

CLASS XII-COMMON PRE-BOARD EXAMINATION

Subject: Mathematics

Time Allotted: 3 Hours.

Max.Marks:100

General Instructions:

- All the questions are compulsory
- The question paper consists of 29 questions divided into three Sections A,B,C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of 4 marks each and , Section C comprises of 7 questions of 6 marks each.
- All questions in Section A are to be answered in one word, one sentence or as per the requirement of the question.
- There is no overall choice. However, internal choice has been provided in 4 questions of 4 marks each and 2 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

SECTION-A

1. If $\begin{bmatrix} 2 & 3 & 1 & -3 \\ 5 & 7 & -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, write the value of x.

2. Find the value of x + y if

$$\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$$

3. What is the degree of the differential equation

$$\frac{dy}{dx}^3 - \frac{d^2y}{dx^2}^3 - 6y = x^2$$

4. Find the principal value of $\sin^{-1} \left(-\frac{\sqrt{3}}{2}\right)$.

5. What is the value of $\cot^{-1}(\tan^{-1} a + \cot^{-1} a)$

6. For any 3 vectors \vec{a}, \vec{b} & \vec{c} find

$$\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b})$$

7. If \vec{a} is a unit vector and $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 24$ then write the value of $|\vec{x}|$.

8. Write the Cartesian equation of the plane bisecting the line segment joining the points A (2, 3, 5) and B (4, 5, 7) at right angles.

9. If A_{ij} is the cofactor of the element a_{ij} of the matrix $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$ then write

the value of $a_{32}A_{32}$.

10. The confidence developed in a player by playing x trial matches of cricket is given by a function $C(x) = 3x^2 + 36x + 5$ then find the marginal confidence gained after playing 5 matches.

Section B

11. Show that the relation R defined by $(a, b)R(c, d) \Rightarrow a + d = b + c$ on the set $N \times N$ is an equivalence relation.

12. Prove the following

$$\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$$

OR

$$\text{Solve: } \tan^{-1} \frac{x-1}{x-2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$$

13. If $x^{13} y^7 = (x + y)^{20}$ prove that $\frac{dy}{dx} = \frac{y}{x}$.

OR

Differentiate $\tan^{-1} \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}$ with respect to $\cos^{-1} x^2$.

14. let $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, & x < 0 \\ k, & x = 0 \\ \frac{2x}{\sqrt{x+4} - 2}, & x > 0 \end{cases}$

Then find k if f(x) is continuous at x=0.

15. Find $\int e^x \left(\frac{\sin 4x - 4}{1 - \cos 4x} \right) dx$

16. If $y\sqrt{x^2 + 1} = \log(\sqrt{x^2 + 1} - x)$ show that

$$x^2 + 1) \frac{dy}{dx} + xy + 1 = 0$$

17. Evaluate $\int_{-1}^{\frac{3}{2}} |x \sin \pi x| dx$

18. Evaluate $\int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}} dx$

19. The magnitude of the vector product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector

along the sum of the vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to $\sqrt{2}$.

Find the value of k.

OR

If $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$ show that the angle between

\vec{a} and \vec{b} is 60° .

20. Find the shortest distance between the following lines

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

OR

Find the equation of the plane through the points (2, 1, -1) and (-1, 3, 4) and perpendicular to the plane $x-2y+4z=10$.

21. In a group of 50 scouts in a camp 30 are well trained in first aid techniques while remaining are well trained in hospitality but not in first aid. Two scouts are selected at random from the group. Find the probability distribution of number of selected scouts who are well trained in first aid. Find the mean of the distribution also. Write value expected from a trained scout.

22. Using properties of determinants, prove the following

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y)$$

Section C

23. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2m and volume $8m^3$. If building of tank costs Rs. 70 per sq. meter for the base and Rs. 45 per sq. meter for sides, what is the cost of least expensive tank?

OR

Find the area of the greatest isosceles triangle that can be inscribed in an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ with its vertex at one end of major axis.}$$

24. Two trusts A & B receive Rs. 70000 and 55000 respectively from central government to award prize to persons of a district in 3 fields agriculture, education and social services. Trust A awarded 10, 5 and 15 persons in the field of agriculture, education and social services respectively while Trust B awarded 15, 10 and 5 persons in the field of agriculture, education and social services respectively. If all three prizes together amount to Rs. 6000, then find amount of each prize by matrix method.

What field you prefer most for award for development of society.? Give answer with justifications.

25. Find the area of region bounded $\{(x, y): y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$.

OR

Evaluate $\int_0^4 (|x - 1| + |x - 2| + |x - 3|) dx$.

26. In a manufacturing company makes two types of teaching aides A & B of mathematics for class XII. Each type of A requires nine labour hours for fabricating and one labour hour for finishing. Each type of B requires twelve labour hours for fabricating and three labour hours for finishing. For fabricating and finishing maximum labour hours available are 180 & 30 respectively. The company makes a profit of Rs. 80 on each piece of type A and Rs. 120 on each piece of type B. How many pieces of Type A and B should be manufactured per week to get maximum profit? What is the maximum profit per week?

Is teaching aid necessary for teaching learning process? If yes, justify your answers.

27. In a test an examinee either knows the answer or guesses or copies the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{6}$ and the probability that he copies the answer is $\frac{1}{9}$. The probability that his answer is correct, given that he has copied is $\frac{1}{8}$. Find the probability that he knew the answer to the question, given that he correctly answered it.

Does the result of this question indicate that most of the students believe in the value of passing an examination with honesty and self-knowledge?

28. Find the equation of the plane passing through the line of intersection of planes $2x + y - 6z = 3$ and $5x - 3y + 4z + 9 = 0$ and parallel to the line

$$\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{4}$$

29. Find the particular solution of the differential equation satisfying the given conditions $x^2 dy + y(x + y) dx = 0$, $y = 1$ when $x = 1$.