z are positive numbers such that x + [y] + {z} = 3.8, [x] + {y} + z = 3.2, {x} + y + [z] = 2.2, where [p] denotes the greatest integer less than or equal to p and {p} denotes the fractional part of p, e.g. [1.23] = 1, $\{1.23\} = \frac{23}{100}$. The numerical value of $[x^2 + y^2 + z^2]$ is
   (a) 7
   (b) 8
   (c) 9
   (d) None of these

   Solution:
   Adding the three equations we get, 2(x + y + z) = 9.2 => x + y + z = 4.6 subtracting first eq. from the above eq. we get {y} + [z] = 0.8 => {y} = 0.8 and [z] = 0, subtracting second eq. from the above eq. {x} + [y] = 1.4 => {x} = 0.4 and [y] = 1, subtracting third eq. from the above eq. [x] + {z} = 2.4 => [x] = 2, and {z} = 0.4 => x = 2.4, y = 1.8 and z = 0.4. Hence, choice (c) is the right answer.

9. Two guys Abhinav and Bineesh are walking downward and upward respectively on a descending escalator. Abhinav takes three steps in the same time when Bineesh takes two steps. When Abhinav covers 90 steps he gets out of the escalator while Bineesh takes 80 steps to get out of the escalator. If they start from opposite ends using the same escalator, find the difference in steps covered by them when they meet.
   (a) 48
   (b) 20
   (c) 30
   (d) 24

   Solution: Let the escalator move x steps when A walks down 90 steps. Total number of steps on a stationary escalator = x + 90.
   When A takes 90 steps, B should have taken 60 steps and the escalator x steps. So when B takes 80 steps, the escalator should have taken 4/3*x steps.
   So, 4/3*x + 80 = x + 90 = Total number of steps in the escalator when it is stationary. So x = 30.
   Hence, total number of steps = 120.
   By the time they meet, together they will 120 steps in the ratio 3:2. i.e. 72 and 48 steps. So difference is 24.

10. A number is said to be a 'zeroth number' if the sum of the squares of its digits ends in a zero. How many two-digit 'zeroth numbers' are there?
   a) 14
   b) 12
   c) 13
   d) 17

   Solution: Let the number be 10x + y.

Then, $1 \leq X \leq 9$, $0, \leq Y \leq 9$ and $x^2 + y^2 = 10k$.

The required number = 17

x	x^2	y^2 must end in	Possible value of y
1	1	9	3, 7
2	4	6	4,6
3	9	1	1,9
4	16	4	2,8
5	25	5	5
6	36	4	2,8
7	49	1	1,9
8	64	6	4,6
9	81	9	3,7

11. Find the minimum value of a + b + c + d in the equation $x^5 - ax^4 + bx^3 - cx^2 + dx - 243 = 0$, if it is given that the roots are positive real numbers

a) 780
b) 660
c) 840
d) 720

Solution: There are 5 roots in this equation.

Here the product of the roots is given as 243. Hence the sum will be minimum when the roots are equal. In fact a , b , c and d will be minimum when the roots are equal.

$243 = 3^5$

Hence, the roots will be 3,3,3,3 and 3

a= sum of the roots = 15

b=Sum of product of roots taken 2 at a time= (3x3) + (3x3)…10 times or = ${}^5C_2 \times 9 = 90$

c= Sum of product of roots taken 3 at a time= (3x3x3) + (3x3x3)…5 times or = ${}^5C_3 \times 27 = 270$

d=Sum of product of roots taken 4 at a time= (3x3x3x3)….5 times= 405

Hence, a + b + c + d = 780

12. Consider two points from which two persons start walking towards each other. They cross each other after time T1. They went further to the opposite points to cross each other again after time T2. Then T2 =

a) 2T1

b) T1

c) 3T1

d) $\frac{3}{2}T_1$

Solution: Suppose the distance between the two points it'd'.

When the persons meet for the first time, total distance covered = d

When persons meet for the second time, total distance covered = d + 2d

Distance = d, time taken = T1; distance = 2d, time taken = 2T1 = T2

Hence Option (a)

13. A shopkeeper keeps 100 marbles in two boxes, A and B, such that Box A contains only red and Box B contains only green marbles. Barring an equal number of marbles from each box, he sold all the others and earned Rs. 25 and Rs. 15 from Box A and Box B respectively. Later, he sold half of the remaining marbles. His total earnings from all the transactions accumulated to Rs. 45. What is the cost of each marble – red or green?

a] 40 paise

b] 25 paise

c] 75 paise

d] 50 paise

Solution: Let the number of red and green marbles be a and b respectively.
Let the selling price of each marble be Rs.e.
Let the equal number of marbles not sold the first time = c.
Then, e(a – c) = 25 … (i)
e(b – c) = 15 … (ii)
The number of marbles that remained with the shopkeeper = 2c
e(a – c) + e(b – c) + ec = 45
=>ec = 5 …(iii) (from (i) and (ii))
Also, (100 – c)e = 45
=> 100e – 5 = 45 (from (iii))
=>100e = 50
=> e = 0.5 = 50 Paise.

14. In how many different ways can the six faces of cube be painted by 6 different colours?

Answer: 30

Solution: Mark a particular face X. (Later on, we will have to divide whatever answer we get by 6 because all 6 faces are identical at this point, and we have chosen one out of 6 randomly).

Put a color on face X. There are 6 ways to do this.

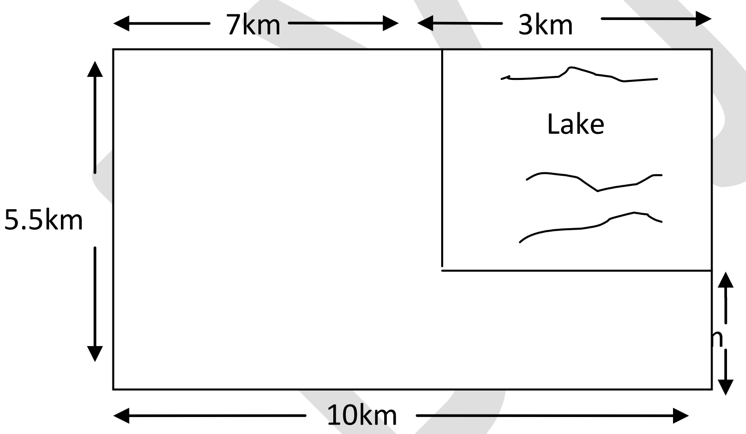
Put another color on the face opposite X. There are 5 ways to do this.

Arrange the remaining 4 colors in the remaining four places - there are 3! ways to do this (circular permutation of 4 colours = (4-1)!

Total = $\frac{[6 .5 .3!]}{6}$

= 30

15. A plane MH370 is reported to have crashed somewhere in the rectangular region shown in fig. The probability that it crashed inside the lake (shown in the figure) is $\frac{P}{110}$. Find the value of 'P'.



Answer: 21

Solution: Area of the entire region where the helicopter can crash = (5.5X10)km^2 =55 km^2

Area of the lake = (3X3.5) km^2 =10.5 km^2

$\therefore$ Probability that helicopter crashed inside the lake = $\frac{10.5}{55} = \frac{21}{110}$

So, P = 21

16. What is the number of digits in 5^{40}? (Given $\log_{10} 2$ = 0.3010)

(a) 28
(b) 27
(c) 14
(d) 13

Solution: Log 5^{40} = 40 log 5 = 40 x $\left[\log\left(\frac{10}{2}\right)\right]$ = 40 $(\log 10 - \log 2)$
= 40 $(1 - 0.3010)$ = 40 x 0.6990 = 27.96.
Characteristic = 27. Hence, the number of digits in 5^{40} is 28.

17. Digit sum (DG) is defined as the sum of the digits of a number till you get a single digit number. Eg) DG(40)=4+0=4. DG(345)=DG(12)=DG(3)=3. How many positive integers (n) are there between 500 and 1500 such that DG(N) =7?
a) 111
b) 222
c) 333
d) 444

Solution:
Approach 1: Logic
The first number which satisfies this is 502. The next number will be 511 and so on. The terms will fall in an Arithmetic Progression with common difference=9. Last term = 1492
Number of terms = $\frac{990}{9} + 1 = 111.$

Approach 2: Shortcut in a Shortcut!
Digit sum of numbers will fall in an Arithmetic progression with a common difference of 9. Thus, there can be $\frac{1000}{9} =$ 111 or 112 such numbers. Since only 111 is there in the answer options, you can mark it directly.

18.) A king took a glass filled with red wine and drank $1/5^{th}$ of its contents. When the king looked away, the court jester refilled the glass by adding water to the remaining red wine and then stirred it. The king drank $¼^{th}$ of this liquid mixture. When the king looked away again, the court jester refilled the cup with more water and stirred it. The king then drank $1/3^{rd}$ of this liquid mixture. When the king looked away for the third time, the court jester refilled the cup with more water. What percent of this final mixture is red wine?
a) 50%
b) 40%
c) 30%
d) 25%

Solution: -
Method-1
Let the capacity of glass be 100 ml.

	Red Wine	Water
Initial	100	0
After the first	80	20

operation		
After the second operation	60	40
After the third operation	40	60

Method 2 – Quantity of red wine left after 3 operations, when the glass originally contains 100% wine from which 1/5, 1/4 and 1/3 of total wine are taken out each time and replaced is

= $100\left(1-\frac{1}{5}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{3}\right)$ = 40%

19. Two clocks are set to show the correct time at 1:00am on a day. One clock loses 3 minutes in an hour and the other clock gains 3 minutes in an hour. Exactly after how many days will both the watches show the correct time? Assume that is a 12 hour clock.

Answer: 30

Solution:

Every clock shall display the correct time when it is late or fast by hours which are a multiple of 12.

As per the question, the first clock loses 3 minutes in an hour. Therefore, to lose 12 hours, it will take $\frac{12x60}{3} = 240$ hours = 10 days.

Also, the second clock gains 2 minutes in an hour. Therefore, to gain 12 hours, it will take: $\frac{12x60}{2} =$ 360 Hours = 15 days.

Therefore, after 30 days (LCM of 15 and 10), both the watches will show the correct time.

20. A function f(x) is defined as:

$f(x) = x^{15} – 2013x^{14} + 2013x^{13} – 2013x^{12} + ... – 2013x^2 + 2013x$. Find the value of f(2012).

(a) 2011

(b) 2012

(c) 2013

(d) 2014

Solution: -

Method 1- Conventional Approach

$f(x) = x^{15} – 2013\{x^{14} – x^{13} + x^{12} – .... + x^2 – x\}$ or

$f(x) = x^{15} – 2013\{–x + x^2 – x^3 + ... – x^{13} + x^{14}\}$

$f(x) = x^{15} – \left(\frac{2013x(x^{14}-1)}{(x+1)}\right)$

Now, Put x = 2012

$f(2012) = (2012)^{15} – \left(\frac{2013.2012(2012^{14}-1)}{(2012+1)}\right)$

$f(2012) = (2012)^{15} – 2012(2012^{14} – 1)$

f(2012) = 2012.

Method 2- Assumption

Assume 2012 = n and 2013 = (n+ 1). Put x=1. Now replace the options in terms of "n" as

a) (n – 1) b) n c) (n + 1) d) (n + 2)

f(1) = 1 – 2 + 2 – 2 + 2 – ………………….–2 + 2

f(1) = 1. Options (a),(c) & (d) can be eliminated as only option (b) gives us 1.

21. A sphere is inscribed in a cube that has a surface area of 24 m^2. A second cube is then inscribed within the sphere. What is the surface area in square metres of the inner cube?

Answer: 8

Solution: Given that surface area of the original cube = 24 units.

∴ Length of each side of the cube = $\sqrt{\frac{24}{6}}$ = 2 units.

The radius of the biggest possible sphere that can be kept inside the cube of each side 2 Units.

= $\left(\frac{2}{2}\right)$ = 1 unit.

Now, the diagonal of the second cube inscribed in the sphere = Diameter of sphere

$\Rightarrow\sqrt{3}$ (length of each side of second cube) = 2 units.

⇒ length of each side of second cube =$\frac{2}{\sqrt{3}}$ unit

∴ surface area of the inner cube = 6 x$\left(\frac{2}{\sqrt{3}}\right)$

=6 x$\frac{4}{3}$ = 8 units

22. When a two digit number is rotated by 180° a different two digit number is obtained. What is the probability of such an occurrence ?

a. 2/15

b. 5/18

c. 8/45

d. 2/9

Solution: Total number of two digit numbers : 9 x 10 = 90.

Digits which are symmetrical : 1, 8, 0.

Digits which form a complementary symmetrical pair (i.e. when one is rotated, the other is obtained) – 6, 9.

We cannot use 0 as a digit in any place as we will be obtaining a single digit number upon reversing.
Total possible numbers using digits 1, 8, 6, 9 : 4 x 4 = 16
But the nos. 11, 88, 69, 96 when rotated by 180° reveals the same number again.
Thus, total no. of possible outcomes = 16-4 = 12
$\therefore$ Probability of such an occurrence = $\frac{12}{90} = \frac{2}{15}$

23. A merchant buys 80 articles, each at Rs. 40. He sells n of them at a profit of n% and the remaining at a profit of (100 – n)%. What is the minimum profit the merchant could have made on this trade?

Answer: 1580

Solution:

CP = 80 × 40
Profit from the n objects = n% × 40 × n.
Profit from the remaining objects = (100 – n)% × 40 × (80 – n).
We need to find the minimum possible value of n% × 40 × n + (100 – n)% × 40 × (80 – n).
Or, we need to find the minimum possible value of $n^2 + (100 - n)(80 - n)$.
Minimum value of $n^2 - 90n + 4000$

To find the minimum value , we can make the above expression as $n^2 - 90n + 2025 - 2025 + 4000 = (n-45)^2 + 1975$
This reaches minimum when n = 45.

When n = 45, the minimum profit made
45% × 40 × 45 + 55% × 40 × 35 = Rs. 1580

24. On mixing three varieties of a cereal costing Rs. 18/kg, Rs. 20/kg and Rs 24/kg, a new variety of cereal costing Rs. 21/kg was obtained. If 2 kg of the variety costing 24/kg is present in the mixture. Then the amount of variety 20/kg in the mixture is

Answer: 3

Solution: (18*m + 20*n + 2*24)/(m + n + 2) = 21

⇨ 18m + 20n + 48 = 21m + 21n + 42

⇨ 3m + n = 6

There are 2 possibilities

If m = 1, then n = 3

Or if m = 2, then n = 0(which is not possible)

Hence, n = 3

25. If x > 0 and (x^2+1/x^2) – 4(x+1/x) + 23/4 = 0, then how many values does (x+1/x) attain?

Answer: 1

Solution: Let (x+1/x) = a, Then $(x+1/x)^2 = x^2 + 1/x^2 + 2$

i.e. $x^2 + 1/x^2 = a^2 - 2$

The given equation becomes, $a^2 - 2 - 4a + 23/4 = 0$

=> $4a^2 - 8 - 16a + 23 = 0$

=> $4a^2 - 16a + 15 = 0$

=> (2a–5)(2a–3) = 0

=> a = 3/2, 5/2

This implies that x + 1/x = 5/2, 3/2

It is given that x > 0

The minimum value of x + 1/x = 2, hence x+1/x = 5/2

Only one solution is possible

26.) If $f(x) = \frac{x}{x+1}$, determine the value of g(2) if $g(x) = f\left[\frac{1}{x+1}\right] - \frac{1}{f(x+1)}$.

a) 1/12

b) 13/12

c) -13/12

d) -1/12

Method 1: Conventional Approach

$$f\left[\frac{1}{x+1}\right] = \frac{\left[\frac{1}{x+1}\right]}{\left[\left\{\frac{1}{x+1}\right\} + 1\right]} = \frac{1}{x\ +\ 2}$$

$$\frac{1}{f(x+1)} = \frac{1}{\left[\frac{x+1}{x+2}\right]} = \frac{x+2}{x+1}$$

$$g(x) = f\left[\frac{1}{x+1}\right] - \frac{1}{f(x+1)} = \left[\frac{1}{x+2}\right] - \left[\frac{x+2}{x+1}\right]$$

$$g(2) = \left(\frac{1}{4}\right) - \left(\frac{4}{3}\right) = -\frac{13}{12}$$

SHORTCUT:-Direct substitution

When x =2

$G(2) = f\left(\frac{1}{3}\right) - \frac{1}{f(3)} = \left(\frac{1}{4}\right) - \frac{4}{3} = -\frac{13}{12}$. Answer is option (c)

27. How many positive integers greater than 100 and less than1260 (1260 included) are relatively prime to 1260?

Method 1: Conventional Approach

1260 = $2^2 * 3^2 * 5 * 7$

We need to find the number of positive integers 1260 which are not divisible by 2, 3, 5 or 7

Let A be the set consisting of multiples of 2 < = 1260; B be the set consisting of multiples of 3; C be the set consisting of multiples of 5 & D of multiples of 7.

$A = \frac{1260}{2} = 630 \quad B = \frac{1260}{3} = 420 \qquad C = \frac{1260}{5} = 252 \qquad D = \frac{1260}{7} = 180$

$A \Pi B = \frac{1260}{2} * 3 = 210 \quad A \Pi C = \frac{1260}{2} * 5 = 126 A \Pi D = \frac{1260}{2} * 7 = 90$

$$B \Pi C = \frac{1260}{3} * 5 = 84$$

$B \Pi D = \frac{1260}{3} * 7 = 60 \quad C \Pi D = \frac{1260}{5} * 7 = 36 \qquad A \Pi B \Pi C = \frac{1260}{2} * 3 * 5 = 42$

$A \Pi B \Pi D = \frac{1260}{2} * 3 * 7 = 30 \quad A \Pi C \Pi D = \frac{1260}{2} * 5 * 7 = 18$

$$B \Pi C \Pi D = \frac{1260}{3} * 5 * 7 = 12$$

$$A \Pi B \Pi C \Pi D = \frac{1260}{2} * 3 * 5 * 7 = 6$$

So, A U B U C U D = 630 + 420 + 252 + 180 – 210 – 126 – 90 – 84 – 60 – 36 + 42 + 30 + 18 + 12 – 6 = 972

So, 1260 – 972 = 288 integers are relatively prime to 1260. Out of these we have 25 prime numbers which are less than 100 and 21 numbers which are coprime to 1260(because 2,3,5,7 are factors). So we get 288 -21 = 267 numbers

Method 2: Shortcut: Using the concept of Euler's number

The number of positive integers <= 1260 are relatively prime to 1260 can be directly found by finding the Euler's number of 1260

$N\left(1 - \frac{1}{2}\right) x \left(1 - \frac{1}{3}\right) x \left(1 - \frac{1}{5}\right) x \left(1 - \frac{1}{7}\right)$ (Where 2,3,5,7 are the prime factors of 1260)

$,1260\ x \frac{1}{2} x \frac{2}{3} x \frac{4}{5} x \frac{6}{7} = 288$

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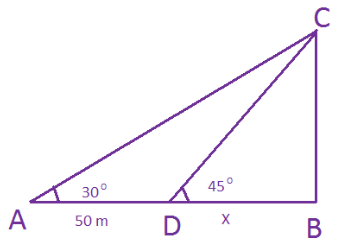
We have 21 primes less than 100 which are coprime to 1260. (25 primes and 4 primes 2,3,5,7 which are factors of 1260)

We have 288-21 = 267 numbers.

28. Anil looked up at the top of a lighthouse from his boat, and found the angle of elevation to be 30^o. After sailing in a straight line 50 m towards the lighthouse, he found that the angle of elevation changed to 45^o. Find the height of the lighthouse.
    a) 25
    b) 25√3
    c) 25(√3-1)
    d) 25(√3+1)

    Solution:



Now since tan 45= 1, BC=DB= x

Also, tan 30 =$\frac{1}{\sqrt{3}}$= $\frac{x}{50+x}$

Thus x(√3 -1)= 50 or x= 25(√3 +1) m

29. x, y, and z are three angles of a triangle. Which of the following set of values of x, y, and z satisfies log(x*y*z) = 3 log x + 4 log 2, given that x, y, and z are integers?
    1. 30^0, 70^0, and 80^0
    2. 30^0, 60^0, and 90^0
    3. **20^0, 80^0, and 80^0**
    4. 100^0, 40^0 ,40^0

Solution: Conventional approach :

logx + logy + logz = 3logx + 4log2

2logx = logy + logz – 4log2

2logx = logyz – 4log2

2logx = log$\frac{yz}{16}$

→ x = $\sqrt{yz}$ /4

It is given that x is an integer, therefore yz must be a perfect square. Only possible combination that satisfies the condition is (c) or (d). Only (c) satisfies the relation.

Alternate Approach :

Substituting the options, we get only (c) satisfies the relation.

30. Let T_n be defined as the sum to n terms of the series T. Find T_{34}

$T = 2^{88} - 2^{87} - 2^{86} - 2^{85} \dots \dots$

a) 2^{54}
b) 2^{55}
c) 2^{56}
d) $2^{34} - 1$

Solution:

$T_1 = 2^{88}$

$$T_2 = 2^{88} - 2^{87} = 2^{87}(2-1) = 2^{87}$$

$$T_3 = 2^{88} - 2^{87} - 2^{86} = 2^{86}\ (2^2 - 2^1 - 1) = 2^{86}$$

Thus, the pattern is

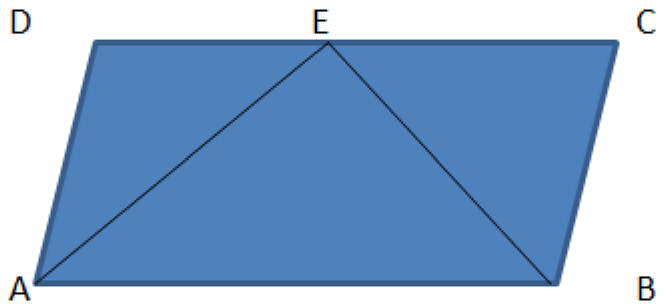
$$T_n = 2^{89-n}$$

Solution : B

31. ABCD is a parallelogram, E is a point on CD such that BE bisects angle ABC. BE = 12 cm, angle BEC = angle AED. Find ED, if CE = 9 cm

Answer :7

Solution:



∠ABE = ∠BEC (alternate angles)
∠AED = ∠EAB
∠AED = ∠BEC
∠EAB = ∠EBA
As BE bisects ∠ABC
∠EBC = ∠ABE
ΔABE ~ ΔBEC
So, BE/CE = AB/BE
$BE^2 = (CE + ED)CE$
Substituting BE = 12cm and CE = 9cm, we get ED = 7cm

32. The number of values of x which satisfies the equation, $-|-\sin x| = |x^2 - 5x + 6|$?

Answer: 0

Solution: Mod of any function is always equal to or greater than zero and $-|-\sin x|$ is always negative or zero, so we just need to check value of both the functions are not zero for same x.

$|x^2 - 5x + 6|$ will be zero for 2,3 which is not the zero of $|-\sin x|$

33. The value of a gold bar is directly proportional to the square of its weight. If a gold bar weighing 8 kg breaks into 2 pieces, its total value decreases by 3/8 times. Find the weights of the two pieces.

(a) 4 kg, 4 kg
(b) 5 kg, 3 kg
(c) 6 kg, 2 kg
(d) 4.5 kg, 3.5 kg

Solution:

Value (V) = $Kx\ 8^2 = 64\ K$
Let weights are x kg and (8 – x) kg.
$V1 = Kx^2$ and $V2 = K\ (8 - x)^2$
New value = $K[x^2 + (8 - x)^2]$
Given $K[x^2 + (8 - x)^2] = 5/8(64k)$
$x^2 + 64 + x^2 - 16x = 40.$
$2x^2 - 16x + 24 = 0$
$x^2 - 8x + 12 = 0 \Rightarrow (x - 6)\ (x - 2) = 0$
x = 6, x = 2.

Alternatively

Assume k=1; the solution becomes simpler

Value is currently 64 and it becomes $5/8^{th}$ of that which is 40 = 36+4 (This is the only possible breakup with 2 square numbers) hence, answer is option (c)

34. If 2a + 3b + 4c = 4608, then what is the maximum value of $(a^2b^3c^4)^{1/81}$?

Answer: 2

Solution: We know that A.M ≥ G.M,

=> $(a + a + b + b + b + c + c + c + c)/9 \geq (a^2b^3c^4)^{1/9}$
=> $2a + 3b + 4c / 9 \geq (a^2b^3c^4)^{1/9}$
=> $4608 / 9 \geq (a^2b^3c^4)^{1/9}$
=> $512 \geq (a^2b^3c^4)^{1/9}$
=> $2^9 \geq (a^2b^3c^4)^{1/9}$

Maximum value is 2