CBSE_2015_SET-1

SECTION – A

1. Given that $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$

We need to find $|\vec{a}\times\vec{b}|$

$$|\vec{a}\times\vec{b}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$$

$$= \hat{i}(-2-15) - \hat{j}(-4-9) + \hat{k}(10-3)$$

$$= -17\hat{i} + 13\hat{j} + 7\hat{k}$$

$$Hence, |\vec{a}\times\vec{b}| = \sqrt{17^2 + 13^2 + 7^2}$$

$$\Rightarrow |\vec{a}\times\vec{b}| = \sqrt{507}$$

2. Let $\hat{i} - \hat{j}$, $\hat{j} - \hat{k}$

$$\vec{a}.\vec{b} = (\hat{i} - \hat{j})(\hat{j} - \hat{k}) = 1\times 0 + (-1)\times 1 + 0\times(-1) = -1$$

$$|\vec{a}| = \sqrt{1^2 + (-1)^2 + 0^2} = \sqrt{2}$$

$$|\vec{b}| = \sqrt{0^2 + 1^2 + (-1)^2} = \sqrt{2}$$

We know that $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$$Thus, \cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{-1}{\sqrt{2}\times\sqrt{2}} = \frac{-1}{2}$$

$$\Rightarrow \cos\theta = \cos 120^\circ$$

$$\Rightarrow \theta = 120^\circ$$

3. Consider the vector equation of the plane.

$$\vec{r}.(6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$$

$$\Rightarrow \left(x\hat{i} + y\hat{j} + z\hat{k}\right).(6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$$

$\Rightarrow$ 6x - 3y + 2z = 4

$\Rightarrow$ 6x - 3y + 2z – 4=0

Thus the Cartesian equation of the plane is 6x - 3y + 2z – 4=0

Let d be the distance between the point (2, 5, -3) to the plane.

$$Thus, d = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

$$\Rightarrow d = \left|\frac{6\times 2 - 3\times 5 + 2\times(-3) - 4}{\sqrt{6^2 + (-3)^2 + 2^2}}\right|$$

$$\Rightarrow d = \left|\frac{12 - 15 - 6 - 4}{\sqrt{36 + 9 + 4}}\right|$$

$$\Rightarrow d = \left|\frac{-13}{\sqrt{49}}\right|$$

$$\Rightarrow d = \frac{13}{7} units$$

4. Given that of $a_{ij} = e^{2ix}\sin(jx)$
Substitute i = 1 and j = 2
Thus, $a_{12}=e^{2\times 1\times x}\sin(2\times x)=e^{2x}\sin(2x)$

5. Consider the equation, y = mx, where m is the parameter.
Thus, the above equation represents the family of lines which pass through the origin.
y=mx ....(1)

$$\Rightarrow \frac{y}{x} = m...............(2)$$

Differentiating the above equation (1) with respect to x,
y=mx

$$\frac{dy}{dx} = m\times 1$$

$$\Rightarrow \frac{dy}{dx} = m$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} \qquad [\because \text{ from equation (2)}]$$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = 0$$

Thus we have eliminated the constant, m.
The required differential equation is

$$\frac{dy}{dx} - \frac{y}{x} = 0$$

6. Consider the given differential equation:

$$x\log x\frac{dy}{dx} + y = 2\log x$$

Dividing the above equation by xlogx, we have,

$$\frac{x\log x}{x\log x}\frac{dy}{dx}+\frac{y}{x\log x}=\frac{2\log x}{x\log x}$$

$$\Rightarrow \frac{dy}{dx}+\frac{y}{x\log x}=\frac{2}{x}\text{.............................}(1)$$

Consider the general linear differential equation.

$\frac{dy}{dx}+Py=Q$ .where P and Q are functions of x

Comparing equation (1) and the general equation, we have,

$$P(x)=\frac{1}{x\log x}\text{ and } Q(x)=\frac{2}{x}$$

The integrating factor is given by the formula $e^{\int Pdx}$

$$Thus,\ I.F.=e^{\int Pdx}=e^{\int\frac{dx}{x\log x}}$$

$$Consider\ \ I=\int\frac{dx}{x\log x}$$

$$Substituting\ \ \log x=t;\frac{dx}{x}=dt$$

$$Thus\ \ I=\int\frac{dt}{t}=\log(t)=\log(\log x)$$

$$Hence,\ I.F.=e^{\int\frac{dx}{x\log x}}=e^{\log(\log x)}=\log x$$

SECTION-B

7.

$$A=\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}$$

$$A^2=\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+2\times2+2\times2 & 1\times2+2\times1+2\times2 & 1\times2+2\times2+2\times1\\2\times1+1\times2+2\times2 & 2\times2+1\times1+2\times2 & 2\times2+1\times2+2\times1\\2\times1+2\times2+1\times2 & 2\times2+2\times1+1\times2 & 2\times2+2\times2+1\times1\end{bmatrix}$$

$$=\begin{bmatrix}1+4+4 & 2+2+4 & 2+4+2\\2+2+4 & 4+1+4 & 4+2+2\\2+4+2 & 4+2+2 & 4+4+1\end{bmatrix}$$

$$=\begin{bmatrix}9&8&8\\8&9&8\\8&8&9\end{bmatrix}$$

Consider $A^2 - 4A - 5I$

$$=\begin{bmatrix}9&8&8\\8&9&8\\8&8&9\end{bmatrix}-4\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}-5\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}9&8&8\\8&9&8\\8&8&9\end{bmatrix}-\begin{bmatrix}4&8&8\\8&4&8\\8&8&4\end{bmatrix}-\begin{bmatrix}5&0&0\\0&5&0\\0&0&5\end{bmatrix}$$

$$=\begin{bmatrix}9-9 & 8-8 & 8-8\\8-8 & 9-9 & 8-8\\8-8 & 8-8 & 9-9\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

Now

$A^2-4A-5I=0$

$A^2-4A=5I$

$A^2A^{-1}-4A\,A^{-1}=5I\,A^{-1}$ (Postmultiply by A^{-1})

$A-4I=5\,A^{-1}$

$$\begin{bmatrix}1 & 2 & 2\\2 & 1 & 2\\2 & 2 & 1\end{bmatrix} - \begin{bmatrix}4 & 0 & 0\\0 & 4 & 0\\0 & 0 & 4\end{bmatrix} = 5A^{-1}$$

$$\begin{bmatrix}-3 & 2 & 2\\2 & -3 & 2\\2 & 2 & -3\end{bmatrix} = 5A^{-1}$$

$$A^{-1} = \begin{bmatrix}\frac{-3}{5} & \frac{2}{5} & \frac{2}{5}\\\frac{2}{5} & \frac{-3}{5} & \frac{2}{5}\\\frac{2}{5} & \frac{2}{5} & \frac{-3}{5}\end{bmatrix}$$

OR

$$|A| = \begin{bmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 21 & 3\end{bmatrix}$$

=2(3-0)-0(15-0)-1(5-0)

=6-0-5

=1

≠0

Hence A^{-1} exists.

$A^{-1}A=I$

$$A^{-1}\begin{bmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$\text{Applying} R_1 \to \left(\frac{1}{2}\right) R_1$$

$$A^{-1}\begin{bmatrix}1 & 0 & \frac{-1}{2}\\ 5 & 1 & 0\\ 0 & 1 & 3\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\text{Applying} R_2 \rightarrow R_2 + (-5)R_1$

$$A^{-1}\begin{bmatrix}1 & 0 & \frac{-1}{2}\\ 0 & 1 & \frac{5}{2}\\ 0 & 1 & 3\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ -\frac{5}{2} & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\text{Applying} R_3 \rightarrow R_3 + (-1)R_2$

$$A^{-1}\begin{bmatrix}1 & 0 & \frac{-1}{2}\\ 0 & 1 & \frac{5}{2}\\ 0 & 1 & \frac{1}{2}\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ -\frac{5}{2} & 1 & 0\\ \frac{5}{2} & -1 & 1\end{bmatrix}$$

$\text{Applying} R_3 \rightarrow (2)R_3$

$$A^{-1}\begin{bmatrix}1 & 0 & \frac{-1}{2}\\ 0 & 1 & \frac{5}{2}\\ 0 & 1 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ -\frac{5}{2} & 1 & 0\\ 5 & -2 & 2\end{bmatrix}$$

$\text{Applying} R_1 \rightarrow R_1 + \left(\frac{1}{2}\right)R_3$

$R_2 \rightarrow R_2 + \left(-\frac{5}{2}\right)R_3$

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}$$

8.

$$Let\ \Delta = \begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$\Delta = \begin{vmatrix} x+2 & 4 & -3 \\ x+6 & -7 & -4 \\ x-1 & 3 & 7 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Delta = \begin{vmatrix} x+2 & 4 & -3 \\ 4 & -11 & -1 \\ -3 & -1 & 10 \end{vmatrix}$$

Applying $R_2 \to R_2 + R_3$

$$\Delta = \begin{vmatrix} x+2 & 4 & -3 \\ 1 & -12 & 9 \\ 0 & -37 & 37 \end{vmatrix}$$

Expanding along C_1

$$\Delta = (x+2)\begin{vmatrix} -12 & 9 \\ -37 & 37 \end{vmatrix} - 1\begin{vmatrix} 4 & -3 \\ -37 & 37 \end{vmatrix}$$

$$\Delta = (x+2)(-444+333) - 1(148-111)$$

$$\Delta = (x+2)(-111) - 1(37)$$

$$\therefore \Delta = 0 = -111x - 259$$

$$\therefore x = \frac{259}{111} = -\frac{7}{3}$$

9. Let

$$I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx \ldots\ldots\ldots\ldots\ldots(i)$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx. \qquad \left[\text{Using Property}, \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^2 x}{\sin x + \cos x} dx......................(ii)$$

Adding, (i) and (ii),

$$\Rightarrow 2I = \int_0^{\pi/2} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x} dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} \frac{dx}{\sin x + \cos x}$$

$$\Rightarrow 2I = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \frac{dx}{\sin x.\frac{1}{\sqrt{2}} + \cos x \frac{1}{\sqrt{2}}}$$

$$\Rightarrow 2I = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \frac{dx}{\sin x.\cos\frac{\pi}{4} + \cos x.\sin\frac{\pi}{4}}$$

$$\Rightarrow 2I = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \frac{dx}{\sin\left(\frac{\pi}{4} + x\right)}$$

$$\Rightarrow 2I = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \cos ec\left(\frac{\pi}{4} + x\right) dx$$

$$\Rightarrow 2I = \frac{1}{\sqrt{2}} \left[\ln\left|\cos ec\left(\frac{\pi}{4} + x\right) - \cot\left(\frac{\pi}{4} + x\right)\right|\right]_0^{\pi/2}$$

$$\Rightarrow 2I = \frac{1}{\sqrt{2}} \left[\ln\left|\cos ec\left(\frac{\pi}{4} + \frac{\pi}{2}\right) - \cot\left(\frac{\pi}{4} + \frac{\pi}{2}\right)\right|\right] - \left[\ln\left|\cos ec\left(\frac{\pi}{4} + 0\right) - \cot\left(\frac{\pi}{4} + 0\right)\right|\right]$$

$$\Rightarrow 2I = \frac{1}{\sqrt{2}} \left[\ln\left|\sqrt{2} - (-1)\right| - \ln\left|\sqrt{2} - 1\right| \right]$$

$$\Rightarrow I = \frac{1}{2\sqrt{2}} \left[\ln\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right| \right]$$

OR

$$\int_{-1}^{2}(e^{3x}+7x-5)dx.$$

$$Here\ \ f(x)=e^{3x}+7x-5$$

$$a=-1, b=2, h=\frac{b-a}{n}=\frac{3}{n}$$

$$\text{By definition} \int_{-1}^{2}(e^{3x}+7x-5)dx=\lim_{n\to\infty}\sum_{r=1}^{n}h.f(a+rh)$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}h.f(-1+rh)=\lim_{n\to\infty}\sum_{r=1}^{n}h..\left(e^{3(-1+rh)}+7(-1+rh)-5\right)$$

$$=\lim_{n\to\infty}\left[h.e^{-3}.e^{3h}\left(1+e^{3h}+e^{6h}+.......+e^{3nh}\right)+7h^2(1+2+3+.........+n)-12nh\right]$$

$$=\lim_{n\to\infty}\left[\left(\frac{he^{3\times\frac{3}{n}}}{ne^3}\times\left(e^{3\times\frac{3}{n}}-1\right)\times\left(\frac{3h}{e^{3h}-1}\right)\times\frac{n}{3\times3}\right)+\frac{63}{n^2}\times\frac{n(n+1)}{2}-12\times3\right]$$

Now applying the limit we get

$$=\frac{e^9-1}{3e^3}+\frac{63}{2}-36$$

$$=\frac{e^9-1}{3e^3}-\frac{9}{2}$$

10.

$$\int\frac{x^2}{x^4+x^2-2}dx$$

$$=\int\frac{x^2}{(x^2-1)(x^2+2)}dx$$

$$=\int\frac{x^2}{(x-1)(x+1)(x^2+2)}dx$$

Using partial fraction,

$$\frac{x^2}{(x-1)(x+1)(x^2+2)}=\frac{A}{(x-1)}+\frac{B}{(x+1)}+\frac{Cx+D}{(x^2+2)}$$

$$\frac{x^2}{(x-1)(x+1)(x^2+2)}=\frac{A(x+1)(x^2+2)+B(x^2+2)(x-1)+(Cx+d)(x-1)(x+1)}{(x-1)(x+1)(x^2+2)}$$

Equating the coefficients from both the numerators we get,

A+B+C=0 ....(1)

A-B+D=1 ....(2)

2A+2B-C=0 ....(3)

2A-2B-D=0 ....(4)

Solving the above equations we get,

A = 1/6 , B= - 1/6, C=0, D=2/3

Our Integral becomes,

$$\int \frac{x^2}{(x-1)(x+1)(x^2+2)}dx = \int \frac{1}{6(x-1)} - \frac{1}{6(x+1)} + \frac{2}{3(x^2+2)}dx$$

$$= \frac{1}{6}\log(x-1) - \frac{1}{6}\log(x+1) + \frac{2}{3} \times \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$$

$$= \frac{1}{6}\left[\log(x-1) - \log(x+1) + 2\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right] + C$$

11. Let E_1 , E_1 and A be the events defined as follows:

E_1= Selecting a coin having head on both the sides

E_1 = Selecting a coin not having head on both the sides

A = Getting all heads when a coin is tossed five times

We have to find P(E_1 /A) .

There are 2 coins having heads on both the sides.

$$P(E_1) = \frac{{}^{2}C_1}{{}^{10}C_1} = \frac{2}{10}$$

There are 8 coins not having heads on both the sides.

$$P(E_2) = \frac{{}^{8}C_1}{{}^{10}C_1} = \frac{8}{10}$$

$$P(A/E_1) = (1)^5 = 1$$

$$P(A/E_2) = \left(\frac{1}{2}\right)^5$$

By Baye's Theorem, we have

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\left(\frac{2}{10}\right)(1)}{\left(\frac{2}{10}\right)(1) + \left(\frac{8}{10}\right)\left(\frac{1}{2}\right)^5}$$

$$= \frac{2}{2 + \left(\frac{8}{32}\right)}$$

$$= \left(\frac{8}{9}\right)$$

OR

Let p denotes the probability of getting heads.

Let q denotes the probability of getting tails.

p = ½

q = 1 – ½ = ½

Suppose the coin is tossed n times.

Let X denote the number of times of getting heads in n trials.

$$P(X=r)={}^{n}C_{r}p^{r}q^{n-r}={}^{n}C_{r}\left(\frac{1}{2}\right)^{r}\left(\frac{1}{2}\right)^{n-r}={}^{n}C_{r}\left(\frac{1}{2}\right)^{n},r=0,1,2,\ldots\ldots\ldots\ldots\ldots n$$

$$P(X\geq 1)>\frac{80}{100}$$

$$\Rightarrow P(X=1)+P(X=2)+\ldots\ldots\ldots\ldots\ldots\ldots+P(X=n)>\frac{80}{100}$$

$$\Rightarrow P(X=1)+P(X=2)+\ldots\ldots\ldots\ldots\ldots\ldots+P(X=n)+P(X=0)-P(X=0)>\frac{80}{100}$$

$$\Rightarrow 1-P(X=0)>\frac{80}{100}$$

$$\Rightarrow P(X=0)<\frac{1}{5}$$

$$\Rightarrow {}^{n}C_{0}\left(\frac{1}{2}\right)^{n}<\frac{1}{5}$$

$$\Rightarrow \left(\frac{1}{2}\right)^{n}<\frac{1}{5}$$

⇒ n = 3, 4, 5 …………

So the fair coin should be tossed for 3 or more times for getting the required probability.

12. Position vector of $\overrightarrow{OA}=4\hat{i}+\hat{j}+2\hat{k}$

Position vector of $\overrightarrow{OB}=5\hat{i}+x\hat{j}+6\hat{k}$

Position vector of $\overrightarrow{OC}=5\hat{i}+\hat{j}-\hat{k}$

Position vector of $\overrightarrow{OD}=7\hat{i}+4\hat{j}+0\hat{k}$

$$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$$

$$=5\hat{i}+x\hat{j}+6\hat{k}-4\hat{i}-\hat{j}-2\hat{k}=\hat{i}+(x-1)\hat{j}+4\hat{k}$$

$$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}$$

$$=5\hat{i}+\hat{j}-\hat{k}-4\hat{i}-\hat{j}-2\hat{k}=\hat{i}-3\hat{k}$$

$$\overrightarrow{AD}=\overrightarrow{OD}-\overrightarrow{OA}$$

$$=7\hat{i}+4\hat{j}+0\hat{k}-4\hat{i}-\hat{j}-2\hat{k}=3\hat{i}+3\hat{j}-2\hat{k}$$

The above three vectors are coplanar

$\Rightarrow \overrightarrow{AB} \cdot \left(\overrightarrow{AC} \times \overrightarrow{AD}\right) = 0$

$\Rightarrow \begin{vmatrix} 1 & x-1 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$

$\Rightarrow 1(0+9) - (x-1)(-2+9) + 4(3-0) = 0$

$\Rightarrow 9 - 7(x-1) + 12 = 0$

$\Rightarrow -7(x-1) = -21$

$\Rightarrow x - 1 = 3$

$\therefore x = 4$

13. Let the equation of the line be $\vec{r} = \vec{a} + \lambda\vec{b}$

Here ,

$\vec{a} = 4\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

∴ Equation of the line is $\vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

Let L be the foot of the perpendicular and P be the required point from which we have to find the length of the perpendicular

$P(\vec{\alpha}) = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{PL}$ = Position vector of L- position vector of P

$= 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$= 3\hat{i} - \hat{k} + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$................$(i)$

$\text{Now}, \overrightarrow{\text{PL}}.\vec{\text{b}} = 0$ $[\text{Since } \overrightarrow{\text{PL}} \text{ is perpendicular to } \vec{\text{b}}]$

$\left[3\hat{i} - \hat{k} + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)\right] \cdot \left(2\hat{i} + 3\hat{j} + 6\hat{k}\right) = 0$

$\Rightarrow \left[\hat{i}(3+2\lambda) + (3\lambda)\hat{j} + \hat{k}(-1+6\lambda)\right]\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right) = 0$

$\Rightarrow (3+2\lambda)2 + (3\lambda)3 + (-1+6\lambda)6 = 0$

$\Rightarrow 6 + 4\lambda + 9\lambda - 6 + 36\lambda = 0$

$\Rightarrow 49\lambda = 0$

$\therefore \lambda = 0$

$\overrightarrow{PL} = 3\hat{i} - \hat{k}[\,from(ii)]$

$|\overrightarrow{PL}| = \left|\sqrt{3^2 + (-1)^2}\right|$

$\therefore |\overrightarrow{PL}| = \sqrt{10}$

Length of the perpendicular drawn on the line from P= $\sqrt{10}$

14. $\sin^{-1}(1 - x) - 2\sin^{-1} x = \dfrac{\pi}{2}$

$\Rightarrow \sin^{-1}(1 - x) = \frac{\pi}{2} + 2\sin^{-1} x$

$\Rightarrow (1-x) = \sin\left(\frac{\pi}{2}\right) + 2\sin^{-1} x$

$\Rightarrow (1 - x) = \cos(2\sin^{-1} x)$

$\Rightarrow (1 - x) = \cos(\cos\text{-}1\ (1 - 2x^2))$

$\Rightarrow (1 - x) = (1 - 2x^2)$

$\Rightarrow 1 - x = 1 - 2x^2$

$\Rightarrow 2x^2 - x = 0$

$\therefore x = 0, x = ½$

OR

$$2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$$

L.H.S.,

$$= \cos^{-1}\left(1 - 2\times\frac{9}{25}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= \cos^{-1}\left(\frac{7}{25}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= \tan^{-1}\left(\frac{\frac{24}{7} - \frac{17}{31}}{1 + \frac{24}{7}\times\frac{17}{31}}\right)$$

$$= \tan^{-1}\left(\frac{24\times31 - 17\times7}{31\times7 + 24\times17}\right)$$

$$= \tan^{-1}\left(\frac{625}{625}\right)$$

$$= \tan^{-1} 1$$

$$= \frac{\pi}{4}$$

Hence Proved

15. $y = e^{ax} .\cos bx$

$$\frac{dy}{dx} = ae^{ax}.\cos bx - be^{ax}.\sin bx.............(i)$$

$$\frac{dy}{dx} = ay - be^{ax}.\sin bx$$

$$\frac{d^2y}{dx^2} = a\frac{dy}{dx} - b\left(ae^{ax}.\sin bx + be^{ax}.\cos bx\right)$$

$$\frac{d^2y}{dx^2} = a\frac{dy}{dx} - bae^{ax}.\sin bx - b^2e^{ax}.\cos bx$$

$$\frac{d^2y}{dx^2} = a\frac{dy}{dx} - a\left(ay - \frac{dy}{dx}\right) - b^2y \quad \text{[Substituting } be^{ax} \text{ sinbx from (i)]}$$

$$\frac{d^2y}{dx^2} = a\frac{dy}{dx} - a^2y + a\frac{dy}{dx} - b^2y$$

$$\therefore \frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)y = 0$$

Hence Proved

16. $x^x + x^y + y^x = ab$ ....(i)

Let $u = x^x$

log u = x log x

$$\frac{1}{u}.\frac{du}{dx} = x.\frac{1}{x} + \log x$$

$$\therefore \frac{du}{dx} = x^x(1 + \log x)$$

Let $v = x^y$

log v = y log x

$$\frac{1}{v}.\frac{dv}{dx} = \left(\frac{y}{x} + \log x\frac{dy}{dx}\right)$$

$$\therefore \frac{dv}{dx} = x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right)$$

Let $w = y^x$

Log w = x log y

$$\frac{1}{w}.\frac{dw}{dx} = \left(\frac{x}{y}\frac{dy}{dx} + \log y\right)$$

$$\therefore \frac{dw}{dx} = y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right)$$

(i) can be written as

u + v + w = ab

$$\frac{du}{dx}+\frac{dv}{dx}+\frac{dw}{dx}=0$$

$$\Rightarrow x^x(1+\log x)+x^y\left(\frac{y}{x}+\log x\frac{dy}{dx}\right)+y^x\left(\frac{x}{y}\frac{dy}{dx}+\log y\right)=0$$

$$\Rightarrow x^x+x^x\log x+x^y.\frac{y}{x}+x^y.\log x.\frac{dy}{dx}+y^x.\frac{x}{y}\frac{dy}{dx}+y^x\log y=0$$

$$\Rightarrow \frac{dy}{dx}\left(x^y.\log x+y^x.\frac{x}{y}\right)=x^x+x^x\log x+x^y.\frac{y}{x}+y^x\log y$$

$$\Rightarrow \frac{dy}{dx}\left(x^y.\log x+xy^{x-1}.\right)=\left(x^x+x^x\log x+yx^{y-1}+y^x\log y\right)$$

$$\therefore \frac{dy}{dx}=\frac{\left(x^x+x^x\log x+yx^{y-1}+y^x\log y\right)}{\left(x^y.\log x+xy^{x-1}.\right)}$$

17. x = a sin 2t(1 + cos 2t),

y = b cos 2t(1 – cos 2t)

$\frac{dx}{dt}$=2acos2t(1 cos2t) + asin2t(– 2sin2t)

=2acos2t+ 2acos^2 2t –2asin^2 2t

=2acos2t + 2acos4t

$\frac{dy}{dt}$ = –2bsin2t(1– cos2t) + bcos2t(2sin2t)

=–2bsin2t + 2bsin2tcos2t + 2bcos2t sin2t

=–2bsin2t + 4bsin2tcos2t

=–2bsin2t +2bsin4t

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{-2b\sin 2t+2b\sin 4t}{2a\cos 2t+2a\cos 4t}$$

$$\Rightarrow \frac{dy}{dt}=\frac{-2b\sin 2t+2b\sin 4t}{2a\cos 2t+2a\cos 4t}$$

$$\Rightarrow \left|\frac{dy}{dt}\right|_{t=\frac{\pi}{4}}=\frac{-2b\sin\frac{2\pi}{4}+2b\sin\frac{4\pi}{4}}{2a\cos\frac{2\pi}{4}+2a\cos\frac{4\pi}{4}}$$

$$\Rightarrow \left|\frac{dy}{dt}\right|_{t=\frac{\pi}{4}}=\frac{-2b\sin\frac{\pi}{2}+2b\sin\pi}{2a\cos\frac{\pi}{2}+2a\cos\pi}$$

$$\Rightarrow \left|\frac{dy}{dt}\right|_{t=\frac{\pi}{4}}=\frac{-2b}{-2a}=\frac{b}{a}$$

$$\therefore \left|\frac{dy}{dt}\right|_{t=\frac{\pi}{4}}=\frac{b}{a}$$

18.

$$\int\frac{(x+3)e^x}{(x+5)^3}dx$$

$$=\int\frac{(x+5-2)e^x}{(x+5)^3}dx$$

$$=\int\left[\frac{(x+5)}{(x+5)^3}-\frac{2}{(x+5)^3}\right]e^x dx$$

$$=\int\left[\frac{1}{(x+5)^2}-\frac{2}{(x+5)^3}\right]e^x dx$$

This is of the form

$\int e^x\,[f(x)+f\,'(x)]dx=e^x f(x)+C$

$$\Rightarrow\int\left[\frac{1}{(x+5)^2}-\frac{2}{(x+5)^3}\right]e^x dx$$

$$=\frac{e^x}{(x+5)^2}+C$$

19.

$$\begin{bmatrix}30 & 12 & 70\\40 & 15 & 55\\35 & 20 & 75\end{bmatrix}\begin{bmatrix}25\\100\\50\end{bmatrix}$$

$$=\begin{bmatrix}30\times25 & 12\times100 & 70\times50\\40\times25 & 15\times100 & 55\times50\\35\times25 & 20\times100 & 75\times50\end{bmatrix}$$

$$=\begin{bmatrix}5450\\5250\\6625\end{bmatrix}=\begin{bmatrix}X\\Y\\Z\end{bmatrix}$$

The funds collected by X = Rs. 5450, Y = Rs. 5250, Z = Rs. 6625
Total funds collected = Rs. 17325
Value generated: team work

SECTION – C

20. Let A = Q × Q, where Q is the set of rational numbers.
Given that * is the binary operation on A defined by (a, b) * (c, d) = (ac, b + ad) for (a, b), (c, d) Î A.
(i)
We need to find the identity element of the operation * in A.
Let (x, y) be the identity element in A.
Thus,
(a, b) * (x, y) = (x, y) * (a, b) = (a, b), for all (a, b) ∈ A
⇒(ax, b + ay) = (a, b)
⇒ ax = a and b + ay =b
⇒ y = 0 and x = 1
Therefore, (1, 0) ∈ A is the identity element in A with respect to the operation *.
(ii)
We need to find the invertible elements of A.
Let (p, q) be the inverse of the element (a, b)
Thus,
(a, b) * (p, q) = (1, 0)
⇒(ap, b + aq) = (1, 0)
⇒ ap = 1 and b + aq = 0

$$\Rightarrow p=\frac{1}{a} \text{ and } q=-\frac{b}{a}$$

Thus the inverse elements of (a, b) is $\left(\frac{1}{a},-\frac{b}{a}\right)$

Now let us find the inverse of (5, 3) and $\left(\frac{1}{2},4\right)$

Hence, inverse of (5, 3) is $\left(\frac{1}{5}, -\frac{3}{5}\right)$

And inverse of $\left(\frac{1}{2}, 4\right)$ is $\left(2, \frac{-4}{\frac{1}{2}}\right)$ =(2, -8)

OR

Let f: W→W be defined as

$$f(n) = \begin{cases} n-1, & \text{if n is odd} \\ n+1, & \text{if n is even} \end{cases}$$

We need to prove that 'f' is invertible.

In order to prove that 'f' is invertible it is sufficient to prove that f is a bijection.

A function f: A→B is a one-one function or an injection, if

f(x) = f(y) ⇒ x = y for all x, y ∈ A.

Case i:

If x and y are odd.

Let f(x) = f(y)

⇒x − 1 = y − 1

⇒x = y

Case ii:

If x and y are even,

Let f(x) = f(y)

⇒x + 1 = y + 1

⇒x = y

Thus, in both the cases, we have,

f(x) = f(y) ⇒ x = y for all x, y ∈ W.

Hence f is an injection.

Let n be an arbitrary element of W.

If n is an odd whole number, there exists an even whole number n − 1 ∈ W such that f(n − 1) = n − 1 + 1 = n.

If n is an even whole number, then there exists an odd whole number n + 1∈ W such that f(n + 1) = n + 1 − 1 = n.

Also, f(1) = 0 and f(0) = 1

Thus, every element of W (co-domain) has its pre-image in W (domain).

So f is an onto function.

Thus, it is proved that f is an invertible function.

Thus, a function g: B→A which associates each element y ∈ B to a unique element x ∈ A such that f(x) = y is called the inverse of f.

That is, f(x) = y ⇔ g(y) = x

The inverse of f is generally denoted by f^{-1}.

Now let us find the inverse of f.

Let x, y ∈ W such that f(x) = y

$\Rightarrow$x + 1 = y, if x is even
And
x − 1 = y, if x is odd

$$\Rightarrow\ x = \begin{cases} y-1, & \text{if y is odd} \\ y+1, & \text{if y is even} \end{cases}$$

$$\Rightarrow\ f^{-1}(y) = \begin{cases} y-1, & \text{if y is odd} \\ y+1, & \text{if y is even} \end{cases}$$

Interchange, x and y, we have,

$$\Rightarrow\ f^{-1}(x) = \begin{cases} x-1, & \text{if x is odd} \\ x+1, & \text{if x is even} \end{cases}$$

Rewriting the above we have,

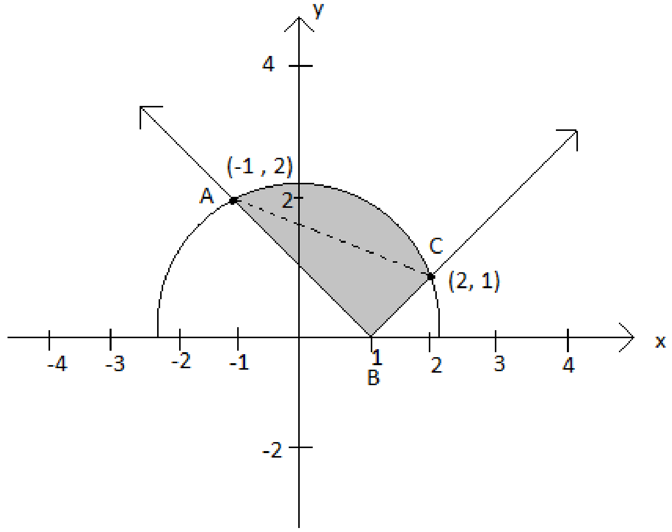
$$\Rightarrow\ f^{-1}(x) = \begin{cases} x+1, & \text{if x is even} \\ x-1, & \text{if x is odd} \end{cases}$$

Thus, $f^{-1}(x)=f(x)$

21. Consider the given equation $y = \sqrt{5-x^2}$
This equation represents a semicircle with centre at the origin and radius = √5 units
Given that the region is bounded by the above semicircle and the line y = |x-1|
Let us find the point of intersection of the given curve meets the line y= |x-1|

$$\Rightarrow \sqrt{5-x^2} = |x-1|$$

Squaring both the sides, we have,
$5-x^2=|x-1|^2$
$\Rightarrow 5-x^2=x^2+1-2x$
$\Rightarrow 2x^2-2x-5+1=0$
$\Rightarrow 2x^2-2x-4=0$
$\Rightarrow 2x^2-x-2=0$
$\Rightarrow 2x^2-2x+x-2=0$
$\Rightarrow x(x-2)+1(x-2)=0$
$\Rightarrow (x+1)(x-2)=0$
$\Rightarrow x=-1, x=2$
When x=-1, y=2
When x=2, y=1
Consider the following figure.
Thus the intersection points are (-1,2) and (2,1)
Consider the following sketch of the bounded region.



$$\text{Required Area, A} = \int_{-1}^{2} (y_2 - y_1)dx$$

$$= \int_{-1}^{1}\left[\sqrt{5-x^2} + (x-1)\right]dx + \int_{1}^{2}\left[\sqrt{5-x^2} - (x-1)\right]dx$$

$$\int_{-1}^{1}\sqrt{5-x^2}\,dx + \int_{-1}^{1} xdx + \int_{1}^{2}\sqrt{5-x^2}\,dx - \int_{1}^{2} xdx + \int_{1}^{2} dx$$

$$\left[\frac{x}{2}\sqrt{5-x^2} + \sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\right]_{-1}^{1} + \left(\frac{x^2}{2}\right)_{-1}^{1} - (x)_{-1}^{1}$$

$$+\left[\frac{x}{2}\sqrt{5-x^2} + \sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\right]_{1}^{2} - \left(\frac{x^2}{2}\right)_{1}^{2} + (x)_{1}^{2}$$

$$= \frac{5}{2}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + \frac{5}{2}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right) - \frac{1}{2}$$

$$\text{Required Area} = \left[\frac{5}{2}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + \frac{5}{2}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right) - \frac{1}{2}\right] sq.units$$

22. $x^2dy = (2xy + y^2)dx$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy + y^2}{x^2} \text{.......................}(i)$$

Let y= vx,

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting in (i), we get

$$v + x\frac{dv}{dx} = \frac{2vx^2 + v^2x^2}{x^2}$$

$$\Rightarrow v + x\frac{dv}{dx} = 2v + v^2$$

$$\Rightarrow x\frac{dv}{dx} = v + v^2$$

$$\Rightarrow \frac{dv}{v^2 + v} = \frac{dx}{x}$$

Integrating both sides.

$$\int \frac{dv}{v^2 + v} = \int \frac{dx}{x}$$

$$\Rightarrow \int \frac{\text{v} + 1 - \text{v}}{\text{v}(\text{v} + 1)}.dv = \int \frac{dx}{x}$$

$$\Rightarrow \log v - \log|v + 1| = \log x + \log C$$

$$\Rightarrow \log\left|\frac{v}{v + 1}\right| = \log|Cx|$$

$$\Rightarrow \log\left|\frac{y/x}{y/x + 1}\right| = \log|Cx|$$

$$\Rightarrow \frac{y}{y + x} = Cx \quad \text{[Removing logarithm in both sides]}$$

$\therefore \text{y} = \text{Cxy} + \text{Cx}^2$, which is the general solution.

Putting $\text{y} = 1$ and $\text{x} = 1$,

$$1 = \text{C} + \text{C}$$

$$\Rightarrow 2\text{C} = 1$$

$$\Rightarrow \text{C} = \frac{1}{2}$$

$$y = \frac{xy}{2} + \frac{x^2}{2}$$

$\therefore$ $2y=xy+x^2$, which is the particular solution.

OR

$$(1+x^2)\frac{dy}{dx} = e^{m\tan^{-1}x} - y$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{m\tan^{-1}x}}{(1+x^2)} - \frac{y}{(1+x^2)}$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{(1+x^2)} = \frac{e^{m\tan^{-1}x}}{(1+x^2)}$$

$P = \frac{1}{(1+x^2)}, Q = \frac{e^{m\tan^{-1}x}}{(1+x^2)}$

$I.F. = e^{\int Pdx}$

$= e^{\int \frac{1}{(1+x^2)}dx}$

$= e^{\tan^{-1}x}$

Thus the solution is

$ye^{\int Pdx} = \int Qe^{\int Pdx}dx$

$\Rightarrow y \times e^{\tan^{-1}x} = \int \frac{e^{m\tan^{-1}x}}{(1+x^2)}.e^{\tan^{-1}x}.dx$

$\Rightarrow y \times e^{\tan^{-1}x} = \int \frac{e^{(m+1)\tan^{-1}x}}{(1+x^2)}dx..........................(i)$

$\int \frac{e(m+1)\tan^{-1}x}{(1+x^2)}dx.........(ii)$

Let $(m+1)\tan^{-1}x = z$

$\frac{(m+1)}{(1+x^2)}dx = dz$

$\frac{dx}{(1+x^2)} = \frac{dz}{(m+1)}$

Substituting in (ii),

$\frac{1}{(m+1)}\int e^z dz$

$= \frac{e^z}{(m+1)}$

$= \frac{e^{(m+1)\tan^{-1}x}}{(m+1)}$

Substituting in (i),

$\Rightarrow y \times e^{\tan^{-1}x} = \frac{e^{(m+1)\tan^{-1}x}}{(m+1)} + C...................(iii)$

Puting y = 1 and x = 1, in the above equation,

$$\Rightarrow y \times e^{\tan^{-1} x} = \frac{e^{(m+1)\tan^{-1} x}}{(m+1)} + C$$

$$\Rightarrow 1 \times e^{\frac{\pi}{4}} = \frac{e^{(m+1)\frac{\pi}{4}}}{(m+1)} + C$$

$$\therefore C = \frac{e^{(m+1)\frac{\pi}{4}}}{(m+1)} - e^{\frac{\pi}{4}}$$

$$\text{Particular solution of the D.E. is } y \times e^{\tan^{-1} x} = \frac{e^{(m+1)\tan^{-1} x}}{(m+1)} + \frac{e^{(m+1)\frac{\pi}{4}}}{(m+1)} - e^{\frac{\pi}{4}}$$

23. f(x) = $\sin^2 x - \cos x$,
f'(x) = 2sin x.cos x + sin x
= sin x(2cos x + 1)
Equating f'(x) to zero.
f'(x) = 0
sin x(2cos x + 1) = 0
sin x = 0
∴ x=0,π
2cos x + 1 = 0
⇒cos x = - 1/2
∴ x= 5π/6
f(0) = $\sin^2 0 - \cos 0 = -1$

$$f\left(\frac{5\pi}{6}\right) = \sin^2\left(\frac{5\pi}{6}\right) - \cos\left(\frac{5\pi}{6}\right)$$

$$= \sin^2\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{6}\right)$$

$$= \frac{1}{4} - \frac{\sqrt{3}}{2}$$

$$= \left(\frac{1-2\sqrt{3}}{4}\right)$$

f(π) = $\sin^2 \pi - \cos \pi = 1$
Of these values, the maximum value is 1, and the minimum value is −1.
Thus, the absolute maximum and absolute minimum values of f(x) are 1 and −1, which it attains at x = 0 and x = π.

24.

$$\vec{r} = \hat{i} + \hat{j} + \hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Convert ing into cartesian form,

$$\frac{x-1}{1}=\frac{y-1}{-1}=\frac{z-1}{1}$$

$$(x_1, y_1, z_1)=(1,1,1)$$

$$\vec{r}=4\hat{j}+2\hat{k}+\mu(2\hat{i}-\hat{j}+3\hat{k})\text{...........}(ii)$$

$$\frac{x-0}{2}=\frac{y-4}{-1}=\frac{z-2}{3}$$

$$(x_2, y_2, z_2)=(0,4,2)$$

$$a_2=2,\ b_2=-1,\ c_2=3$$

Condition for the lines to be coplanar is

$$\begin{vmatrix} 0-1 & 4-1 & 2-1 \\ 1 & -1 & 1 \\ 2 & -1 & 3 \end{vmatrix}=\begin{vmatrix} -1 & 3 & 1 \\ 1 & -1 & 1 \\ 2 & -1 & 3 \end{vmatrix}=0$$

∴ the lines are coplanar.

Intersection of the two lines is Let the equation be a(x – x_1) b(y- y_1) c(z- z_1) = 0.....(iii)

Direction ratio of the plane is

a – b + c = 0

2a – b + 3c = 0

Solving by cross-multiplication,

$$\frac{a}{-3+1}=\frac{b}{2-3}=\frac{c}{-1+2}$$

a = -2λ , b = - λ, c = λ

Since the plane passes through (0,4,2) from line (ii)

a (x – 0) + b (y – 4) + c (z – 2) = 0

⟹ -2 λ – λ(y – 4) + λ(z – 2) = 0

⟹ - 2x – y + 4 + z – 2 = 0

⟹ -2x – y + z = -2

⟹ 2x + y – z = 2

25. x – 2y ≤ 2

3x + 2y ≤ 12

-3x + 2y ≤ 3

x = ≥ 0, y ≥ 0

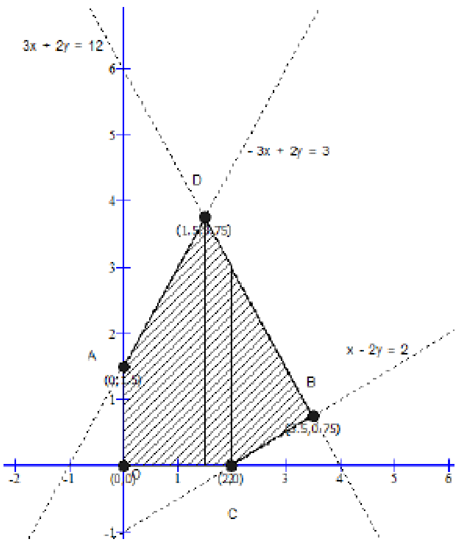
Converting the inequations into equations, we obtain the lines

X – 2y = 2 . . . . (i)

3x + 2y = 12 . . . . (ii)

-3x + 2y = 3 . . . .(iii)

X = 0, y = 0



From the graph, we get the corner points as
A(0, 5), B(3.5, 0.75), C(2, 0), D(1.5, 3.75), 0(0.0)
The values of the objective function are:

Point (x, y)	Values of the objective function Z = 5x + 2y
A(0, 5)	5 × 0 + 2 × 5 = 10
B(3.5, 0.75)	5 × 3.5 + 2 × 0.75 = 19 (Maximum)
C(2, 0)	5 × 2 + 2 × 0= 10
D(1.5, 3.75)	5 × 1.5 + 2 × 3.75 = 15
O(0, 0)	5 × 0 + 2 × 0 = 0 (Minimum)

The maximum value of Z is 19 and its minimum value is 0.

26. First six positive integers are {1, 2, 3, 4, 5, 6}
No, of ways of selecting 2 numbers from 6 numbers without replacement = ${}^6C_2 = 15$
X denotes the larger of the two numbers, so X can take the values 2, 3, 4, 5, 6.
Probability distribution of X:

x	2	3	4	5	6
P(x)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

Computation of Mean and Variance:

Xi	P(x = xi)	piXi	$piXi^2$
2	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{2}{15}$
3	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{18}{15}$
4	$\frac{3}{15}$	$\frac{12}{15}$	$\frac{48}{15}$

5	$\frac{4}{15}$	$\frac{20}{15}$	$\frac{100}{15}$
6	$\frac{5}{15}$	$\frac{30}{15}$	$\frac{180}{15}$
		$\sum piXi = \frac{70}{15} = \frac{14}{3}$	$\sum piXi^2 = \frac{350}{15} = \frac{70}{3}$

Mean $\sum piXi = \frac{70}{15} = 4.67$

Variance = $\sum piXi^2 - \left(\sum piXi\right)^2 = \frac{70}{3} - \frac{196}{9} = \frac{210-196}{9} = \frac{14}{9}$