 MATHEMATICS

निर्धारित समय : 3 प्रतिये  
Time allowed : 3 hours

अधिकतम अंक : 100  
Maximum Marks : 100

65/1/1 1 [P.T.O.]
General Instructions:

(i) All questions are compulsory.

(ii) This question paper contains 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of one mark each, Section B comprises of 8 questions of two marks each, Section C comprises of 11 questions of four marks each and Section D comprises of 6 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 1 question of Section A, 3 questions of Section B, 3 questions of Section C and 3 questions of Section D. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may ask logarithmic tables, if required.

खण्ड – अ

SECTION – A

प्रश्न संख्या 1 से 4 तक के प्रत्येक प्रश्न 1 अंक का है।

Question numbers 1 to 4 carry 1 mark each.

1. यदि A और B एक ही कोटि 3 के वर्ग आव्यूह हैं और |A| = 2 तथा AB = 2I है, तो |B| का मान लिखिए।

If A and B are square matrices of the same order 3, such that |A| = 2 and AB = 2I, write the value of |B|. 
2. If \( f(x) = x + 1 \) is, then find \( \frac{d}{dx} (f \circ f)(x) \) and evaluate it.

If \( f(x) = x + 1 \), find \( \frac{d}{dx} (f \circ f)(x) \).

3. Differentiate \( x^2 \frac{d^2 y}{dx^2} = \left( 1 + \left( \frac{dy}{dx} \right)^2 \right)^4 \) to find the order and the degree of the differential equation.

Find the order and the degree of the differential equation \( x^2 \frac{d^2 y}{dx^2} = \left( 1 + \left( \frac{dy}{dx} \right)^2 \right)^4 \).

4. If a line makes angles 90°, 135°, 45° with the x, y and z axes respectively, find its direction cosines.

Or

Find the vector equation of the line which passes through the point \((3, 4, 5)\) and is parallel to the vector \(2\hat{i} + 2\hat{j} - 3\hat{k}\).

खण्ड – ब

SECTION – B

प्रश्न संख्या 5 से 12 तक के प्रश्न के 2 अंक हैं।

Question numbers 5 to 12 carry 2 marks each.

5. (i) If \( a * b = ab + 1 \) is defined on \( \mathbb{R} \), then examine whether the operation * is a binary or not.

(ii) If \( a * b = ab + 1 \) is a binary operation, is it associative or not?
6. Find a matrix $A$ such that $2A - 3B + 5C = O$, where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$.

7. Find: $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} \, dx$.

8. Find: $\int \sqrt{1 - \sin 2x} \, dx$, $\frac{\pi}{4} < x < \frac{\pi}{2}$

OR

Find: $\int \sin^{-1}(2x) \, dx$.

9. Form the differential equation representing the family of curves $y = e^{2x} (a + bx)$, where 'a' and 'b' are arbitrary constants.
10. यदि दो मात्रक सदिशों का योग एक मात्रक सदिश हो, तो सिद्ध कीजिए कि उन दो सदिशों के अन्तर का परिमाण $\sqrt{3}$ होगा।

अथवा

यदि $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ तथा $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$ है, तो [$\vec{a} \cdot \vec{b} \cdot \vec{c}$] ज्ञात कीजिए।

If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

OR

If $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$, find [$\vec{a} \cdot \vec{b} \cdot \vec{c}$].

11. एक पौँसा जिस पर 1, 2, 3 लाल रंग से तथा 4, 5, 6 हरे रंग से लिखा गया है, को उठाला जाता है।

“संख्या सम होने” की घटना को $A$ से और “संख्या लाल रंग में लिखी है” की घटना $B$ से परिभाषित है। ज्ञात कीजिए कि क्या ये दो घटनाएँ $A$ तथा $B$ स्वतंत्र हैं या नहीं।

A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let $A$ be the event “number is even” and $B$ be the event “number is marked red”. Find whether the events $A$ and $B$ are independent or not.

12. एक पासे को छः बार उठाला जाता है। यदि “पासे पर चित्र संख्या प्राप्त होना” एक सफलता है, तो

(i) 5 सफलताएँ, (ii) अधिकतम 5 सफलताएँ, की प्रायिकताएँ क्या-क्या होंगी?

अथवा

एक यादृच्छिक चर $X$ का प्रायिकता बंटन $P(X)$ निम्न प्रकार से है, जहाँ ‘$k$’ कोई संख्या है:

$$P(X = x) = \begin{cases} k, & \text{यदि } x = 0 \\ 2k, & \text{यदि } x = 1 \\ 3k, & \text{यदि } x = 2 \\ 0, & \text{अन्यथा} \end{cases}$$

‘$k$’ का मान ज्ञात कीजिए।

A die is thrown 6 times. If “getting an odd number” is a “success”, what is the probability of (i) 5 successes? (ii) atmost 5 successes?

OR

The random variable $X$ has a probability distribution $P(X)$ of the following form, where ‘$k$’ is some number.

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of ‘$k$’. 
 COLUMN – C

Question numbers 13 to 23 carry 4 marks each.

13. दिखाइए कि समुच्चय \( R \) में \( R = \{(a, b) : a \leq b\} \) द्वारा परिभाषित संबंध \( R \) स्वतंत्र व संक्रामक है, परन्तु सममित नहीं है।

अथवा

सिद्ध कीजिए कि फलन \( f : N \rightarrow N, f(x) = x^2 + x + 1 \), द्वारा परिभाषित है, एक एकैकी फलन है किंतु आच्छादक नहीं।

फलन \( f : N \rightarrow S, \) जहाँ \( S \) फलन \( f \) का परिसर है, का प्रतिलोम भी ज्ञात कीजिए।

Show that the relation \( R \) on \( \mathbb{R} \) defined as \( R = \{(a, b) : a \leq b\} \), is reflexive, and transitive but not symmetric.

OR

Prove that the function \( f : N \rightarrow N \), defined by \( f(x) = x^2 + x + 1 \) is one-one but not onto. Find inverse of \( f : N \rightarrow S \), where \( S \) is range of \( f \).

14. हल कीजिए: \( \tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{4} \)

Solve: \( \tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{4} \)

15. सारणिकों के गुणधर्मों का प्रयोग करके, सिद्ध कीजिए कि

\[
\begin{vmatrix}
  a^2 + 2a & 2a + 1 & 1 \\
  2a + 1 & a + 2 & 1 \\
  3 & 3 & 1 \\
\end{vmatrix}
= (a - 1)^3.
\]

Using properties of determinants, prove that

\[
\begin{vmatrix}
  a^2 + 2a & 2a + 1 & 1 \\
  2a + 1 & a + 2 & 1 \\
  3 & 3 & 1 \\
\end{vmatrix}
= (a - 1)^3.
\]

16. यदि \( \log (x^2 + y^2) = 2 \tan^{-1} \left(\frac{y}{x}\right) \) हो, तो दर्शाइए कि \( \frac{dy}{dx} = \frac{x + y}{x - y} \).

अथवा

यदि \( x^y - y^x = a^b \) है, तो \( \frac{dy}{dx} \) ज्ञात कीजिए।

If \( \log (x^2 + y^2) = 2 \tan^{-1} \left(\frac{y}{x}\right) \), show that \( \frac{dy}{dx} = \frac{x + y}{x - y} \).

OR

If \( x^y - y^x = a^b \), find \( \frac{dy}{dx} \).
17. If \( y = (\sin^{-1}x)^2 \), prove that \( (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2 = 0 \).

18. Find the equation of tangent to the curve \( y = \sqrt{3x-2} \) which is parallel to the line \( 4x - 2y + 5 = 0 \). Also, write the equation of normal to the curve at the point of contact.

19. Find: \( \int \frac{3x+5}{x^2+3x-18} \, dx \).

20. Prove that \( \int_{0}^{a} f(x) \, dx = \int_{0}^{a} f(a-x) \, dx \), hence evaluate \( \int_{0}^{\pi} \frac{x \sin x}{1 + \cos^2 x} \, dx \).
21. Solve the differential equation: \( x \, dy - y \, dx = \sqrt{x^2 + y^2} \, dx \) given that \( y = 0 \) when \( x = 1 \).

**Answer**

Solve the differential equation: \((1 + x^2) \frac{dy}{dx} + 2xy - 4x^2 = 0 \), subject to the initial condition \( y(0) = 0 \).

**OR**

Solve the differential equation: \((1 + x^2) \frac{dy}{dx} + 2xy - 4x^2 = 0 \), subject to the initial condition \( y(0) = 0 \).

22. If \( \hat{i} + \hat{j} + \hat{k}, \ 2\hat{i} + 5\hat{j}, \ 3\hat{i} + 2\hat{j} - 3\hat{k} \) and \( \hat{i} - 6\hat{j} - \hat{k} \) respectively are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) are collinear or not.

**Answer**

23. Find the value of \( \lambda \), so that the lines \( \frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2} \) and \( \frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5} \) are at right angles. Also, find whether the lines are intersecting or not.
24. If \( A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \) is a matrix, then find the inverse of \( A \). Hence, solve the system of equations:

\[
\begin{align*}
x + y + z &= 6 \\
x + 2z &= 7 \\
3x + y + z &= 12
\end{align*}
\]

**OR**

Find the inverse of the following matrix using elementary operations:

\[
A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}
\]

25. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m\(^3\). If building of tank costs ₹70 per square metre for the base and ₹45 per square metre for the sides, what is the cost of least expensive tank ?
26. Using integration, find the area of triangle ABC, whose vertices are A(2, 5), B(4, 7) and C(6, 2).

OR

Find the area of the region lying above x-axis and included between the circle \(x^2 + y^2 = 8x\) and inside of the parabola \(y^2 = 4x\).

27. Find the vector and Cartesian equations of the plane passing through the points (2, 2, –1), (3, 4, 2) and (7, 0, 6). Also find the vector equation of a plane passing through (4, 3, 1) and parallel to the plane obtained above.

OR

Find the vector equation of the plane that contains the lines \(\vec{r} = (\hat{i} + \hat{j}) + \lambda (\hat{i} + 2\hat{j} – \hat{k})\) and the point (–1, 3, –4). Also, find the length of the perpendicular drawn from the point (2, 1, 4) to the plane, thus obtained.
28. A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by A?

29. A manufacturer has 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer’s profit on an item of model A is ₹15 and on an item of model B is ₹10. How many of items of each model should be made per day in order to maximize daily profit? Formulate the above LPP and solve it graphically and find the maximum profit.