

Senior Secondary School Certificate Examination

July'2018

Marking Scheme — Mathematics 65/3

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.
2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.
3. Alternative methods are accepted. Proportional marks are to be awarded.
4. In question (s) on differential equations, constant of integration has to be written.
5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.
6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
7. Separate Marking Scheme for all the three sets has been given.
8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/ Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

65/3 VALUE POINTS
SECTION A

1. $[\hat{i} \ \hat{k} \ \hat{j}] = \hat{i} \cdot (\hat{k} \times \hat{j}) = -\hat{i} \cdot (\hat{j} \times \hat{k})$ $\frac{1}{2}$
- $= -1$ $\frac{1}{2}$
2. $\frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$ $\frac{1}{2}$ for any one of $\frac{\pi}{3}$ or $\frac{2\pi}{3}$ $\frac{1}{2} + \frac{1}{2}$
3. Writing $\frac{3ae}{2} = a$ and finding $e = \frac{2}{3}$ $\frac{1}{2} + \frac{1}{2}$
4. $A' = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & x & -1 \end{pmatrix}$ and getting $x = -2$ $\frac{1}{2} + \frac{1}{2}$

SECTION B

5. $R'(x) = 6x + 36$. 1
- $R'(5) = 66$ 1
6. Let $y = \tan^{-1} \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right) = \tan^{-1} \left(\frac{1 - \tan x}{1 + \tan x} \right)$ $\frac{1}{2}$
- $= \tan^{-1} \left(\tan \left(\frac{\pi}{4} - x \right) \right)$ $\frac{1}{2}$
- $= \frac{\pi}{4} - x$ $\frac{1}{2}$
- $\Rightarrow \frac{dy}{dx} = -1$ $\frac{1}{2}$
7. $P(A/B) = \frac{P(A \cap B)}{P(B)}$ gives $P(A \cap B) = \frac{2}{13}$ 1
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $= \frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{11}{26}$ 1

$$8. \quad \vec{a} + \vec{b} + \vec{c} = 0$$

$$\vec{a} + \vec{b} = -\vec{c}$$

$$\vec{a}^2 + \vec{b}^2 + 2\vec{a} \cdot \vec{b} = \vec{c}^2$$

$$\Rightarrow \vec{a} \cdot \vec{b} = \frac{|\vec{c}|^2 - |\vec{a}|^2 - |\vec{b}|^2}{2} \quad \frac{1}{2}$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{|\vec{c}|^2 - |\vec{a}|^2 - |\vec{b}|^2}{2|\vec{a}| |\vec{b}|} \quad \frac{1}{2}$$

$$= \frac{9^2 - 5^2 - 6^2}{2(5)(6)}$$

$$\cos \theta = \frac{81 - 25 - 36}{60} = \frac{1}{3} \quad \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{3}\right) \quad \frac{1}{2}$$

$$9. \quad \frac{dy}{dx} = \cos^{-1} a \Rightarrow \int dy = \cos^{-1} a \cdot \int dx \quad \frac{1}{2} + \frac{1}{2}$$

$$y = x \cos^{-1} a + c \quad 1$$

$$10. \quad \frac{3 - 5 \sin x}{\cos^2 x} dx = 3 \int \sec^2 x dx - 5 \int \sec x \tan x dx \quad 1$$

$$= 3 \tan x - 5 \sec x + C \quad \frac{1}{2} + \frac{1}{2}$$

$$11. \quad \text{Finding } A^{-1} = \frac{-1}{19} \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} \quad 1$$

$$\Rightarrow \frac{-1}{19} \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 2k & 3k \\ 5k & -2k \end{bmatrix} \quad \frac{1}{2}$$

$$\Rightarrow k = \frac{1}{19} \quad \frac{1}{2}$$

$$12. \text{ Put } x = \cos \theta \text{ in R.H.S} \quad \frac{1}{2}$$

$$\text{as } \frac{1}{2} \leq x \leq 1, \text{ RHS} = \cos^{-1} (4 \cos^3 \theta - 3 \cos \theta) = \cos^{-1} (\cos 3\theta) = 3\theta \quad \frac{1}{2} + \frac{1}{2}$$

$$= 3 \cos^{-1} x = \text{LHS} \quad \frac{1}{2}$$

$$13. \text{ Getting } \overrightarrow{AB} = (5-4)\hat{i} + (x-4)\hat{j} + (8-4)\hat{k} = \hat{i} + (x-4)\hat{j} + 4\hat{k}$$

$$\overrightarrow{AC} = \hat{i} + 0\hat{j} - 3\hat{k} \text{ and } \overrightarrow{AD} = 3\hat{i} + 3\hat{j} - 2\hat{k} \quad 1 \frac{1}{2}$$

$$\text{for coplanarity } [\overrightarrow{AB} \quad \overrightarrow{AC} \quad \overrightarrow{AD}] = 0 \quad \frac{1}{2}$$

$$\Rightarrow \begin{vmatrix} 1 & x-4 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0 \quad \frac{1}{2}$$

$$\Rightarrow x = 7 \quad 1 \frac{1}{2}$$

SECTION C

$$14. \frac{4}{(x-2)(x^2+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+4} \quad 1$$

$$4 = A(x^2+4) + (Bx+C)(x-2)$$

$$\text{gives } A = \frac{1}{2}, B = -\frac{1}{2}, C = 1 \quad \frac{1}{2} \times 3$$

$$\int \frac{4 dx}{(x-2)(x^2+4)} = \frac{1}{2} \int \frac{dx}{x-2} - \int \frac{(x+2)}{2(x^2+4)} dx$$

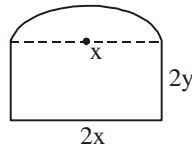
$$= \frac{1}{2} \log |x-2| - \frac{1}{4} \log |x^2+4| - \frac{1}{2} \tan^{-1} \left(\frac{x}{2} \right) + C \quad \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

15.
$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 1$$
 1+1

$$\sqrt{(b_1 c_2 - b_2 c_1)^2 + (a_2 c_1 - a_1 c_2)^2 + (a_1 b_2 - a_2 b_1)^2} = \sqrt{1 + 4 + 1} = \sqrt{6} \quad 1 + \frac{1}{2}$$

$$d = \frac{1}{\sqrt{6}} \quad \frac{1}{2}$$

16. Let the dimensions of window be $2x$ and $2y$



$$2x + 4y + \pi x = 10 \quad \frac{1}{2}$$

$$A = 4xy + \frac{1}{2}\pi x^2 = 4x\left(\frac{10 - \pi x - 2x}{4}\right) + \frac{1}{2}\pi x^2 \quad 1$$

$$= 10x - \frac{\pi x^2}{2} - 2x^2 \Rightarrow \frac{dA}{dx} = 10 - (\pi + 4)x$$

$$\frac{dA}{dx} = 0 \Rightarrow x = \frac{10}{\pi + 4} \quad \frac{1}{2}$$

$$\frac{d^2 A}{dx^2} = -(\pi + 4) < 0 \quad \frac{1}{2}$$

$$\text{Getting, } y = \frac{5}{\pi + 4}, \text{ so the dimensions are } \frac{20}{\pi + 4} \text{ m and } \frac{10}{\pi + 4} \text{ m} \quad \frac{1}{2}$$

Any relevant explanation. 1

17. Let X denote the number of defective bulbs. $\frac{1}{2}$

$$X = 0, 1, 2, 3 \quad \frac{1}{2}$$

$$\left. \begin{aligned} P(X=0) &= \left(\frac{15}{20}\right)^3 = \frac{27}{64} \\ P(X=1) &= 3\left(\frac{5}{20}\right)\left(\frac{15}{20}\right)^2 = \frac{27}{64} \\ P(X=2) &= 3\left(\frac{5}{20}\right)^2\left(\frac{15}{20}\right) = \frac{9}{64} \\ P(X=3) &= \left(\frac{5}{20}\right)^3 = \frac{1}{64} \end{aligned} \right\}$$

$$\frac{1}{2} \times 4$$

$$\text{Mean} = \sum XP(X) = \frac{27}{64} + \frac{18}{64} + \frac{3}{64} = \frac{3}{4}$$

1

$$18. \quad \frac{dy}{dx} = \frac{y^2 - x^2}{2xy} = \frac{\frac{y^2}{x^2} - 1}{\frac{2y}{x}}$$

1

$$\text{Put } \frac{y}{x} = v \Rightarrow y = vx \text{ and so } \frac{dy}{dx} = v + x \frac{dv}{dx}$$

1

$$v + x \frac{dv}{dx} = \frac{v^2 - 1}{2v} \Rightarrow \frac{xdv}{dx} = -\frac{(1 + v^2)}{2v}$$

$$\frac{1}{2}$$

$$\int \frac{dx}{x} = -\int \frac{2v dv}{1 + v^2} \Rightarrow \log x = -\log(1 + v^2) + \log C$$

$$\frac{1}{2} + \frac{1}{2}$$

$$\Rightarrow x(1 + v^2) = C \text{ so } x \left(1 + \frac{y^2}{x^2}\right) = C \text{ or } x^2 + y^2 = Cx$$

$$\frac{1}{2}$$

OR

$$\frac{dy}{dx} + \frac{2x}{1 + x^2} y = \frac{1}{(1 + x^2)^2}$$

1

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = (1+x^2)$$

1

$$\text{Solution is } y(1+x^2) = \int \frac{1}{1+x^2} dx = \tan^{-1} x + C \quad 1$$

$$\text{getting } C = -\frac{\pi}{4} \quad \frac{1}{2}$$

$$\therefore y(1+x^2) = \tan^{-1} x - \frac{\pi}{4}$$

$$\text{or } y = \frac{\tan^{-1} x}{1+x^2} - \frac{\pi}{4(1+x^2)} \quad \frac{1}{2}$$

$$19. \text{ Let } E_1 = \text{First group wins, } E_2 = \text{Second group wins} \quad 1$$

H = Introduction of new product.

$$P(E_1) = 0.6, P(E_2) = 0.4, \quad \frac{1}{2}$$

$$P(H/E_2) = 0.3, P(H/E_1) = 0.7 \quad \frac{1}{2}$$

$$\text{Now, } P(E_2/H) = \frac{P(E_2) P(H/E_2)}{P(E_2) P(H/E_2) + P(E_1) P(H/E_1)} \quad \frac{1}{2}$$

$$= \frac{0.4 \times 0.3}{0.4 \times 0.3 + 0.6 \times 0.7} = \frac{2}{9} \quad 1 + \frac{1}{2}$$

$$20. \quad x = \frac{\sin y}{\cos(a+y)} \text{ gives } \frac{dx}{dy} = \frac{\cos(a+y) \cos y + \sin y \sin(a+y)}{\cos^2(a+y)} \quad \frac{1}{2} + 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\cos(a+y-y)} = \frac{\cos^2(a+y)}{\cos a} \quad 1 + \frac{1}{2}$$

$$\text{Hence } \frac{dy}{dx} = \cos a \text{ when } x = 0 \text{ i.e. } y = 0 \quad 1$$

$$21. \quad C_1 \rightarrow C_1 + C_2 + C_3 \text{ gives L.H.S. as}$$

$$\begin{vmatrix} a+b+c & -2a+b & -2a+c \\ a+b+c & 5b & -2b+c \\ a+b+c & -2c+b & 5c \end{vmatrix} \quad 1$$

$$= (a+b+c) \begin{vmatrix} 1 & -2a+b & -2a+c \\ 1 & 5b & -2b+c \\ 1 & -2c+b & 5c \end{vmatrix} \quad \frac{1}{2}$$

$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$ gives

$$= (a+b+c) \begin{vmatrix} 1 & -2a+b & -2a+c \\ 0 & 2a+4b & 2a-2b \\ 0 & 2a-2c & 4c+2a \end{vmatrix} \quad 1$$

$$= (a+b+c) \begin{vmatrix} 2a+4b & 2a-2b \\ 2a-2c & 4c+2a \end{vmatrix} \quad \frac{1}{2}$$

$$= 4(a+b+c) \begin{vmatrix} a+2b & a-b \\ a-c & 2c+a \end{vmatrix} = 4(a+b+c) 3(ab+bc+ac) \quad \frac{1}{2} + \frac{1}{2}$$

$$= 12(a+b+c)(ab+bc+ac)$$

22. Point of intersection = $(1, \sqrt{3})$ 1

$$x^2 + y^2 = 4 \Rightarrow 2x + 2y \frac{dy}{dx} = 0 \quad \frac{dy}{dx} \Big|_{(1, \sqrt{3})} = -\frac{1}{\sqrt{3}} = m_1 \quad \frac{1}{2} + \frac{1}{2}$$

$$(x-2)^2 + y^2 = 4 \Rightarrow 2(x-2) + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} \Big|_{[1, \sqrt{3}]} = \frac{1}{\sqrt{3}} = m_2 \quad \frac{1}{2} + \frac{1}{2}$$

$$\text{So, } \tan \phi = \frac{\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}}{1 - 1/3} = \sqrt{3} \Rightarrow \phi = \frac{\pi}{3} \quad 1$$

OR

$$f'(x) = -6(x+1)(x+2) \quad 1$$

$$f'(x) = 0 \Rightarrow x = -2, x = -1 \quad \frac{1}{2}$$

$$\Rightarrow \text{Intervals are } (-\infty, -2), (-2, -1) \text{ and } (-1, \infty) \quad \frac{1}{2}$$

$$\text{Getting } f'(x) > 0 \text{ in } (-2, -1) \text{ and } f'(x) < 0 \text{ in } (-\infty, -2) \cup (-1, \infty) \quad 1$$

$\Rightarrow f(x)$ is strictly increasing in $(-2, -1)$
 and strictly decreasing in $(-\infty, 2) \cup (-1, \infty)$

1

23. Writing $\frac{dy}{d\theta} = 3a \tan^2 \theta \sec^2 \theta$

1

$$\frac{dx}{d\theta} = 3a \sec^3 \theta \tan \theta$$

1

$$\frac{dy}{dx} = \frac{\tan \theta}{\sec \theta} = \sin \theta$$

 $\frac{1}{2}$

$$\frac{d^2 y}{dx^2} = \frac{d}{d\theta} \left(\frac{dy}{dx} \right) \frac{d\theta}{dx} = \cos \theta \times \frac{1}{3a \sec^3 \theta \tan \theta}$$

1

$$\left. \frac{d^2 y}{dx^2} \right)_{\theta=\frac{\pi}{3}} = \frac{\frac{1}{2}}{3a \times 8 \times \sqrt{3}} = \frac{1}{48\sqrt{3}a}$$

 $\frac{1}{2}$

OR

$$y = e^{\tan^{-1} x}$$

$$\frac{dy}{dx} = e^{\tan^{-1} x} \left(\frac{1}{1+x^2} \right) = \frac{y}{1+x^2}$$

 $1 + \frac{1}{2}$

$$(1+x^2) \frac{dy}{dx} = y \Rightarrow (1+x^2) \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} = \frac{dy}{dx}$$

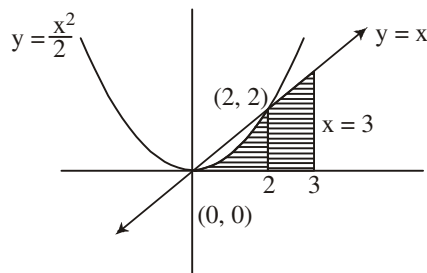
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$$\Rightarrow (1+x^2) \frac{d^2 y}{dx^2} + (2x-1) \frac{dy}{dx} = 0$$

 $\frac{1}{2}$

SECTION D

24.



1

Point of intersection of $x^2 = 2y$ and $y = x$ are $(0, 0)$ and $(2, 2)$. 2

$$\text{Required area} = \int_0^2 \frac{x^2}{2} dx + \int_2^3 x dx \quad 2$$

$$= \frac{8}{6} + \frac{5}{2} = \frac{23}{6} \quad 1$$

25. $|A| = 5(-1) + 4(1) = -1$ 1

$$\begin{array}{lll} C_{11} = -1 & C_{21} = 8 & C_{31} = -12 \\ C_{12} = 0 & C_{22} = 1 & C_{32} = -2 \\ C_{13} = 1 & C_{23} = -10 & C_{33} = 15 \end{array} \quad 2$$

$$A^{-1} = \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix} \quad 1$$

$$(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix} \quad 1$$

$$= \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix} \quad 1$$

OR

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad 1$$

$$R_1 \rightarrow R_1 + 2R_3$$

$$\begin{bmatrix} 1 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad 1$$

$$R_2 \rightarrow R_2 + R_1$$

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} A \quad 1$$

$$R_3 \rightarrow R_3 + 2R_2$$

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A \quad 1$$

$$R_1 \rightarrow R_1 + 2R_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A \quad 1$$

$$\text{So, } A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \quad 1$$

26. $(x - x) = 0$ is divisible by 3 for all $x \in z$. So, $(x, x) \in R$ 1

$\therefore R$ is reflexive.

$(x - y)$ is divisible by 3 implies $(y - x)$ is divisible by 3.

So $(x, y) \in R$ implies $(y, x) \in R$, $x, y \in z$ $1 \frac{1}{2}$

$\Rightarrow R$ is symmetric.

$(x - y)$ is divisible by 3 and $(y - z)$ is divisible by 3.

So $(x - z) = (x - y) + (y - z)$ is divisible by 3. $1 + 1 + \frac{1}{2}$

Hence $(x, z) \in R \Rightarrow R$ is transitive

$\Rightarrow R$ is an equivalence relation 1

OR

*	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

Table Format 1

Values of each correct row,

$$\frac{1}{2} \times 6 = 3$$

$$a * 0 = a + 0 = a \quad \forall a \in A \Rightarrow 0 \text{ is the identity for } *.$$

$$\frac{1}{2}$$

$$\text{Let } b = 6 - a \text{ for } a \neq 0$$

$$\frac{1}{2}$$

$$\text{Since } a + b = a + 6 - a < 6$$

$$\Rightarrow a * b = b * a = a + 6 - a - 6 = 0$$

$$\frac{1}{2}$$

$$\text{Hence } b = 6 - a \text{ is the inverse of } a.$$

$$\frac{1}{2}$$

27. Since the line is parallel to the two planes.

$$\therefore \text{Direction of line } \vec{b} = (\hat{i} - \hat{j} + 2\hat{k}) \times (3\hat{i} + \hat{j} + \hat{k})$$

$$1$$

$$= -3\hat{i} + 5\hat{j} + 4\hat{k}$$

$$1$$

\therefore Equation of required line is

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k}) \quad \dots(i)$$

$$1$$

$$\text{Any point on line (i) is } (1 - 3\lambda, 2 + 5\lambda, 3 + 4\lambda)$$

$$1$$

$$\text{For this line to intersect the plane } \vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 4$$

$$\text{we have } (1 - 3\lambda)2 + (2 + 5\lambda)1 + (3 + 4\lambda)1 = 4$$

$$\Rightarrow \lambda = -1$$

$$1$$

$$\therefore \text{Point of intersection is } (4, -3, -1)$$

$$1$$

28. Let number of units of type A be x and that of type B be y

LPP is Maximize $P = 40x + 50y$

1

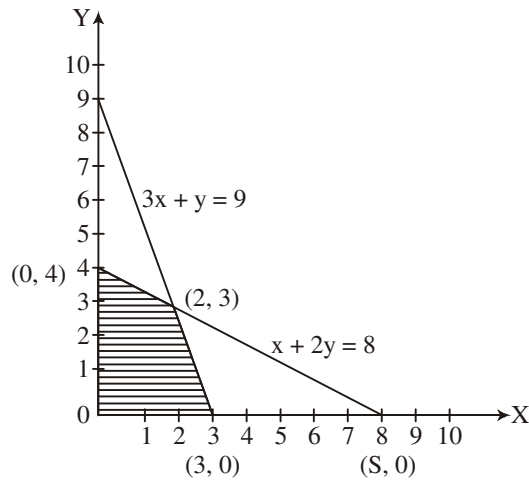
subject to constraints

$$3x + y \leq 9$$

2

$$x + 2y \leq 8$$

$$x, y \geq 0$$



2

$$P(3, 0) = 120$$

$$P(2, 3) = 230$$

$$P(0, 4) = 200$$

$$\therefore \text{Max profit} = ₹ 230 \text{ at } (2, 3)$$

1

So to maximise profit, number of units of A = 2 and number of units of B = 3

29.
$$I = \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

$$= \int_0^{\pi/2} \frac{(\pi/2 - x) \sin(\pi/2 - x) \cos(\pi/2 - x)}{\sin^4(\pi/2 - x) + \cos^4(\pi/2 - x)} dx = \int_0^{\pi/2} \frac{(\pi/2 - x) \cos x \sin x}{\cos^4 x + \sin^4 x} dx$$

1

$$2I = \pi/2 \int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx = \frac{\pi}{2} \int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + (1 - \sin^2 x)^2} dx$$

1

$$\text{Let } \sin^2 x = t \Rightarrow \sin x \cos x dx = \frac{1}{2} dt \quad \frac{1}{2}$$

$$2I = \frac{\pi}{2} \frac{1}{2} \int_0^1 \frac{dt}{t^2 + (1-t)^2} \quad 1$$

$$\Rightarrow I = \frac{\pi}{8} \int_0^1 \frac{dt}{2t^2 - 2t + 1} = \frac{\pi}{16} \int_0^1 \frac{dt}{(t - 1/2)^2 + (1/2)^2} \quad 1$$

$$I = \frac{\pi}{16} \frac{2}{1} \cdot \tan^{-1}(2t - 1) \Big|_0^1 = \frac{\pi}{8} \cdot \left[\frac{\pi}{4} - \left(-\frac{\pi}{4} \right) \right] = \frac{\pi^2}{16} \quad 1 + \frac{1}{2}$$

OR

$$a = 1, b = 3, h = \frac{2}{n} \Rightarrow nh = 2 \quad 1$$

$$\int_1^3 (3x^2 + 2x + 1) dx = \lim_{h \rightarrow 0} h[f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)] \quad 1$$

$$= \lim_{h \rightarrow 0} h[6 + \{3(1+h^2 + 2h) + 2(1+h) + 1\} + \{3(1+4h^2 + 4h) + 2(1+2h) + 1\} \\ + \dots \{3(1+(n-1)^2h^2 + 2(n-1)h + 2(1+(n-1)h) + 1\}] \quad 1$$

$$= \lim_{h \rightarrow 0} h[6n + 8h(1 + 2 + \dots + (n-1)) + 3h^2(1^2 + 2^2 + \dots + (n-1)^2)] \quad \frac{1}{2}$$

$$= \lim_{h \rightarrow 0} 6nh + \frac{8(nh-h)(nh)}{2} + \frac{3(nh-h)(nh)(2hn-h)}{6} \quad 1 \frac{1}{2}$$

$$= 6(2) + \frac{8(2)(2)}{2} + \frac{3(2-0)(2)(4)}{6} \quad \frac{1}{2}$$

$$= 12 + 16 + 8 = 36 \quad \frac{1}{2}$$