MATHS QUESTION PAPER

CLASS-XII

Time : 3.00 Hours ***Maximum Marks : 100***

Instructions :

1. Answer **all** the questions.
2. Write your answers according to the instructions given below with the questions.
3. Begin each **section** on a **new page**.

SECTION - A

*Given below are **1** to **15** multiple choice questions. Each carries **one** mark. Write the letter (A) , (B), (C) or (D) in your answer book of the alternative which you feel is the correct answer of the questions.*

1. The origin be shifted to $(-2, 3)$ so that the new co-ordinates of .... would be $(3, -2)$.

(A) $(-1, 1)$ (B) $(1, 1)$

(C) $(1, -1)$ (D) $(-1, -1)$

2. For all $a, b, c \in R$, $2a + 3b + 5c = 0$, the line $ax + by + c = 0$ passes through fixed point .... $(a^2 + b^2 \neq 0)$

(A) $(2, 3)$ (B) $(-2, -3)$

(C) $\left(\frac{-2}{5}, \frac{-3}{5}\right)$ (D) $\left(\frac{2}{5}, \frac{3}{5}\right)$

3. Circle $x^2 + y^2 - 2ax - 2ay + a^2 = 0, \quad a \neq 0$ ....

(A) passes through origin. (B) touches only X-axis.

(C) touches only Y-axis. (D) touches both the axes.

4. One of the end point of the focal chord of Parabola $y^2 = 16x$ is $\left(\frac{1}{4}, 2\right)$, then the other end point is ....

(A) $\left(2, \frac{1}{4}\right)$ (B) $\left(\frac{1}{4}, -2\right)$

(C) $(64, -32)$ (D) $(-64, 32)$

5. Equation of a tangent to $\frac{x^2}{3} - \frac{y^2}{2} = 1$ and parallel to $y = x$ is ....

(A) $x - y + 1 = 0$ (B) $x + y - 1 = 0$

(C) $x - y + 2 = 0$ (D) $x + y + 2 = 0$

6. If $|\bar{x}| = |\bar{y}| = |\bar{x} - \bar{y}|$, then $|\bar{x} + \bar{y}| =$ ....

(A) $\sqrt{3}\,\bar{x}$ (B) $\sqrt{3}\,|\bar{x}|$

(C) $3\,\bar{x}$ (D) $3|\bar{x}|$

7. For a parallelogram ABCD, $\overrightarrow{AB} = \bar{a}$ and $\overrightarrow{BC} = \bar{b}$, then its area = ...

(A) $\frac{1}{2}\left|\bar{a} \times \bar{b}\right|$ (B) $\left|\bar{a} \times \bar{b}\right|$

(C) $\left|(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})\right|$ (D) None of these

8. A plane cuts axes at A, B, C such that the centroid of ΔABC is (1, 3, 1), the equation of this plane is ....

(A) $\frac{x}{3} + \frac{y}{1} + \frac{z}{3} = 3$ (B) $\frac{x}{1} + \frac{y}{3} + \frac{z}{1} = 3$

(C) $3x + 3y + z = 3$ (D) None of these

9. $x \in N^*(-2, \delta) \Rightarrow f(x) \in N(9, 0.01)$, then the maximum value of δ is ...... , where $f(x) = 5 - 2x$.

(A) 0.001 (B) 0.005

(C) 0.009 (D) None of these

10. If $\frac{d}{dx} f(x) = g(x)$, then $\frac{d}{dx}\left(-\frac{1}{f(x)}\right) = \ldots\ldots\ldots$

(A) $\frac{-1}{(f(x))^2}$ (B) $\frac{1}{(f(x))^2}$

(C) $\frac{-f(x)}{(g(x))^2}$ (D) $\frac{g(x)}{(f(x))^2}$

11. $\int \{\sin(\log x) + \cos(\log x)\}\, dx = \ldots\ldots\ldots + c.$

(A) $x \sin(\log x)$ (B) $x \cos(\log x)$

(C) $\sin(\log x)$ (D) $\cos(\log x)$

12. $\int \frac{1}{\sqrt{\left(\log \frac{1}{2}\right)^2 - x^2}}\, dx = \ldots\ldots\ldots + c.$

(A) $\sin^{-1}\left(\frac{x}{\log \frac{1}{2}}\right)$ (B) $-\sin^{-1}\left(\frac{x}{\log 2}\right)$

(C) $\sin^{-1}\left(\frac{x}{\log 2}\right)$ (D) None of these

13. $\int_0^{2a} \frac{f(x)}{f(x) + f(2a - x)}\, dx = \ldots\ldots\ldots\ldots$.

(A) a (B) $-a$

(C) $\frac{a}{2}$ (D) $\frac{-a}{2}$

14. Degree of a differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{2}{3}} = \left(y + \frac{dy}{dx}\right)^{\frac{1}{2}}$ is ....

(A) 1 (B) 2

(C) 3 (D) 4

15. A particle is projected vertically upward with a velocity of 24.5 m/sec., then velocity of that particle after 2 sec. is .... m/sec.

(A) 4.9 (B) – 4.9

(C) –14.7 (D) 14.7

SECTION - B

*Answer the following **16** to **30** questions. Each carries **one** mark.*

16. Find the incentre of the triangle whose vertices are $(\sqrt{3}, 1)$, $(0, 0)$, $(0, 2)$.

17. Obtain the location of point $(a \cos \alpha,\ a \sin \alpha)$ in the plane relative to a circle $x^2 + y^2 = r^2$, where $\alpha \in (-\pi, \pi]$, $|a| < r$, $a \neq 0$.

18. There is a point on the Parabola $y^2 = 8x$ whose Y- coordinate is two times the X- coordinate. If this point is not the vertex of the Parabola, find that point.

19. Let L and L′ be the feet of perpendicular drawn from the foci S and S′ respectively to the tangent at any point $P(x, y)$ of the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$, then find $SL \cdot S'L'$.

OR

Find the measure of eccentric angle of point $(-2, -2\sqrt{2})$ on the ellipse $2x^2 + y^2 = 16$.

20. If α, β, γ are the direction angles of the vector $\bar{r}$, then find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$.

21. Force $2\bar{i} + 2\bar{j} + 2\bar{k}$ is applied at B(1, 2, 3); find the torque around A(–1, 2, 0).

22. Find the equation of the line through (4, 3, 2) and parallel to the line $\frac{x-10}{15} = \frac{y-2}{5} = \frac{z-1}{3}$.

23. If the position vectors of the end points of a diameter of a sphere are $4\bar{i}$ and $2\bar{j}$, find the Cartesian equation of the Sphere.

24. The formula connecting the periodic time T and length l of a pendulum is $T = 2\pi\sqrt{l/g}$. If there is an error of 2% in measuring the length l, what will be the percentage error in T ?

25. Discuss the validity of Rolle's Theorem for $f(x) = x^{\frac{1}{4}},\ x \in [-1,\ 1]$.

OR

The radius of a right circular cone is constant. If there is an error δh in measuring its height, what will be the error in measurement of its volume ?

26. Evaluate : $\int \frac{e^x - 1}{e^x + 1}\, dx$.

27. Obtain the value of $\int_0^{\pi} \sin^3 x \cdot \cos^3 x\, dx$.

OR

If $\int_n^{n+1} f(x)\, dx = n^3$, then find the value of $\int_{-3}^{3} f(x)\, dx$.

28. Obtain the differential equation representing the family of curves $y = a\cos^{-1} x + b$, where a and b are arbitrary constants.

29. A body projected in vertical direction attains maximum height 16 m. Find its initial velocity.

30. Range of a projectile is $\frac{4}{\sqrt{3}}$ times its maximum height $\frac{u^2 \sin^2 \alpha}{2g}$. Find measure of angle of projection.

SECTION - C

*Answer the following questions from **31** to **40**.*
*Each carries **TWO** marks, as directed in the question.*

31. The equation of a perpendicular bisector of $\overline{AB}$ is $5x + 2y - 18 = 0$, if A is $(-3, 2)$; then find the co-ordinates of the midpoint of $\overline{AB}$.

OR

Find the co-ordinates of the foot of the perpendicular from $A(a, 0)$ to the line $y = mx + \frac{a}{m};\ m \neq 0$.

32. Find the locus of point P such that the slopes of the tangents drawn from P to a Parabola have *(i)* constant sum *(ii)* constant non zero product.

OR

Find the co-ordinates of the points of contact of the tangents drawn from (1, 5) to the Parabola $y^2 = 24x$.

33. If the difference between measures of the eccentric angles of P and Q is $\frac{\pi}{2}$ and if $\overleftrightarrow{PQ}$ cuts intercepts c and d on the axes, prove that $\frac{a^2}{c^2} + \frac{b^2}{d^2} = 2$.

34. Find the equation of a curve from every point of which the tangents to the Hyperbola $\frac{x^2}{144} - \frac{y^2}{36} = 1$ intersect at right angles.

OR

If the chord of the Hyperbola joining P(α) and Q(β) on the hyperbola subtends a right angle at the centre C(0, 0); prove that $a^2 + b^2 \sin\alpha \cdot \sin\beta = 0$.

35. If $\bar{a} \neq \bar{0}$, $\bar{b} + \bar{c} \neq \bar{0}$ and $\bar{a} + \bar{b} + \bar{c} \neq \bar{0}$; show that $\bar{a}$, $\bar{b} + \bar{c}$, $\bar{a} + \bar{b} + \bar{c}$ are coplanar.

36. The dot product with $\bar{i} + \bar{j} + \bar{k}$ of the unit vector having the same direction as the vector sum of $2\bar{i} + 4\bar{j} - 5\bar{k}$ and $\lambda \bar{i} + 2\bar{j} + 3\bar{k}$ is 1, find λ .

37. Find the equation of the sphere passing through the point O(0, 0, 0), A($-a$, b, c), B(a, $-b$, c), C(a, b, $-c$).

38. If $y = \tan^{-1}\left(\frac{3 - 2x}{2 + 3x}\right)$, then find $\frac{dy}{dx}$.

OR

If $y = \left(\cos^{-1} x\right)^2$, then prove that $\left(1 - x^2\right)y_2 - xy_1 = 2$.

39. Obtain the intervals in which function $f(x) = x^3 - 6x^2 - 36x + 2$ in increasing and decreasing.

40. Evaluate : $\int_0^1 x^2(1-x)^{\frac{1}{2}}\,dx$.

SECTION - D

Answer the following questions from ***41*** *to* ***50****, each carrying* ***THREE*** *marks as directed in the question.*

41. If A is $(-2, 1)$ and B is $(1, -7)$; find a point on $\overleftrightarrow{AB}$ such that $5AP = 3AB$.

OR

If $P(at^2, 2at)$, $Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$ and $S(a, 0)$ are three points, show that $\frac{1}{SP}+\frac{1}{SQ}$ is independent of t.

42. Find the co-ordinates of points which are at minimum and maximum distance from the point $(-7, 2)$ on the circle $x^2 + y^2 - 10x - 14y - 151 = 0$.

OR

Find the equation of the circle that touches the Y-axis and passes through $(-2, 1)$ and $(-4, 3)$.

43. Prove by using vectors that the perpendicular bisectors of the sides of a triangle are concurrent.

44. Find the measure of the angle between two lines if their direction cosines l, m, n satisfy $l + m + n = 0$, $l^2 - m^2 + n^2 = 0$.

45. Obtain the foot of perpendicular, perpendicular distance and equation of perpendicular line from $A(2, 3, 2)$ on $\bar{r}\cdot(1,-2,1) = -5$.

46. Find : $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{16r^2+8r-3}$.

47. By using mean value theorem for $\log(1 + x)$ in $[0, x]$, prove that

$$0 < \frac{1}{\log(1+x)} - \frac{1}{x} < 1, \quad \text{where } x > 0.$$

OR

The slope of the tangent at the point $(1, 1)$ on the curve $xy + ax + by = 2$ is 2, find a and b.

48. Evaluate : $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\, dx$.

49. Evaluate : $\int \frac{6x+7}{\sqrt{(4-x)(5-x)}}\, dx \quad (x < 4)$

50. Solve : $\frac{dy}{dx} + \frac{4xy}{x^2+1} = \frac{1}{\left(x^2+1\right)^2}$.

SECTION - E

*Answer the following questions from **51 to 54**, each carrying **FIVE** marks.*

51. The lines $x - 2y + 2 = 0$, $3x - y + 6 = 0$ and $x - y = 0$ contain the three sides of a triangle. Determine the co-ordinates of the orthocentre without finding the co-ordinates of the vertices of the triangle.

OR

Find the equation of the line passing through $\left(\sqrt{3}, -1\right)$ if its perpendicular distance from the origin is $\sqrt{2}$.

52. Find : $\lim_{x\to 1}\left\{\frac{25}{x^{25}-1} - \frac{15}{x^{15}-1}\right\}$.

53. If $f(x+y) = f(x).f(y)$, then find $f'(3)$; where $f(x) = \log(e + x)$, $x > 0$.

54. Evaluate : $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$.

OR

Prove that the area of the region bounded by the circle $x^2 + y^2 = 16$ and the Parabola $y^2 = 6x$ is $\frac{4}{3}\left(4\pi + \sqrt{3}\right)$.