MATHS QUESTION PAPER
CLASS-XII

Time : 3.00 Hours] ***[Maximum Marks : 100***

Instructions :

1. **All** the questions are **compulsory.**
2. Write your answers according to the instructions given below with the questions.
3. Begin each section from a new page.

SECTION - A

*Given below are **1** to **15** multiple choice questions, each carrying **ONE** mark. Write the letter of the correct option (A) or (B) or (C) or (D).* **15**

1. Find the value of a, if P(2, 3) is circumcentre of the triangle with vertices A(a, 6), B(5, 1) and C(4, 6).

(A) -4 (B) 1

(C) 4 (D) 0

2. Find α if a line $x + y + 1 = 0$ is converted in the form of a line $x \cos\alpha + y \sin\alpha = p$.

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$

(C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$

3. If the circle $x^2 + y^2 + 4x + Ky - 4 = 0$ touches both the axes, then find out K.

(A) $\pm$ 8 (B) $\pm$ 4

(C) $\pm$ 2 (D) $\pm$ 1

4. Obtain the equation of a Parabola having focus (0, –2) and the equation of directrix is $y = 2$ and (0, 0) is the vertex of Parabola.

(A) $x^2 = -8y$ (B) $y^2 = 8x$

(C) $x^2 = 8y$ (D) $y^2 = -8x$

5. Find the radius of a director-circle of an ellipse $4x^2 + 9y^2 = 36$.

(A) $\sqrt{5}$ (B) $\sqrt{13}$

(C) $\sqrt{10}$ (D) 5

6. If $|\bar{a}| = 10$, $|\bar{b}| = 2$ and $\bar{a} \cdot \bar{b} = 12$, then find $|\bar{a} \times \bar{b}|$.

(A) 12 (B) 14

(C) 16 (D) 18

7. Find magnitude of projection of vector $\bar{i} + \bar{j} + \bar{k}$ on $\bar{j}$.

(A) –1 (B) 0

(C) 1 (D) 2

8. Find the measure of the angle between plane $\bar{r} \cdot (1, 2, 1) = 1$ and $\frac{x}{2} = \frac{y}{1} = \frac{z}{-1}$.

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$ (D) None of them

9. Find $\lim\limits_{x \to 0} \dfrac{(1+x)^{1/3} - 1}{x}$

(A) 0 (B) 1

(C) $1/3$ (D) None of them

10. Find $\dfrac{d}{dx}\left[\tan^{-1}\left(\dfrac{1-\cos x}{1+\cos x}\right)^{\frac{1}{2}}\right]$; $\pi < x < 2\pi$.

(A) 0 (B) $1/2$

(C) $-1/2$ (D) 1

11. Find c applying Rolle's theorem to $f(x) = 1 + \sin x$, $x \in [0, \pi]$

(A) 0 (B) $\pi/4$

(C) π (D) $\pi/2$

12. Evaluate : $\int_1^{\sqrt{3}} \frac{1}{1+x^2}\,dx$

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

13. Find the area of the region bounded by the curve $y = \tan x$, X-axis and the lines $x = 0$ and $x = \pi/4$.

(A) $\log 2$ (B) $\frac{3}{2}\log 2$

(C) $\frac{1}{2}\log 2$ (D) $2 \log 2$

14. Determine the degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$

(A) 1 (B) 2

(C) 0 (D) not defined

15. A stone falls from a tower of height 40 m. What will be its velocity, when it reaches on the land ?

(A) 14 m/s (B) 28 m/s

(C) 21 m/s (D) 7 m/s

SECTION - B

*Answer the following **15** questions. (No. **16** to **30**)*
*Each question carries **ONE** mark.* **15**

16. Find the point A on the X-axis which is at the distance of 5 units from point B(2, –3).

17. Obtain the equation of a circle which touches the X-axis, given that the equations of lines containing two of the diameters of the circle are $3x - 2y - 5 = 0$ and $x + y - 5 = 0$.

18. Find the focus of a Parabola $y^2 + 6y - 2x + 5 = 0$.

19. The equations of the asymptotes of Hyperbola are $3x + 4y = 2$ and $4x - 3y = 2$. Find the eccentricity.

20. Find the unit vector in the direction of vector (1, 2, 3).

21. Find the area of a Parallelogram, if its diagonals are $2\bar{i} + \bar{k}$ and $\bar{i} + \bar{j} + \bar{k}$.

22. Represent the equation of line $\frac{3-x}{1} = \frac{2-y}{3} = \frac{1-z}{4}$ in the vector-form.

23. Find the length of a chord, cut by sphere $x^2 + y^2 + z^2 - x - y - z = 0$ on any axis.

24. If $f'(x) = f(x)$ and $f(0) = 1$, then find out the value of $\lim_{x \to 0} \frac{f(x)-1}{x}$.

25. Evaluate :

$\int x^{4x}(1 + \log x)\,dx, \quad x > 0$.

26. Evaluate :

$\int \left(\frac{1+x}{x^2}\right) e^{-x}\,dx$.

27. If $\int_1^k f(x)\,dx = 47$; $\quad f(x) = \begin{cases} 2x + 8, & \text{if } 1 \le x \le 2 \\ 6x, & \text{if } 2 \le x \le k \end{cases}$

then find k .

28. Find the length of sub tangent of $y = e^{x/c}$.

29. If a distance of 150 cm. is travelled in 30 seconds with an initial velocity of 10 cm/s, find the constant accleration (retardation).

30. If the maximum horizontal range is 200 m, find the minimum velocity for that.

SECTION - C

Answer the following 10 questions ***(31 to 40).***
Each question carries ***TWO*** *marks. Do as directed :* **20**

31. A line passing through (2, 4) intersects the X-axis and Y-axis at A and B respectively. Find the equation of the locus of the mid-point of $\overline{AB}$.

32. For the Parabola $x^2 = 12y$, find the area of the triangle, whose vertices are the vertex of the parabola and the two end-points of its latus rectum.

33. Find the equation of Ellipse, which is passing through the points (1, 4) and (– 6, 1).

34. Find the equation of Hyperbola for which the distance from one vertex to two foci are 9 and 1.

OR

Find the measure of angle between the asymptotes of hyperbola $3x^2 - 2y^2 = 1$.

35. If $\bar{x}.\bar{y} = \bar{x}.\bar{z}$, $\bar{x} \times \bar{y} = \bar{x} \times \bar{z}$ and $\bar{x} \neq \bar{0}$, then prove that $\bar{y} = \bar{z}$.

36. If $\bar{a}.\bar{b} = \bar{a}.\bar{c} = 0$, $|\bar{a}| = |\bar{b}| = |\bar{c}| = 1$, then prove that

$\bar{a} = \pm 2(\bar{b} \times \bar{c})$, where $(\bar{b} \wedge \bar{c}) = \pi/6$.

37. Find the equation of a sphere given that its centre is (1, 1, 0) and that it touches the plane $2x + 2y + z + 5 = 0$.

38. If $y = \tan^{-1}\left(\frac{5x}{1-6x^2}\right)$, then find $\frac{dy}{dx}$.

OR

$f(x) = [x]$. Is f continuous and differentiable at $x = 1$?

39. Find the measure of the angle between the curves
$y = \sin x$ and $y = \cos x$, $0 < x < \pi$.

40. Obtain $\int \frac{\sqrt{\tan x}}{\sin x \cos x}\, dx$; $x \neq \frac{k\pi}{2}$, $\tan x > 0$.

OR

Obtain $\int \frac{1}{\sin^4 x + \cos^4 x}\, dx$.

SECTION - D

*Answer the following 10 questions (**41** to **50**) .*

*Each question carries **3 (THREE)** marks. Do as directed.* **30**

41. A is $\left(2\sqrt{2},\ 0\right)$ and B is $\left(-2\sqrt{2},\ 0\right)$. If $|AP - PB| = 4$, find the equation of locus of P.

42. Find the equation of the incircle of the triangle formed by the following lines –

$x = 2$, $4x + 3y = 5$ and $4x - 3y + 13 = 0$.

OR

Get the equation of the circle that passes through the origin and that cuts chords of length 5 on the lines $y = \pm x$.

43. Prove by vectors, that if the median on the base of a triangle is also altitude on the base, the triangle is isosceles.

OR

There are two forces (2, 5, 6) and (–1, 2, 1) that act on a particle and as a result of which the particle moves from A(4, –3, –2) to B(6, 1, –3). Find the work done.

44. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect each other and also find the point of intersection.

45. Obtain the equation of a plane that passes through the points (2, 3, – 4) and (1, –1, 3), and that is parallel to X-axis.

46. Find $\lim_{x \to e^3} \frac{\log x - 3}{x - e^3}$.

47. Prove that of all the rectangles having the same area, the square has minimum perimeter.

OR

$y = ax^3 + bx^2 + cx + 5$ touches X-axis at (–2, 0) and the slope of the tangent where it meets Y-axis is 3, then find a, b, c.

48. Evaluate :

$$\int_0^1 \frac{\log(1+x)}{(1+x)^2}\, dx .$$

49. Find the area of the region bounded by the curve $y = 2\sqrt{1 - x^2}$ and X-axis.

OR

Evaluate :

$\int_2^3 e^{-x}\, dx$ as a limit of the sum.

50. Solve the differential equation.

$x\, dy + y\, dx = xy\, dx,\ y(1) = 1.$

SECTION - E

*Answer the following 4 questions (**51** to **54**).*

*Each question carries **FIVE** marks. Do as directed* **20**

51. The equation of the line containing one of the sides of an equilateral triangle is $x + y = 2$ and one of the vertices of the triangle is (2, 3). Find the equations of lines containing the remaining sides of the triangle.

OR

A is (1, 3) in ΔABC and the lines $x - 2y + 1 = 0$ and $y - 1 = 0$ contain two medians of the triangle. Find the co-ordinates of B and C.

52. Find $\lim_{x \to 1} \frac{x^n - 1 - n(x-1)}{(x-1)^2}$; $x \neq 1$.

53. If $y = \log(1 + \sin x)$, then prove that $e^y \cdot \frac{d^2y}{dx^2} + 1 = 0$.

54. Evaluate :

$$\int \left(\frac{2007x + 2008}{2008x + 2007} \right) dx .$$

OR

Evaluate :

$$\int \frac{1}{\sin x + \sec x} \, dx .$$