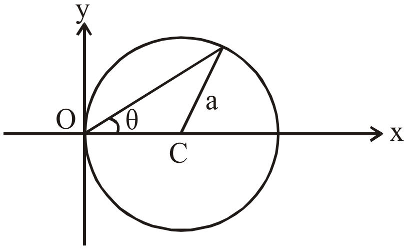
PART A – PHYSICS

JEE (Main) – 2014 (ON LINE)

1. A particle is moving in a circular path of radius a, with a constant velocity v as shown in the figure. The center of circle is marked by 'C. The angular momentum from the origin O can be written as :-



(1) va cos 2θ
(2) va (1 + cos θ)
(3) va (1 + cos 2θ)
(4) va

Ans. (3)

Sol. (v cos θ) × 2 turns

2cos va –1 = cos 2w

2rv $\cos^2$v

va (1 + cos 2θ)

2. A lamp emits monochromatic green light uniformly in all directions. The lamp is 3% efficient in converting electrical power to electromagnetic waves and consumes 100 W of power. The amplitude of the electric field associated with the electromagnetic radiation at a distance of 5 m from the lamp will be nearly :-

(1) 4.02 V/m (2) 2.68 V/m
(3) 5.36 V/m (4) 1.34 V/m

Ans. (2)

Sol. $I = \langle u \rangle c = \frac{P}{4\pi r^2}$

$$\frac{\epsilon_0 \epsilon_0^2}{2} C = \frac{P}{4\pi r^2}$$

$$\epsilon_0^2 = \frac{P}{2\pi r^2 \epsilon_0 C}$$

$$\epsilon_0 = \sqrt{\frac{P}{2\pi r^2 \epsilon_0 C}} = \sqrt{\frac{3}{2\pi(5)^2 \times 8.85 \times 10^{-12} \times 3 \times 10^8}}$$

ϵ_0 = 2.68 volt/metre

3. Hot water cools from 60°C to 50°C in the first 10 minutes and to 42°C in the next 10 minutes. The temperature of the surroundings is :-

(1) 25°C (2) 20°C
(3) 10°C (4) 15°C

Ans. (3)

Sol. $\frac{10}{10}$ = K [55 – T]

$\frac{8}{10}$ = K [46 – T]

T = 10°C

for average, interval should be small.

$\frac{60-50}{10}$ = K [55 – T]

$\frac{18}{20}$ = K [51 – T]

$$\frac{10 \times 20}{10 \times 18} = \frac{55-T}{51-T}$$

510 – 101 = 55 × 9 – 91

15 = 1

4. In an experiment of single slit diffraction pattern, first minimum for red light coincides with first maximum of some other wavelength. If wavelength of red light is 6600 Å , then wavelength of first maximum will be :-

(1) 5500 Å (2) 3300 Å
(3) 6600 Å (4) 4400 Å

Ans. (4)

Sol. $\frac{\lambda_R b}{a} = \frac{3}{2}\lambda\frac{b}{a}$

$\lambda = \frac{2\lambda_R}{3} = \frac{2}{3} \times 6600$

λ = 4400 Å

5. A 4 g bullet is fired horizontally with a speed of 300 m/s into 0.8 kg block of wood at rest on a table. If the coefficient of friction between the block and the table is 0.3, how far will the block slide approximately :-

(1) 0.758 m (2) 0.19 m
(3) 0.569 m (4) 0.379 m

Ans. (4)

Sol. $\frac{4}{1000}\times 300=\frac{0.8}{10}v$

$v = \frac{3}{2} = 1.5 m/s$

$v^2 - Cl^2 = 2as$

$\frac{g^3}{4} = 2 \times 3 \times 5$

$\frac{3}{8} = 5 = 0.375$ m

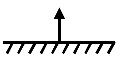
6. A source of sound A emitting waves of frequency 1800 Hz is falling towards ground with a terminal speed v. The observer B on the ground directly beneath the source receives waves of frequency 2150 Hz. The source A receives waves, reflected from ground, of frequency nearly : (Speed of sound =343 m/s)

(1) 2400 Hz (2) 2500 Hz

(3) 1800 Hz (4) 2150 Hz

Ans. (2)

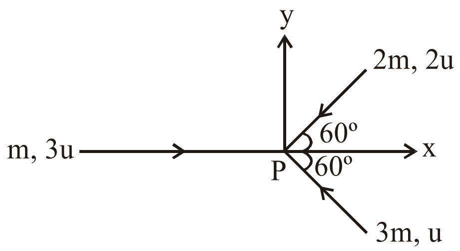
Sol. v ↓ O source $\left|\frac{343}{343+v}\right|1800 = 2150$



$343 = 1.2 \times 343 + 1.2v = 57.16$

$\left|\frac{343+57}{343}\right|\times 2150$

7. Three masses m, 2m and 3m are moving in x-y plane with speed 3u, 2u, and u respectively as shown in figure. The three masses collide at the same point at P and stick together. The velocity of resulting mass will be :-



(1) $\frac{u}{12}\left(-\hat{i}-\sqrt{3}\,\hat{j}\right)$ (2) $\frac{u}{12}\left(-\hat{i}+\sqrt{3}\,\hat{j}\right)$

(3) $\frac{u}{12}\left(\hat{i}-\sqrt{3}\,\hat{j}\right)$ (4) $\frac{u}{12}\left(\hat{i}+\sqrt{3}\,\hat{j}\right)$

Ans. (1)

Sol. $p_i = p_y$

$m3u\hat{i} + 3m\,[-u \cos 60°\hat{i} + u \sin 60°\hat{j}]$

$+ 2m\,[-2u \cos 60°\hat{i} - 2u \sin 60°\hat{j}]$

$3mu\hat{i} + 3m\left[-\frac{v}{2}\hat{i}+\frac{0\sqrt{3}}{2}\hat{j}\right]+2m\left[-\frac{2u}{2}\hat{i}-\frac{2u\sqrt{3}}{2}\hat{j}\right]$

$3u - \frac{3v}{2} - 2u$ $\frac{3\sqrt{3}}{2}\hat{j}-2\sqrt{3}$

$-\frac{u}{2}$ $-\frac{\sqrt{3}}{2}\hat{j}$

8. For sky wave propagation, the radio waves must have a frequency range in between :-

(1) 45 MHz to 50 MHz

(2) 35 MHz to 40 MHz

(3) 1 MHz to 2 MHz

(4) 5 MHz to 25 MHz

Ans. (4)

9. In the experiment of calibration of voltmeter, a standard cell of e.m.f. 1.1 volt is balanced against 440 cm of potentiometer wire. The potential difference across the ends of resistance is found to balance against 220 cm of the wire. The corresponding reading of voltmeter is 0.5 volt. The error in the reading of voltmeter will be :-

(1) – 0.15 volt (2) 0.5 volt

(3) 0.15 volt (4) – 0.05 volt

Ans. (4)

Sol. Potential gradient

$\eta = \frac{1.1}{440}$

$V_{actual} = \frac{1.1}{440}\times 220$

$= 0.55$ volt

Error = 0.5 – 0.55

= – 0.05 volt

10. A piece of bone of an animal from a ruin is found to have ^{14}C activity of 12 disintegrations per minute per gm of its carbon content. The ^{14}C activity of a living animal is 16 disintegrations per minute per gm. How long ago nearly did the animal die ? (Given half life of ^{14}C is $t_{1/2}$ = 5760 years):-

(1) 3291 years (2) 1672 years

(3) 4453 year (4) 2391 years

Ans. (4)

Sol. $R = \dfrac{R_0}{2^{t/T_H}}$

$$2^{t/T_H} = \frac{R_0}{R} = \frac{16}{12} = \frac{4}{3}$$

$$\frac{t}{T_H} = \log\frac{4}{3}$$

$$t = \frac{T_H}{\log 2}\ [2 \log 2 - \log 3]$$

$$= \frac{5760\ y\ [2 \times 0.30 - 0.48]}{0.3010}$$

$$= \frac{5760\ y \times 0.12}{0.3} \approx 2391\ y$$

11. The space between the plates of a parallel plate capacitor is filled with a 'dielectric' whose 'dielectric constant' varies with distance as per the relation :

$K(x) = K_0 + \lambda x$ (λ = a constant) The capacitance C, of this capacitor, would be related to its 'vacuum' capacitance C_0 as per the relation :-

(1) $C = \dfrac{\lambda}{d.\ell n(1 + K_0/\lambda d)} C_0$

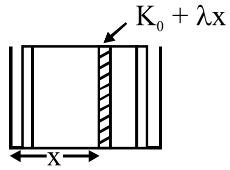
(2) $C = \dfrac{\lambda}{d.\ell n(1 + K_0 \lambda d)} C_0$

(3) $C = \dfrac{\lambda d}{\ell n(1 + K_0 \lambda d)} C_0$

(4) $C = \dfrac{\lambda d}{\ell n(1 + \lambda d/K_0)} C_0$

Ans. (4)

Sol.



$$d_C = \frac{(K_0 + \lambda_x)A}{dx}$$

$$\int \frac{1}{d_C} = \int_0^{\phi} \frac{dx}{A \cdot (K_0 + \lambda x)}$$

$$C_0 = \frac{\varepsilon_0 A}{d} \quad \frac{1}{\lambda} \int_{K_0}^{K_0+\lambda d} \frac{dt}{At}$$

$K_0 + \lambda x = t \qquad \lambda dx = dt$

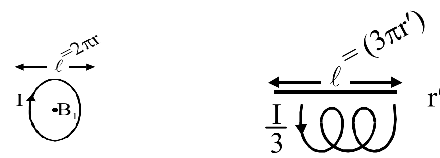
$$\lambda A \qquad \frac{1}{\lambda A} \ell n\left[\frac{K_0 + \lambda d}{K_0}\right]$$

12. Consider two thin identical conducting wires covered with very thin insulating material. One of the wires is bent into a loop and produces magnetic field B_1, at its centre when a current (I) passes through it. The second wire is bent into a coil with three identical loops adjacent to each other and produces magnetic field B_2 at the centre of the loops when current 1/3 passes through it. The ratio $B_1 : B_2$ is :-

(1) 1 : 9 (2) 1 : 3 (3) 9 : 1 (4) 1 : 1

Ans. (2)

Sol. $r = \dfrac{\ell}{2\pi}$



$r' = \dfrac{\ell}{6\pi}$

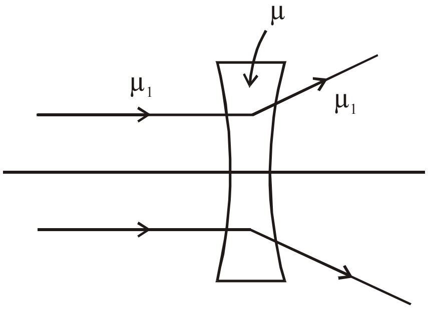
$$B_1 = \frac{\mu_0 I}{2\pi\ell} \times 2\pi \qquad B_2 = 3 \cdot \frac{\mu_0 I/3}{2\pi\ell} \cdot 6\pi$$

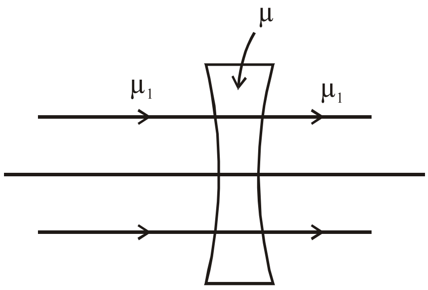
$$= \frac{\mu_0 I}{\ell} \qquad = 3 \cdot \frac{\mu_0 I}{\ell} \cdot \frac{1}{3} \cdot 3$$

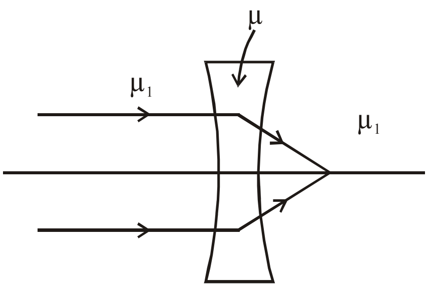
$$= 3$$

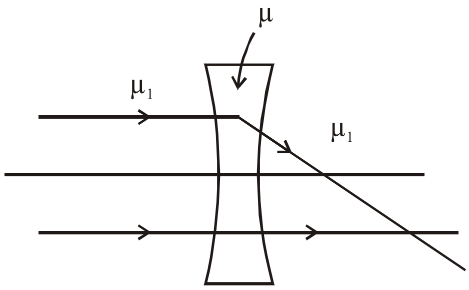
$B_1 : B_2 = 1 : 3$

13. The refractive index of the material of a concave lens is μ. It is immersed in a medium of refractive index μ_1. A parallel beam of light is incident on the lens. The path of the emergent rays when $\mu_1 > \mu$ is :-

(1)


(2)


(3)


(4)


Ans. (3)

Sol. bc = $\mu_1 > \mu$

So, option (3) is correct.

14. A person climbs up a stalled escalator in 60 s. If standing on the same but escalator running with constant velocity he takes 40 s. How much time is taken by the person to walk up the moving escalator ?

(1) 27 s (2) 45 s (3) 37 s (4) 24 s

Ans. (4)

Sol. $60 = \dfrac{d}{v_{man}}$

$40 = \dfrac{d}{v_{es}}$

$t = \dfrac{d}{v_{man} + v_{es}}$

$\dfrac{1}{t} = \dfrac{v_{man}}{d} + \dfrac{v_{es}}{d} = \dfrac{1}{60} + \dfrac{1}{40}$

t = 24 sec

15. A Carnot engine absorbs 1000 J of heat energy from a reservoir at 127°C and rejects 600 J of heat energy during each cycle. The efficiency of engine and temperature of sink will be :-

(1) 70% and –10ºC (2) 50% and –20ºC

(3) 40% and –33ºC (4) 20% and –43ºC

Ans. (3)

Sol. Q_1 = 1000 J

Q_2 = 600

$\eta = 1 - \dfrac{Q_2}{Q_1}$

= 40%

$\dfrac{T_1}{T_2} = \dfrac{Q_1}{Q_2}$

$\dfrac{400}{T_2} = \dfrac{1000}{600}$

T_2 = 240 K – 273 = – 33°C

16. A spring of unstretched length ℓ has a mass m with one end fixed to a rigid support. Assuming spring to be made of a uniform wire, the kinetic energy possessed by it if its free end is pulled with uniform velocity v is :-

(1) $\dfrac{1}{6}mv^2$ (2) mv^2 (3) $\dfrac{1}{3}mv^2$ (4) $\dfrac{1}{2}mv^2$

Ans. (1)

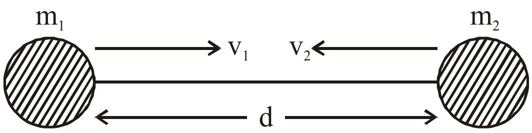
Sol. $KE = \int \frac{1}{2} dm \left(\frac{v}{\ell} x\right)^2$

$= \frac{1}{2}\int \frac{m}{\ell} dx \frac{v^2}{\ell^2} x^2$

$= \frac{mv^2}{2\ell^2}\left(\frac{x^3}{3}\right)_0^{\ell}$

$= \frac{mv^2}{2\ell^3} \times \frac{\ell^3}{3}$

$= \frac{mv^2}{6}$

17. 

Two hypothetical planets of masses m_1 and m_2 are at rest when they are infinite distance apart. Because of the gravitational force they move towards each other along the line joining their centres. What is their speed when their separation is 'd' ? (Speed of m_1 is v_1 and that of m_2 is v_2):-

(1) $v_1 = m_2\sqrt{\frac{2G}{m_1}}$

$v_2 = m_1\sqrt{\frac{2G}{m_2}}$

(2) $v_1 = v_2$

(3) $v_1 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$

$v_2 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$

(4) $v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$

$v_2 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$

Ans. (4)

Sol. From M.E conservation

$O = -\frac{GM_1M_2}{d} + KE$

$KE = \frac{GM_1M_2}{d}$

Since momentum is constant

So $KE \propto \frac{1}{m}$

K.E. of $m_1 = \left(\frac{m_2}{m_1+m_2}\right)\frac{GM_1M_2}{d} = \frac{1}{2}m_1v_1^2$

$v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$

KE of $m_2 = \left(\frac{m_1}{m_1+m_2}\right)\left(\frac{GM_1M_2}{d}\right) = \frac{1}{2}m_2v_2^2$

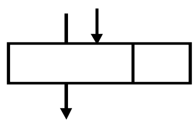
$v = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$

18. Steel ruptures when a shear of $3.5 \times 10^8\ Nm^{-2}$ is applied. The force needed to punch a 1 cm diameter hole in a steel sheet 0.3 cm thick is nearly :-

(1) 3.3×10^4 N
(2) 1.4×10^4 N
(3) 1.1×10^4 N
(4) 2.7×10^4 N

Ans. (1)

Sol. $\frac{F}{A}$ = stress = $3.5 \times 10^8\ N/m^2$



$A = 2\pi rt$

$= 2\pi \times \frac{1}{2} \times 0.3$

$= 0.3\ \pi \times 10^{-4}\ m^2$

$F = A \times$ stress

$= 0.3\pi \times 10^{-4} \times 3.5 \times 10^8$

$= 1.05\ \pi \times 10^4$

$= 3.3 \times 10^4$ N

19. From the following combinations of physical constants (expressed through their usual symbols) the only combination, that would have the same value in different systems of units, is :-

(1) $\frac{e^2}{2\pi \epsilon_0 G m_e^2}$ (m_e = mass of electron)

(2) $\frac{ch}{2\pi \epsilon_0^2}$

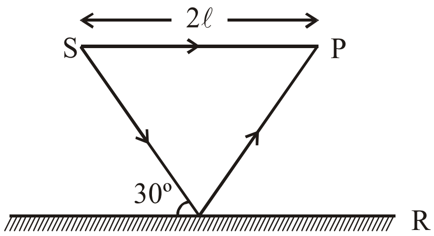
(3) $\frac{2\pi\sqrt{\mu_0 \epsilon_0}}{ce^2}\frac{h}{G}$

(4) $\frac{\mu_0 \epsilon_0}{c^2}\frac{G}{he^2}$

Ans. (1)

Sol. A dimensionless & unitless terms will have same value is all systems.

20. Interference pattern is observed at 'P' due to superimposition of two rays coming out from a source 'S' as shown in the figure. The value of ℓ for which maxima is obtained at 'P' is (R is perfect reflecting surface) :-



(1) $\ell = \frac{(2n-1)\lambda}{2(\sqrt{3}-1)}$ (2) $\ell = \frac{(2n-1)\lambda}{\sqrt{3}-1}$

(3) $\ell = \frac{(2n-1)\lambda\sqrt{3}}{4(2-\sqrt{3})}$ (4) $\ell = \frac{2n\lambda}{\sqrt{3}-1}$

Ans. (3)

Sol. $x \cos 30° = \ell$

$x\frac{\sqrt{3}}{2} = \ell$ $\quad$ $x = \frac{\sqrt{2\ell}}{\sqrt{3}}$

$$\frac{2\ell}{\sqrt{3}} + \frac{2\ell}{\sqrt{3}} + \frac{\lambda}{2} - 2\ell = n\lambda$$

$$n\lambda - \frac{\lambda}{2}$$

$$2\ell\left(\frac{2}{\sqrt{3}} - 1\right) = (2n-1)\frac{\lambda}{2}$$

$$2\ell\left(2-\sqrt{3}\right) = \frac{(2n-1)\lambda\sqrt{3}}{2}$$

21. At room temperature a diatomic gas is found to have an r.m.s. speed of 1930 ms^{-1}. The gas is :-

(1) F_2 (2) O_2

(3) Cl_2 (4) H_2

Ans. (4)

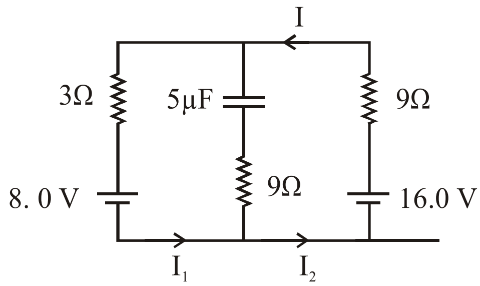
Sol. $\frac{1}{3}\rho v_{rms}^2 = p$ $\quad$ $pv = \frac{MRT}{M_0}$

$v_{rms} = \sqrt{\frac{3p}{\rho}}$ $\quad$ $\frac{p}{\rho} = \frac{RT}{M_0}$

$$1930 = \sqrt{\frac{3RT}{M_0}}$$

$$\frac{3 \times 8.314 \times 300}{1930 \times 1930}$$

22. The circuit shown here has two batteries of 8.0V and 16.0 V and three resistors 3Ω, 9Ω and 9Ω and a capacitor 5.0 μF



How much is the current I in the circuit in steady state ?

(1) 1.6 A (2) 2.5 A (3) 0.67 A (4) 0.25 A

Ans. (3)

Sol. $\frac{8}{12}\cdot\frac{2}{3} = 0.67$

23. A beam of light has two wavelengths 4972 Å and 6216 Å with a total intensity of 3.6×10^{-3} Wm^{-2} equally distributed among the two wavelengths. The beam falls normally on an area of 1 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each capable photon ejects one electron. The number of photo electrons liberated in 2s is approximately :-

(1) 15×10^{11} (2) 6×10^{11}

(3) 9×10^{11} (4) 11×10^{11}

Ans. (3)

Sol. $\frac{I\Delta t\lambda}{hc}$

$$\frac{1.8\times10^{-3}\times1\times10^{-4}}{2.5\times10^{-10}}\cdot10^{3}$$

$$\frac{18}{25}\times10^{3}\ \text{photon}$$

$$\frac{1.8\times10^{-3}\times1\times10^{-4}}{2\times10^{-10}}$$

$$\frac{12400}{6216}\cdot\frac{1.8}{2}\times10^{3}$$

24. A positive charge 'q' of mass 'm' is moving along the +x axis. We wish to apply a uniform magnetic field B for time Δt so that the charge reverses its direction crossing the y axis at a distance d. Then :-

(1) $B = \frac{2mv}{qd}$ and $\Delta t = \frac{\pi d}{2v}$

(2) $B = \frac{mv}{qd}$ and $\Delta t = \frac{\pi d}{v}$

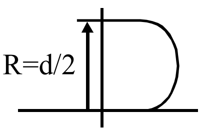
(3) $B = \frac{2mv}{qd}$ and $\Delta t = \frac{\pi d}{v}$

(4) $B = \frac{mv}{2qd}$ and $\Delta t = \frac{\pi d}{2v}$

Ans. (1)

Sol. $\frac{1.8\times10^{-3}\times10^{-4}}{2.5\times1.6\times10^{-19}}$

R=d/2



$$\frac{18}{2.5\times1.6}10^{12}$$

$$\frac{mv^2}{r}=qvB$$

$$r = \frac{mv}{qB}$$

$$\frac{d}{2}=\frac{mv}{qB} \Rightarrow B = \frac{2mv}{qd}$$

$$\frac{\pi d}{2v}$$

25. A spherically symmetric charge distribution is characterised by a charge density having the following variation

$$\rho(r)=\rho_0\left(1-\frac{r}{R}\right) \text{ for } r < R$$

$$\rho(r)=0 \qquad \text{for } r \geq R$$

Where r is the distance from the centre of the charge distribution and ρ_0 is a constant. The electric field at an internal point (r < R) is :-

(1) $\frac{\rho_0}{12\epsilon_0}\left(\frac{r}{3}-\frac{r^2}{4R}\right)$ (2) $\frac{\rho_0}{3\epsilon_0}\left(\frac{r}{3}-\frac{r^2}{4R}\right)$

(3) $\frac{\rho_0}{4\epsilon_0}\left(\frac{r}{3}-\frac{r^2}{4R}\right)$ (4) $\frac{\rho_0}{\epsilon_0}\left(\frac{r}{3}-\frac{r^2}{4R}\right)$

Ans. (4)

Sol. $\oint \varepsilon.ds = \frac{\Sigma a}{\varepsilon_0}$

$$\varepsilon(4\pi r^2) = \rho\cdot\int_0^r\left(1-\frac{r}{R}\right)4\pi r^2dr$$

$$4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^2}{4R}\right]_0^q = \varepsilon(4\pi r^2)$$

$$\varepsilon = \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3}-\frac{r^2}{4R}\right)$$

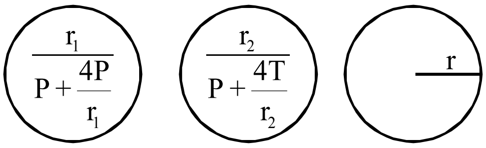
26. Two soap bubbles coalesce to form a single bubble. If V is the subsequent change in volume of contained air and S the change in total surface area, T is the surface tension and P atmospheric pressure, which of the following relation is **correct** ?

(1) 3PV + 2ST = 0 (2) 4PV + 3ST = 0

(3) 2PV + 3ST = 0 (4) 3PV + 4ST = 0

Ans. (4)

Sol.



$$\left(P+\frac{4T}{r_1}\right)\frac{4}{3}\pi r_1^3 + \left(P+\frac{4T}{r_2}\right)\frac{4}{3}\pi r_2^3$$

$$= \left(P+\frac{4T}{r}\right)\frac{4}{3}\pi r^3$$

$$PV + \frac{4T}{3}(4\pi r_1^2 + 4\pi r_2^2 - 4\pi r^2)$$

3pv + 4TS

27. For LED's to emit light in visible region of electromagnetic light, it should have energy band gap in the range of :-

(1) 0.9 eV to 1.6 eV (2) 0.5eVto 0.8eV

(3) 0.1 eV to 0.4 eV (4) 1.7eV to 3.0eV

Ans. (4)

Sol. $\frac{12400}{3000}$ to $\frac{12400}{7800}$ or 3.26eV to 1.6eV

28. Which of the following expressions corresponds to simple harmonic motion along a straight line, where x is the displacement and a, b, c are positive constants ? :-

(1) $-bx$ (2) $a - bx + cx^2$

(3) $a + bx - cx^2$ (4) bx^2

Ans. (1)

29. A cylindrical vessel of cross-section A contains water to a height h. There is a hole in the bottom of radius 'a'. The time in which it will be emptied is :-

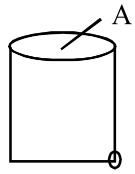
(1) $\frac{2\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$

(2) $\frac{\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$

(3) $\frac{A}{\sqrt{2}\pi a^2}\sqrt{\frac{h}{g}}$

(4) $\frac{2A}{\pi a^2}\sqrt{\frac{h}{g}}$

Ans. (2)



Sol. $-Adh = a\cdot\sqrt{2gh}.dt$

$$\int_H^0 \frac{dh}{\sqrt{2gh}} = -\frac{q}{A}\int_0^t dt$$

$$\Rightarrow \frac{2}{\sqrt{2g}}\sqrt{h^{-1/2+1}} \quad \frac{2}{\sqrt{2g}}\left|\sqrt{h}\right| = \frac{q}{A}t$$

$$t = \frac{\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$$

30. A sinusoidal voltage V(t)=100 sin (500t) is applied across a pure inductance of L = 0.02 H. The current through the coil is : :-

(1) 10 cos (500t) (2) – 10 cos (500t)

(3) 10 sin (500t) (4) – 10 sin (500t)

Ans. (2)

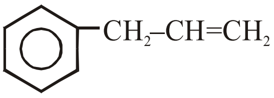
Sol. $x_L = \omega L$

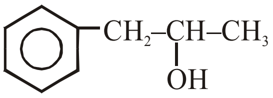
$$= \frac{2\pi\times 500}{500\times\frac{.02}{100}}$$

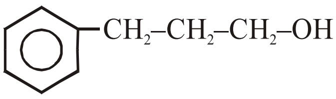
$i = 10 \sin[\omega t - \pi/2]$

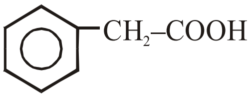
$= -10\cos 500t \quad \Rightarrow 10$

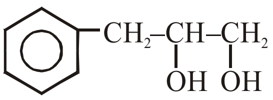
PART B – CHEMISTRY

1.  on mercuration-demercuration produces the major product :-

(1) 

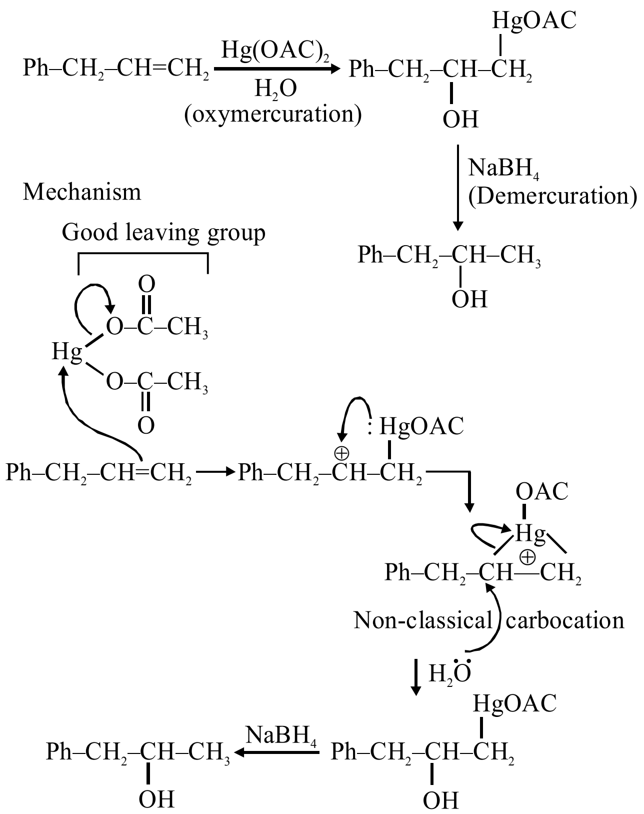
(2) 

(3) 

(4) 

Ans. (1)

Sol.



Rearrangement of carbocation formed is not possible due to formatiion of cyclic non-classical carbocation.

2. How many electrons would be required to deposit 6.35 g of copper at the cathode during the electrolysis of an aqueous solution of copper sulphate ?

(Atomic mass of copper = 63.5 u,

N_A = Avogadro's constant) :-

(1) $\frac{N_A}{10}$ (2) $\frac{N_A}{2}$

(3) $\frac{N_A}{5}$ (4) $\frac{N_A}{20}$

Ans. (3)

Sol. $W = \frac{E}{96500} \times Q$

$\Rightarrow 6.35 = \frac{63.5}{2 \times 96500} \times Q$

$\Rightarrow Q = 2 \times 9650$ coulomb

$\Rightarrow$ 1F = charge of 1 mol of e^- = 96500

$\therefore$ No. of $e^- = \frac{N_A}{10} \times 2$

$= \frac{N_A}{5}$

3. The amount of $BaS0_4$ formed upon mixing 100 mL of 20.8% $BaCl_2$ solution with 50 mL of 9.8% H_2SO_4 solution will be :

(Ba = 137, Cl = 35.5, S=32, H = 1 and O = 16)

(1) 33.2 g (2) 11.65 g (3) 23.3 g (4) 30.6 g

Ans. (3)

Sol. $BaCl_2 + H_2SO_4 \longrightarrow BaSO_4 + 2\ HCl$

100ml 50 ml $\frac{1}{20}$ ml

20.8% 9.8%

20.8 gm 4.9 gm $\frac{1}{20} \times 233$ gm

$\frac{1}{10}$ mol $\frac{4.9}{98} = \frac{1}{20}$ mol = 11.65 gm

4. Which of the following will **not** show mutarotation?

(1) Sucrose (2) Lactose

(3) Maltose (4) Glucose

Ans. (1)

Sol. Sucrose does not exhibit mutarotation because the glycosidic bond is between the anomeric carbon of glucose and anomeric carbon of fructose. Hence hemiacetal is not present in sucrose, while it is there in lactose, maltose and glucose.

5. The standard enthalpy of formation ($\Delta_f H^o_{298}$) for methane, CH_4 is– 74.9 kJ mol^{-1}. In order to calculate the average energy given out in the formation of a C–H bond from this it is necessary to know which one of the following?

(1) the dissociation energy of the hydrogen molecule, H_2.

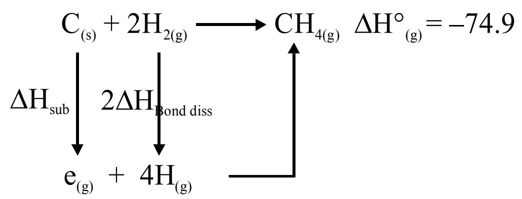
(2) the dissociation energy of H_2 and enthalpy of sublimation of carbon (graphite).

(3) the first four ionisation energies of carbon and electron affinity of hydrogen.

(4) the first four ionisation energies of carbon.

Ans. (2)

Sol.



To calculate average bond energy of (C–H) bond, dissociation energy of H_2 and enthalpy of sublimation of carbon (graphite) is needed.

6. Among the following species the one which causes the highest CFSE, Δo as a ligand is :-

(1) CN^- (2) NH_3

(3) CO (4) F^-

Ans. (3)

Sol. CO is the strongest ligand therefore it has maximum CFSE value.

7. In a monoclinic unit cell, the relation of sides and angles are respectively :-

(1) $a \neq b \neq c$ and $\alpha \neq \beta \neq \gamma \neq 90^o$

(2) $a \neq b \neq c$ and $\beta = \gamma = 90^o \neq \alpha$

(3) $a = b \neq c$ and $\alpha = \beta = \gamma = 90^o$

(4) $a \neq b \neq c$ and $\alpha = \beta = \gamma = 90^o$

Ans. (2)

8. What happens when an inert gas is added to an equilibrium keeping volume unchanged :-

(1) Less product will form

(2) Equilibrium will remain unchanged

(3) More product will form

(4) More reactant will form

Ans. (2)

9. The major product obtained in the photo catalysed bromination of 2-methylbutane is :-

(1) 2-bromo-2-methylbutane

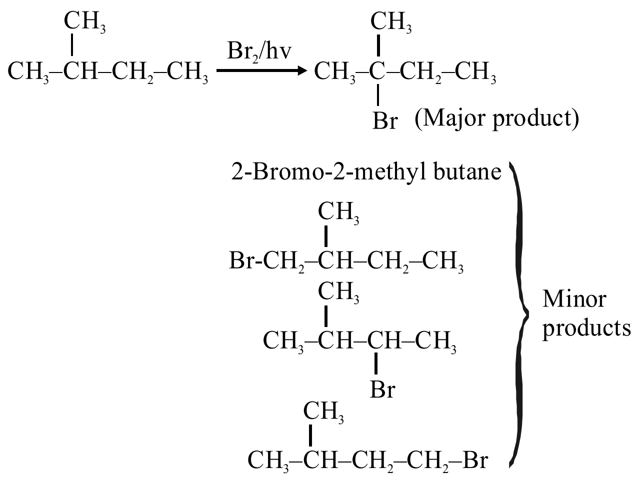
(2) 2-bromo-3-methylbutane

(3) l-bromo-2-methylbutane

(4) l-bromo-3-methylbutane

Ans. (1)

Sol.



relectivity ratio for bromination is

1° : 2° : 3° : : 1 : 82 : 1600

Hence 3° product will be major product.

10. Which one of the following complexes will most likely absorb visible light ?

(At nos. Sc = 21, Ti = 22, V = 23, Zn = 30) :-

(1) $[Ti(NH_3)_6]^{4+}$ (2) $[V(NH_3)_6]^{3+}$

(3) $Zn(NH_3)_6]^{2+}$ (4) $[Sc(H_2O)_6]^{3+}$

Ans. (2)

11. The conjugate base of hydrazoic acid is :-

(1) HN_3^- (2) N_3^-

(3) N_2^- (4) N^{-3}

Ans. (2)

Sol. $N_3H \longrightarrow N_3^{\ominus} + H^{\oplus}$

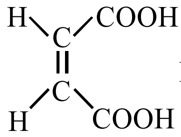
Hydrazoic conjugate lase Azide ion

12. Which one of the following acids does not exhibit optical isomerism ?

(1) Lactic acid (2) α-amino acids

(3) Tartaric acid (4) Maleic acid

Ans. (4)

Sol. 

Maleic Acid

Maleic acid shows geometrical isomerism not optical isomerism.

13. Which of the following arrangements represents the increasing order (smallest to largest) of ionic radii of the given species O^{2-}, S^{2-}, N^{3-}, P^{3-} ? :-

(1) $N^{3-} < O^{2-} < P^{3-} < S^{2-}$

(2) $O^{2-} < N^{3-} < S^{2-} < P^{3-}$

(3) $O^{2-} < P^{3-} < N^{3-} < S^{2-}$

(4) $N^{3-} < S^{2-}\ O^{2-} < P^{3-}$

Ans. (2)

Sol. $O^{2-} < N^{3-}$

$10e^{\ominus}$ $10e^{\ominus}$ isoelectronic

$S^{-2} < P^{3-}$

$O^{2-} < N^{3-} < S^{-2} < P^{3-}$

14. In the Victor-Meyer's test, the colour given by 1°, 2° and 3° alcohols are respectively :-

(1) Red, blue, colourless

(2) Colourless, red, blue

(3) Red, blue, violet

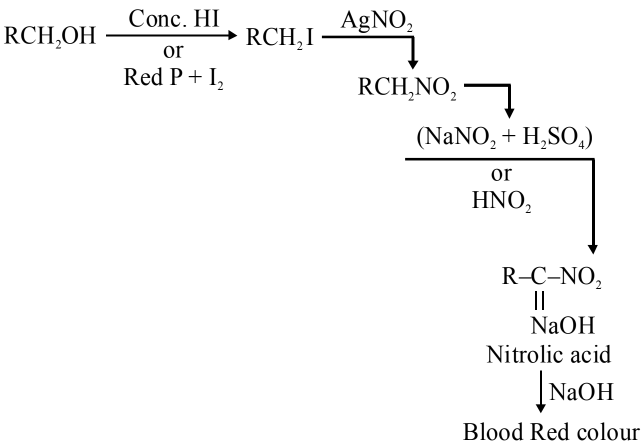
(4) Red, colourless, blue

Ans. (1)

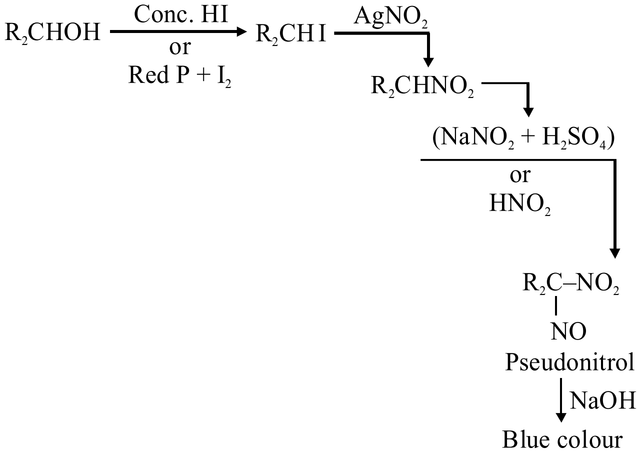
Sol.

Victor Meyer's Test :

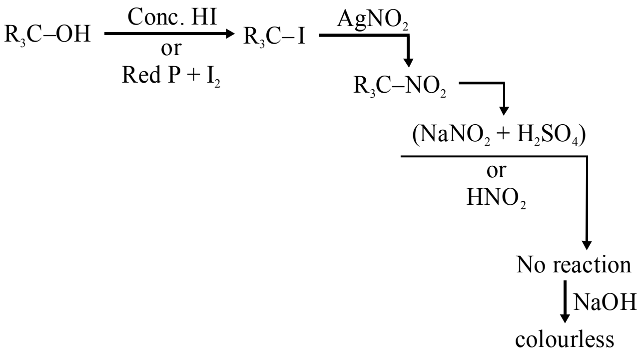
1°Alcohol



2°Alcohol



3°Alcohol



15. In the presence of peroxide, HCℓ and HI do not give anti-Markownikoff s addition to alkenes because :-

(1) All the steps are exothermic in HCl and HI

(2) One of the steps is endothermic in HCl and HI

(3) HCl is oxidizing and the HI is reducing

(4) Both HCl and HI are strong acids

Ans. (2)

16. Aminoglycosides are usually used as :-

(1) hypnotic

(2) antifertility

(3) antibiotic

(4) analgesic

Ans. (3)

Sol. Aminoglycosides is bactericidal antibiotic.

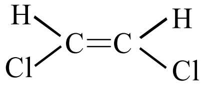
17. Which of the following molecules has two sigma(σ) and two pi(π) bonds :-

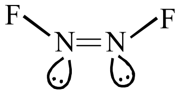
(1) HCN (2) $C_2H_2Cl_2$

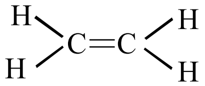
(3) N_2F_2 (4) C_2H_4

Ans. (1)

Sol. H–C≡N 2σ and 2π bond


5σ and 1π bond


3σ and 1π bond


5σ and 1π bond

18. Excited hydrogen atom emits light in the ultraviolet region at 2.47×10^{15} Hz. With this frequency, the energy of a single photon is :-
($h = 6.63 \times 10^{-34}$ Js)
(1) 2.680×10^{-19} J
(2) 6.111×10^{-17} J
(3) 1.640×10^{-18} J
(4) 8.041×10^{-40} J

Ans. (3)

Sol. E = hv = Energy of single photon

$$= 6.63 \times 10^{-34}\ \text{J. sec} \times 2.47 \times 10^{15}\ \frac{1}{\text{sec.}}$$
$$= 16.37 \times 10^{-19}\ \text{J}$$
$$= 1.637 \times 10^{-18}\ \text{J}$$

19. The rate coefficient (k) for a particular reactions is $1.3 \times 10^{-4}\ M^{-1}\ s^{-1}$ at 100°C, and $1.3 \times 10^{-3}\ M^{-1}\ s^{-1}$ at 150°C, What is the energy of activation (E_A) (in kJ) for this reaction ?
(R = molar gas constant = 8.314 $JK^{-1}\ mol^{-1}$)

(1) 132 (2) 60
(3) 99 (4) 16

Ans. (2)

Sol.
$$\log \frac{K_2}{K_1} = \frac{E_a}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\Rightarrow \log \frac{1.3 \times 10^{-3}}{1.3 \times 10^{-4}} = \frac{E_a}{2.303R}\left[\frac{1}{373} - \frac{1}{423}\right]$$

$$\Rightarrow 1 = \frac{E_a}{2.303\ R}\left[\frac{50}{373 \times 423}\right]$$

$$\Rightarrow E_a = \frac{2.303 \times 8.314 \times 373 \times 423}{1000 \times 50}$$
$$= 60.42\ \text{kJ}$$

20. Which of the following xenon-OXO compounds may not be obtained by hydrolysis of xenon fluorides ?

(1) XeO_2F_2 (2) XeO_3
(3) XeO_4 (4) $XeOF_4$

Ans. (3)

Sol. $XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF$
$XeF_6 + H_2O \longrightarrow XeOF_4 + 2HF$
$XeF_6 + 2H_2O \longrightarrow XeO_2F_2 + 4HF$

21. Global warming is due to increase of :-
(1) methane and nitrous oxide in atmosphere
(2) methane and CO in atmosphere
(3) methane and CO_2 in atmosphere
(4) methane and O_3 in atmosphere

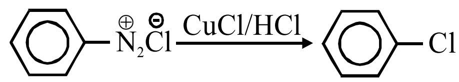
Ans. (3)

22. Conversion of benzene diazonium chloride to chloro benzene is an example of which of the following reactions ?

(1) Wurtz (2) Claisen
(3) Sandmeyer (4) Friedel-craft

Ans. (3)

Sol. Sandmeyer reaction



23. The de-Brogile wavelenths of a particle of mass 6.63 g moving with a velocity 100 ms^{-1} is :-

(1) 10^{-35} m (2) 10^{-33} m
(3) 10^{-25} m (4) 10^{-31} m

Ans. (2)

Sol. $\lambda = \frac{h}{mv}$

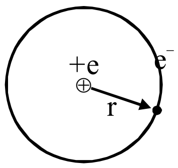
$$= \frac{6.63 \times 10^{-34} \times 1000}{6.63 \times 100} = 10^{-33}\ \text{m}$$

24. If m and e are the mass and charge of the revolving electron in the orbit of radius r for hydrogen atom, the total energy of the revolving electron will be :-

(1) $\frac{me^2}{r}$ (2) $-\frac{e^2}{r}$

(3) $\frac{1}{2}\frac{e^2}{r}$ (4) $-\frac{1}{2}\frac{e^2}{r}$

Ans. (4)

Sol. 

$$TE = PE + KE \qquad \left\{\because \frac{mv^2}{r} = \frac{ke^2}{r^2}\right.$$

$$= -\frac{ke^2}{r} + \frac{1}{2}mv^2 \qquad \left.\therefore \frac{1}{2}mv^2 = \frac{1}{2}\frac{ke^2}{r}\right\}$$

$$= -\frac{ke^2}{r} + \frac{1}{2}\frac{ke^2}{r}$$

$$= -\frac{1}{2}\frac{ke^2}{r}$$

$$= -\frac{1}{2}\frac{e^2}{r} \qquad \text{[In CGS system K = 1]}$$

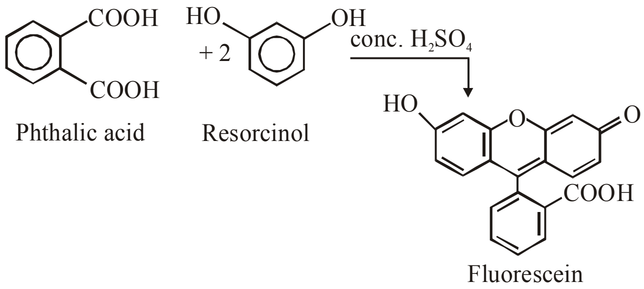
25. Phthalic acid reacts with resorcinol in the presence of concentrated H_2SO_4 to give :-

(1) Fluorescein (2) Coumarin

(3) Alizarin (4) Phenolphthalein

Ans. (1)

Sol.



26. The entropy (S°) of the following substances are :

CH_4 (g) 186.2 J K^{-1} mol^{-1}

O_2 (g) 205.0 J K^{-1} mol^{-1}

CO_2 (g) 213.6 J K^{-1} mol^{-1}

H_2O (l) 69.9 J K^{-1} mol^{-1}

The entropy change (ΔS°) for the reaction

$CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O\ (\ell)$ is:-

(1) –312.5 JK^{-1} mol^{-1}

(2) – 37.6 JK^{-1} mol^{-1}

(3) – 108.1 JK^{-1} mol^{-1}

(4) – 242.8 JK^{-1} mol^{-1}

Ans. (4)

Sol. $CH_{4(g)} + 2\ O_{2(g)} \longrightarrow CO_{2(g)} + 2H_2O(\ell)$

186.2 205 213.6 69.9

$\Delta S^\circ = \Sigma(s^\circ)_p - \Sigma(s^\circ)_R$

$= [213.6 + (2 \times 69.9)] - [186.2 + 2 \times 205]$

$= 353.4 - 596.2$

$= -\ 242.8$ J/mol K

27. Copper becomes green when exposed to moist air for a long period. This is due to :-

(1) the formation of a layer of cupric oxide on the surface of copper.

(2) the formation of basic copper sulphate layer on the surface of the metal

(3) the formation of a layer of cupric hydroxide on the surface of copper.

(4) the formation of a layer of basic carbonate of copper on the surface of copper.

Ans. (4)

Sol. $CuCO_3\ \ Cu(OH)_2$

malachite green

1 : 1 ratio.

28. Which one of the following exhibits the largest number of oxidation states ?

(1) Mn(25) (2) V(23)

(3) Cr (24) (4) Ti (22)

Ans. (1)

Sol. Mn $\longrightarrow$ + 7 oxidation state

29. Hydrogen peroxide acts both as an oxidising and as a reducing agent depending upon the nature of the reacting species. In which of the following cases H_2O_2 acts as a reducing agent in acid medium ? :-

(1) MnO_4^- (2) SO_3^{2-}

(3) KI (4) $Cr_2O_7^{2-}$

Ans. (1)

Sol. $H_2O_2 + MnO_4^- \xrightarrow{\overset{\oplus}{H}} O_2 + Mn^{+2}$

30. Similarity in chemical properties of the atoms of elements in a group of the Periodic table is most closely related to :-

(1) number of principal energy levels

(2) atomic numbers

(3) number of valence electrons

(4) atomic masses

Ans. (3)

Sol. According to modern periodic law.

PART C – MATHEMATICS

1. Two tangents are drawn from a point (–2,–1) to the curve , $y^2 = 4x$. If α is the angle between them, then | tan α | is equal to :-

(1) $\sqrt{3}$ (2) 3

(3) $\frac{1}{3}$ (4) $\frac{1}{\sqrt{3}}$

Ans. (2)

Sol. Let equation of tangent from (– 2, –1) be
$y + 1 = m(x + 2)$
$\Rightarrow y = mx + (2m - 1)$

Condition of tangency, $C = \frac{a}{m}$

i.e., $2m - 1 = \frac{1}{m}$

$\Rightarrow 2m^2 - m - 1 = 0$
$(2m + 1)(m - 1) = 0$

$m = -\frac{1}{2}, 1$

Now, $|\tan\alpha| = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{1 + \frac{1}{2}}{1 - \frac{1}{2}}\right| = 3$

2. The general solution of the diffferential equation, $\sin 2x\left(\frac{dy}{dx} - \sqrt{\tan x}\right) - y = 0$, is :-

(1) $y\sqrt{\tan x} = \cot x + c$

(2) $y\sqrt{\cot x} = x + c$

(3) $y\sqrt{\tan x} = x + c$

(4) $y\sqrt{\cot x} = \tan x + c$

Ans. (2)

Sol. $\sin 2x\left(\frac{dy}{dx} - \sqrt{\tan x}\right) - y = 0$

$\frac{dy}{dx} - \frac{y}{\sin 2x} = \sqrt{\tan x}$

I.F. $= e^{-\int \text{cosec}\, 2x\, dx} = e^{-\frac{1}{2}\ln\tan x} = \frac{1}{\sqrt{\tan x}}$

$\Rightarrow$ General solution

$y.\frac{1}{\sqrt{\tan x}} = \int \sqrt{\tan x}.\frac{1}{\sqrt{\tan x}} + c$

$y\sqrt{\cot x} = x + c$

3. If the three distinct lines $x + 2ay + a = 0$, $x + 3by + b = 0$ and $x + 4ay + a = 0$ are concurrent, then the point (a, b) lies on a :-

(1) circle (2) straight line

(3) parabola (4) hyperbola

Ans. (2)

Sol. $x + a(2y + 1) = 0$
$x + b(3y + 1) = 0$
$x + a(4y + 1) = 0$

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4a & a \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & 2a & a \\ 0 & 3b - 2a & b - a \\ 0 & 2a & 0 \end{vmatrix} = 0$$

$\Rightarrow 2a(b - a) = 0$
$2a = 0$ or $b = a$
Locus of (a, b) $\Rightarrow x = 0$ or $y = x$

4. If

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$$

$= k\lambda \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$ $\lambda \neq 0$, then k is equal to :-

(1) $4\lambda abc$ (2) $-4\lambda^2$

(3) $4\lambda^2$ (4) $-4\lambda abc$

Ans. (3)

Sol. $R_2 \to R_2 - R_1$, $R_1 \to R_1 - R_3$

$$\begin{vmatrix} \lambda(2a-\lambda) & \lambda(2b-\lambda) & \lambda(2c-\lambda) \\ 4a\lambda & 4b\lambda & 4c\lambda \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$$

$= R_3 \to R_3 + R_1,\ R_1 \to R_1 - \frac{1}{2}R_2$

$$= \begin{vmatrix} -\lambda^2 & -\lambda^2 & -\lambda^2 \\ 4a\lambda & 4b\lambda & 4c\lambda \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$= -4\lambda^3 \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$= +4\lambda^3 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

$\therefore\ K = 4\lambda^2$

5. The sum of the roots of the equation, $x^2 + |2x - 3| - 4 = 0$, is :-

(1) -2 (2) $\sqrt{2}$

(3) $-\sqrt{2}$ (4) 2

Ans. (2)

Sol. $x^2 + |2x - 3| - 4 = 0$

Case-1 : $x \geq \frac{3}{2}$

$\Rightarrow x^2 + 2x - 3 - 4 = 0$

$\Rightarrow x = -1 \pm 2\sqrt{2}$

$\Rightarrow x = -1 + 2\sqrt{2}$

Case-2 : $x < \frac{3}{2}$

$\Rightarrow x^2 - 2x - 1 = 0$

$\Rightarrow x = 1 \pm \sqrt{2}$

$\Rightarrow x = 1 - \sqrt{2}$

$\Rightarrow$ Sum $= \sqrt{2}$

6. Let $z \neq -i$ be any complex number such that $\frac{z-i}{z+i}$ is a purely imaginary number. Then $z + \frac{1}{z}$ is :-

(1) any non-zero real number other than 1.

(2) a purely imaginary number.

(3) 0

(4) any non- zero real number

Ans. (4)

Sol. Let $Z = x + iy$

$\frac{z-i}{z+i}$ is a purely imaginary number

$\Rightarrow \frac{x+i(y-1)}{x+i(y+1)} \times \frac{x-i(y+1)}{x-i(y+1)}$ is a purely imaginary

$\Rightarrow \frac{(x^2+y^2-1)-i(2x)}{x^2+(y+1)^2}$ is purely imaginary

$\Rightarrow x^2 + y^2 - 1 = 0 \Rightarrow x^2 + y^2 = 1$ ...(i)

$z + \frac{1}{z} = x + iy + \frac{1}{x+iy}$

$= (x+iy) + \frac{1}{(x+iy)} \times \frac{(x-iy)}{(x-iy)}$

$= (x+iy) + \frac{(x-iy)}{x^2+y^2} = 2x$

$x \neq 1$

($\because$ if $x = 1$ then $y = 0$, from (i) & z won't be complex number)

If $x = 1 \Rightarrow y = 0$

$\Rightarrow z = 1$

$\Rightarrow \frac{z-i}{z+i} = \frac{1-i}{1+i}$ cannot be purely imaginary.

7. If $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$ be such that $AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$, then :-

(1) $y = -2x$ (2) $y = 2x$

(3) $y = x$ (4) $y = -x$

Ans. (2)

Sol. $AB = \begin{bmatrix} y+3x \\ 3y-x+2 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

y + 3x = 6

3y − x = 6

y + 3x = 3y − x

4x = 4y

$\boxed{x = y}$

8. Let $\overline{X}$ and M.D. be the mean and the mean deviation about $\overline{X}$ of n observation x_i, i = 1,2,...., n. If each of the observation is increased by 5, then the new mean and the mean deviation about the new mean respectively , are :-

(1) $\overline{X}$ + 5, M.D. (2) $\overline{X}$ + 5, M.D. + 5

(3) $\overline{X}$, M.D. (4) $\overline{X}$, M.D. + 5

Ans. (1)

Sol. If all the observations are increased by K then mean is increased by K but M.D. remains same.

9. If for a continuous function f(x), $\int_{-\pi}^{t}(f(x)+x)dx = \pi^2 - t^2$, for all t ≥ − π, then $f\left(-\frac{\pi}{3}\right)$ is equal to :-

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{2}$ (4) π

Ans. (4)

Sol. $\int_{-\pi}^{t}(f(x)+x)dx = \pi^2 - t^2$

$\Rightarrow \int_{-\pi}^{t} f(x)dx + \int_{-\pi}^{t} xdx = \pi^2 - t^2$

$\Rightarrow \int_{-\pi}^{t} f(x)dx = \frac{3}{2}(\pi^2 - t^2)$

$\Rightarrow \int_{-\pi}^{t} f(x)dx = \int_{-\pi}^{t} -3xdx \Rightarrow f(x) = -3x$

$f\left(-\frac{\pi}{3}\right) = -3\left(-\frac{\pi}{3}\right) = \pi$

10. A number x is chosen at random from the set {1, 2, 3, 4, ..., 100}. Define the event : A = the chosen number x satisfies $\frac{(x-10)(x-50)}{(x-30)} \geq 0$

Then P (A) is :-

(1) 0.20 (2) 0.51

(3) 0.71 (4) 0.70

Ans. (3)

Sol. S = {1, 2, 3...........100}

A : Chosen no. x satisfies

$\frac{(x-10)(x-50)}{(x-30)} \geq 0$

$\therefore$ x ∈ {10, 11, 12......29} ∪ {50, 51.....100}

$P(A) = \frac{71}{100} = 0.71$

11. Let f and g be two diffrentiable functions on **R** such that f'(x) > 0 and g' (x) < 0, for all x ∈ **R**. Then for all x :-

(1) g(f(x)) > g (f(x −1))

(2) f(g(x)) > f (g(x −1))

(3) g(f(x)) < g (f(x + 1))

(4) f(g(x)) > f (g (x − 1))

Ans. (4)

Sol. f′(x) > 0 ⇒ f(x) is increasing function

g′(x) < 0 ⇒ g(x) is decreasing function

Now,

(i) x > x − 1

f(x) > f(x − 1)

g(f(x)) < g(f(x − 1))

and

(ii) x + 1 > x

f(x + 1) > f(x)

g(f(x + 1)) < g(f(x))

(iii)x > x − 1

g(x) < g(x − 1)

f(g(x)) < f(g(x − 1))

(iv)x + 1 > x

g(x + 1) < g(x)

f(g(x + 1)) < f(g(x))

12. If $f(x) = x^2 - x + 5, x > \frac{1}{2}$, and g(x) is its inverse function, then g'(7) equals :-

(1) $\frac{1}{3}$ (2) $-\frac{1}{3}$

(3) $-\frac{1}{13}$ (4) $\frac{1}{13}$

Ans. (1)

Sol. $f(x) = x^2 - x + 5$

$g(f(x)) = x$

$$g'(f(x)) = \frac{1}{f'(x)}$$

$$\Rightarrow g'(f(2)) = \frac{1}{f'(2)}$$

$$\Rightarrow g'(7) = \frac{1}{3}$$

13. **Statement I :** The equation $(\sin^{-1}x)^3 + (\cos^{-1}x)^3 - a\pi^3 = 0$ has a solution for all $a \geq \frac{1}{32}$.

Statement II : For any $x \in R$,

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ and

$$0 \leq \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \leq \frac{9\pi^2}{16}$$

(1) Both statement I and II are true

(2) Statement I is true and statement II is false

(3) Statement I is false and statement II is true

(4) Both statements I and II are false

Ans. (3)

Sol. **Statement-I**

$(\sin^{-1}x)^3 + (\cos^{-1}x)^3 - a\pi^3 = 0$

$\Rightarrow (\sin^{-1}x)^3 + (\cos^{-1}x)^3 = a\pi^3$

$\Rightarrow (\sin^{-1}x + \cos^{-1}x)^3 - 3\sin^{-1}x\cos^{-1}x(\sin^{-1}x + \cos^{-1}x) = a\pi^3$

$$\Rightarrow \frac{\pi^3}{8} - \frac{3\pi}{2}\left(\frac{\pi}{2} - \cos^{-1} x\right)\cos^{-1} x = a\pi^3$$

$$\Rightarrow \frac{\pi^3}{8} - \frac{3\pi^2}{4}\cos^{-1} x + \frac{3\pi}{2}(\cos^{-1} x)^2 = a\pi^3$$

$$\Rightarrow \frac{3\pi}{2}\left[(\cos^{-1} x)^2 - \frac{\pi}{2}\cos^{-1} x\right] + \frac{\pi^3}{8} = a\pi^3$$

$$\Rightarrow \frac{3\pi}{2}\left[\left(\cos^{-1} x - \frac{\pi}{4}\right)^2 - \frac{\pi^2}{16}\right] + \frac{\pi^3}{8} = a\pi^3$$

$$\left(\cos^{-1} x - \frac{\pi}{4}\right)^2 = \left(\frac{2a}{3} - \frac{1}{48}\right)\pi^2 \quad ...(1)$$

$\because\ 0 \leq \cos^{-1}x \leq \pi$

$$-\frac{\pi}{4} \leq \left(\cos^{-1} x - \frac{\pi}{4}\right) \leq \frac{3\pi}{4}$$

$$0 \leq \left(\cos^{-1} x - \frac{\pi}{4}\right)^2 \leq \frac{9\pi^2}{16}$$

From equation (1)

$$\frac{1}{32} \leq a \leq \frac{7}{8}$$

$\therefore$ Statement-I is false

Statement-II

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\ \forall\ x \in [-1, 1]$

not for any $x \in R$ so Statement-II is false

14. Let f, g: $R \to R$ be two functions defined by

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$, and g(x) = xf (x) :-

Statement I : f is a continuous function at x = 0.

Statement II : g is a differentiable function at x = 0.

(1) Statement I is false and statement II is true

(2) Statement I is true and statement II is false

(3) Both statement I and II are true

(4) Both statements I and II are false

Ans. (3)

Sol. **Statement-I**

$$f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right) & x\neq 0 \\ 0 & x=0\end{cases}$$

$$\lim_{x\to 0} x\sin\left(\frac{1}{x}\right)=0=f(0)$$

Hence f(x) is continuous function

$g(0) = 0 = \lim_{x\to 0} g(x)$

LHD :

$$\lim_{x\to 0}\frac{g(o-h)-g(0)}{-h}$$

$$\lim_{x\to 0}\frac{h^2\sin\left(-\frac{1}{h}\right)-0}{-h}$$

LHD = 0

RHD :

$$\lim_{h\to 0}\frac{g\left(0+h\right)-g(0)}{h}$$

$$\lim_{h\to 0}\frac{h^2\sin\left(\frac{1}{n}\right)-0}{h}$$

RHD = 0

LHD = RHD

Hence g(x) at x = 0 is diff. function.

15. For the two circles $x^2 + y^2 = 16$ and $x^2 + y^2 - 2y = 0$, there is / are:-

(1) no common tangent

(2) two pairs of common tangents

(3) one pair of common tangents

(4) three common tangents

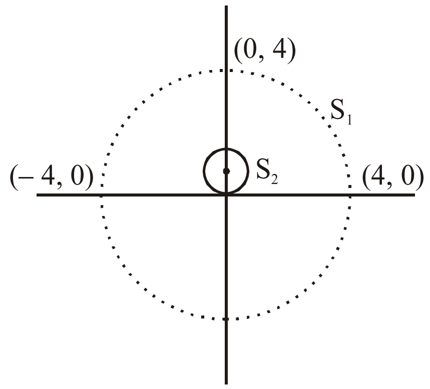
Ans. (1)

Sol. S-I : $x^2 + y^2 = 4^2$

S-II : $x^2 + (y - 1)^2 = 1$

Aliter : Distance between their centres is 1 units and sum of their radii is 3 so one of them lie completely inside the other, hence no common tangent.

Also, from figure we can say no common tangent :



16. If $\left(2+\frac{x}{3}\right)^{55}$ is expanded in the ascending powers of x and the coefficients of powers of x in two consecutive terms of the expansion are equal, then these terms are :-

(1) 28^{th} and 29^{th} (2) 8^{th} and 9^{th}

(3) 7^{th} and 8^{th} (4) 27^{th} and 28^{th}

Ans. (2)

Sol. $\left(2+\frac{x}{3}\right)^{55}$

General term

$$^{55}C_r\times 2^{55-r}\times\left(\frac{x}{3}\right)^r$$

Let T_{r+1} and T_{r+2} are having some co-efficients

$\Rightarrow$ Coff. of T_{r+1} = Coff. of T_{r+2}

$$^{55}C_r\times 2^{55-r}\times\left(\frac{1}{3}\right)^r={}^{55}C_{r+1}\times\left(2\right)^{54-r}\times\left(\frac{1}{3}\right)^{r+1}$$

$\Rightarrow$ r = 6

$\Rightarrow$ Coff. of T_7 = Coff. of T_8

17. If the distance between planes, $4x - 2y - 4z + 1 = 0$ and $4x - 2y - 4z + d = 0$ is 7, then d is :-

(1) 41 or – 42 (2) – 42 or 44

(3) 42 or – 43 (4) – 41 or 43

Ans. (4)

Sol. $\left|\frac{d-1}{\sqrt{4^2+2^2+4^2}}\right|=7\Rightarrow\left|\frac{d-1}{6}\right|=7$

d– 1 = ± 42

d = + 43, – 41

18. The minimum area of a triangle formed by any tangent to the ellipse $\frac{x^2}{16}+\frac{y^2}{81}$ = 1 and the co-ordinate axes is :-

(1) 26 (2) 18

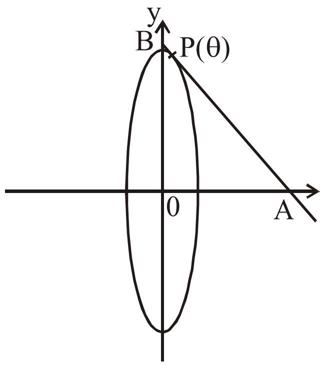
(3) 36 (4) 12

Ans. (3)

Sol. Let P(4cosθ, 9sinθ) be a point on ellipse

equation of tangent $\frac{x}{4}\cos\theta+\frac{y}{9}\sin\theta=1$

Let A & B are point of intersection of tangent at P with co-ordinate axes.



$A\left(\frac{4}{\cos\theta},0\right)$ $B\left(0,\frac{9}{\sin\theta}\right)$

Area of ΔOAB = $\frac{1}{2}\left(\frac{4}{\cos\theta}\right)\left(\frac{9}{\sin\theta}\right)=\frac{36}{\sin 2\theta}$

$(\text{Area})_{\min}$ = 36 as sin2θ = 1

19. If $\hat{x},\hat{y}$ and $\hat{z}$ are three unit vectors in three-dimensional space, then the minimum value of $|\hat{x}+\hat{y}|^2+|\hat{y}+\hat{z}|^2+|\hat{z}+\hat{x}|^2$ is :-

(1) $\frac{3}{2}$ (2) $3\sqrt{3}$ (3) 3 (4) 6

Ans. (3)

Sol. $\left|\hat{x}+\hat{y}\right|^2+\left|\hat{y}+\hat{z}\right|^2+\left|\hat{z}+\hat{x}\right|^2=K$ let

$K=2\hat{x}^2+2\hat{y}^2+2\hat{z}^2+2\hat{x}.\hat{y}+2\hat{y}.\hat{z}+2\hat{z}.\hat{x}$

K = 2 + 2 + 2 + 2[$\cos\theta_1$ + $\cos\theta_2$ + $\cos\theta_3$]

When $\theta_1 = \theta_2 = \theta_3 = \frac{2\pi}{3}$

Then $K_{\min} = 6 + 2\left(-\frac{3}{2}\right)$ = 6 – 3 = 3

20. Let G be the geometric mean of two positive numbers a and b, and M be the arthemetic mean of $\frac{1}{a}$ and $\frac{1}{b}$. If $\frac{1}{M}$: G is 4 : 5 , then a : b can be :-

(1) 2 : 3 (2) 1 : 4 (3) 1 : 2 (4) 3 : 4

Ans. (2)

21. The least positive integer n such that $1-\frac{2}{3}-\frac{2}{3^2}-\ldots\ldots-\frac{2}{3^{n-1}}<\frac{1}{100}$, is :-

(1) 7 (2) 4 (3) 5 (4) 6

Ans. (4)

Sol. $1-2\left(\frac{1}{3^1}+\frac{1}{3^2}\ldots\ldots\frac{1}{3^{n-1}}\right)<\frac{1}{100}$

$$1-2\left(\frac{1}{3}\right)\frac{\left[1-\frac{1}{3^{n-1}}\right]}{\left(\frac{2}{3}\right)}<\frac{1}{100}$$

$\Rightarrow\ 1-1+\frac{1}{3^{n-1}}<\frac{1}{100}$

$100 < 3^{n-1}$

n – 1 = 5

n = 6

22. If a line intercepted between the coordinate axes is trisected at a point A (4, 3), which is nearer to x-axis, then its equation is :-

(1) 3x + 8y = 36

(2) 4x – 3y = 7

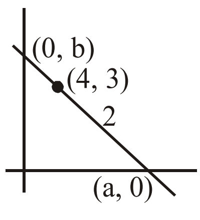
(3) x + 3y = 13

(4) 3x + 2y = 18

Ans. (4)

Sol. $4=\frac{a}{3}$ a = 12

$3=\frac{2b}{3}$ $b=\frac{9}{2}$



$\frac{x}{12}+\frac{y}{9}(2)=1$

$\boxed{3x+8y=36}$

23. If $f(\theta) = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\sin\theta & 1 & -\cos\theta \\ -1 & \sin\theta & 1 \end{vmatrix}$ and

A and B are respectively the maximum and the minimum values of f (θ), then (A, B) is equal to :-

(1) $(4, 2-\sqrt{2})$

(2) $(2+\sqrt{2}, 2-\sqrt{2})$

(3) (3, –1)

(4) $(2+\sqrt{2}, -1)$

Ans. (2)

Sol. Expanding the determinant, we get

$f(\theta)= 1(1 + \sin\theta\cos\theta) + \cos\theta(\sin\theta + \cos\theta) + 1(1-\sin^2\theta)$

$= 1 + 2\sin\theta\cos\theta + 2\cos^2\theta$

$= 1 + \sin2\theta + (1 + \cos2\theta)$

$= 2 + \sin2\theta + \cos2\theta$

Now,

$\sin2\theta + \cos2\theta$ lies between $-\sqrt{2}$ to $\sqrt{2}$

$$\left[\sqrt{2}\left(\sin\left(\frac{\pi}{4}+2\theta\right)\right) \to \pm\sqrt{2}\right]$$

$\therefore$ A = $2+\sqrt{2}$; $\quad B = 2-\sqrt{2}$

24. The integral $\int \frac{\sin^2 x\cos^2 x}{(\sin^3 x+\cos^3 x)^2}dx$ is equal to :-

(1) $-\frac{\cos^3 x}{3(1+\sin^3 x)}+c$ (2) $\frac{1}{(1+\cot^3 x)}+c$

(3) $-\frac{1}{3(1+\tan^3 x)}+c$ (4) $\frac{\sin^3 x}{(1+\cos^3 x)}+c$

Ans. (3)

Sol. $$\int \frac{\sin^2 x\cos^2 x dx}{\left(\sin^3 x+\cos^3 x\right)^2} = \int \frac{\tan^2 x\sec^2 x dx}{\left(1+\tan^3 x\right)^2}$$

Put $\tan^3 x = t$

$3\tan^2 x.\sec^2 x dx = dt$

$$\int \frac{dt}{3(1+t)^2} = \frac{-1}{3(1+t)}+C$$

$$= \frac{-1}{3(1+\tan^3 x)}+C$$

25. A symmetrical form of the line of intersection of the planes x = ay + b and z = cy + d is :-

(1) $\frac{x-a}{b}=\frac{y-0}{1}=\frac{z-c}{d}$

(2) $\frac{x-b-a}{b}=\frac{y-1}{0}=\frac{z-d-c}{d}$

(3) $\frac{x-b-a}{a}=\frac{y-1}{1}=\frac{z-d-c}{c}$

(4) $\frac{x-b}{a}=\frac{y-1}{1}=\frac{z-d}{c}$

Ans. (3)

Sol. $\frac{x-b}{a}=y=\frac{z-d}{c}$

$$\Rightarrow \frac{x-b}{a}-1=y-1=\frac{z-d}{c}-1$$

$$\frac{x-b-a}{a}=\frac{y-1}{1}=\frac{z-d-c}{c}$$

26. 8-digit numbers are formed using the digits 1,1,2,2,2,3,4,4. The number of such number in which the odd digits do not occupy odd places, is :-

(1) 60 (2) 48 (3) 160 (4) 120

Ans. (4)

Sol. No. of ways of selecting 3 odd places out of 4 odd places.

$${}^4C_3 \times \frac{3!}{2!} \times \frac{5!}{3!2!}$$

$= 4 \times 3 \times 5 \times 2$

$= 120$

27. If $1 + x^4 + x^5 = \sum_{i=0}^{5} a_i\,(1 + x)^i$, for all x in **R**, then a_2 is :-

(1) – 8 (2) 6 (3) 10 (4) – 4

Ans. (4)

Sol. $1 + x^4 + x^5 = a_0 + a_1(1 + x) + a_2(1 + x)^2 + a_3(1 + x)^3 + a_4(1 + x)^4 + a_5(1 + x)^5$

$= a_0 + a_1(1 + x) + a_2(1 + 2x + x^2) + a_3(1 + 3x + 3x^2 + x^3) + a_4(1 + 4x + 6x^2 + 4x^3 + x^4) + a_5(1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5)$

So, Coeff. of x^i in LHS = Coeff. of x^i on RHS

$i = 5 \Rightarrow 1 = a_5$ ...(i)

$i = 4 \Rightarrow 1 = a_4 + 5a_5 = a_4 + 5$

$\Rightarrow a_4 = -4$ ...(ii)

$i = 3 \Rightarrow 0 = a_3 + 4a_4 + 10a_5$

$\Rightarrow a_3 - 16 + 10 = 0$

$\Rightarrow a_3 = 6$ ...(iii)

$i = 2 \Rightarrow 0 = a_2 + 3a_3 + 6a_4 + 10a_5$

$\Rightarrow a_2 + 18 - 24 + 10 = 0$

$\Rightarrow a_2 = -4$

Put $x = -1$

$1 = a_0$

Now differentiate w.r.t. x.

$4x^3 + 5x^4 = a_1 + 2a_2(1 + x) + 3a_3(1 + x)^2 + ...$

Put $x = -1$

$\Rightarrow 1 = a_1$

Again differentiate w.r.t. x

$12x^2 + 20x^3 = 2xa_2 + 6a_3\,(1 + x)$

Put $x = -1$

$12 - 20 = 2a_2 \Rightarrow a_2 = -4$

28. Let p, q, r denote arbitrary statements. Then the logically equivalent of the statement $p \Rightarrow (q \vee r)$ is :-

(1) $(p \vee q) \Rightarrow r$

(2) $(p \Rightarrow \sim q) \wedge (p \Rightarrow r)$

(3) $(p \Rightarrow q) \wedge (p \Rightarrow \sim r)$

(4) $(p \Rightarrow q) \vee (p \Rightarrow r)$

Ans. (4)

Sol. $p \rightarrow (q \vee r)$

$\sim p \vee (q \vee r)$

$(\sim p \vee q) \vee (\sim p \vee r)$

$(p \Rightarrow q) \vee (p \Rightarrow r)$

29. If [ ] denote the greatest integer function, then the integral $\int_0^{\pi} [\cos x]\,dx$ is equal to :-

(1) 0 (2) $\frac{\pi}{2}$ (3) $-\frac{\pi}{2}$ (4) –1

Ans. (3)

Sol. $\int_0^{\pi} [\cos x]dx$

$= \int_0^{\pi/2} 0.\,dx + \int_{\frac{\pi}{2}}^{\pi} -1\,dx$

$= [-x]_{\frac{\pi}{2}}^{\pi} = -\frac{\pi}{2}$

30. A relation on the set $A = \{x : |x| < 3, x \in Z\}$, where Z is the set of integers is defined by $R = \{(x, y) : y = |x|, x \neq -1\}$. Then the number of elements in the power set of R is : :-

(1) 32 (2) 64 (3) 16 (4) 8

Ans. (3)

Sol. $A = \{-2, -1, 0, 1, 2\}$

$R = \{(-2, 2)\ (0, 0)\ (1, 1), (1, 2)\}$

$n(P(R)) = 2^4 = 16$