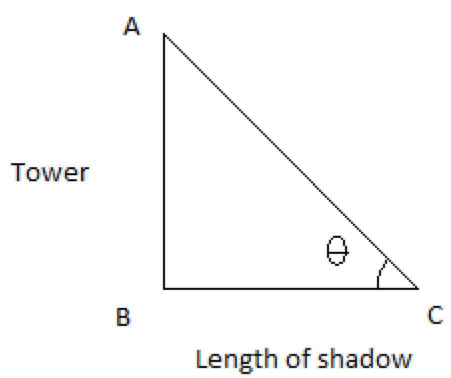
CBSE – CLASS X MATHS - 2012

General Instructions:

1. All questions are compulsory.
2. The question paper consists of 34 questions divided into four sections A, B, C and D.
3. Section A contains 10 questions of 1 mark each, which are multiple choices type Questions, Section B contains 8 questions of 2 marks each, Section C contains 10 questions of 3 marks each, Section D contains 6 questions of 4 marks each.
4. There is no overall choice in the paper. However, internal choice is provided in one question of 2 marks, 3 questions of 3 marks each and two questions of 4 marks each.
5. Use of calculators is not permitted.

Q1



Let AB be the tower and BC be the length of the shadow of the tower.

Here, θ is the angle of elevation of the sun.

Given, length of shadow of tower = $\sqrt{3}$ x Height of the tower.

BC = $\sqrt{3}$ AB …. (1)

In right $\triangle$ABC,

tan θ = $\frac{AB}{BC}$ $\left[\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}\right]$

$\tan\theta = \frac{AB}{\sqrt{3}AB}$ [Using (1)]

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \tan 30^0$ [$\therefore \tan 30^0 = \frac{1}{\sqrt{3}}$]

$\Rightarrow \theta = 30^0$

Thus, the angle of elevation of the sun is 30^0.

Hence, the correct answer is B.

Q2

Let r_1 and r_2 be the radii of the two given circles.

Given, $2r_1 = 10$ cm

$\therefore r_1 = 5$ cm

Also, $2r_2 = 24$ cm

$\therefore r_2 = 12$ cm

Let R be the radius of the larger circle.

Given, area of larger circle = Sum of areas of two given circles

$\therefore \pi R2 = \pi r_1^2 + \pi r_2^2$

$\Rightarrow R^2 = (5 \text{ cm})^2 + (12 \text{ cm})^2$

$\Rightarrow R^2 = 25 \text{ cm}^2 + 144 \text{ cm}^2$

$\Rightarrow R^2 = 169 \text{ cm}^2$

$\Rightarrow R = \sqrt{169}$ cm

$\Rightarrow R = 13$ cm

Thus, the diameter of the larger circle is (2 x 13) cm = 26 cm

Hence, the correct answer is B.

Q3

Let the radius and height of the original cylinder be r and h respectively.

∴ Volume of the original cylinder = $\pi r^2 h$

According to the question, radius of the new cylinder is halved keeping the height same.

$\Rightarrow$ Radius of the new cylinder = $\frac{r}{2}$

Also, height of the new cylinder = h

∴ Volume of the new cylinder = $\pi \left(\frac{r}{2}\right)^2 h = \frac{\pi r^2 h}{4}$

∴ $\frac{\text{Volume of the new cylinder}}{\text{Volume of the original cylinder}}$ = $(\pi r^2h/4)/(\pi r^2h)$ = (1/4) = 1 : 4

Hence, the correct answer is C.

Q4

Elementary events associated with random experiment of the given two dice are:

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

∴ Total number of outcomes = 36

Let A be the event of getting same number on both dice.

Elementary events favourable to event A are (1,1),(2,2)(3,3),(4,4),(5,5) and (6,6).

$\Rightarrow$ Favourable number of outcomes = 6

∴ $P(A) = \frac{6}{36} = \frac{1}{6}$

So, required probability is $\frac{1}{6}$

Hence, the correct answer is C.

Q5

Let P(x,y) divides line segment joining A(1,3) and B(4,6) in the ratio 2 : 1.

We know that , the coordinates of a point(x,y) dividing the line segment joining the points

(x_1, y_1) and (x_2,y_2) in the ratio $m_1 : m_2$ are given by

x = $\frac{m_1x_2+m_2x_1}{m_1+m_2}$ and y = $\frac{m_1y_2+m_2y_1}{m_1+m_2}$

∴ Here x = $\frac{2(4)+1(1)}{2+1}$ and y = $\frac{2(6)+1(3)}{2+1}$

$\Rightarrow$ x = $\frac{9}{3}$ and y = $\frac{15}{3}$

$\Rightarrow$ x = 3 and y = 5

Thus, (3, 5) divides the line segment AB in the ratio 2: 1

Hence, the correct answer is B.

Q6

Let AB be the diameter and O be the centre of the circle.

We are given co-ordinates of one end point of circle and co-ordinates of its centre.

So, co-ordinates of A are (2, 3) and centre O are (-2, 5).

Let co-ordinates of point B be (x, y).

We know that centre of a circle is the midpoint of the diameter.

∴ By midpoint formula,

-2 = $\frac{2+x}{2}$ and 5 = $\frac{3+y}{2}$

$\Rightarrow -4 = 2 + x$ and $10 = 3 + y$

$\Rightarrow$ x = -6 and y = 7

So, other end of the diameter is (-6, 7).

Hence, the correct answer is A.

Q7

Odd natural numbers are in the pattern 1,3,5,7,9…..

These numbers form an A.P. where a = 1, d = 3 – 1 = 2

We know that, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore s_{20} = \frac{20}{2}[2 \times 1 + (20-1) \times 2]$

$= \frac{20}{2}[2 + 19 \times 2]$

$=10[2 + 38]$

$=10 \times 40$

$=400$

Thus, the sum of first 20 odd natural numbers is 400.

Hence, the correct answer is C.

Q8

The given equations are $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$

Given, 1 is the root of both the equations

Therefore, y = 1 will satisfy both these equations.

Putting y = 1 in $ay^2 + ay + 3 = 0$, we get

$a(1)^2 + a \times 1 + 3 = 0$

$\therefore a + a + 3 = 0$

$\Rightarrow 2a + 3 = 0$

$\Rightarrow 2a = -3$

$\Rightarrow a = -\frac{3}{2}$ ……..(1)

Putting y = 1 in $y^2 + y + b = 0$, we get

$(1)^2 + 1 + b = 0$

∴ 1+1+b = 0

⇒ 2 + b = 0

⇒ b = −2 ......(2)

∴ ab = $-\frac{3}{2} * (-2) = 3$ [Using (1) & (2)]

Thus, the value of ab is 3.

Hence, the correct answer is A.

Q9

Given, AP = 4 cm, BP = 3 cm and AC = 11 cm.

The lengths of tangents drawn from an external point to the cirlce are equal.

AP = AR, BP = BQ, CQ = CR .... (1)

AC = 11 cm

⇒ AR + RC = 11 cm

⇒ AP + CQ = 11 cm [From equation (1)]

⇒ 4 cm + CQ = 11 cm

⇒ CQ = (11 −4) cm

⇒ CQ = 7 cm

BP = BQ = 3 cm

Now, BC = BQ + QC

⇒ BC = (3+7) cm

⇒ BC = 10 cm

Hence, the correct option is B.

Q10

EK = 9 cm

As length of tangents drawn from an external point to the circle are equal.

∴ EK = EM = 9 cm

Also, DH = DK and FH = FM ……(1)

EK = EM = 9 cm

$\Rightarrow$ ED + DK = 9 cm and EF + FM = 9 cm

$\Rightarrow$ ED + DH = 9 cm and EF + FM = 9 cm [From equation (i)] ….(ii)

Perimeter of $\triangle$EDF = ED + DF + EF

= ED + DH + HF + EF

= (9 + 9) cm [From equation (ii)]

= 18 cm

Hence, the correct option is A.

Q11

The given points are A (0, 2), B (3, p) and C (p, 5).

According to the question, A is equidistant from point B and C.

∴ AB = AC

$\Rightarrow \sqrt{(3-0)^2+(p-2)^2} = \sqrt{(p-0)^2+(5-2)^2}$

$\Rightarrow \sqrt{(3)^2+(p-2)^2} = \sqrt{(p)^2+(3)^2}$

$\Rightarrow \sqrt{9+p^2+4-4p} = \sqrt{p^2+9}$

$\Rightarrow \sqrt{p^2-4p+13} = \sqrt{p^2+9}$

On squaring both sides, we obtain:

$\Rightarrow p^2 - 4p + 13 = p^2 + 9$

$\Rightarrow -4p = -4$

$\Rightarrow$ p = 1

Q12

Total number of outcomes = 50

Multiples of 3 and 4 which are less than or equal to 50 are:

12,24,36,48

Favourable number of outcomes = 4

Probability of the number being a multiple of 3 and 4

$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

$= \frac{4}{50}$

$= \frac{2}{25}$

Q13

Given, volume of hemisphere = $2425\frac{1}{2}\ cm^3 = \frac{4851}{2}\ cm^3$

Let the radius of the hemisphere be 'r' cm.

Volume of hemisphere = $\frac{2}{3}\pi r^3$

$\Rightarrow \frac{2}{3}\pi r^3 = \frac{4851}{2}$

$\Rightarrow \frac{2}{3} \times \frac{22}{7} \times r^3 = \frac{4851}{2}$

$\Rightarrow r^3 = \frac{4851 \times 3 \times 7}{2 \times 2 \times 22}$

$\Rightarrow r^3 = \frac{441 \times 21}{2 \times 2 \times 2}$

$\Rightarrow r^3 = \frac{21 \times 21 \times 21}{2 \times 2 \times 2}$

$\Rightarrow r = \sqrt[3]{\frac{21 \times 21 \times 21}{2 \times 2 \times 2}}$

$\Rightarrow r = \frac{21}{2}$ cm .....(1)

$\therefore$ Curved surface area of hemisphere = $2\pi r^2$

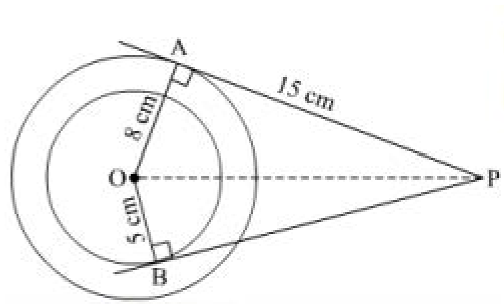
$= 2 \times \frac{22}{7} \times \left(\frac{21}{2}\right)^2$ [Using (1)]

$= 2 \times \frac{22}{7} \times \frac{21 \times 21}{2 \times 2}$

$= 693\ cm^2$

Q14

Given that : OA = 8 cm, OB = 5 cm and AP = 15 cm



To find : BP

Construction : Join OP.

Now, $OA \perp AP$ and $OB \perp BP$ $\because$ $\left[\begin{array}{l}\text{Tangent to a circle is perpendicular to the} \\ \text{Radius through the point of contact}\end{array}\right]$

$\Rightarrow \angle OAP = \angle OBP = 90^0$

On applying Pythagoras theorem in $\triangle OAP$, we obtain:

$(OP)^2 = (OA)^2 + (AP)^2$

$\Rightarrow (OP)^2 = (8)^2 + (15)^2$

$\Rightarrow (OP)^2 = 64 + 225$

$\Rightarrow OP = \sqrt{289}$

$\Rightarrow (OP)^2 = 289$

$\Rightarrow OP = 17$

Thus, the length of OP is 17 cm.

On appling Pythagoras theorem in $\triangle OBP$, we obtain :

$(OP)^2 = (OB)^2 + (BP)^2$

$\Rightarrow (17)^2 = (5)^2 + (BP)^2$

$\Rightarrow 289 = 25 + (BP)^2$

$\Rightarrow (BP)^2 = 289 - 25$

$\Rightarrow (BP)^2 = 264$

$\Rightarrow BP = 16.25$ cm (approx.)

Hence, the length of BP is 16.25 cm.

Q15

Given : An isoceles $\triangle$ ABC with AB = AC, circumscribing a circle.

To prove : P bisects BC

Proof : AR and AQ are the tangents drawn from an external point A to the circle.

∴AR = AQ (Tangents drawn from an external point to the circle are equal)

Similarly, BR = BP and CP = CQ.

It is given that in $\triangle$ ABC, AB = AC.

$\Rightarrow$ AR + RB = AQ + QC.

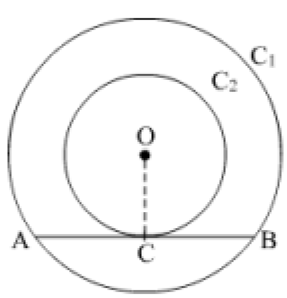
$\Rightarrow$ BR = QC (As AR = AQ)

$\Rightarrow$ BP = CP (As BR = BP and CP = CQ)

$\Rightarrow$ P bisects BC.

Hence, the result is proved.

OR



Given : Two concentric circles C_1 and C_2 with centre O, and AB is the chord of C_1 touching C_2 at C.

To prove : AC = CB

Construction : Join OC.

Proof : AB is the chord of C_1 touching C_2 at C, then AB is the tangent to C_2 at C with OC as its radius.

We know that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore$ OC $\perp$ AB

Considering, AB as the chord of the circleC_1. So, OC $\perp$AB.

$\therefore$ OC is the bisector of the chord AB.

Hence, AC = CB (Perpendicular from the centre to the chord bisects the chord).

Q16

It is given that OABC is a square of side 7 cm.

$\therefore$ Area of square OABC $= (7)^2 \text{ cm}^2 = 49 \text{ cm}^2$

Also, it is given that OAPC is a quadrant of circle with centre O.

$\therefore$ Radius of the quadrant of the circle = OA = 7 cm

$\therefore$Area of the quadrant of circle = $\frac{1}{4}$ (πr^2)

$= \frac{1}{4}(\pi * 7^2) \text{ cm}^2$

$= \frac{49\pi}{4} \text{ cm}^2$

$= \frac{49}{4} \times \frac{22}{7} \text{ cm}^2$

$= \frac{77}{2} \text{ cm}^2$

$\therefore$ Area of the shaded region = Area of Square — Area of Quadrant of circle.

$= [\, 49 - \frac{77}{2} \,] \text{ cm}^2$

$= [\, \frac{98-77}{2} \,] \text{ cm}^2$

$= \frac{21}{2} \text{ cm}^2$

$= 10.5 \text{ cm}^2$

Thus, the area of the shaded region is 10.5 cm^2.

Q17

Three digit natural numbers which are multiples of 7 are 105, 112, 119,…..,994.

105, 112, 119,……..994 are in A.P.

First term (a) = 105

Commom difference (d) = 7

Let 994 be the n^{th} term of A.P.

$\therefore a_n = 994$

$\Rightarrow 105 + (n-1) \times 7 = 994$ [$\because a_n = a + (n-1)d$]

$\Rightarrow 7(n-1) = 994 - 105$

$\Rightarrow 7(n-1) = 889$

$\Rightarrow n-1 = 127$

$\Rightarrow n = 128$

Sum of all the terms of A.P. = $\frac{128}{2}(105 + 994)$ [$\because S_n = \frac{n}{2}(a + l)$, l being last term]

$= 64 \times 1099$

$= 70336$

Thus, the sum of all three digit natural numbers which are multiples of 7 is 70336.

Q18

The given quadratic equation is $3x^2 - 2kx + 12 = 0$

On comparing it with the general quadratic equation $ax^2 + bx + c = 0$, we obtain

a = 3, b = -2k and c = 12

discriminant, 'D' of the given quadratic equation is given by

$D = b^2 - 4ac$

= $(-2k)^2 - 4 * 3 * 12$

= $4k^2 - 144$

For equal roots of the given quadratic equations, Discriminant will be equal to 0.

i.e., D = 0

$\Rightarrow 4k^2 - 144 = 0$

$\Rightarrow 4\,(k^2 - 36) = 0$

$\Rightarrow k^2 = 36$

$\Rightarrow k = \pm 6$

Thus, the values of k for which the quadratic equation $3x^2 - 2kx + 12 = 0$ will have equal roots are 6 and -6.

Q19

The given points are A (3,-5) and B (-4, 8).

Here, $x_1 = 3$, $y_1 = -5$, $x_2 = -4$ and $y_2 = 8$.

Since $\frac{AP}{PB} = \frac{K}{1}$, the point P divides the line segment joining the points A and B in the ratio K : 1. The coordinates of P can be found using the section formula $\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}$n

here, m = K and n = 1

Co-ordinates of P = $\left(\frac{\mathrm{K\ x\ }(-4)+1\mathrm{\ x\ }3}{\mathrm{K}+1}, \frac{\mathrm{K\ x\ }8+1\mathrm{\ x\ }(-5)}{\mathrm{K}+1}\right) = \left(\frac{-4\mathrm{K}+3}{\mathrm{K}+1}, \frac{8\mathrm{K}-5}{\mathrm{K}+1}\right)$

It is given that, P lies on the line x+y = 0.

$\therefore \frac{-4K+3}{K+1} + \frac{8K-5}{K+1} = 0$

$\Rightarrow \frac{-4K+3+8K-5}{K+1} = 0$

$\Rightarrow 4K - 2 = 0$

$\Rightarrow 4K = 2$

$\Rightarrow K = \frac{1}{2}$

Thus, the required value of K is $\frac{1}{2}$.

Q20

Given, vertices of a triangle are (1,-3), (4, p) and (-9, 7).

$x_1 = 1$, $y_1 = -3$

$x_2 = 4$, $y_2 = p$

$x_3 = -9$, $y_3 = 7$

Area of given triangle

$= \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

$= \frac{1}{2} [1(p-7) + 4(7+3) + (-9) (-3-p)]$

$= \frac{1}{2} [p-7+40+27+9p]$

$= \frac{1}{2} [10p + 60]$

$= 5 (p+6)$

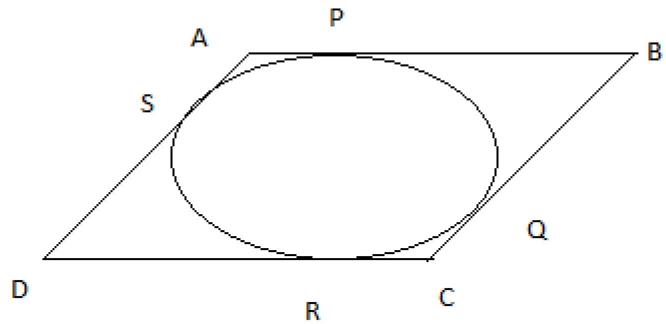
Here, the obtained expression may be positive or negative.

$\therefore 5(p+6) = 15$ or $5(p+6) = -15$

$\Rightarrow p + 6 = 3$ or $p + 6 = -3$

$\Rightarrow p = -3$ or $p = -9$

Q21



Since ABCD is a parallelogram,

AB = CD ...(1)

BC = AD ...(2)

It can be observed that

DR = DS (Tangents on the circle from point D)

CR = CQ (Tangents on the circle from point C)

BP = BQ (Tangents on the circle from point B)

AP = AS (Tangents on the circle from point A)

Adding all these questions, we obtain

DR + CR +BP + AP = DS + CQ + BQ + AS

(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC

On putting the values of equations (1) and (2) in this equation, we obtain

2AB = 2BC

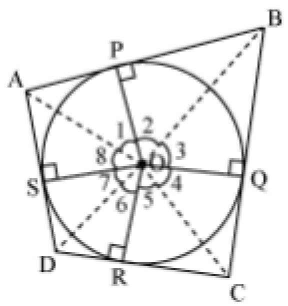
AB = BC ...(3)

Comparing equations (1),(2) and (3), we obtain

AB = BC = CD = DA

Hence, ABCD is a rhombus.

OR



Let ABCD be a quadrilateral circumscribing a circle centered at O such that it touches the circle at point P, Q, R, S. let us join the vertices of the quadrilateral ABCD to the center of the circle.

Consider $\triangle$OAP and $\triangle$ OAS,

AP = AS (Tangents from the same point)

OP = OS (Radii of the same circle)

OA = OA (Common side)

$\triangle$OAP $\cong$ $\triangle$ OAS (SSS congruence criterion)

Therefore, A $\longleftrightarrow$ A, P $\longleftrightarrow$ S, O $\longleftrightarrow$ O

And thus, $\angle$POA = $\angle$AOS

$\angle 1 = \angle 8$

Similarly,

$\angle 2 = \angle 3$

$\angle 4 = \angle 5$

$\angle 6 = \angle 7$

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^0$

$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^0$

$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^0$

$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^0$

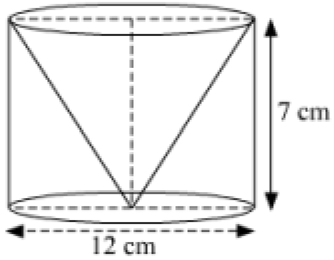
$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180^0$

$\angle AOB + \angle COD = 180^0$

Similarly, we can prove that $\angle BOC + \angle DOA = 180^0$

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Q22



It is given that, height (h) of cylindrical part = height (h) of the conical part = 7 cm

Diameter of the cylindrical part = 12 cm

$\therefore$ Radius (r) of the cylindrical part = $\frac{12}{2}$ cm = 6 cm

$\therefore$ Radius of conical part = 6 cm

Slant height (l) of conical part = $\sqrt{r^2 + h^2}$ cm

= $\sqrt{6^2 + 7^2}$ cm

= $\sqrt{36 + 49}$ cm

= $\sqrt{85}$ cm

= 9.22 cm (approx.)

Total surface area of the remaining solid

= CSA of cylindrical part + CSA of conical part + Base area of the circular part

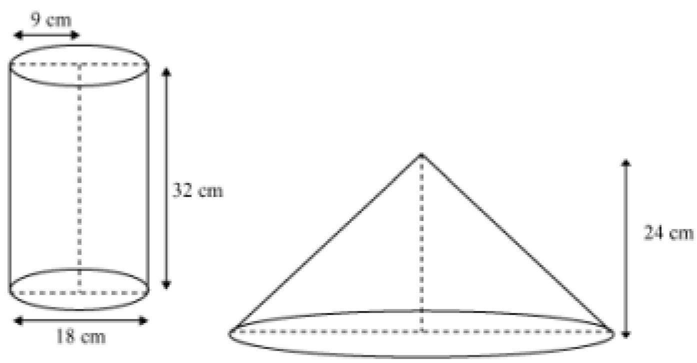
$= 2\,\pi r h + \pi r l + \pi \mathrm{r}^2$

= 2 x $\frac{22}{7}$ x 6 x 7 cm^2 + $\frac{22}{7}$ x 6 x 9.22 cm^2 + $\frac{22}{7}$ x 6 x 6 cm^2

= 264 cm^2 + 173.86 cm^2 + 113.14 cm^2

= 551 cm^2

OR



Height (h_1) of cylindrical bucket = 32 cm

Radius (r_1) of circular end of bucket = 18 cm

Height (h_2) of conical heap = 24 cm

Let the radius of the circular end of conical heap be r_2.

The volume of sand in the cylindrical bucket will be equal to the volume of sand in the conical heap.

Volume of sand in the cylindrical bucket = Volume of sand in conical heap

$\pi * r_1^2 * h_1 = \frac{1}{3}\pi * r_2^2 * h_2$

$\pi * 18^2 * 32\,\pi \frac{1}{3} * 24$

$\pi * 18^2 * 32 = \frac{1}{3}\pi * r_2^2 * 24$

$r_2^2 = \frac{3 * 18^2 * 32}{24} = 18^2$ x 4

r_2 = 18 x 2 = 36 cm

slant height = $\sqrt{36^2 + 24^2} = 12^2 + (3^2 + 2^2) = 12\sqrt{13}$ cm therefore, the radius and slant height of the conical heap are 36 cm and 12 $\sqrt{13}$ cm respectively.

Q23

PQ and AB are the arcs of two concentric circles of radii 7 cm and 3.5 cm respectively.

Let r_1 and r_2 be the radii of the outer and the inner circle respectively.

Suppose θ be the angle subtended by the arcs at the centre O.

Then r_1 = 7 cm, r_2 = 3.5 cm and θ = 30^0

Area of the shaded region

= Area of sector OPQ − Area of sector OAB

$= \frac{\theta}{360^0}\pi r_1^2 - \frac{\theta}{360^0}\pi r_2^2$

$= \frac{\theta}{360^0}\pi\,(r_1^2 - r_2^2)$

$= \frac{30^0}{360^0} \times \frac{22}{7}$ [(7 cm)^2 – (3.5 cm)^2]

$= \frac{1}{12} \times \frac{22}{7} \times (49 - 12.25)$ cm^2

$= \frac{1}{12} \times \frac{22}{7} \times 36.75$ cm^2

= 9.625 cm^2

thus, the area of the shaded region is 9.625 cm^2.

Q24

The given quadratic equation is $4x^2 - 4ax + (a^2 - b^2) = 0$

$4x^2 - 4ax + (a^2 - b^2) = 0$

$\therefore 4x^2 - 4ax + (a-b)(a+b) = 0$

$\Rightarrow 4x^2 + [-2a - 2a + 2b - 2b]x + (a-b)(a+b) = 0$

$\Rightarrow 4x^2 + (2b-2a)x - (2a+2b)x + (a-b)(a+b) = 0$

$\Rightarrow 4x^2 + 2(b-a)x - 2(a+b)x + (a-b)(a+b) = 0$

$\Rightarrow$ 2x [2x – (a-b)] – (a+b) [2x – (a-b)] = 0

$\Rightarrow$ [2x – (a – b)] or [2x – (a + b) = 0

$\Rightarrow$ 2x = a – b or 2x = a + b

$\Rightarrow$ $x = \frac{a-b}{2}$ or $x = \frac{a+b}{2}$

Thus, the solution of the given quadratic equation is given by $x = \frac{a-b}{2}$ or $x = \frac{a+b}{2}$

OR

The given quadratic equation is $3x^2 - 2\sqrt{6x} + 2 = 0$

Comparing with the quadratic equation $ax^2 + bx + c = 0$, we have

$a = 3$, $b = -2\sqrt{6}$ and $c = 2$

discriminant of the given quadratic equation

$D = b^2 - 4\,ac = (2\sqrt{6})^2 - 4\times3\times2 = 24 - 24 = 0$

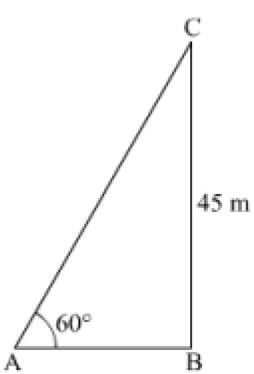
$\therefore x = \frac{-(-2\sqrt{6})\pm\sqrt{0}}{2 \text{ x } 3}$ $\left[\because x = \frac{-b \pm\sqrt{D}}{2a}\right]$

$\Rightarrow$ $x = \frac{2\sqrt{6}}{6}$

$\Rightarrow$ $x = \frac{\sqrt{6}}{3}$

Thus, the solution of the given quadratic equation is $x = \frac{\sqrt{6}}{3}$.

Q25



Let C be the position of kite above the ground such that it subtends an angle of 60^0 at point A on the ground.

Suppose the length of the string. AC be l m.

Given, BC = 45 m and $\angle$ BAC = 60^0.

In $\triangle$ ABC :

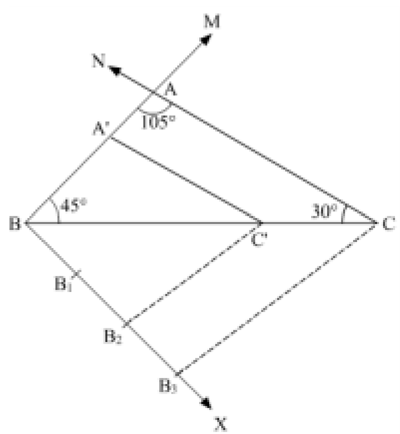
sin 60^0 = $\frac{BC}{AC}$ $\because$ sin θ = $\frac{\text{Perpendicular}}{\text{Hypotensue}}$

therefore, $\frac{\sqrt{3}}{2}$ = $\frac{45}{l}$

⇨ l = $\frac{45 \times 2}{\sqrt{3}} = \frac{90}{\sqrt{3}} = 30\sqrt{3}$

Thus, the length of the string is $30\sqrt{3}$ m.

Q26



Given:

BC = 6 cm, $\angle$ C = 30^0 and $\angle$ A = 105^0

Therefore $\angle$ B = 180^0– ($\angle$ A + $\angle$ C) (In $\triangle$ABC, $\angle$ A + $\angle$ B + $\angle$ C = 180^0)

= 180^0 – (105^0 + 30^0)

= 45^0

Steps of construction :

1. draw a line BC = 6 cm.

2. draw a ray CN making an angle of 30^0 at C.
3. draw a ray BM making an angle of 45^0 at B.
4. locate the point of intersection of rays CN and BM and name it as A.
5. ABC is the triangle whose similar triangle is to be drawn.
6. Draw any ray BX making anacute angle with BC on the side opposite to the vertex A.
7. Locate 3 (Greater of 2 and 3 in 2/3) points B_1,B_2 and B_3 on BX so that $BB_1 = B_1B_2 = B_2B_3$.
8. Join B_3C and draw a line through B_2 (smaller of 2 and 3 in 2/3) parallel to B_3C to intersect BC at C'.
9. Draw a line through C' parallel to the line CA to intersect BA at A'.
10. A 'BC' is the required similar triangle whose sides are 2/3 times the corresponding sides of $\triangle$ABC.

Q27

Let a be the first term and d be the common difference of the given A.P.

According to the given question,

16^{th} term of the AP = 2 x 8^{th} term of the AP + 1

i.e., $a_{16} = 2a_8 + 1$

a+ (16-1)d = 2[a+(8-1)d] +5 (because a_n = a+(n-1)d)

- ⇨ a+15d = 2[a+7d]+5
- ⇨ a+15d = 2a+14d+5
- ⇨ d = a+1 .....(1)

Also, 12^{th} term, a_{12} = 47

- ⇨ a+(12-1)d = 47
- ⇨ a+11d = 47
- ⇨ a+11(a+1) = 47 [Using (1)]
- ⇨ a+11a+11 = 47
- ⇨ 12 a = 36
- ⇨ a = 36

On putting the value of a in (1), we get d = 3+1 = 4

Thus, n^{th} term of the AP, a_n = a + (n-1)d

On putting the respective values of a and d, we get

a_n = 3+(n-1) 4 = 3 + 4n- 4 = 4n – 1

hence, n^{th} term of the given AP is 4n – 1.

Q28

Total number of cards in a deck of cards = 52

Therefore, Total number of outcomes = 52

(i) let A denote the event of getting a king of red colour.

There are two cards in favour of getting a king of red colour i.e., king of heart and king of diamond.

Number of outcomes in favour of event A =2

$\therefore$ P(A) = $\frac{\text{Outcomes in favour of event A}}{\text{Total number of outcomes}} = \frac{2}{52} = \frac{1}{26}$

(ii) let B denote the event of getting a face card.

There are 12 cards in favour of getting a face card i.e., 4 King, 4 Queen and 4 Jack cards.

Number of outcomes in favour of event B = 12

P(B) = $\frac{\text{Outcomes in favour of event B}}{\text{Total number of outcomes}} = \frac{12}{52} = \frac{3}{13}$

(iii) let C denote the event of getting a queen of diamond.

There is one queen of diamond in the deck of cards.

P(C) = $\frac{\text{Outcomes in favour of event C}}{\text{Total number of outcomes}} = \frac{1}{52}$

Q29

Let the height of the bucket be h cm.

Suppose r_1 and r_2 be the radii of the circular ends of the bucket.

Given, r_1 = 28 cm and r_2 = 21 cm

Capacity of bucket = 28.49 litres

$\therefore$Volume of the bucket = 28.49 ×1000 cm^3 [1 litre = 1000 cm^3]

⇨ $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$ = 28.49 ×1000 cm^3

⇨ $\frac{1}{3} \times \frac{22}{7} \times \mathrm{h}\,[(28)^2 + (21)^2 + (28 \times 21)]\mathrm{cm}^2$ = 28490 cm^3

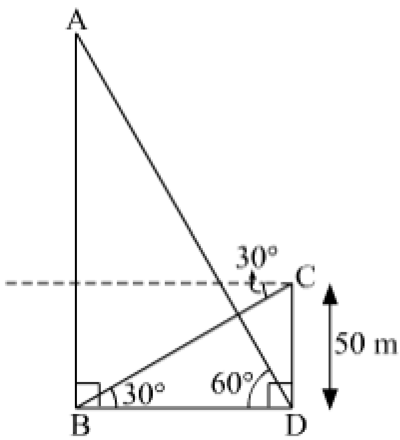
⇨ $\frac{22}{21}$ x h x 1813 = 28490 cm

⇨ h = $\frac{28940 * 21}{22 * 1813}$

⇨ h = 15 cm

Thus, the height of the bucket is 15 cm.

Q30



Let AB be the hill and CD be the tower.

Angle of elevation of the hill at the foot of the tower is 60^0. i.e., $\angle$ADB = 60^0 and the angle of depression of the foot of hill from the top of the tower is 30^0, i.e., $\angle$CBD = 30^0.

Now in right angled $\triangle$CBD :

tan 30^0 = $\frac{CD}{BD}$

⇨ BD = $\frac{CD}{\tan 30^0}$

⇨ BD = $\frac{50}{[\frac{1}{\sqrt{3}}]}$

⇨ BD = 50$\sqrt{3}$m

In right $\triangle$ ABD :

tan 60^0 = $\frac{AB}{CD}$

⇨ AD = BD X tan 60^0

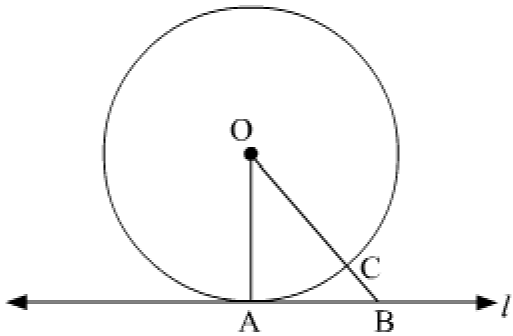
= 50$\sqrt{3}$ ×$\sqrt{3}$ m

= 50 ×3 m

= 150 m

Hence, the height of the hill is 150 m.

Q31



Given : A circle (O,r) and a tangent l at point A.

To prove : OA $\perp$ l

Construction : take any point B other than A on the tangent l. Join OB.

Suppose OB meets the cirlce at C.

Proof : Among all line segments joining the centre O to any point on l, the perpendicular is the shortest to l.

So, in order to prove OA $\perp$l we need to prove that OA is shorter than OB.

OA = OC (Radius of same circle)

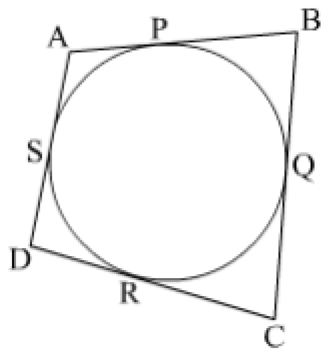
Now, OB = OC + BC

- ⇨ OB > OC
- ⇨ OB > OA
- ⇨ OA > OB

B is an arbitrary point on the tangent l. thus, OA is shorter than any other line segment joining O to any point on l.

Hence OA is perpendicular to l.

OR



Let the sides of the quadrilateral ABCD touch the circle at points P, Q, Rand S as shown in the figure.

We know that, tangents drawn from an external point to the circle are equal in length.

Therefore,

$$\left.\begin{array}{l} AP = AS \\ BP = BQ \\ CQ = CR \\ DR = DS \end{array}\right\} \quad .....(1)$$

Therefore,
AB + CD = (AP+BP) + (CR+DR)
= (AS+BQ) + (CQ+DS)   [Using (1)]
= (AS+DS) + (BQ+CQ)
= AD + BC

Hence, AB + CD = AD + BC

Q32

Let the number of books purchased by the shopkeeper be x.

Cost price of x books = Rs 80

∴ Original cost price of one book = Rs $\frac{80}{x}$

If the shopkeeper had pruchased 4 more books, then the number of books purchased by him would be (x+4).

∴ New cost price of one book = Rs $\frac{80}{x+4}$

Given, Original cost price of one book – New cost price of one book = Rs 1

$\therefore \frac{80}{x} - \frac{80}{x+4} = 1$

⇨ $\frac{80(x+4) - 80x}{x(x+4)} = 1$

⇨ $80x + 320 - 80x = x(x+4)$

⇨ $x^2 + 4x = 320$

⇨ $x^2 + 4x - 320 = 0$

⇨ $x^2 + 20x - 16x - 320 = 0$

⇨ $x(x+20) - 16(x+20) = 0$

⇨ $(x-16)(x+20) = 0$

⇨ $x - 16 = 0$ or $x + 20 = 0$

⇨ $x = 16$ or $x = -20$

$\therefore x = 16$ (because number of books cannot be negative)

OR

Let the first number be x.

Given : First number + Second number = 9

$\therefore$ x + Second number = 9

⇨ Second number = $9 - x$

Given, $\frac{1}{\text{First number}} + \frac{1}{\text{Second number}} = \frac{1}{2}$

$$\therefore \frac{1}{x} + \frac{1}{9-x} = \frac{1}{2}$$

⇨ $\frac{9-x+x}{x(9-x)} = \frac{1}{2}$

⇨ $9 \times 2 = 9x - x^2$

⇨ $x^2 - 9x + 18 = 0$

⇨ $x^2 - 6x - 3x + 18 = 0$

⇨ $x(x-6) - 3(x-6) = 0$

⇨ $(x-3)(x-6) = 0$

⇨ $x - 3 = 0$ or $x - 6 = 0$

⇨ $x = 3$ or $x = 6$

when x = 3, we have

$9-x = 9-3 = 6$

When x=6, we have

$9-x = 9-6 = 3$

Thus, the two numbers are 3 and 6.

Q33

First term of the A.P (a) = 7; sum of first 20 terms = -240.

The sum of first in terms of an AP, Sn $=\frac{n}{2}$ $[\,2a+(n-1)d]$, where a is the first term and d is the common difference.

$\therefore S_{20} = \frac{20}{2}$ [2 × 7 + (20-1)d] = -240

- ⇨ 10 [14 + 19d] = -240
- ⇨ 14 + 19 d = - 24
- ⇨ 19d = - 24 – 14
- ⇨ 19d = - 38
- ⇨ d = -2

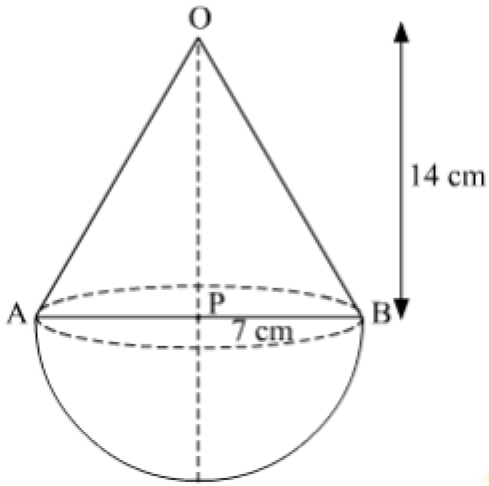
now, 24^{th} term of the AP, a_{24} = a+(24-1)d
on putting respective values of a and d, we get
a_{24} + 7 + 23 × (-2) = 7 – 46 = - 39
hence, 24^{th} term of the given AP is -39.

Q34

Let r and h be radius and height of the cone respectively.

Radius of cone (r) = 7 cm (Given)

Diameter of cone = 2 × r = (2 × 7)cm = 14 cm



According to the question, height of the cone is equal to its diameter.

∴ Height of cone (h) = 14 cm

Radius of hemisphere = Radius of cone = 7cm

∴ Volume of solid = Volume of cone + Volume of hemisphere

$= \frac{1}{3}\ \pi r^2 h + \frac{2}{3}\ \pi r^3$

$= \frac{\pi r^2}{3}\ [h + 2r]$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times [14 + (2 \times 7)] cm^3$

$= \frac{22}{3} \times 7 \times 28 cm^3$

$= \frac{4312}{2} cm^3$

= 1437.33 cm^3

Thus, the volume of the solid is 1437.33 cm^3.