Marks: 100
Time: 3 Hrs.

Instructions : **(1) This section contains 50 questions and all questions are compulsory**
(2) All questions carry 1 mark.
(3) Read each question carefully and select proper alternatives and answer in the OMR Answer Sheet.

PART-A [50]

(1) The number of commutative binary operations on $\{1,2\}$ is ______

(a) 8 (b) 4 (c) 16 (d) 27

(2) If $a * b = \frac{ab}{100}$ on Q^+, inverse of 0.1 is

(a) 100000 (b) 1000 (c) 10 (d) 10000

(3) If $f(x) = \frac{x-1}{x+1}$, then $f(2x) =$

(a) $\frac{2f(x)+1}{f(x)+3}$ (b) $\frac{2f(x)+3}{f(x)+1}$ (c) $\frac{3f(x)+1}{f(x)+3}$ (d) $\frac{f(x)+2}{3f(x)+1}$

(4) $f: N \to N, f(x) = 2x + 3$ & $g(x) = 5x + 7$ then $fog(x) - gof(x) =$______

(a) $x + 5$ (b) $2x + 5$ (c) $3x + 5$ (d) -5

(5) If $f(x - 1) = x^2 - 3x + 1$, then $f(x + 1) =$______

(a) $x^2 - 3x + 1$ (b) $x^2 + x - 1$ (c) $x^2 - x + 1$ (d) $x^2 + 3x - 1$

(6) If $tan^{-1}x - tan^{-1}y = 0$ & $sin^{-1}x + sin^{-1}y = \frac{\pi}{2}$ then $x + y =$______

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) 0

(7) If $m\angle A = 90°$ in ΔABC, then $tan^{-1}\left(\frac{c}{a+b}\right) + tan^{-1}\left(\frac{b}{a+c}\right) =$______

(a) 0 (b) 1 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(8) If $\sin(2tan^{-1}x) = 1$, then $x =$_______

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\sqrt{3}$

(9) If $sin^{-1}x + sin^{-1}y = \frac{\pi}{2}$ then $x\sqrt{1-y^2} + y\sqrt{1-x^2} =$______

(a) 0 (b) 1 (c) -1 (d) 2

(10) $\sin(x + y) = \cos(x + y) \Rightarrow \frac{dy}{dx} =$______

(a) 1 (b) -1 (c) 2 (d) -2

(11) $\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix} =$______

a) 5! (b) 4! (c) 3! (d) 2!

(12) If $\begin{vmatrix} 6i & 3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + yi$, then--------

(a)$x = 3, y = 1$ (b)$x = 1, y = 3$ (c)$x = 0, y = 3$ (d)$x = y = 0$

(13) If $A + B = \begin{bmatrix} -2 & 1 \\ 4 & 3 \end{bmatrix}$ & $A - B = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$ then A =______

(a)$\begin{bmatrix} -1 & 0 \\ 2 & 0 \end{bmatrix}$ (b)$\begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix}$ (c)$\begin{bmatrix} -1 & -2 \\ 2 & 2 \end{bmatrix}$ (d)$\begin{bmatrix} 0 & 2 \\ 2 & 1 \end{bmatrix}$

(14) If we apply the Rolle's theorem to $f(x) = x(x-3)^2, x \in [0,3]$, then $c =$______

(a)1 (b)$\frac{1}{2}$ (c)2 (d)$\frac{3}{2}$

(15) If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \infty$, then $\frac{dy}{dx} =$______

(a)yy' (b)y (c)$\frac{1}{y}$ (d)$-y$

(16) If $f(x) = |x-1| + |x-3|$, then $f'(2) =$______

(a)-2 (b)0

(c)2 (d)$f'(2)$does not exist

(17) If $x^2 + y^2 = t + \frac{1}{t}$&$x^4 + y^4 = t^2 + \frac{1}{t^2}$, then $\frac{dy}{dx} =$______

(a)$\frac{x}{y}$ (b)$\frac{y}{x}$ (c)$-\frac{x}{y}$ (d)$-\frac{y}{x}$

(18) If $P(A') = 0.3$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$ then $P(B/A \cup B') =$______

(a) 0.375 (b) 0.32 (c) 0.31 (d) 0.28

(19) The probability that a student is not a swimmer is 4/5. The probability that out of 5 students exactly 4 are swimmers is ______

(a)$\left(\frac{1}{5}\right)^3$ (b) $4\left(\frac{1}{5}\right)^4$ (c)$\binom{5}{4}\left(\frac{4}{5}\right)^4$ (d)$\left(\frac{4}{5}\right)^4$

(20) The mean and variance of a random variable x having a binomial distribution are 4 & 2 respectively. Then $P(x = 1)$ is ______

(a)$\frac{1}{16}$ (b)$\frac{1}{8}$ (c)$\frac{1}{4}$ (d)$\frac{1}{32}$

(21) A problem in mathematics is given to three students A,B,C and their respective probability of solving the problem is $\frac{1}{2}$, $\frac{1}{3}$ & $\frac{1}{4}$. Probability that the problem is solved is ______

(a)$\frac{3}{4}$ (b)$\frac{1}{2}$ (c)$\frac{2}{3}$ (d)$\frac{1}{3}$

(22) Objective function of an LP problems is ______

(a)a constant (b) a function to be optimized

(c)an inequality (d) a quadratic equation

(23) In solving the LP problem minimize $z = 6x + 10y$ subject to $x \geq 6, y \geq 2, 2x + y \geq 10, x \geq 0, y \geq 0$ Redundant constraints are ______

(a) $x \geq 6, y \geq 2$
(b) $2x + y \geq 10, x \geq 0, y \geq 0$
(c) $x \geq 6$
(d) $x \geq 6, y \geq 0$

(24) $\int \frac{\sqrt{tanx}}{sinx.cosx} dx =$ ______ $+c$

(a) $\frac{\sqrt{tanx}}{2}$
(b) $\frac{\sqrt{cotx}}{2}$
(c) $2\sqrt{cotx}$
(d) $2\sqrt{tanx}$

(25) $\int \frac{1+cosx}{sinx.cosx} dx =$ ______ $+ c$

(a) $\log|cosx| + \log|sinx|$
(b) $\log|tanx.tan\frac{x}{2}|$
(c) $\log|1 + tan\frac{x}{2}|$
(d) $\log|\sec\frac{x}{2} + tan\frac{x}{2}|$

(26) Approximate value of $(31)^{\frac{1}{5}}$ is ______

(a) 2.01
(b) 2.1
(c) 2.0125
(d) 1.9878

(27) The line $y = mx + 1$ touches $y^2 = 4x$ if $m =$ ______

(a) 0
(b) 1
(c) -1
(d) 2

(28) $\int \left(logx + \frac{1}{x^2}\right) e^x dx =$ ______ $+ c$

(a) $e^x\left(logx + \frac{1}{x^2}\right)$
(b) $e^x\left(logx + \frac{1}{x}\right)$
(c) $e^x\left(logx - \frac{1}{x^2}\right)$
(d) $e^x\left(logx - \frac{1}{x}\right)$

(29) $\int \cos(logx)\, dx =$ ______ $+ c$

(a) $\frac{x}{2}[\cos(logx) + \sin(logx)]$
(b) $\frac{x}{4}[\cos(logx) + \sin(logx)]$
(c) $\frac{x}{2}[\cos(logx) - \sin(logx)]$
(d) $\frac{x}{2}[\sin(logx) - \cos(logx)]$

(30) $\int \frac{1}{x^2(x^4+1)^{\frac{3}{4}}} dx =$ ______ $+ c$

(a) $\left(1 + \frac{1}{x^4}\right)^{\frac{1}{4}}$
(b) $((x^4 + 1))^{\frac{1}{4}}$
(c) $\left(1 - \frac{1}{x^4}\right)^{\frac{1}{4}}$
(d) $-\left(1 + \frac{1}{x^4}\right)^{\frac{1}{4}}$

(31) $\int x\, sinx\, sec^3x\, dx =$ ______ $+ c$

(a) $\frac{1}{2}[sec^2x - tanx]$
(b) $\frac{1}{2}[x\, sec^2x - tanx]$
(c) $\frac{1}{2}[x\, sec^2x + tanx]$
(d) $\frac{1}{2}[sec^2x + tanx]$

(32) $\int_0^{\infty} \frac{x\, dx}{(1+x)(1+x^2)} =$ ______

(a) $\frac{\pi}{4}$
(b) $\frac{\pi}{2}$
(c) π
(d) ∞

(33) The area bounded by $y = |x - 5|$, x-axis and the lines $x = 0$, $x = 1$ is ______

(a) $\frac{9}{2}$ (b) $\frac{7}{2}$ (c) 9 (d) 5

(34) Area bounded by the line $y = 3x + 2$, Xaxis and the line $x = -1$ & $x = 1$ is --------

(a) 4 (b) 3 (c) $\frac{13}{3}$ (d) $\frac{25}{6}$

(35) The area of the region between the curve $y^2 = 4x$ and the line $x = 3$ is ______

(a) $4\sqrt{3}$ (b) $8\sqrt{3}$ (c) $16\sqrt{3}$ (d) $5\sqrt{3}$

(36) Area bounded by the curves $y = x^2$ and $x = y^2$ is ______

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{12}$ (d) 1

(37) The order and degree of $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^3 + y = 0$ are ______ respectively.

(a) 3 , 2 (b) 2 , 3 (c) 3 , not defined (d) 3 , 3

(38) $f(x, y) = \frac{x^3 - y^3}{x + y}$ is a homogeneous function of degree ______.

(a) 1 (b) 2 (c) 3 (d) not defined

(39) An integrating factor of differential equation $\frac{dy}{dx} = \frac{1}{x+y+2}$ is ______.

(a) e^x (b) e^{x+y+2} (c) e^{-y} (d) $\log|x + y + 2|$

(40) The solution of the differential equation $\frac{dy}{dx} + \frac{2y}{x} = 0$ with $y(1) = 1$ is given by -----

(a) $y = \frac{1}{x}$ (b) $y = \frac{1}{x^2}$ (c) $x = \frac{1}{y^2}$ (d) $x^2 = \frac{1}{y^2}$

(41) $|\bar{x}| = |\bar{y}| = 1$, $\bar{x} \perp \bar{y}$, $|\bar{x} + \bar{y}| =$ ____

(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 1 (d) 0

(42) If $\bar{a} = (-3, 1, 0)$ and $\bar{b} = (1, -1, -1)$, then magnitude of projection of $\bar{a}$ on $\bar{b}$

(a) $\frac{4}{\sqrt{10}}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{-4}{\sqrt{10}}$ (d) $\frac{-\sqrt{3}}{4}$

(43) The area of the parallelogram whose adjacent sides are $\hat{\imath} + \hat{k}$ and $\hat{\imath} + \hat{\jmath}$ is ______.

(a) 3 (b) $\sqrt{3}$ (c) $\frac{3}{2}$ (d) $\frac{\sqrt{3}}{2}$

(44) If $\bar{x} = \hat{\imath} - \hat{\jmath} + \hat{k}$, $\bar{y} = 4\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\bar{z} = \hat{\imath} + a\hat{\jmath} + b\hat{k}$ are coplanar and $|z| = \sqrt{3}$, then ______.

(a) $a = 1 , b = -1$ (b) $a = 1 , b = \pm 1$ (c) $a = -1 , b = \pm 1$ (d) $a = \pm 1, b = 1$

(45) If $A(3,-1),\ B(2,3)$and $= (5,1)$, then $m\angle A =$______

(a) $cos^{-1}\frac{3}{\sqrt{34}}$ (b) $\pi - cos^{-1}\frac{3}{\sqrt{34}}$ (c) $sin^{-1}\frac{5}{\sqrt{34}}$ (d) $\frac{\pi}{2}$

(46) The measure of the angle between the lines $x = k+1\ , y = 2k-1\ , z = 2k+3\ , k \in R$ and $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-1}{-2}$ is _____

(a)$sin^{-1}\frac{4}{3}$ (b)$cos^{-1}\frac{4}{9}$ (c)$sin^{-1}\frac{\sqrt{5}}{3}$ (d)$\frac{\pi}{2}$

(47) Plane $2x + 3y + 6z - 15 = 0$ makes angle of measure ______ with X-axis

(a)$cos^{-1}\frac{3\sqrt{5}}{7}$ (b)$sin^{-1}\frac{3}{7}$ (c)$sin^{-1}\frac{2}{7}$ (d)$tan^{-1}\frac{2}{7}$

(48) If $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane $2x - 4y + z = 7$, then $k =$______

(a) 7 (b) 6 (c) -7 (d) any value of k

(49) Perpendicular distance of $(2,-3,6)$from $3x - 6y + 2z + 10 = 0$ is ______.

(a)$\frac{13}{7}$ (b)$\frac{46}{7}$ (c) 7 (d)$\frac{10}{7}$

(50) Line passing through $(2,-3,1)$ and $(3,-4,-5)$ intersects ZX –plane at ______.

(a)$(-1,0,13)$ (b)$(-1,0,19)$ (c) $\left(\frac{13}{6},0,\frac{-19}{6}\right)$ (d)$(0,-1,13)$

PART-B

Instructions : (1) There are three sections in part B and total 18 questions are there.
(2) All questions are compulsory.
(3) The numbers at right side represent the marks of the questions.

Section-A **[16]**

(1) Show that the semi vertical angle of the right circular cone of maximum volume and given slant height is $tan^{-1}\sqrt{2}$.

(2) Obtain $\int_2^3 x^3 dx$ as a limit of a sum.

(3) Prove by vector method that in any $\Delta ABC\ \ c^2 = a^2 + b^2 - 2abcosC$.

OR

If A(3, 2, -4), B(4, 3, -4), C(3, 3, 3) and D(4, 2, -3), find projection of $\overrightarrow{AD}$ on $\overrightarrow{AB}$ x $\overrightarrow{AC}$.

(4) Find the area of the region included between the parabola $y^2 = 4ax$ and $x^2 = 4ay$, $a > 0$.

OR

Find the area of the region bounded by the curves $y = 4 - x^2$, x = 0, x = 3 and X-axis.

(5) Prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

(6) Show that $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ satisfies the equation $x^2 + 4x - 42 = 0$

(7) $f: R - \left\{-\frac{3}{2}\right\} \to R - \left\{\frac{3}{2}\right\}, f(x) = \frac{3x+2}{2x+3}$. Find f^{-1}.

(8) Minimize z= $2x + 4y$ subject to $x + 2y \geq 10\,, 3x + y \geq 10\,, x \geq 0\,, y \geq 0$.

Section-B [18]

(1) If $cos^{-1}x - cos^{-1}\frac{y}{2} = \propto$, then prove that $4x^2 - 4xycos \propto + y^2 = 4sin^2 \propto$.

(2) Prove that $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$.

(3) A bag X contains 2-white and 3-red balls and a bag Y contains 4-white and 5-red balls. One ball is drawn at Random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y.

(4) Obtain $\int \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}\,dx$. **OR** $\int \frac{cosx+x\sin x}{x^2+x\cos x}\,dx$.

(5) Find the general solution of the differential equation of $\frac{dy}{dx} + sin\frac{x+y}{2} = sin\frac{x-y}{2}$.

OR

Solve $\frac{dy}{dx} + \frac{y\,(x+y)}{x^2} = 0$

(6) Find the foot of the perpendicular drawn from the points $A(1,0,3)$ to the join of the points $B(4,7,1)$ and $C(3,5,3)$.

Section-C [10]

(1) If $A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 8 & 1 \\ 0 & 2 & 1 \\ 4 & 3 & -1 \end{bmatrix}$ find $(AB)^{-1}$.

(2) Evaluate $\int \sqrt{\frac{\sin(x-\theta)}{\sin(x+\theta)}}\,d\theta$

(3) $\int_{-\pi}^{\pi} \frac{2x(1+sinx)}{1+cos^2x}\,dx$. **OR** $\int_0^{\infty} \frac{x\log x}{(1+x^2)^2}$ dx.

(4) Find the maximum area of a rectangle inscribed in a semi-circle of radius r.