

SAMPLE QUESTION PAPER

CLASS-XII (2016-17)

MATHEMATICS (041)

Time allowed: 3 hours

Maximum Marks: 100

General Instructions:

- (i) All questions are compulsory.
- (ii) This question paper contains 29 questions.
- (iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
- (iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
- (v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
- (vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

SECTION-A

Questions from 1 to 4 are of 1 mark each.

- 1. What is the principal value of $(\tan^{-1} \frac{1}{\sqrt{3}})$?
- 2. A and B are square matrices of order 3 each, $|A| = 2$ and $|B| = 3$. Find $|AB|$
- 3. What is the distance of the point (p, q, r) from the x-axis?
- 4. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 3x^2 - 5$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $g(x) = \frac{1}{x}$. Find $g \circ f$

SECTION-B

Questions from 5 to 12 are of 2 marks each.

- 5. How many equivalence relations on the set $\{1,2,3\}$ containing (1,2) and (2,1) are there in all? Justify your answer.
- 6. Let l_i, m_i, n_i ; $i = 1, 2, 3$ be the direction cosines of three mutually perpendicular vectors in space. Show that $AA' = I_3$, where $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$.
- 7. If $e^y (x+1) = 1$, show that $\frac{dy}{dx} + \frac{1}{x+1} = 0$
- 8. Find the sum of the order and the degree of the following differential equations:

$$\frac{dy}{dx} + \frac{1}{x+1} = 0$$

9. Find the Cartesian and Vector equations of the line which passes through the point $(-2, 4, -5)$ and parallel to the line given by — — —
10. Solve the following Linear Programming Problem graphically:
 Maximize $Z = 3x + 4y$
 subject to
 $x + y$
11. A couple has 2 children. Find the probability that both are boys, if it is known that (i) one of them is a boy (ii) the older child is a boy.
12. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which its area increases, when side is 10 cm long.

SECTION-C

Questions from 13 to 23 are of 4 marks each.

13. If $A + B + C = \pi$, then find the value of

$$\begin{vmatrix} \sin(A) & \cos(B) & \cos(C) \\ \cos(A) & \sin(B) & \sin(C) \\ \sin(A) & \sin(B) & \sin(C) \end{vmatrix}$$

OR

Using properties of determinant, prove that

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc -$$

14. It is given that for the function $f(x) = x^3 - 6x^2 + ax + b$ Rolle's theorem holds in $[1, 3]$ with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of 'a' and 'b'
15. Determine for what values of x, the function $f(x) = x^3 + \frac{1}{x}$ ($x \neq 0$) is strictly increasing or strictly decreasing

OR

Find the point on the curve $y = x^2$ at which the tangent is $y = x - 11$

16. Evaluate $\int_0^2 (x^2 + 3) dx$ as limit of sums.
17. Find the area of the region bounded by the y-axis, $y = \cos x$ and $y = \sin x$, $0 \leq x \leq \frac{\pi}{2}$
18. Can $y = ax + b$ be a solution of the following differential equation?
 $y = x^2 + \frac{1}{x} + \dots (*)$
 If no, find the solution of the D.E. (*).

OR

Check whether the following differential equation is homogeneous or not

$$x^2 y' - xy = 1 + \cos\left(\frac{y}{x}\right), x \neq 0$$

Find the general solution of the differential equation using substitution $y=vx$.

19. If the vectors $\vec{p} = a\hat{i} + \hat{j} + \hat{k}$, $\vec{q} = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + c\hat{k}$ are coplanar, then for a, b, c $\neq 1$ show that

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

20. A plane meets the coordinate axes in A, B and C such that the centroid of ΔABC is the point (α, β, γ) . Show that the equation of the plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$
21. If a 20 year old girl drives her car at 25 km/h, she has to spend Rs 4/km on petrol. If she drives her car at 40 km/h, the petrol cost increases to Rs 5/km. She has Rs 200 to spend on petrol and wishes to find the maximum distance she can travel within one hour. Express the above problem as a Linear Programming Problem. Write any one value reflected in the problem.
22. The random variable X has a probability distribution P(X) of the following form, where k is some number:
- $$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$
- (i) Find the value of k (ii) Find $P(X < 2)$ (iii) Find $P(X \leq 2)$ (iv) Find $P(X \geq 2)$
23. A bag contains $(2n + 1)$ coins. It is known that 'n' of these coins have a head on both its sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, find the value of 'n'.

SECTION-D

Questions from 24 to 29 are of 6 marks each

24. Using properties of integral, evaluate $\int_0^{\pi} \frac{x}{1 + \sin x} dx$

OR

Find: $\int \frac{\sin x}{\sin^3 x + \cos^3 x} dx$

25. Does the following trigonometric equation have any solutions? If Yes, obtain the solution(s):

$$\tan^{-1} \left(\frac{x+1}{x-1} \right) + \tan^{-1} \left(\frac{x-1}{x} \right) = -\tan^{-1} 7$$

OR

Determine whether the operation * define below on $\mathbb{Q}$ is binary operation or not.

$$a * b = ab + 1$$

If yes, check the commutative and the associative properties. Also check the existence of identity element and the inverse of all elements in $\mathbb{Q}$.

26. Find the value of x, y and z, if $A = \begin{bmatrix} x & \\ & z \end{bmatrix}$ satisfies $A' = A^{-1}$

OR

Verify: $A(\text{adj } A) = (\text{adj } A)A = |A| I$ for matrix $A = \begin{bmatrix} 3 & \\ & 2 \end{bmatrix}$

27. Find — if $y = \left\{ 2 \tan^{-1} \sqrt{\quad} \right\}$

28. Find the shortest distance between the line $x - y + 1 = 0$ and the curve $y^2 = x$

29. Define skew lines. Using only vector approach, find the shortest distance between the following two skew lines:

$$\begin{aligned} &= (8 + 3\lambda) \hat{i} - (9 + 16\lambda) \hat{j} + (10 + 7\lambda) \hat{k} \\ &= 15 \hat{i} + 29 \hat{j} + 5 \hat{k} + \mu (3 \hat{i} + 8 \hat{j} - 5 \hat{k}) \end{aligned}$$

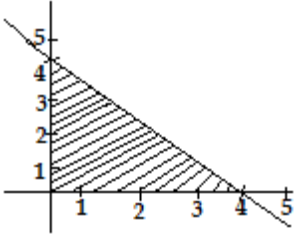
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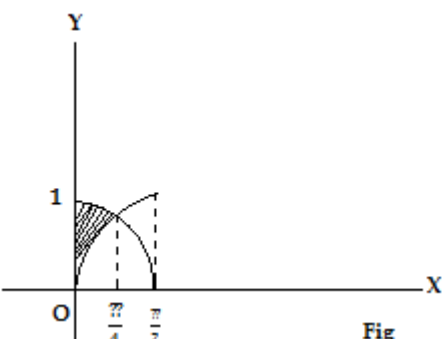
MATHEMATICS (041)

Marking Scheme

1.	$\tan \left(\tan^{-1} \frac{1}{2} \right) = \frac{1}{2}$	1
2.	$ 3AB = 3^3 A B = 27 \times 162$	1
3.	Distance of the point (p, q, r) from the x-axis = Distance of the point (p, q, r) from the point (p,0,0) $= \sqrt{q^2 + r^2}$	1
4.	$g \circ f(x) = g\{f(x)\} = g(3x^2 - 5) = \frac{1}{(3x^2 - 5)^2}$	1
5.	Equivalence relations could be the following: $\{(1,1), (2,2), (3,3), (1,2), (2,1)\}$ and (1) $\{(1,1), (2,2), (3,3), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2)\}$ (1) So, only two equivalence relations.(Ans.)	2
6.	$AA' = \begin{bmatrix} l & m & n \end{bmatrix} \begin{bmatrix} l \\ m \\ n \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} = I_3 \longrightarrow (1)$ because $a_{ii} = 1$, for each $i = 1, 2, 3 \longrightarrow 1/2$ $a_{ij} = 0$ ($i \neq j$) for each $i, j = 1, 2, 3 \longrightarrow 1/2$	2
7.	On differentiating $e^y (x + 1) = 1$ w.r.t. x, we get $e^y + (x + 1) e^y \frac{dy}{dx} = 0 \longrightarrow (1)$ $\Rightarrow e^y + \frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -e^{-y} \longrightarrow (1)$	2
8.	Here, $\left\{ \frac{d}{dx} \left(\frac{y}{2} + (1+x) \right) \right\}^3 = - \frac{1}{2} \longrightarrow (1)$ Thus, order is 2 and degree is 3. So, the sum is 5 $\longrightarrow (1)$	2
9.	Here, $\vec{r} = \vec{a} + \lambda \vec{b}$ Cartesian equation of the line is $\frac{x-1}{-2} = \frac{y+4}{5} = \frac{z-6}{3} \longrightarrow (1)$ Vector equation of the line is $= (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda (3\hat{i} + 5\hat{j} + 6\hat{k}) \longrightarrow (1)$	2

10.	The feasible region is a triangle with vertices $O(0,0)$, $A(4,0)$ and $B(0,4)$ $Z_0 = 3 \times 0 + 4 \times 0 = 0$ $Z_A = 3 \times 4 + 4 \times 0 = 12$ $Z_B = 3 \times 0 + 4 \times 4 = 16$ } (1) Thus, maximum of Z is at $B(0,4)$ and the maximum value is 16 $\longrightarrow \frac{1}{2}$ 	2
11.	Sample space = $\{ B_1B_2, B_1G_2, G_1B_2, G_1G_2 \}$, B_1 and G_1 are the older boy and girl respectively. Let E_1 = both the children are boys; E_2 = one of the children is a boy ; E_3 = the older child is a boy Then, (i) $P(E_1/E_2) = P(\text{---}) = \text{---} = \text{---} \longrightarrow (1)$ (ii) $P(E_1/E_3) = P(\text{---}) = \text{---} = \text{---} \longrightarrow (1)$	2
12.	Here, $\text{Area}(A) = \frac{\sqrt{3}}{4} x^2$, where 'x' is the side of the equilateral triangle $\longrightarrow \frac{1}{2}$ So. $\frac{\sqrt{3}}{4} \times \text{---} \longrightarrow (1)$ $\frac{\sqrt{3}}{4} (10)^2 = 10\sqrt{3} \text{ cm}^2/\text{sec} \longrightarrow \frac{1}{2}$	2
13.	As $A + B + C = \pi$ $\begin{vmatrix} \sin(A+C) & \cos(A+C) \\ \sin(A+B) & \cos(A+B) \end{vmatrix}$ $A = \begin{vmatrix} \sin(A+C) & \cos(A+C) \\ \sin(A+B) & \cos(A+B) \end{vmatrix}$ $\longrightarrow (2)$ $= 0 \times \begin{vmatrix} \sin(A+C) & \cos(A+C) \\ \sin(A+B) & \cos(A+B) \end{vmatrix}$ $- \sin B \times \begin{vmatrix} \sin(A+C) & \cos(A+C) \\ \sin(A+B) & \cos(A+B) \end{vmatrix}$ $+ \cos C \times \begin{vmatrix} \sin(A+C) & \cos(A+C) \\ \sin(A+B) & \cos(A+B) \end{vmatrix}$ $= 0 - \sin B \tan A \cos C + \cos C \sin B \tan A = 0 \text{ (Ans.)} \longrightarrow (2)$ OR	4

	Let $\Delta = \begin{vmatrix} c & b \end{vmatrix}$ Applying $C_1 \rightarrow C_1 + C_3$, we get $\Delta = (a + b + c) \begin{vmatrix} 1 & b \end{vmatrix} \longrightarrow (1)$ Applying $R_2 \rightarrow R_2 - R_1$, and $R_3 \rightarrow R_3 - R_1$, we get $\Delta = (a + b + c) \begin{vmatrix} 0 & a \end{vmatrix} \longrightarrow (1)$ Expanding Δ along first column, we have the result $\longrightarrow (2)$	4
14.	Since Rolle's theorem holds true, $f(1) = f(3)$ i.e., $(1)^3 - 6(1)^2 + a(1) + b = (3)^3 - 6(3)^2 + a(3) + b$ i.e., $a + b + 22 = 3a + b$ $\Rightarrow a = 11 \longrightarrow (2)$ Also, $f'(x) = 3x^2 - 12x + a$ or $3x^2 - 12x + 11$ As $f'(c) = 0$, we have $3\left(2 + \frac{1}{\sqrt{3}}\right)^2 - 12\left(2 + \frac{1}{\sqrt{3}}\right) + 11 = 0$ As it is independent of b, b is arbitrary. $\longrightarrow (2)$	4
15.	Here, $f'(x) = 3x^2 - 3x^{-4} = \frac{3(x-1)}{4} \longrightarrow (1)$ $= \frac{3(x+x+1)}{4}(x+1)(x-1)$ Critical points are -1 and $1 \longrightarrow (1)$ $\Rightarrow f'(x) > 0$ if $x > 1$ or $x < -1$; and $f'(x) < 0$ if $-1 < x < 1$ $\therefore \frac{3(x-1)}{4} \in \{e\}$ Hence, $f(x)$ is strictly increasing for $x > 1$ $\longrightarrow (1)$ or $x < -1$; and strictly decreasing for $(-1, 0) \cup (0, 1) [1] \longrightarrow (1)$ OR Here, $\frac{3(x-1)}{4} = -11 \longrightarrow \frac{1}{2}$ So, slope of the tangent is -11	4

	Slope of the given tangent line is 1. Thus, $3x^2 - 11 = 1 \longrightarrow (1)$ that gives $x = \pm 2$ When $x = 2, y = 2 -$ When $x = -2, y = -2$ Out of the two points $(2, -9)$ and $(-2, -13) \longrightarrow (2)$ only the point $(2, -9)$ lies on the curve Thus, the required point is $(2, -9) \longrightarrow \frac{1}{2}$	
16.	Here, $f(x) =$ $a = 0, b = 2$ and $nh = b - a = 2 \longrightarrow (1)$ $\int_0^2 (x^2 - 1) dx = h[f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)] \longrightarrow (1)$ $= h[1 + (n-1) \cdot 3]$ $= h[1 + (n-1) \cdot 3]$ $= \left[\frac{(n-1)n(2-1)}{3} \right]$ $= \left[\frac{(n-1)nh(2-h)}{3} \right] \longrightarrow (1)$ $= \left[\frac{(2-h)2(4-h)}{3} \right]$ $= 6 + \dots, \text{i.e., } \dots \longrightarrow (1)$	4
17.	The rough sketch of the bounded region is shown on the right. $\longrightarrow (1)$ Required area $= \int \dots dx - \int \dots dx \longrightarrow (1)$ $= (\sin x - \cos x) \longrightarrow (1)$ $= \sin x - \cos x$ $= \sqrt{2} - 1, \text{ i.e., } (\sqrt{2} - 1) \longrightarrow (1)$ 	4
18.	$y = ax + \dots (1)$ $\dots = a \longrightarrow (1-)$ Substituting this value of 'a' in (1), we get	4

	$y = x - + \frac{\cos(-)}{x^2} \longrightarrow (1-)$ Thus, $y = ax + \frac{\cos(-)}{x^2}$ is a solution of the following differential equation $y = x - + \frac{\cos(-)}{x^2} \longrightarrow 1$ OR Given differential equation can be written as $\frac{dy}{dx} = - + \left[\frac{\cos(-)}{x^2} \right] \dots\dots(1)$ Let $F(x,y) = - + \left[\frac{\cos(-)}{x^2} \right]$. Then $F(\lambda x, \lambda y) = - + \left[\frac{\cos(-)}{(\lambda x)^2} \right]$ $= - + \left[\frac{\cos(-)}{x^2} \right] \neq F(x,y)$ Hence, the given D.E. is not a homogeneous equation. $\longrightarrow (1)$ Putting $y = vx$ and $\frac{dy}{dx} = v + x \frac{dv}{dx}$ in (1), we get $\frac{dv}{dx} = - + \frac{\cos(-)}{x^2} - v$ $\Rightarrow \frac{dv}{dx} = - + \frac{\cos(-)}{x^2} - v$ $\Rightarrow \sec^2(-) dv = -\frac{1}{x} dx \longrightarrow (1)$ Integrating both sides, we get $2 \tan - = -\frac{1}{x} + C \longrightarrow 1 -$ $\text{or } 2 \tan - = -\frac{1}{x} + C \longrightarrow -$	4
19.	Since the vector $\vec{p}, \vec{q}$ and $\vec{r}$ are coplanar $\therefore [\vec{p}, \vec{q}, \vec{r}] = 0$ $[\vec{p}, \vec{q}, \vec{r}] \longrightarrow (1)$ i.e., $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$ $\begin{matrix} \longrightarrow \\ \longrightarrow \end{matrix} \longrightarrow (1)$ or $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$	4

	$\Rightarrow a(b-1)(c-1) - 1(1-a)(c-1) - 1(1-a)(b-1) = 0$ i.e., $a(1-b)(1-c) + (1-a)(1-c) + (1-a)(1-b) = 0 \longrightarrow (1)$ Dividing both the sides by $(1-a)(1-b)(1-c)$, we get $\frac{a}{1-a} + \frac{1}{1-a} + \frac{1}{1-b} = 0$ i.e., $-\left(\frac{1}{1-a}\right) = -\frac{1}{1-b} - \frac{1}{1-a}$ i.e., $\frac{1}{1-a} = \frac{1}{1-b} + \frac{1}{1-a} \longrightarrow (1)$									
20.	We know that the equation of the plane having intercepts a, b and c on the three coordinate axes is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \longrightarrow (1)$ Here, the coordinates of A, B and C are $(a,0,0), (0,b,0)$ and $(0,0,c)$ respectively. The centroid of ΔABC is $(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}) \longrightarrow (1)$ Equating $(\frac{a}{3}, \frac{b}{3}, \frac{c}{3})$ to (α, β, γ) , we get $a = 3\alpha, b = 3\beta$ and $c = 3\gamma \longrightarrow (1)$ Thus, the equation of the plane is $\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3\gamma} = 1$ or $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3 \longrightarrow (1)$	4								
21.	Let the distance covered with speed of 25 km/h = x km and the distance covered with speed of 40 km/h = y km $(\frac{1}{2})$ Total distance covered = z km The L.P.P. of the above problem, therefore, is $z = x + y \longrightarrow (1)$ Maximize $z = x + y$ subject to constraints $\begin{cases} 4x + 5y \leq 20 \\ x \geq 0 \\ y \geq 0 \end{cases} \longrightarrow (1)$ $x \geq 0$ $\longrightarrow (1)$ Any one value $\longrightarrow (\frac{1}{2})$	4								
22.	Here, <table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$P(X)$</td><td>k</td><td>$2k$</td><td>$3k$</td></tr></table> (i) Since $P(0) + P(1) + P(2) = 1$, we have	X	0	1	2	$P(X)$	k	$2k$	$3k$	4
X	0	1	2							
$P(X)$	k	$2k$	$3k$							

	$k + 2k + 3k = 1$ $\text{i.e., } 6k = 1, \text{ or } k = \frac{1}{6} \longrightarrow (1)$ $\text{(ii) } P(X < 2) = P(0) + P(1) = k + 2k = 3k = \frac{1}{2}; \longrightarrow (1)$ $\text{(iii) } P(X \leq 2) = P(0) + P(1) + P(2) = k + 2k + 3k = 6k = 1 \longrightarrow (1)$ $\text{(iv) } P(X \geq 2) = P(2) = 3k = \frac{1}{2} \longrightarrow (1)$	
23.	Let the events be described as follows: E_1 : a coin having head on both sides is selected. E_2 : a fair coin is selected. A : head comes up in tossing a selected coin $P(E_1) = \frac{1}{2^{n+1}}; P(E_2) = \frac{1}{2^n}; P(A/E_1) = \frac{1}{2}; P(A/E_2) = \frac{1}{2} \longrightarrow (2)$ It is given that $P(A) = \frac{1}{2}$ $P(E_1) P(A/E_1) + P(E_2) P(A/E_2) = \frac{1}{2}$ $\Rightarrow \frac{1}{2^{n+1}} \times \frac{1}{2} + \frac{1}{2^n} \times \frac{1}{2} = \frac{1}{2} \longrightarrow (1)$ $\Rightarrow \frac{1}{2^{n+1}} [n + 1] = \frac{1}{2}$ $\Rightarrow 42(3n + 1) = 62(1)$ $\Rightarrow \frac{42(3n + 1)}{62} = 1 \longrightarrow (1)$	4
24.	$I = \int_0^{\pi} \frac{dx}{n(\pi - x)} \quad (1)$ $= \pi \int_0^{\pi} \frac{dx}{n(\pi - x)} - \int_0^{\pi} \frac{dx}{n(\pi - x)}$ $\Rightarrow 2I = \int_0^{\pi} \frac{dx}{n(\pi - x)} \quad (1)$ $= \int_0^{\pi} \frac{dx}{\cos(\pi - x)}$ $\Rightarrow - \int_0^{\pi} \frac{dx}{(-\cos x)}$ $= - \int_0^{\pi} \frac{dx}{\cos x} \quad (1)$ $\Rightarrow I = - \left[-2 \tan \left[\left(\frac{\pi}{2} - x \right) \right] \right] \quad (2)$ $\Rightarrow I = - \left[2(-2) \right] = 4 \quad (1)$ OR	6

	Let $I = \int \frac{1}{\tan x} dx = \int \frac{1}{\tan x} dx$ (½) On substituting $\tan x = t$ and dt, we get (1) $I = \int \frac{1}{t} dt = \int \frac{1}{t} dt$ (½) $= \int \frac{1}{t} dt + \int \frac{1}{t} dt$ $= \log t + \frac{1}{\sqrt{3}} \int \frac{1}{t^2 + (\frac{\sqrt{3}}{3})} dt$ (1) $= \log t + \frac{1}{\sqrt{3}} \int \frac{1}{t^2 + (\frac{\sqrt{3}}{3})} dt + \int \frac{1}{t} dt$ $= \log t + \frac{1}{\sqrt{3}} \int \frac{1}{(t^2 + (\frac{\sqrt{3}}{3})^2 + (\frac{\sqrt{3}}{3})^2)} dt$ $= \log t + \frac{1}{\sqrt{3}} \int \frac{1}{(t^2 + (\frac{\sqrt{3}}{3})^2 + (\frac{\sqrt{3}}{3})^2)} dt$ (2) $= \log \tan x + \frac{1}{\sqrt{3}} \int \frac{1}{(t^2 + (\frac{\sqrt{3}}{3})^2 + (\frac{\sqrt{3}}{3})^2)} dt + \log \tan x + \frac{1}{\sqrt{3}} \int \frac{1}{(t^2 + (\frac{\sqrt{3}}{3})^2 + (\frac{\sqrt{3}}{3})^2)} dt$ (1)	6
25.	$\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right) =$ $\Rightarrow \left[\frac{\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right) - \left(\frac{1}{x}\right)}\right] =$, if $\left(\frac{1}{x}\right) \left(\frac{1}{x}\right) < 1$(*) (2) $\Rightarrow \left[\frac{x(x-1) + (x-1)^2}{(x-1)x - (x-1)(x-1)}\right] =$ $\Rightarrow \frac{(x-x) + (x-x)}{(x-x) - (x-1)} = [-7]$ $\Rightarrow \frac{0}{-1} =$ (1) $\Rightarrow 2x^2 - 8x + 8 =$ $\Rightarrow (x-2)^2 = 0$ $\Rightarrow x = 2$ (1) Let us now verify whether $x = 2$ satisfies the condition (*) For $x = 2$, $\left(\frac{1}{x}\right) \left(\frac{1}{x}\right) = 3 \times - = -$ which is not less than 1 Hence, this value does not satisfy the condition (*) (1) i.e., there is no solution to the given trigonometric equation. (1) OR Given * on $\mathbb{Q}$, defined by $a*b = ab+1$ Let, $a \in \mathbb{Q}, b \in \mathbb{Q}$ then $ab \in \mathbb{Q}$	6

	and $(ab+1) \in \mathbb{Q}$ $\Rightarrow a*b = ab+1$ is defined on $\mathbb{Q}$ $\therefore *$ is a binary operation on $\mathbb{Q}$ (1) Commutative: $a*b = ab+1$ $b*a = ba+1$ $= ab+1 \quad (\because ba = ab \text{ in } \mathbb{Q})$ $\Rightarrow a*b = b*a$ So $*$ is commutative on $\mathbb{Q}$ (1) Associative: $(a*b)*c = (ab+1)*c = (ab+1)c+1$ $= abc+c+1$ $a*(b*c) = a*(bc+1)$ $= a(bc+1)+1$ $= abc+a+1$ $\therefore (a*b)*c \neq a*(b*c)$ So $*$ is not associative on $\mathbb{Q}$ (1) Identity Element : Let $e \in \mathbb{Q}$ be the identity element, then for every $a \in \mathbb{Q}$ $a*e = a$ and $e*a = a$ $ae+1 = a$ and $ea+1 = a$ $\Rightarrow e = \frac{a-1}{a}$ and $e = \frac{a-1}{a}$ (1) e is not unique as it depend on 'a', hence identity element does not exist for $*$ (1) Inverse: since there is no identity element hence, there is no inverse. (1)	6
26.	The relation $A' = A^{-1}$ gives $A'A = A^{-1}A = I$ (1) Thus, $\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \left(1 \frac{1}{2}\right)$ $\Rightarrow \begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-zx+zx & 2yz-yz-yz & z^2+z^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$ $\Rightarrow 2x^2 = 1; 6y^2 = 1 \text{ and } 3z^2 = 1$ $\Rightarrow x = \pm \frac{1}{\sqrt{2}}; y = \pm \frac{1}{\sqrt{6}}; z = \pm \frac{1}{\sqrt{3}} \quad \left(1 \frac{1}{2}\right)$ OR	6

	Here, $A = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix} = 1(0+0) + 1(9+2) + 2(0-0) = 11$ (1) $\Rightarrow A I = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$(1) (1/2) $\text{adj } A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$ (2) Now, $A(\text{adj } A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ (1) and $(\text{adj } A)A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ (1) Thus, it is verified that $A(\text{adj } A) = (\text{adj } A)A = A I$ (1/2)	6
27.	Putting $x = \cos 2\theta$ in $\left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\}$, we get (1) $2 \tan^{-1} \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$ i.e., $2 \tan^{-1} \sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}} = 2 \tan^{-1}(\tan \theta) = 2\theta = \cos^{-1} x$ (2) Hence, $y = e^{\sin^2 x} \cos^{-1} x$ $\Rightarrow \log y = \sin^2 x + \log (\cos^{-1} x)$ $\Rightarrow \frac{1}{y} \times \frac{dy}{dx} = 2 \sin x \cos x + \frac{1}{\cos^{-1} x} \times \frac{-1}{\sqrt{1-x^2}} = \sin 2x - \frac{1}{\cos^{-1} x \sqrt{1-x^2}}$ (2) $\Rightarrow \frac{dy}{dx} = e^{\sin^2 x} \cos^{-1} x \left[\sin 2x - \frac{1}{\cos^{-1} x \sqrt{1-x^2}} \right]$ (1)	6
28.	Let (t^2, t) be any point on the curve $y^2 = x$. Its distance (S) from the line $x - y + 1 = 0$ is given by $\frac{1}{\sqrt{2}}$ $S = \left \frac{t^2 - t - 1}{\sqrt{1+1}} \right \quad \frac{1}{\sqrt{2}}$ $= \frac{t^2 - t + 1}{\sqrt{2}} \quad \left\{ \because t^2 - t + 1 = \left(t - \frac{1}{2}\right)^2 + \frac{3}{4} > 0 \right\} \quad (1)$ $\Rightarrow \frac{dS}{dt} = \frac{1}{\sqrt{2}} (2t-1) \quad (1)$ and $\frac{d^2S}{dt^2} = \sqrt{2} > 0$ (1) Now, $\frac{dS}{dt} = 0 \Rightarrow \frac{1}{\sqrt{2}} (2t-1) = 0$,i.e., $t = \frac{1}{2}$ (1) Thus, S is minimum at $t = \frac{1}{2}$	6

So, the required shortest distance is $\frac{(-)^2 - (-) +}{\sqrt{2}} = \frac{1}{4\sqrt{2}}$, or $\frac{\sqrt{1}}{4\sqrt{2}}$ (1)

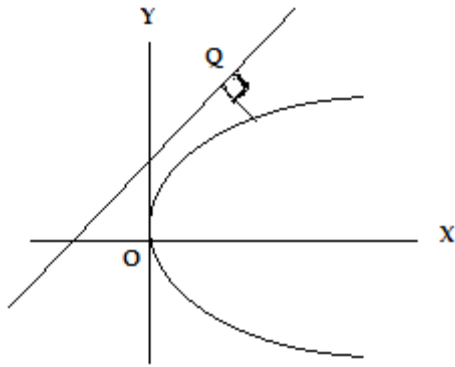


Fig. 1

29. 1) the line which are neither intersecting nor parallel. (1)

2) The given equations are

$$= 8\hat{i} - 9\hat{j} + 10\hat{k} + \mu(3\hat{i} - 16\hat{j} + 7\hat{k}) \dots\dots\dots(1) \quad (\frac{1}{2})$$

$$= 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k}) \dots\dots\dots(2)$$

$$\text{Here, } \vec{a_1} = 8\hat{i} - 9\hat{j} + 10\hat{k}; \quad \vec{a_2} = 15\hat{i} + 29\hat{j} + 5\hat{k}$$

$$\vec{b_1} = 3\hat{i} - 16\hat{j} + 7\hat{k} \quad \vec{b_2} = 3\hat{i} + 8\hat{j} - 5\hat{k}$$

$$\text{Now, } \vec{a_2} - \vec{a_1} = (15 - 8)\hat{i} + (29 + 9)\hat{j} + (5 - 10)\hat{k} \quad \hat{i} \quad \hat{j} \quad \hat{k} \quad (\frac{1}{2})$$

and

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = \hat{i} \quad \hat{j} \quad \hat{k} \quad (1)$$

$$\Rightarrow (\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = (\hat{i} \quad \hat{j} \quad \hat{k}) \cdot (7\hat{i} + 38\hat{j} - 5\hat{k}) = 1176 \quad (1)$$

$$\text{Shortest distance} = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right| \quad (1)$$

$$= \left| \frac{1176}{\sqrt{24^2 + 36^2 + 72^2}} \right| = \frac{1176}{\sqrt{6048}} = \frac{1176}{72\sqrt{11}} = \frac{14\sqrt{11}}{9} \quad (1)$$