1. Given that $A^2 = A$.

We need to find the value of $7A – (I + A)^3$, where I is the identity matrix.

Thus,

$7A - (I + A)^3 = 7A - (I^3 + 3I^2A + 3I^2A + A^3)$

$\Longrightarrow 7A - (I + A)^3 = 7A – (I^3 + 3A + 3A^2 + A^2 \times A)$ $[I^3 = I, I^2A = A, IA^2 = A^2]$

$\Longrightarrow 7A – (I + A)^3 = 7A – (I + 3A + 3A + A)$ $[\because A^2 = A]$

$\Longrightarrow 7A – (I + A)^3 = 7A – I – 3A – 3A – A$

$\Longrightarrow 7A – (I + A)^3 = 7A – I – 7A$

$\Longrightarrow 7A – (I + A)^3 = - I.$

2. Given that $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$

We need to find the value of x + y.

$$\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$$

Two matrices A and B are equal to each other, if they have the same dimensions and the same elements $a_{ij} = b_{ij}$, for I = 1, 2, …, n and j = 1, 2, …,m.

$x – y = -1$…(1)

$2x – y = 0$…(2)

Equation (2) – (1) is $x = 1$

Substituting the value of x = 1 in equation (1), we have

$1 – y = -1$

$\Longrightarrow y = 2$

Therefore, $x + y = 1 + 2 = 3$

3. Given that $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$ and $xy < 1$.

We need to find the value of $x + y + xy$.

$\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \frac{\pi}{4} [\because xy < 1]$$

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right] = \tan\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \frac{x+y}{1-xy} = 1$$

$$\Rightarrow x+y = 1-xy \Rightarrow x+y+xy = 1$$

4. Given that $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$

We need to find the value of x

$$\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$$

$\Rightarrow$ 12x – (– 14) = 32 – 42

$\Rightarrow$ 12x + 14 = – 10

$\Rightarrow$ 12x = – 10 – 14

$\Rightarrow$12x = – 24

$\Rightarrow$ x = – 2

5. Since differentiation operation is the inverse operation of integration, we have $f'(x) = x\sin x$

Let $f(x) = \int_0^x t\sin t dt$

Let us do this by integration by parts.

Therefore assume u = t; du = dt

$$\int \sin t dt = \int dv$$

−cost = v

Therefore,

$$f(x) = [t-(-\cos t)]_0^x - \int_0^x (-\cos t)dt$$

$$f(x) = -x\cos x + \sin x + C$$

Differentiating the above function with respect to x,

$$f'(x) = -[x(-\sin x) + \cos x] + \cos x = x\sin x$$

6. Since the vectors are parallel, we have

$\vec{a} = \lambda\vec{b}$

$\Rightarrow 3\hat{i} + 2\hat{j} + 9\hat{k} = \lambda(\hat{i} - 2p\hat{j} + 3\hat{k})$

$\Rightarrow 3\hat{i} + 2\hat{j} + 9\hat{k} = \lambda\hat{i} - 2\lambda p\hat{j} + 3\lambda\hat{k}$

Comparing the respective coefficients, we have

$\Rightarrow \lambda = 3$;

$-2\lambda p = 2$

$\Rightarrow -2 \times 3 \times p = 2$

$\Rightarrow p = \frac{-1}{3}$

7. The set of natural numbers, N = {1,2,3,4,5,6…}

The relation is given as

R = {(x, y) : x + 2y = 8}

Thus, R = {(6,1),(4,2),(2,3)}

Domain = {6,4,2}

Range = {1,2,3}.

8. Given that the cartesian equation of the line as

$$\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$$

That is,

$$\frac{-(x-3)}{5} = \frac{y-(-4)}{7} = \frac{2(z-3)}{4}$$

$$\Rightarrow \frac{x-3}{-5} = \frac{y-(-4)}{7} = \frac{z-3}{2} = \lambda$$

Any point on the line is of the form :

$-5\lambda + 3, 7\lambda - 4, 2\lambda + 3$

Thus, the vector equation is of the form:

$\vec{r} = \vec{a} + \lambda\vec{b}$, where $\vec{a}$ is the position vector of any point on the line and $\vec{b}$ is the vector parallel to the line.

Therefore, the vector equation is

$$\vec{r} = (-5\lambda + 3)\hat{i} + (7\lambda - 4)\hat{j} + (2\lambda + 3)\hat{k}$$

$$\Rightarrow \vec{r} = -5\lambda\hat{i} + 7\lambda\hat{j} + 2\lambda\hat{k} + 3\hat{i} - 4\hat{j} + 3\hat{k}$$

$$\Rightarrow \vec{r} = 3\hat{i} - 4\hat{j} + 3\hat{k} + \lambda\left(-5\hat{i} + 7\hat{j} + 2\hat{k}\right)$$

9. Given that $\int_0^a \frac{dx}{4+x^2} = \frac{\pi}{8}$

We need to find the value of a.

Let I = $\int_0^a \frac{dx}{4+x^2} = \frac{\pi}{8}$

Thus , I = $\frac{1}{2}\left(\tan^{-1}\frac{x}{2}\right)_0^a = \frac{\pi}{8}$

$\Rightarrow \frac{1}{2}\tan^{-1}\frac{a}{2} = \frac{\pi}{8}$

$\Rightarrow \tan^{-1}\frac{a}{2} = 2\times\frac{\pi}{8}$

$\Rightarrow \tan^{-1}\frac{a}{2} = \frac{\pi}{4}$

$\Rightarrow \frac{a}{2} = 1 \Rightarrow a = 2$

10. Given that $\vec{a}$ and $\vec{b}$ are two perpendicular vectors.

Thus, $\vec{a}.\vec{b} = 0$

Also given that, $\left|\vec{a}+\vec{b}\right| = 13$ and $\left|\vec{a}\right| = 5$.

We need to find the value of $\vec{b}$.

Consider $\left|\vec{a}+\vec{b}\right|^2$:

$\left|\vec{a}+\vec{b}\right|^2 = \left|\vec{a}\right|^2 + 2\left|\vec{a}.\vec{b}\right| + \left|\vec{b}\right|^2$

$13^2 = 5^2 + 2\times 0 + \left|\vec{b}\right|^2$

$169 = 25 + \left|\vec{b}\right|^2$

$\left|\vec{b}\right|^2 = 169 - 25$

$\left|\vec{b}\right|^2 = 144$

$\vec{b} = 12$

SECTION – B

11. Given differential equation is:

$(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

$\Rightarrow \frac{dy}{dx} + \frac{y}{(1+x^2)} = \frac{e^{\tan^{-1}x}}{(1+x)^2}$

This a linear equation of the form

$$\frac{dy}{dx}+Py=Q$$

Where $P=\frac{1}{\left(1+x^2\right)}$ and $Q=\frac{e^{\tan^{-1}x}}{\left(1+x\ \right)}$

Therefore,

I.F = $e^{\int Pdx}=e^{\tan^{-1}x}$

Thus the solution is,

y(I.F) = $\int Q(I.F)dx$

$$\Rightarrow y\left(e^{\tan^{-1}x}\right)=\int\frac{e^{\tan^{-1}x}}{\left(1-x^2\right)}\times e^{\tan^{-1}x}dx$$

Substitute $e^{\tan^{-1}x}=t;$

Thus,

$$y\left(e^{\tan^{-1}x}\right)=\int tdt$$

$$\Rightarrow y\left(e^{\tan^{-1}x}\right)=\frac{t^2}{2}+C$$

$$\Rightarrow y\left(e^{\tan^{-1}x}\right)=\frac{\left(e^{\tan^{-1}x}\right)^2}{2}+C$$

12. Given position vectors of four A, B, C and D are:

$\overrightarrow{OA}=4\hat{i}+5\hat{j}+\hat{k}$

$\overrightarrow{OB}=-\hat{j}-\hat{k}$

$\overrightarrow{OC}=3\hat{i}+9\hat{j}+4\hat{k}$

$\overrightarrow{OD}=4\left(-\hat{i}+\hat{j}+\hat{k}\right)$

These points are coplanar, if the vectors, $\overrightarrow{AB},\overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar.

$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$

$=-\hat{j}-\hat{k}-\left(4\hat{i}+5\hat{j}+\hat{k}\right)=-4\hat{i}-6\hat{j}-2\hat{k}$

$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}$

$=3\hat{i}+9\hat{j}+4\hat{k}-\left(4\hat{i}+5\hat{j}+\hat{k}\right)=-\hat{i}+4\hat{j}+3\hat{k}$

$\overrightarrow{AD}=\overrightarrow{OD}-\overrightarrow{OA}$

$=4\left(-\hat{i}+\hat{j}+\hat{k}\right)-\left(4\hat{i}+5\hat{j}+\hat{k}\right)=-8\hat{i}-\hat{j}+3\hat{k}$

These vectors are coplanar if and only if, they can be expressed as a linear combination of other two.

$\overrightarrow{AB} = x\overrightarrow{AC} + \overrightarrow{AD}$

$\Rightarrow -4\hat{i} - 6\hat{j} - 2\hat{k} = (-x - 8y)\hat{i} + (4x - y)\hat{j} + (3x + 3y)\hat{k}$

Comparing the coefficeints, we have,

$-x - 8y = -4; 4x - y = -6; 3x + 3y = -2$

Thus, solving the first two equations, we get

$x = \frac{-4}{3}$ and $y = \frac{2}{3}$

These values of x and y satisfy the equation 3x + 3y = –2.

Hence the vectors are coplanar.

OR

Given that

$\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

$\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$

Now consider the sum of the vectors $\vec{b} + \vec{c}$:

$\vec{b} + \vec{c} = (2\hat{i} + 4\hat{j} - 5\hat{k}) + (\lambda\hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow \vec{b} + \vec{c} = (2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}$

Let $\hat{n}$ be the unit vector along the sum of vectors $\vec{b} + \vec{c}$:

$$\hat{n} = \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 6^2 + 2^2}}$$

The scalar product of $\vec{a}$ and $\vec{n}$ is 1. Thus,

$$\vec{a}.\hat{n} = (\hat{i} + \hat{j} + \hat{k}).\left(\frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 6^2 + 2^2}}\right)$$

$$\Rightarrow 1 = \frac{1(2+\lambda) + 1.6 - 1.2}{\sqrt{(2+\lambda)^2 + 6^2 + 2^2}}$$

$$\Rightarrow \sqrt{(2+\lambda)^2 + 6^2 + 2^2} = 2 + \lambda + 6 - 2$$

$$\Rightarrow \sqrt{(2+\lambda)^2 + 6^2 + 2^2} = \lambda + 6$$

$$\Rightarrow (2+\lambda)^2 + 40 = (\lambda + 6)^2$$

$$\Rightarrow \lambda^2 + 4\lambda + 4 + 4 = \lambda^2 + 12\lambda + 36$$

$$\Rightarrow 4\lambda + 44 = 12\lambda + 36$$

$$\Rightarrow 8\lambda = 8$$

$$\Rightarrow \lambda = 1$$

Thus, $\hat{n}$ is :

$$\hat{n} = \frac{(2+1)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+1)^2 + 6^2 + 2^2}}$$

$$\Rightarrow \hat{n} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{3^2 + 6^2 + 2^2}}$$

$$\Rightarrow \hat{n} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{49}}$$

$$\Rightarrow \hat{n} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$$

$$\Rightarrow \hat{n} = \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$$

13. We need to evaluate the integral

$$I = \int_0^{\pi} \frac{4x\sin x}{1+\cos^2 x} dx \ldots(1)$$

Using the property $\int f(a-x)dx = \int f(x)dx$, we have

$$I = \int_0^{\pi} \frac{4(\pi - x)\sin(\pi - x)}{1+\cos^2 x(\pi - x)} dx$$

$$\Rightarrow I = \int_0^{\pi} \frac{4\pi \sin x}{1+\cos^2 x} dx - \int_0^{\pi} \frac{4\pi \sin x}{1+\cos^2 x} dx \ldots(2)$$

Adding equations (1) and (2), we have,

$$\Rightarrow 2I = \int_0^{\pi} \frac{4x\sin x}{1+\cos^2 x} dx + \int_0^{\pi} \frac{4\pi \sin x}{1+\cos^2 x} dx - \int_0^{\pi} \frac{4\pi \sin x}{1+\cos^2 x} dx -$$

$$\Rightarrow 2I = \int_0^{\pi} \frac{4\pi \sin x}{1+\cos^2 x} dx$$

$$\Rightarrow 2I = 4\pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx$$

Subsititute t = cosx; dt = - sinxdx

When x = , t = 1

When x = π, t = -1

$$\Rightarrow 2I = 4\pi\int_{1}^{-1}\frac{(-1)dt}{1+t^2}$$

$$\Rightarrow I = 2\pi\int_{-1}^{1}\frac{dt}{1+t^2}$$

$$\Rightarrow I = 2\times 2\pi\int_{0}^{1}\frac{dt}{1+t^2}$$

$$\Rightarrow I = 2\times 2\pi\left(\tan^{-1} t\right)_0^1$$

$$\Rightarrow I = 4\pi\tan^{-1}(1)$$

$$\Rightarrow I = 4\pi\times\frac{\pi}{4} = \pi^2$$

OR

We need to evaluate the integral

$$\int\frac{x+2}{\sqrt{x^2+5x+6}}dx$$

$$\text{Let I} = \int\frac{x+2}{\sqrt{x^2+5x+6}}dx$$

Consider the integrand as follows:

$$\frac{x+2}{\sqrt{x^2+5x+6}} = \frac{A\frac{d}{dx}\left(x^2+5x+6\right)+B}{\sqrt{x^2+5x+6}}$$

$$\Rightarrow x+2 = A(2x+5)+B \Rightarrow x+2 = (2A)x+5A+B$$

Comparing the coefficients, we have

2A = 1; 5A + B = 2

Solving the above equations, we have

A = $\frac{1}{2}$ and B = $\frac{1}{2}$

Thus,

$$I = \int \frac{x+2}{\sqrt{x^2+5x+6}}\,dx$$

$$= \int \frac{\frac{2x+5}{2} - \frac{1}{2}}{\sqrt{x^2+5x+6}}\,dx$$

$$= \frac{1}{2}\int \frac{2x+5}{\sqrt{x^2+5x+6}}\,dx - \frac{1}{2}\int \frac{1}{\sqrt{x^2+5x+6}}\,dx$$

$$I = \frac{1}{2}I_1, - \frac{1}{2}I_2,$$

Where $I_1 = \int \frac{2x+5}{\sqrt{x^2+5x+6}}\,dx$

and $I_2 = \int \frac{1}{\sqrt{x^2+5x+6}}\,dx$

Now consider I_1 :

$$I_1 = \int \frac{2x+5}{\sqrt{x^2+5x+6}}\,dx$$

Subsitute

$$x^2+5x+6 = t; (2x+5)dx = dt$$

$$I_1 = \int \frac{dt}{\sqrt{t}}$$

$$= 2\sqrt{t}$$

$$= 2\sqrt{x^2+5x+6}$$

Now consider I_2 :

$$I_2 = \int \frac{1}{\sqrt{x^2+5x+6}}\,dx$$

$$= \int \frac{1}{\sqrt{x^2 + 5x\left(\frac{5}{2}\right)^2 + 6 - \left(\frac{5}{2}\right)^2}} dx$$

$$= \int \frac{1}{\sqrt{\left(x + \frac{5}{2}\right)^2 + 6 - \frac{25}{4}}} dx$$

$$= \int \frac{x+2}{\sqrt{\left(x + \frac{5}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} dx$$

$$I_2 = \log\left| x + \frac{5}{2} - \sqrt{x^2 + 5x + 6} \right| + C$$

$$\text{Thus, } I = \frac{1}{2} I_1 - \frac{1}{2} I_2$$

$$I = \sqrt{x^2 + 5x + 6} - \frac{1}{2}\log\left| x + \frac{5}{2} - \sqrt{x^2 + 5x + 6} \right| + C$$

14. Given function is

f(x) = [x(x – 2)]^2

⟹f’(x) = x2 x 2(x – 2) + (x – 2)2 x 2x

⟹f’ (x) = 2x (x – 2)[x+(x – 2)]

⟹f’(x) = 2x(x – 2)[2x – 2]

⟹f’(x) = 2x (x – 2)[2(x – 1)]

⟹f’(x) = 4x(x – 1) (x – 2)

Since f’(x) is an increasing function, f’(x) > 0.

⟹ f’(x) = 4x (x – 1) (x – 2) > 0

⟹x(x – 1) (x – 2) > 0

⟹0 < x < 1 or x > 2

⟹ x ∈(0, 1) ∪ (2, ∞)

OR

Let $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be the equation of the curve.

Rewriting the above equation as,

$$\frac{y^2}{b^2} = \frac{x^2}{a^2} - 1$$

$$\Rightarrow y^2 = \frac{b^2}{a^2}x^2 - b^2$$

Differentiating the above function w.r.t.x, we get,

$$2y\frac{dy}{dx}=\frac{b^2}{a^2}2x$$

$$\Rightarrow \frac{dy}{dx}=\frac{b^2}{a^2}\frac{x}{y}$$

$$\Rightarrow \left[\frac{dy}{dx}\right]_{(\sqrt{2}a,b)}=\frac{b^2}{a^2}\frac{\sqrt{2}a}{b}=\frac{\sqrt{2}b}{a}$$

Slope of the tangent is m = $\frac{\sqrt{2}b}{a}$

Equation of the tangent is

$(y - y_1) = m\ (x - x_1)$

$$\Rightarrow (y-b)=\frac{\sqrt{2}b}{a}\left(x-\sqrt{2}a\right)$$

$$\Rightarrow a(y-b)=\sqrt{2}b\left(x-\sqrt{2}a\right)$$

$$\Rightarrow \sqrt{2}bx-ay+ab-2ab=0$$

$$\Rightarrow \sqrt{2}bx-ay-ab=0$$

Slope of the normal is $-\frac{1}{\frac{\sqrt{2}b}{a}}$

Equation of the normal is

$$(y-y_1)=m(x-x_1)$$

$$\Rightarrow (y-b)=\frac{-a}{\sqrt{2}b}\left(x-\sqrt{2}a\right)$$

$$\Rightarrow \sqrt{2}b(y-b)=-a\left(x-\sqrt{2}a\right)$$

$$\Rightarrow ax+\sqrt{2}by-\sqrt{2}b^2+\sqrt{2}a^2=0$$

$$\Rightarrow ax+\sqrt{2}by+\sqrt{2}\left(a^2-b^2\right)=0$$

15. Given that f(x) = x^2 + 2 and g(x) = $\frac{x}{x-1}$

Let us find f °g:

f °g = f(g(x))

$\Longrightarrow$ f °g = $(g(x))^2 + 2$

$\Longrightarrow$ f °g = $\left(\frac{x}{x-1}\right)^2+2$

$\Longrightarrow$ f °g = $\frac{x^2+2(x-1)^2}{(x-1)^2}$

$\Rightarrow$ f °g = $\dfrac{x^2+2(x^2-2x+1)}{x^2-2x+1}$

$\Rightarrow$ f °g = $\dfrac{3x^2-4x+2}{x^2-2x+1}$

Therefore, (f°g)(2) = $\dfrac{3\times 2^2-4\times 2+2}{2^2-2\times 2+1}$

$\Rightarrow$ (f °g)(2) = $\dfrac{12-8+2}{4-4+1}=6$

Now let us find g °f:

g °f = g (f(x))

$\Rightarrow$ g °f = $\dfrac{f(x)}{f(x)-1}$

$\Rightarrow$ g °f = $\dfrac{x^2+2}{x^2+2-1}$

$\Rightarrow$ g °f = $\dfrac{x^2+2}{x^2+1}$

Therefore, (g°f)(-3) = $\dfrac{(-3)^2+2}{(-3)^2+1}=\dfrac{9+2}{9+1}=\dfrac{11}{10}$

16. We need to prove that

$$\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x, -\frac{1}{\sqrt{2}}\le x\le 1$$

Consider $x\cos 2t$;

$$\text{L.H.S}=\tan^{-1}\left[\frac{\sqrt{1+\cos 2t}-\sqrt{1-\cos 2t}}{\sqrt{1+\cos 2t}+\sqrt{1-\cos 2t}}\right]$$

$$=\tan^{-1}\left[\frac{\sqrt{2}\cos t-\sqrt{2}\sin t}{\sqrt{2}\cos t+\sqrt{2}\sin t}\right]$$

$$=\tan^{-1}\left[\frac{1-\tan t}{1+\tan t}\right]$$

$$=\tan^{-1}\left[\frac{\tan\frac{\pi}{4}-\tan t}{1+\tan\frac{\pi}{4}\times\tan t}\right]$$

$$=\tan^{-1}\left[\tan\left(\frac{\pi}{4}-t\right)\right]$$

$$=\frac{\pi}{4}-t$$

$$=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x=\text{R.H.S}$$

OR

Given that $\tan^{-1}\left(\frac{x-2}{x-4}\right)+\tan^{-1}\left(\frac{x+2}{x+4}\right)=\frac{\pi}{4}$

We need to find the value of x.

$$\tan^{-1}\left(\frac{x-2}{x-4}\right)+\tan^{-1}\left(\frac{x+2}{x+4}\right)=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{x-2}{x-4}+\frac{x+2}{x+4}}{1-\left(\frac{x-2}{x-4}\right)+\left(\frac{x+2}{x+4}\right)}\right)=\frac{\pi}{4}$$

$$\Rightarrow \frac{\frac{x-2}{x-4}+\frac{x+2}{x+4}}{1-\left(\frac{x-2}{x-4}\right)+\left(\frac{x+2}{x+4}\right)}=\tan\frac{\pi}{4}$$

$$\Rightarrow \frac{(x-2)(x-4)+(x+2)(x+4)}{(x-4)(x+4)-(x-2)(x+2)}=1$$

$$\Rightarrow \frac{\left(x^2+2x-8\right)+\left(x^2-2x-8\right)}{\left(x^2-16\right)-\left(x^2-4\right)}=1$$

$$\Rightarrow \frac{2x^2-16}{-12}=1$$

$$\Rightarrow 2x^2-16=-12$$

$$\Rightarrow 2x^2=4$$

$$\Rightarrow x^2=2$$

$$x=\pm\sqrt{2}$$

17. An experiment succeeds thrice as often as it fails. Therefore, there are 3 successes and 1 failure.

Thus the probability of success = $\frac{3}{4}$

And the probability of failure = $\frac{1}{4}$

We need to find the probability of atleast 3 successes in the next five trials.

Required Probability=P(x = 3) + P(x = 4) + P(x = 5)

= $^5C_3p^3q^2 + {}^5C_4p^4q^1 + {}^5C_5p^5q^0$

$$= {}^5C_3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 + {}^5C_4\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)^1 + {}^5C_5\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)^0$$

$$= 10\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 + 5\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^1 + \left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)^0$$

$$= \frac{918}{1024}$$

$$= \frac{459}{512}$$

18. Given that

y = $Pe^{ax} + Qe^{bx}$

Differentiating the above function w.r.t.x,

$$\frac{dy}{dx} = Pae^{ax} + Qbe^{bx}$$

Differentiating once again, we have,

$$\frac{d^2y}{dx^2} = Pa^2e^{ax} + Qb^2e^{bx}$$

Let us now find (a + b) $\frac{dy}{dx}$:

(a + b) $\frac{dy}{dx}$ = (a + b) $\left(Pae^{ax} + Qbe^{bx}\right)$

$$\Rightarrow (a+b)\frac{dy}{dx} = Pa^2e^{ax} + Qabe^{bx} + Pabe^{bx} + Qb^2e^{bx}$$

$$\Rightarrow (a+b)\frac{dy}{dx} = Pa^2e^{ax} + (P+Q)abe^{bx} + Qb^2e^{bx}$$

Also we have,

aby = ab($Pe^{ax} + Qe^{bx}$)

Thus, $\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby$

= $Pa^2e^{ax} + Qb^2e^{bx} - Pa^2e^{ax} - (P + Q)abe^{bx} - Qb^2e^{bx} + abPe^{ax} + abQe^{bx}$

= 0

19. Consider the determinant

$$\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$$

Taking abc common outside, we have

$$\Delta = abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b}+1 & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix}$$

Apply the transformation , $C_1 \rightarrow C_1 + C_2 + C_3$,

$$\Delta = abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b}+1 & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix}$$

$$\Rightarrow \Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b}+1 & \frac{1}{c} \\ 1 & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix}$$

Apply the transformations, $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$

$$\Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along C_1, we have

$$\Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\times 1\times\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc + ab + bc + ca$$

20. $x = \cos t(3 - 2\cos^2 t)$

and

$y = \sin t\ (3 - 2\sin^2 t)$

We need to find $\frac{dy}{dx}$:

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Let us find $\frac{dx}{dt}$:

X = cost (3 – 2cos^{2}t)

$\frac{dx}{dt}$=cost(4costsint) + (3 – 2cos^{2}t) (- sint)

$$\Rightarrow \frac{dy}{dt}=3\cos t-4\sin^2 t\cos t-2\sin^2 t\cos t$$

$$\text{Thus,}\frac{dy}{dt}=\frac{3\cos t-4\sin^2 t\cos t-2\sin^2 t\cos t}{-3\sin t+4\cos^2 t\sin t+2\cos^2 t\sin t}$$

$$\Rightarrow \frac{dy}{dx}=\frac{3\cos t-6\sin^2 \cos t}{-3\sin t+6\cos^2 t\sin t}$$

$$\Rightarrow \frac{dy}{dx}=\frac{3\cos(1-2\sin^2 t)}{-3\sin t(1-2\cos^2 t)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\cos t}{\sin t}\left[\because 2\cos^2 t-1=1-2\sin^2 t\right]$$

$$\Rightarrow \frac{dy}{dx}=\cot t$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}}=\cot\frac{\pi}{4}=1$$

21. Consider the differential equation,

$$\log\left(\frac{dy}{dx}\right)=3x+4y$$

Taking exponent on both sides, we have

$$e^{\log\left(\frac{dy}{dx}\right)}=e^{3x+4y}$$

$$\Rightarrow \frac{dy}{dx}=e^{3x+4y}$$

$$\Rightarrow \frac{dy}{dx}=e^{3x}.e^{4y}$$

$$\Rightarrow \frac{dy}{e^{4y}}=e^{3x}dx$$

Integration in both the sides, we have

$$\int \frac{dy}{e^{4y}} = \int e^{3x} dx$$

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$$

We need to find the particular solution.

We have, y = 0, when x = 0

$$\frac{1}{-4} = \frac{1}{3} + C$$

$$\Rightarrow C = -\frac{1}{4} - \frac{1}{3}$$

$$\Rightarrow C = \frac{-3-4}{12} = -\frac{7}{12}$$

Thus, the solution is $\frac{e^{3x}}{3} + \frac{e^{-4y}}{4} = \frac{7}{12}$

22. The equation of line L_1:

$$\frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$$

$$\Rightarrow \frac{x-1}{-3} = \frac{y-2}{\frac{p}{7}} = \frac{z-3}{2} \quad ....(1)$$

The equation of line L_2:

$$\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

$$\Rightarrow \frac{x-1}{\frac{-3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5} \quad ...(2)$$

Since line L_1 and L_2 are perpendicular to each other, we have

$$-3 \times \left(\frac{-3p}{7}\right) + \frac{p}{7} \times 1 + 2 \times (-5) = 0$$

$$\Rightarrow \frac{9p}{7} + \frac{p}{7} = 10$$

$$\Rightarrow 10p = 70$$

$$\Rightarrow p = 7$$

Thus equations of lines L_1 and L_2 are:

$$\frac{x-1}{-3} = \frac{y-2}{1} = \frac{z-3}{2}$$

$$\frac{x-1}{-3} = \frac{y-5}{1} = \frac{z-6}{-5}$$

Thus the equation of the line passing through the point (3, 2, -4) and parallel to the line L_1 is:

$$\frac{x-3}{-3}=\frac{y-2}{1}=\frac{z+4}{2}$$

SECTION – C

23. Equation of the plane passing through the intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 is:

(x + y + z – 1) + λ (2x + 3y + 4z – 5)

(1 +2 λ) x + (1 + 3 λ) y + (1 + 4 λ) z – (1+5 λ) = 0

This plane has to be perpendicular to the plane x – y + z = 0

Thus,

(1 +2 λ) x + (1 + 3 λ)(-1) + (1 + 4 λ)1 = 0

1 +2 λ – 1 – 3 λ + 1 + 4 λ = 0

1 + 3 λ = 0

$\lambda = -\frac{1}{3}$

Thus, the equation of the plane is:

$$\left|\frac{-(-2)}{\sqrt{1^2+0^2+1^2}}\right|=\left|\frac{2}{\sqrt{2}}\right|=\sqrt{2}$$

OR

Any point in the line is

2 + λ, -4 + 4 λ, 2 + 2 λ

The vector equation of the plane is given as

$$\vec{r}.\left(\hat{i}-2\hat{j}+\hat{k}\right)=0$$

Thus the cartesian equation of the plane is x – 2y + z = 0

Since the point lies in the plane

(2 + 3 λ)1 + (-4 + 4 λ)(-2)+(2 + 2 λ) = 0

⟹ 2 + 8 +2 + 3 λ – 8 λ + 2 λ = 0

⟹12 - 3 λ

⟹ λ = 4

Thus, the point of intersection of the line and the plane is: 2 + 3 x 4, -4 + 4 x 4, 2 + 2 x 4

⟹ 14, 12, 10

Distance between (2, 12,5) and (14, 12, 10)is:

$$d=\sqrt{(14-2)^2+(12-12)^2+(10-5)^2}$$

$$\Rightarrow d=\sqrt{144+25}$$

$$\Rightarrow d=\sqrt{169}$$

$$\Rightarrow d=13\,\text{units}$$

24. Consider the vertices, A(-1, 2), B(1, 5) and C(3, 4)

Let us find the equation of the sides of ΔABC.

Thus, the equation of AB is:

$$\frac{y-2}{5-2}=\frac{x+1}{1+1}$$

$$\Rightarrow 3x-2y+7=0$$

Similarly, the equation of BC is:

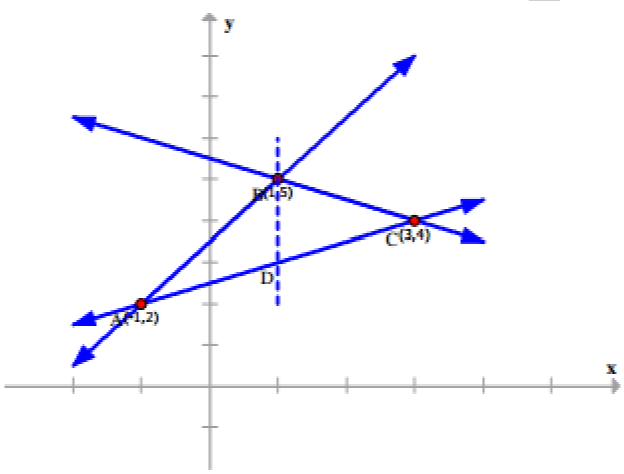
$$\frac{y-5}{4-5}=\frac{x-1}{3-1}$$

$$\Rightarrow x+2y-11=0$$

Also, the equation of CA is:

$$\frac{y-4}{2-4}=\frac{x-3}{-1-3}$$

$$\Rightarrow x-2y+5=0$$



Now the area of ABC=Area of ΔADB + Area of ΔBDC

$$\therefore \text{Area of } \Delta ADB=\int_{-1}^{1}\left[\frac{3x+7}{2}-\frac{x+5}{2}\right]dx$$

Similarly, Area of ΔADB = $\int_{1}^{3}\left[\frac{11-x}{2}-\frac{x+5}{2}\right]dx$

Thus, Area of ΔADB + Areaof ΔBDC

$$= \int_{-1}^{1}\left[\frac{3x+7}{2}-\frac{x+5}{2}\right]dx+\int_{1}^{3}\left[\frac{11-x}{2}-\frac{x+5}{2}\right]dx$$

$$= \int_{-1}^{1}\left[\frac{2x+2}{2}\right]dx+\int_{1}^{3}\left[\frac{6-2x}{2}\right]dx$$

$$= \int_{-1}^{1}[x+1]dx+\int_{1}^{3}[3-x]dx$$

$$= \left[\frac{x^2}{2}+x\right]_{-1}^{1}+\left[3x-\frac{x^2}{2}\right]_{1}^{3}$$

$$= 2+9-\frac{9}{2}-3+\frac{1}{2}$$

$$= 2+\frac{9}{2}-\frac{5}{2}$$

$$= 4 \text{ square units}$$

25. Let x be the number of pieces manufactured of type A and y be the number of pieces manufactured of type B. Let us summarise the data given in the problem as follows:

Product	Time for Fabricating (in hours)	Time for Finishing (in hours)	Maximum labour hours available
Type A	9	1	180
Type B	12	3	30
Maximum Profit (in Rupees)	80	120	

Thus,the mathematical form of above L.P.P. is

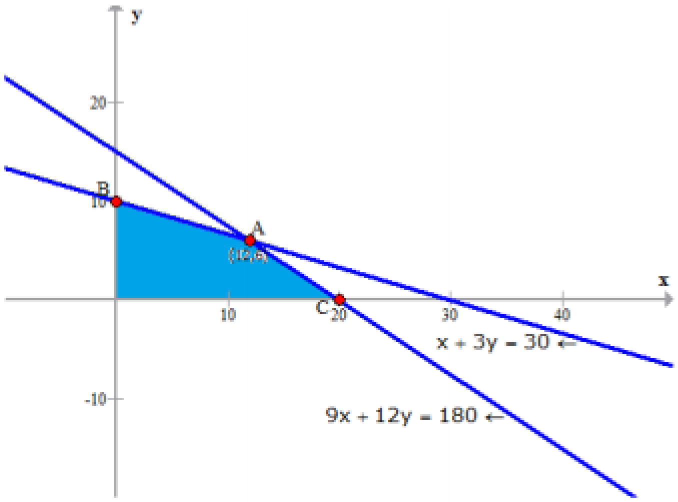
Maximise Z = 80x + 120y subject to

$9x + 12y \leq 180$

$x + 3y \leq 30$

Also, we have $x \geq 0$, $y \geq 0$

Let us now find the feasible region, which is the set of all points whose coordinates satisfy all constraints. Consider the following figure.



Thus, the feasible region consists of the points A, B and C. The values of the objective function at the corner points are given below in the following table:

Points	Value of Z
A(12, 6)	Z = 80 x 12+120 x 6 = Rs. 1680
B(0, 10)	Z = 80 x 0+12 x 10 = Rs. 1200
C(20, 0)	Z = 80 x 20+120 x 0 = Rs. 1600

Clearly,Z is maximum at x = 12 and y = 6 and the maximum profit is Rs. 1680.

26. Let E_1 ,E_2 ,E_3 and A be the events defined as follows:

E_1 = Choosing 2 headed coin

E_2 = Choosing coin with 75% chance of getting heads

E_3 = Choosing coin with 40% chance of getting heads

A= Getting heads

Then $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

Also, $P(A/E_1) = 1$, $P(A / E_2) = \frac{75}{100} = \frac{3}{4}$, $P(A / E_3) = \frac{40}{100} = \frac{2}{5}$

Required probability

$= P(E_1 / A)$

$$= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times\frac{3}{4}+\frac{1}{3}\times\frac{2}{5}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{4}+\frac{2}{15}}$$

$$= \frac{\frac{1}{3}}{\frac{43}{60}} = \frac{20}{43}$$

OR

If 1 is the smallest number,
the other numbers are:2,3,4,5,6
If 2 is the smallest number,
the other numbers are:3,4,5,6
If 3 is the smallest number,
the other numbers are:4,5,6
If 4 is the smallest number,
the other numbers are:5,6
If 5 is the smallest number,
the other number is:6

Thus, the sample space is S = $\begin{Bmatrix} 12,13,14,15,16 \\ 23,24,25,26 \\ 34,35,36 \\ 45,46 \\ 56 \end{Bmatrix}$

Thus, there are 15 set of numbers in the sample space.
Let X be
X : 2 3 4 5 6
P(X) = $\frac{1}{15}$ $\frac{2}{15}$ $\frac{3}{15}$ $\frac{4}{15}$ $\frac{5}{15}$
We know that,
E(X) = $X_iP(X_i)$

$$= 2\times\frac{1}{15}+3\times\frac{2}{15}+4\times\frac{3}{15}+5\times\frac{4}{15}+6\times\frac{5}{15}$$
$$= \frac{2+6+12+20+30}{15}$$
$$= \frac{70}{15}$$
$$= 4.66$$

27. From the given data, we write the following equations:

$$\begin{pmatrix} x & y & z \end{pmatrix}\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = 1600$$

$$\begin{pmatrix} x & y & z \end{pmatrix}\begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} = 2300$$

$$\begin{pmatrix} x & y & z \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 900$$

From above system, we get:

3x+ 2y + z = 1600

4x + y + 3z = 2300

x + y + z = 900

Thus we get:

$$\begin{pmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1600 \\ 2300 \\ 900 \end{pmatrix}$$

AX = B, where $A = \begin{pmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}; X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $B = \begin{pmatrix} 1600 \\ 2300 \\ 900 \end{pmatrix}$

$$|A| = \begin{vmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 3(1-3)-2(4-3)+1(4-1) = -6-2+3 = -5 \neq 0$$

We need to find A^{-1} :

$C_{11} = -2$; $C_{12} = -1$; $C_{13} = 3$

$C_{21} = -1$; $C_{22} = 2$; $C_{23} = -1$

$C_{31} = 5$; $C_{32} = -5$; $C_{33} = -5$

Therefore, adj A = $\begin{pmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{pmatrix}^T = \begin{pmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{pmatrix}$

Thus, $A^{-1} = \frac{adjA}{|A|} = -\frac{1}{5}\begin{pmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{pmatrix}$

Therefore, $X = A^{-1}B$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{pmatrix}\begin{pmatrix} 1600 \\ 2300 \\ 900 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -2\times 1600 - 1\times 2300 + 5\times 900 \\ -1\times 1600 + 2\times 2300 - 5\times 900 \\ 3\times 1600 - 1\times 2300 - 5\times 900 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -1000 \\ -1500 \\ -2000 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 200 \\ 300 \\ 400 \end{pmatrix}$$

Awards can be given for discipline.

28. Let ΔABC be the right angled triangle with base b and hypotenuse h.

Given that b + h = k

Let A be the area of the right triangle.

$$A = \frac{1}{2}\times b \times \sqrt{h^2 - b^2}$$

$$\Rightarrow A^2 = \frac{1}{4}b^2\left(h^2 - b^2\right)$$

$$\Rightarrow A^2 = \frac{b^2}{4}\left((k-b)^2 - b^2\right) \quad [\because h = k - b]$$

$$\Rightarrow A^2 = \frac{b^2}{4}\left(k^2 + b^2 - 2kb - b^2\right)$$

$$\Rightarrow A^2 = \frac{b^2}{4}\left(k^2 - 2kb\right) \Rightarrow A^2 = \frac{b^2k^2 - 2kb^3}{4}$$

Differentiating the above function w.r.t.x, we have

$$2A = \frac{dA}{db} = \frac{2bk^2 - 6kb^2}{4} ....(1)$$

$$\Rightarrow \frac{dA}{db} = \frac{bk^2 - 3kb^2}{2A}$$

For the area to be maximum, we have

$$\frac{dA}{db} = 0$$

$$\Rightarrow bk^2 - 3kb^2 = 0$$

$$\Rightarrow bk = 3b^2$$

$$\Rightarrow b = \frac{k}{3}$$

Again differentiating the function in equation (1), with respect to b, we have

$$2\left(\frac{dA}{bd}\right)^2 + 2A\frac{d^2A}{db^2} = \frac{2k^2 - 12kb}{4} ....(2)$$

Now substituting $\frac{dA}{db}$ =0 and b = $\frac{k}{3}$ in equation (2), we have

$$2A\frac{d^2A}{db^2} = \frac{2k^2 - 12k\left(\frac{k}{3}\right)}{4}$$

$$\Rightarrow 2A\frac{d^2A}{db^2} = \frac{6k^2 - 12k^2}{12}$$

$$\Rightarrow 2A\frac{d^2A}{db^2} = -\frac{k^2}{2}$$

$$\Rightarrow \frac{d^2A}{db^2} = -\frac{k^2}{4A} < 0$$

Thus area is maximum at $b = \frac{k}{3}$.

Now, h = k $- \frac{k}{3} = \frac{2k}{3}$

Let θ be the angle between the base of the triangle and the hypotenuse of the right triangle.

Thus, cosθ = $\frac{b}{h} = \frac{\frac{k}{3}}{\frac{2k}{3}} = \frac{1}{2}$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

29. We need to evaluate $\int \frac{dx}{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}$

Let I = $\int \frac{dx}{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}$

Multiply the numerator and the denominator by sec ^{4}x, we have

$$I = \int \frac{\sec^4 x dx}{\tan^4 x + \tan^2 x + 1}$$

$$I = \int \frac{\sec^4 x \times \sec^2 x dx}{\tan^4 x + \tan^2 x + 1}$$

We know that $\sec^2 x = 1 + \tan^2 x$

Thus,

$$I = \int \frac{(1 + \tan^2 x)\sec^2 x dx}{\tan^4 x + \tan^2 x + 1}$$

Now substitute t = tanx; dt = $\sec^2 x dx$

Therefore,

$$I = \int \frac{(1 + t^2)dt}{1 + t^2 + t^4}$$

Let us rewrite the integrand as

$$\frac{(1+t^2)}{1+t^2+t^4} = \frac{(1+t^2)}{(t^2-t+1)(t^2+t+1)}$$

Using partial fractions, we have

$$\frac{(1+t^2)}{1+t^2+t^4} = \frac{At+B}{t^2-t+1} + \frac{Ct+D}{t^2+t+1}$$

$$\Rightarrow \frac{(1+t^2)}{1+t^2+t^4} = \frac{(At+B)(t^2+t+1)+(Ct+D)(t^2-t+1)}{(t^2-t+1)(t^2+t+1)}$$

$$\Rightarrow \frac{(1+t^2)}{1+t^2+t^4} = \frac{At^3 + At^2 + At + Bt^2 + Bt + Ct^3 - Ct^2 + Ct + Dt^2 - Dt + D}{(t^2-t+1)(t^2+t+1)}$$

$$\Rightarrow \frac{(1+t^2)}{1+t^2+t^4} = \frac{t^3(A+C)+t^2(A+B-C+D)+t(A+B+C-D)+(B+D)}{(t^2-t+1)(t^2+t+1)}$$

So we have,

A + C = 0; A + B – C + D = 1; A + B+ C – D = 0; B + D = 1

Solving the above equations, we have

A = C = 0 and B = D = $\frac{1}{2}$

$$I = \int \frac{\left(1+t^2\right)dt}{1+t^2+t^4}$$

$$= \int \left[\frac{1}{2\left(t^2-t+1\right)} + \frac{1}{/\left(t^2+t+1\right)} \right]$$

$$= \int \frac{dt}{2\left(t^2-t+1\right)} + \int \frac{dt}{2\left(t^2+t+1\right)}$$

$$= I_2 + I_2$$

Where, $I_1 = \frac{1}{2}\int \frac{dt}{t^2-t+1}$ and $I_2 = \frac{1}{2}\int \frac{dt}{t^2+t+1}$

Consider I_1 :

$$I_1 = \frac{1}{2}\int \frac{dt}{t^2+t+1}$$

$$= \frac{1}{2}\int \frac{dt}{t^2 - t + \frac{1}{4} + 1 - \frac{1}{4}}$$

$$= \frac{1}{2}\int \frac{dt}{\left(t-\frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{\frac{3}{4}}} \tan^{-1}\left(\frac{t-\frac{1}{2}}{\sqrt{\frac{3}{4}}} \right)$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{2t-1}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{2\tan x - 1}{\sqrt{3}}$$

Similarly,

Consider I_2:

$$I_2 = \frac{1}{2}\int \frac{dt}{t^2 + t + 1}$$

$$= \frac{1}{2}\int \frac{dt}{t^2 + t + \frac{1}{4} + 1 - \frac{1}{4}}$$

$$= \frac{1}{2}\int \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{\frac{3}{4}}} \tan^{-1}\left(\frac{t + \frac{1}{2}}{\sqrt{\frac{3}{4}}}\right)$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{2t + 1}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{2\tan x + 1}{\sqrt{3}}$$

Thus, $I = I_1 + I_2$

$$\Rightarrow I = \frac{1}{\sqrt{3}} \tan^{-1} \frac{2\tan x - 1}{\sqrt{3}} + \frac{1}{\sqrt{3}} \tan^{-1} \frac{2\tan x + 1}{\sqrt{3}}$$

$$I = \frac{1}{\sqrt{3}}\left[\tan^{-1} \frac{2\tan x - 1}{\sqrt{3}} + \tan^{-1} \frac{2\tan x + 1}{\sqrt{3}}\right] + C$$