MATHS QUESTION PAPER
CLASS-XII

Time : 3.00 Hours
Maximum Marks : 100

Instructions :
1. Answer all the questions.
2. Write your answers according to the instructions given below with the questions.
3. Begin each section on a new page.

SECTION - A

Given below are 1 to 15 multiple choice questions. Each carries one mark. Write the letter (A), (B), (C) or (D) in your answer book of the alternative which you feel is the correct answer of the questions.

1. The origin be shifted to (-2, 3) so that the new co-ordinates of ....... would be (3, -2).
   (A) (-1, 1)  (B) (1, 1)  
   (C) (1, -1)  (D) (-1, -1)

2. For all \(a, b, c \in \mathbb{R}\), \(2a + 3b + 5c = 0\), the line \(ax + by + c = 0\) passes through fixed point ........ \((a^2 + b^2 \neq 0)\)
   (A) (2, 3)  (B) (-2, -3)  
   (C) \(\left(\frac{-2}{5}, \frac{-3}{5}\right)\)  (D) \(\left(\frac{2}{5}, \frac{3}{5}\right)\)

3. Circle \(x^2 + y^2 - 2ax - 2ay + a^2 = 0\), \(a \neq 0\) ............
   (A) passes through origin.  (B) touches only X-axis.
   (C) touches only Y-axis.  (D) touches both the axes.
4. One of the end point of the focal chord of Parabola $y^2 = 16x$ is \( \left( \frac{1}{4}, 2 \right) \), then the other end point is ...........

(A) \( \left( 2, \frac{1}{4} \right) \) \hspace{1cm} (B) \( \left( \frac{1}{4}, -2 \right) \) \\
(C) \((64, -32)\) \hspace{1cm} (D) \(( -64, 32)\)

5. Equation of a tangent to \( \frac{x^2}{3} - \frac{y^2}{2} = 1 \) and parallel to \( y = x \) is ...........

(A) \( x - y + 1 = 0 \) \hspace{1cm} (B) \( x + y - 1 = 0 \) \\
(C) \( x - y + 2 = 0 \) \hspace{1cm} (D) \( x + y + 2 = 0 \)

6. If \(|\bar{x}| = |\bar{y}| = |\bar{x} - \bar{y}|\), then \(|\bar{x} + \bar{y}| = ...........

(A) \( \sqrt{3} \bar{x} \) \hspace{1cm} (B) \( \sqrt{3} |\bar{x}| \) \\
(C) \( 3 \bar{x} \) \hspace{1cm} (D) \( 3|\bar{x}| \)

7. For a parallelogram ABCD, \( \overrightarrow{AB} = \overrightarrow{a} \) and \( \overrightarrow{BC} = \overrightarrow{b} \), then its area = .......

(A) \( \frac{1}{2} |\overrightarrow{a} \times \overrightarrow{b}| \) \hspace{1cm} (B) \( |\overrightarrow{a} \times \overrightarrow{b}| \) \\
(C) \( |(\overrightarrow{a} + \overrightarrow{b}) \times (\overrightarrow{a} - \overrightarrow{b})| \) \hspace{1cm} (D) None of these

8. A plane cuts axes at A, B, C such that the centroid of \( \triangle ABC \) is \((1, 3, 1)\), the equation of this plane is ............

(A) \( \frac{x}{3} + \frac{y}{1} + \frac{z}{3} = 3 \) \hspace{1cm} (B) \( \frac{x}{1} + \frac{y}{3} + \frac{z}{1} = 3 \) \\
(C) \( 3x + 3y + z = 3 \) \hspace{1cm} (D) None of these

9. \( x \in \mathbb{N}^* \) \((-2, \delta) \Rightarrow f(x) \in \mathbb{N}(9, 0.01)\), then the maximum value of \( \delta \) is ....... , where \( f(x) = 5 - 2x \).

(A) 0.001 \hspace{1cm} (B) 0.005 \\
(C) 0.009 \hspace{1cm} (D) None of these
10. If \( \frac{d}{dx} f(x) = g(x) \), then \( \frac{d}{dx} \left( -\frac{1}{f(x)} \right) = \ldots \ldots \) 

(A) \( -\frac{1}{(f(x))^2} \) \hspace{1cm} (B) \( \frac{1}{(f(x))^2} \) 

(C) \( -\frac{f(x)}{(g(x))^2} \) \hspace{1cm} (D) \( \frac{g(x)}{(f(x))^2} \) 

11. \( \int \{\sin(\log x) + \cos(\log x)\} \, dx = \ldots \ldots + c. \) 

(A) \( x \sin(\log x) \) \hspace{1cm} (B) \( x \cos(\log x) \) 

(C) \( \sin(\log x) \) \hspace{1cm} (D) \( \cos(\log x) \) 

12. \( \int \frac{1}{\sqrt{(\log \frac{1}{2})^2 - x^2}} \, dx = \ldots \ldots + c. \) 

(A) \( \sin^{-1} \left( \frac{x}{\log \frac{1}{2}} \right) \) \hspace{1cm} (B) \( -\sin^{-1} \left( \frac{x}{\log 2} \right) \) 

(C) \( \sin^{-1} \left( \frac{x}{\log 2} \right) \) \hspace{1cm} (D) None of these 

13. \( \int_0^{2a} \frac{f(x)}{f(x) + f(2a - x)} \, dx = \ldots \ldots \) 

(A) \( a \) \hspace{1cm} (B) \( -a \) 

(C) \( \frac{a}{2} \) \hspace{1cm} (D) \( -\frac{a}{2} \) 

14. Degree of a differential equation \( \left( \frac{d^2 y}{dx^2} \right)^{\frac{2}{3}} = \left( y + \frac{dy}{dx} \right)^{\frac{1}{2}} \) is \( \ldots \ldots \) 

(A) 1 \hspace{1cm} (B) 2 

(C) 3 \hspace{1cm} (D) 4
15. A particle is projected vertically upward with a velocity of 24.5 m/sec., then
velocity of that particle after 2 sec. is ........... m/sec.

(A) 4.9  (B) - 4.9  
(C) -14.7  (D) 14.7

SECTION - B

Answer the following 16 to 30 questions. Each carries one mark.

16. Find the incentre of the triangle whose vertices are \((\sqrt{3}, 1), (0, 0), (0, 2)\).

17. Obtain the location of point \((a \cos \alpha, a \sin \alpha)\) in the plane relative to a circle \(x^2 + y^2 = r^2\), where \(\alpha \in (-\pi, \pi]\), \(|a| < r\), \(a \neq 0\).

18. There is a point on the Parabola \(y^2 = 8x\) whose \(Y\)-coordinate is two times the \(X\)-coordinate. If this point is not the vertex of the Parabola, find that point.

19. Let \(L\) and \(L'\) be the feet of perpendicular drawn from the foci \(S\) and \(S'\) respectively to the tangent at any point \(P(x, y)\) of the ellipse \(\frac{x^2}{9} + \frac{y^2}{16} = 1\), then find \(SL \cdot S'L'\).

OR

Find the measure of eccentric angle of point \((-2, -2\sqrt{2})\) on the ellipse \(2x^2 + y^2 = 16\).

20. If \(\alpha, \beta, \gamma\) are the direction angles of the vector \(\vec{r}\), then find the value of \(\cos 2\alpha + \cos 2\beta + \cos 2\gamma\).

21. Force \(2\vec{i} + 2\vec{j} + 2\vec{k}\) is applied at \(B(1, 2, 3)\); find the torque around \(A(-1, 2, 0)\).

22. Find the equation of the line through \((4, 3, 2)\) and parallel to the line \(\frac{x-10}{15} = \frac{y-2}{5} = \frac{z-1}{3}\).

23. If the position vectors of the end points of a diameter of a sphere are \(4\vec{i}\) and \(2\vec{j}\), find the Cartesian equation of the Sphere.
24. The formula connecting the periodic time $T$ and length $l$ of a pendulum is 

$$T = 2\pi\sqrt{\frac{l}{g}}.$$ 

If there is an error of 2% in measuring the length $l$, what will be the percentage error in $T$?

25. Discuss the validity of Rolle’s Theorem for $f(x) = x^{\frac{1}{4}}$, $x \in [-1, 1]$. 

OR 

The radius of a right circular cone is constant. If there is an error $\delta h$ in measuring its height, what will be the error in measurement of its volume?

26. Evaluate: \(\int \frac{e^x - 1}{e^x + 1} \, dx\).

27. Obtain the value of \(\int_0^{\pi} \sin^3 x \cdot \cos^3 x \, dx\).

OR 

If \(\int_n^{n+1} f(x) \, dx = n^3\), then find the value of \(\int_{-3}^{3} f(x) \, dx\).

28. Obtain the differential equation representing the family of curves 

$$y = a \cos^{-1} x + b,$$ 

where $a$ and $b$ are arbitrary constants.

29. A body projected in vertical direction attains maximum height 16 m. Find its initial velocity.

30. Range of a projectile is \(\frac{4}{\sqrt{3}}\) times its maximum height \(\frac{u^2 \sin^2 \alpha}{2g}\). Find measure of angle of projection.

SECTION - C

Answer the following questions from 31 to 40.

Each carries TWO marks, as directed in the question.

31. The equation of a perpendicular bisector of $\overline{AB}$ is $5x + 2y - 18 = 0$, if $A$ is $(-3, 2)$; then find the co-ordinates of the midpoint of $\overline{AB}$.

OR 

Find the co-ordinates of the foot of the perpendicular from $A(a, 0)$ to the line 

$$y = mx + \frac{a}{m};\ m \neq 0.$$
32. Find the locus of point P such that the slopes of the tangents drawn from P to a
Parabola have (i) constant sum (ii) constant non zero product.

OR

Find the co-ordinates of the points of contact of the tangents drawn from (1, 5)
to the Parabola \(y^2 = 24x\).

33. If the difference between measures of the eccentric angles of P and Q is \(\frac{\pi}{2}\) and
if \(\overrightarrow{PQ}\) cuts intercepts \(c\) and \(d\) on the axes, prove that \(\frac{a^2}{c^2} + \frac{b^2}{d^2} = 2\).

34. Find the equation of a curve from every point of which the tangents to the
Hyperbola \(\frac{x^2}{144} - \frac{y^2}{36} = 1\) intersect at right angles.

OR

If the chord of the Hyperbola joining P(\(\alpha\)) and Q(\(\beta\)) on the hyperbola subtends
a right angle at the centre C(0, 0); prove that \(a^2 + b^2 \sin \alpha \cdot \sin \beta = 0\).

35. If \(\bar{a} \neq \bar{0}, \bar{b} + \bar{c} \neq \bar{0}\) and \(\bar{a} + \bar{b} + \bar{c} \neq \bar{0}\); show that \(\bar{a}, \bar{b} + \bar{c}, \bar{a} + \bar{b} + \bar{c}\)
are coplanar.

36. The dot product with \(\bar{i} + \bar{j} + \bar{k}\) of the unit vector having the same direction as
the vector sum of \(2\bar{i} + 4\bar{j} - 5\bar{k}\) and \(\lambda \bar{i} + 2\bar{j} + 3\bar{k}\) is 1, find \(\lambda\).

37. Find the equation of the sphere passing through the point O(0, 0, 0),
A(-\(a, b, c\)), B(\(a, -b, c\)), C(\(a, b, -c\)).

38. If \(y = \tan^{-1}\left(\frac{3-2x}{2+3x}\right)\), then find \(\frac{dy}{dx}\).

OR

If \(y = (\cos^{-1} x)^2\), then prove that \((1-x^2)y_2 - xy_1 = 2\).

39. Obtain the intervals in which function \(f(x) = x^3 - 6x^2 - 36x + 2\) in
increasing and decreasing.
40. Evaluate: \( \int_{0}^{1} x^2 (1-x)^{\frac{1}{2}} \, dx \).

**SECTION - D**

Answer the following questions from 41 to 50, each carrying THREE marks as directed in the question.

41. If A is (-2, 1) and B is (1, -7); find a point on \( \overrightarrow{AB} \) such that 5AP = 3AB.

OR

If \( P(at^2, 2at), \; Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right) \) and \( S(a, 0) \) are three points, show that \( \frac{1}{SP} + \frac{1}{SQ} \) is independent of \( t \).

42. Find the co-ordinates of points which are at minimum and maximum distance from the point (-7, 2) on the circle \( x^2 + y^2 - 10x - 14y - 151 = 0 \).

OR

Find the equation of the circle that touches the Y-axis and passes through (-2, 1) and (-4, 3).

43. Prove by using vectors that the perpendicular bisectors of the sides of a triangle are concurrent.

44. Find the measure of the angle between two lines if their direction cosines \( l, m, n \) satisfy \( l + m + n = 0, \; l^2 - m^2 + n^2 = 0 \).

45. Obtain the foot of perpendicular, perpendicular distance and equation of perpendicular line from A(2, 3, 2) on \( \vec{r} \cdot (1, -2, 1) = -5 \).

46. Find: \( \lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{16r^2 + 8r - 3} \).

47. By using mean value theorem for \( \log(1 + x) \) in \([0, x]\), prove that

\[
0 < \frac{1}{\log(1 + x)} - \frac{1}{x} < 1, \quad \text{where} \; x > 0.
\]

OR

The slope of the tangent at the point (1, 1) on the curve \( xy + ax + by = 2 \) is 2, find a and b.
48. Evaluate: \( \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} \, dx \).

49. Evaluate: \( \int \frac{6x + 7}{\sqrt{(4 - x)(5 - x)}} \, dx \) \( x < 4 \)

50. Solve: \( \frac{dy}{dx} + \frac{4xy}{x^2 + 1} = \frac{1}{(x^2 + 1)^2} \).

SECTION - E

Answer the following questions from 51 to 54, each carrying FIVE marks.

51. The lines \( x - 2y + 2 = 0, \) \( 3x - y + 6 = 0 \) and \( x - y = 0 \) contain the three sides of a triangle. Determine the co-ordinates of the orthocentre without finding the co-ordinates of the vertices of the triangle.

OR

Find the equation of the line passing through \((\sqrt{3}, -1)\) if its perpendicular distance from the origin is \(\sqrt{2} \).

52. Find: \( \lim_{x \to 1} \left\{ \frac{25}{x^{25} - 1} - \frac{15}{x^{15} - 1} \right\} \).

53. If \( f(x + y) = f(x) \cdot f(y) \), then find \( f'(3) \); where \( f(x) = \log(e + x), \) \( x > 0 \).

54. Evaluate: \( \int_0^1 \sin^{-1}\left(\frac{2x}{1 + x^2}\right) \, dx \).

OR

Prove that the area of the region bounded by the circle \( x^2 + y^2 = 16 \) and the Parabola \( y^2 = 6x \) is \( \frac{4}{3} (4\pi + \sqrt{3}) \).