CBSE_2015_SET-1

General Instructions:

1. All questions are compulsory.
2. Please check that this question paper contains 26 questions.
3. Question 1 – 6 in Section A are very short – answer type questions carrying 1 mark each.
4. Questions 7 – 19 in Section B are long – answer I type question carrying 4 marks each.
5. Questions 20 – 26 in Section B are long – answer II type question carrying 6 marks each.
6. Please write down the serial number of the question before attempting it.

SECTION-A

Qustion numbers 1 to 6 carry 1 mark each.

1. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$, then find $|\vec{a} \times \vec{b}|$.
2. Find the angle between the vectors $\hat{i} - \hat{j}$ and $\hat{j} - \hat{k}$.
3. Find the distance of a point (2, 5, −3) from the plane $\vec{r}.(6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$.
4. Write the element a12 of the matrix A = $[a_{ij}]_{2\times 2}$, whose elements aij are given by $a_{ij} = e^{2ix}$ sin jx.
5. Find the differential equation of the family of lines passing through the origin.
6. Find the integrating factor for the following differential equation: $x \log x \dfrac{dy}{dx} + y = 2\log x$

Section-B

Question numbers 7 to 19 carry 4 marks each.

7. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then show that $A^2 - 4A - 5I = O$, and hence find A^{-1} or $A = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix}$, then find A^{-1} using elementary row operations.
8. Using the properties of determinants, solve the following for x: $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$
9. Evaluate: $\int_0^{\pi/2} \dfrac{\sin^2 x}{\sin x + \cos x} dx.$ OR Evaluate $\int_{-1}^{2} (e^{3x} + 7x - 5) dx.$ as a limit of sums.
10. Evaluate: $\int \dfrac{x^2}{x^4 + x^2 - 2} dx$
11. In a set of 10 coins, 2 coins are with heads on both the sides. A coin is selected at random from this set and tossed five times. If all the five times, the result was heads, find the probability that the selected coin had heads on both the sides. OR How many times must a fair coin be tossed so that the probability of getting at least one head is more than 80%?

12. Find x such that the four points A(4, 1, 2), B(5, x, 6) , C(5, 1, -1) and D(7, 4, 0) are coplanar.

13. A line passing through the point A with position vector $\vec{a} = 4\hat{i} + 2\hat{j} + 2\hat{k}$ is parallel to the vector $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$. Find the length of the perpendicular drawn on this line from a point P with vector $\vec{r}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$.

14. Solve the following for x: $\sin^{-1}(1 - x) - 2\sin^{-1} x = \pi/2$ OR Show that:

$$2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$$

15. If $y = e^{ax}$. cos bx, then prove that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)y = 0$

16. If $x^x + x^y + y^x = a^b$, then find $\frac{dy}{dx}$.

17. If x = a sin 2t (1 + cos 2t) and y = b cos 2t (1 – cos 2t) then find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

18. Evaluate: $\int \frac{(x+3)e^x}{(x+5)^3} dx$

19. Three schools X, Y, and Z organized a fete (mela) for collecting funds for flood victims in which they sold hand-helds fans, mats and toys made from recycled material, the sale price of each being Rs. 25, Rs. 100 andRs. 50 respectively. The following table shows the number of articles of each type sold:

School/Article	SchoolX	SchoolY	SchoolZ
Hand - held fans	30	40	35
Mats	12	15	20
Toys	70	55	75

Using matrices, find the funds collected by each school by selling the above articles and the total funds collected. Also write any one value generated by the above situation.

SECTION-C

Question numbers 20 to 26 carry 6 marks each.

20. Let A = Q × Q, where Q is the set of all rational numbers, and * be a binary opearation on A defined by (a, b) * (c, d) = (ac, b + ad) for (a. b), (c, d) Î A. Then find

(i) The identify element of * in A.

(ii) Invertible elements of A, and hence write the inverse of elements (5, 3) and $\left(\frac{1}{2}, 4\right)$.

OR

Let f : W →W be defined as $f(n) = \begin{cases} n-1, & \text{if n is odd} \\ n+1, & \text{if n is even} \end{cases}$ Show that f is invertible a nd find the inverse of f. Here, W is the set of all whole numbers.

21. Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and y = |x-1| and find its area using intergration .

22. Find the particular solution of the differential equation $x^2dy = (2xy + y^2)\,dx$, given that y= 1 when x = 1.

OR

Find the particular solution of the differential equation $(1 + x^2)\frac{dy}{dx} = \left(e^{m\tan^{-1}x} - y\right)$, given that y =1 when x = 0.

23. Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \sin^2 x - \cos x$, x Î (0, π)

24. Show that lines:

$\vec{r} = \hat{i} + \hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$

$\vec{r} = 4\hat{j} + 2\hat{k} + \mu(2\hat{i} - \hat{j} + 3\hat{k})$

are coplanar. Also, find the equation of the plane containing these lines.

25. Minimum and maximum z = 5x + 2y subject to the following constraints:

$x - 2y \le 2$

$3x + 2y \le 12$

$-3x + 2y \le 3$

$x \ge 0, y \ge 0$

26. Two the numbers are selected at random (without replacement) from first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of X. Find the mean and variance of this distribution.