EXAMINATION PAPER : 2011

Time : 3 hours **Maximum Marks : 70**

General Instructions :

(i) All questions are compulsory.

(ii) There are 30 questions in total. Questions 1 to 8 carry one mark each, questions 9 to 18 carry two marks each, questions 19 to 27 carry three marks each and question 28 to 30 carry five marks each.

(iii) There is no overall choice. However, an internal choice has been provided in the one question of two marks; one question of three marks and all three questions of five marks each. You have to attempt only one of the choices in such questions.

(iv) Use of calculators is not permitted.

(v) Please write down the serial number of question before attempting it.

(vi) You may use the following values of physical constants wherever necessary.

Boltzmann's constant $K = 1.38 \times 10^{-23}$ JK^{-1}

Avogadro's number $N_A = 6.022 \times 10^{23}$/mol

Radius of Earth $R_e = 6400$ km.

$= 1.013 \times 10^5$ Pa

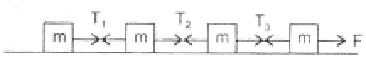
1 Atmospheric Pressure $g = 9.8$ m/s^2

$R = 8.3$ J mol^{-1} K^{-1}

1. Write the dimensional formula of torque. 1

2. Draw velocity-time graph for an object, starting from rest. Acceleration is constant and remains positive. 1

3. Arrange increasing order the tension T_1, T_2 and T_3 in the figure.



4. Why there is lack of atmosphere on the surface of moon? 1

5. The triple point of carbon dioxide is 216.55 K. Express this temperature on Fahrenheit scale. 1

6. In an open organ pipe, third harmonic is 450 Hz. What is the frequency of fifth harmonic?

7. Which type of substances are called elastomers? Give one example 1

8. A simple harmonic motion is described by $a = -16\,x$ where a is acceleration, $x \rightarrow$ displacement in m. What is the time period? 1

9. Percentage error in the measurement of height and radius of cylinder are x and y respectively. Find percentage error in the measurement of volume. Which of the two measurements height or radius need more attention? 2

OR

The length and breadth of a rectangle are measured as $(a \pm \Delta a)$and$(b \pm \Delta b)$ respectively. Find (i) relative error. (ii) absolute error in the measurement of area. 2

10. An object moving on a straight line covers first half of the distance at speed v and second half of the distance at speed 2v. Find (i) average speed, (ii) mean speed.

11. An object moving on a circular path in horizontal plane. Radius of the paths is *r* and constant speed is v. Deduce expression for centripetal acceleration. 2

12. Find the height from the surface of earth at which weight of a body of mass *m* will be reduced by 36% of its weight on the surface. (R_e = 6400 km) 2

13. Define gravitational potential. Give its S.I. unit.

14. An engine has been designed to work between source and sink at temperature 177°C and 27°C respectively. If energy input is 3600 J. What is the work done by the engine? 2

15. Explain :

    (i) Why does the air pressure in a car tyre during driving increase?

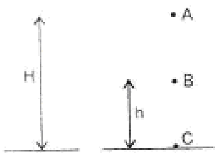
    (ii) Why coolant used in a chemical plant should have high specific heat? 2

16. Calculate the work done in blowing a soap bubble from a radius of 2 cm to 3 cm. The surface tension of the soap solution is 30 dynes cm^{-1}. 2

17. Show that Newton's second law of motion is the real law of motion. 2

18. A block initially at rest breaks into two parts of masses in the ratio 2 : 3. The velocity of smaller part is $\left(8\hat{i} + 6\hat{j}\right)$ m/s. Find the velocity of bigger part. 2

19. A body of mass *m* is released in vacuum from the position A at a height H above the ground. Prove that sum of kinetic and potential energies at A, B and C remains constant. 3



20. Give two points of difference between elastic and inelastic collisions. Two balls A and B with A in motion initially and B at rest. Find their velocities after collision (perfectly elastic). Each ball is of mass "m". 3

21. A liquid is in streamlined flow through a tube of non-uniform cross-section. Prove that sum of its kinetic energy, pressure energy and potential energy per unit volume remains constant. 3

22. Give reason :

   (i) fog particles appear suspended in atmosphere.

   (ii) two boats being moved parallel to each other attract.

   (iii) bridges are declared unsafe after long use. 3

23. State Kepler's law of Planetary motion. Name the physical quantities which remain constant during the planetary motion. 3

24. What is the law of equipartition of energy? Determine the value of γ for diatomic gas N_2 at moderate temperature. 3

25. Show that for small oscillations the motion of a simple pendulum is simple harmonic. Drive an expression for its time period. Does it depend on the mass of the bob? 3

OR

A SHM is described by $y = r \sin \omega t$. What is :

(i) the value of displacement y at which speed of the body executing SHM is half of the maximum speed?

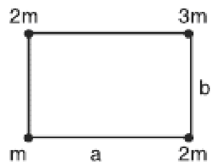
(ii) the time at which kinetic and potential energies are equally shared? 3

26. A solid sphere of mass *m* and radius *r* is impure rolling on a horizontal surface. What fraction of total energy of rotation?

(a) kinetic energy of rotation?

(b) kinetic energy of translation?

27. Four bodies have been arranged at the corners of a rectangle shown in figure. Find the centre of mass of the system. 3



28. A body is projected with velocity m at angle θ_0 upward from horizontal. Prove that the trajectory is parabolic. Deduce expression for 5

(i) horizontal range, (ii) maximum height attained.

OR

A body is projected horizontally from the top of a building of height *h*. Velocity of projection is *u*. Find : 5

(i) the time it will take to reach the ground.

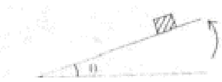
(ii) horizontal distance from foot of building where it will strike the ground.

(iii) velocity with which the body reach the ground.

29. Drive an expression for maximum speed a vehicle should have, to take a turn on a banked road. Hence deduce expression for angle of banking at which there is minimum wear and tear to the tyres of the vehicle. 5

OR

Define angle of friction. The inclination θ of a rough plane is increased gradually. The body on the plane just comes into motion when inclination θ becomes 30°. Find coefficient of friction the inclination is further increased to 45°. Find acceleration of the body along the plane (g = 10 m/s²) 5



30. A progressive wave is given by y(x,t) = 8 cos(300t – 0.15x). Where x in metre y in cm and t in second. What is the

(i) direction of propagation

(ii) wavelength

(iii) frequency

(iv) wave speed

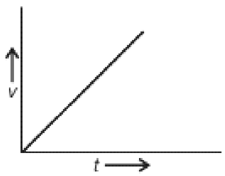
(v) phase difference between two points 0.2 in apart?

OR

Give any three differences between progressive waves and stationary waves. A stationary wave is y = 12 sin300t cos 2x. What is the distance between two nearest nodes. 5

ANSWERS

1. $[ML^2T^{-2}]$

2. 

3. $T_1 < T_2 < T_3$

4. Since, the value of acceleration due to gravity 'g' is less on moon, escape velocity on surface of the moon is small and so the molecules of gases escape from the surface of the moon.

5. $K = °C + 273$

$\Rightarrow 216.55 - 273 = °C$

$\Rightarrow °C = -56.55$

$\because \frac{9}{5}C + 32 = °F$

$\Rightarrow °F = -69.8$

OR

Use $\frac{°F - 32}{180} = \frac{°C}{100} = \frac{216.55 - 273.15}{100}$

$\frac{°F - 32}{180} = -0.566$

$°F - 32 = -0.566 \times 180$

$F = -101.88 + 32 = 69.8$

6. $\because \quad v_3 = 3v_1$

$v_3 = 450 \text{ Hz}$

$\therefore \quad 450 = 3v_1$

$\Rightarrow \quad v_1 = 150 \text{ Hz}$

Fifth harmonic, $v_5 = 5v_1$

$= 5 \times 150$

$v_5 = 750 \text{ Hz}$

7. Those materials for which stress-strain variation is not a straight line within elastic limit *e.g.* Rubber.

8. For S.H.M., $a = -\omega^2 x$

Comparing with $a = -16\ x$

$\because \quad \omega = \frac{2\pi}{T} = \sqrt{16} = 4$

$\therefore \qquad T = \frac{\pi}{2}$ second

9. Height of cylinder = x

Radius of cylinder = y

Volume of cylinder $V = \pi y^2 x$

Percentage error in measurement of volume

$$\frac{\Delta V}{V} \times 100 = \pm\left(2\frac{\Delta y}{y} + \frac{\Delta x}{x}\right) \times 100$$

Hence, radius needs more attention because any error in its measurement is multiplied two times.

OR

(i) Relative error in area

$$\frac{\Delta A}{A} = \left[\frac{\Delta a}{a} + \frac{\Delta b}{b}\right]$$

as $\qquad A = ab$

(ii) Absolute error in area

$$\Delta A = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)A = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)ab$$

$$\Delta A = [(\Delta a)b + (\Delta b)a]$$

10. Let total distance be x. Distance of first half $= \frac{x}{2}$

Speed = v

Time taken $t_1 = \frac{\frac{x}{2}}{v} = \frac{x}{2v}$

Distance of second half $= \frac{x}{2}$

Speed = 2v

$$\text{Time taken } t_2 = \frac{\frac{x}{2}}{2v} = \frac{x}{4v}$$

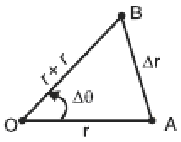
(i) $\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$$= \frac{x}{\frac{x}{2v} + \frac{x}{4v}} = \frac{4v}{3}$$

(ii) $\text{Mean speed} = \frac{v + 2v}{2} = \frac{3v}{2}$

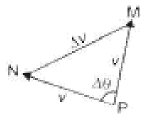
11. Consider a body in a circular path of radius *r*, with a speed V. The velocity direction is tangential at any point in the path.

The position vectors at A and B are represented by two sides of an isosceles triangle first.



(i) The change in position vector is indicated by AB = AB. The velocity at A and B are along the tangenis at these points and the change in velocity will complete an isosceles triangle of velocities.

(ii) $\overline{MN} = \overline{\Delta v}$ Since the triangles are similar,



$$\frac{\Delta V}{\Delta r} = \frac{V}{r} \Rightarrow \Delta V = \frac{V}{r}, \Delta r$$

$$\underset{\Delta t \to 0}{Lt} \frac{\Delta V}{\Delta r} = \underset{\Delta t \to 0}{Lt} \frac{V}{r} \frac{\Delta r}{\Delta t}$$

$\therefore \quad \frac{dv}{dt} = \frac{v}{r}.v \Rightarrow a = \frac{v^2}{r}$

12. h = ?, R_e = 6400 km

$$g' = g\left(1 - \frac{2h}{R}\right) = g - \frac{2hg}{R}$$

$$\Rightarrow \qquad g - g' = \frac{2gh}{R}$$

Percentage decrease in weight $= \frac{mg - mg'}{mg} \times 100 = \frac{g - g'}{g} \times 100$

$$\frac{g - g'}{g} \times 100 = \frac{2gh}{gR} \times 100 = \frac{2h}{R} \times 100$$

$$36 = \frac{2 \times h}{6400} \times 100$$

$$\Rightarrow \qquad h = 1.152 \text{ km}$$

13. Gravitational potential at a point in gravitational field of a body is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration. Its SI unit is J-kg^{-1}

14. $Q_1 = 3600$ J

$T_1 = 177$ °C = 177 + 273 = 450 K

$T_2 = 27$ °C = 27 + 273 = 300 K

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$Q_2 = Q_1 \times \frac{T_2}{T_1} = 3600 \times \frac{300}{450} = 2400 \text{ J}$$

15. (i) Because work done against friction is converted into heat. Due to which the gas in tyre gets heated and hence pressure of gas increases as $P \propto T$ at constant volume.

(ii) Because heat absorbed by a substance is directly proportional to specific heat of substance.

16. $\sigma = 30$ dynes/cm, $r_1 = 2$ cm, $r_2 = 3$ cm

Since, bubble has two surfaces, initial surface area of bubble = $2 \times 4\pi r_1^2$

$= 2 \times 4\pi \times (2)^2$

$= 32\pi \text{ cm}^2$

Final surface area of bubble = $2 \times 4\pi r_2^2 = 2 \times 4\pi \times (3)^2 = 72\pi \text{ cm}^2$

Increase in surface area = $72\pi - 32\pi = 40\pi \text{ cm}^2$

Work done = s × Increase in surface area = $30 \times 40\pi = 3768$ ergs

17. **I law from II law :** According to II law, force experienced is the product of mass and acceleration. When there is no force, the mass does not accelerate and retains the same status. So the view of I law, *i.e.,* when there is no force the body maintains the status of motion is confirmed.

III law and II law : Consider two masses m_1 and m_2 exerting force on each other (internal forces). If their change in momentum are *dp,* and dp_2, then,

$dp_1 + dp_2 = 0$. Since no external force acts on the system of two masses.

$$\therefore \quad \frac{dp_1}{dt} = -\frac{dp_2}{dt}$$

i.e., $f_1 = -f_2$

i.e., force experienced by m_1 due to m_2 and by m_1 are equal and opposite, confirming action and reaction.

18. Let mass of the block = m

After breaking, $m_1 = \frac{2}{5}m$ and $m_2 = \frac{3}{5}m$

Initial momentum $P_i = 0$

Final momentum $P_f = m_1\vec{\nu}_1 + m_2\vec{\nu}_2$

According to law of conservation of momentum

$P_f = P_i \Rightarrow m_1\vec{\nu}_1 + m_2\vec{\nu}_2 = 0$

$\vec{\nu}_1$ = Velocity of smaller part, $\vec{\nu}_2$ = Velocity of bigger part

$$\Rightarrow \frac{2}{5}m\left(8\hat{i} + 6\hat{j}\right) + \frac{3}{5}m\left(\vec{\nu}_2\right) = 0$$

$$\Rightarrow \frac{3}{5}m\vec{\nu}_2 = -\frac{1}{5}m\left(16\hat{i} + 12\hat{j}\right)$$

$$\vec{\nu}_2 = -\left(\frac{16}{3}\hat{i} + 4\hat{j}\right)$$

19. Let a body of mass *m* be dropped from a point A at a height *H.*

P.E. at A = *mgh*

K.E. = 0

Total energy at A = *mgh*

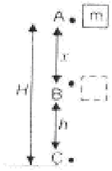
As it reaches B, it would have lost some P.E. and gained K.E.

Velocity on reaching B = $\sqrt{2gx}$

P.E. at B = mg(h − x)

$$\text{K.E.} = \frac{1}{2}m\nu_B^2 = \frac{1}{2}m2gx = mgx$$

Total energy at B = mg(h − x) + mgx = mgh



On reaching the ground C the mass must have gained a velocity $\sqrt{2}$ gh and the P E must be zero

PE at C = 0

$$\text{K.E. at C} = \frac{1}{2}mv^2 = \frac{1}{2}m(2gh) = mgh$$

Total energy at C = mgh

Thus it is proved that the energy at any point in its path is mgh.

20. **Elastic collisions :** (i) K.E. is conserved, (ii) Forces involved must be conservative.

Inelastic collisions : (i) K.E. is not conserved., (ii) Some or all forces involved may be non-conservative.

Mass of ball A and B = m

Initial velocity of ball A = $u_1 = v$

Initial velocity of ball B = $u_2 = 0$

Final velocity ball A = v_1 and of ball B = v_2

According to conservation of momentum

$mv + m_2(0) = mv_1 + mv_2$

$mv = mv_1 + mv_2$........(i)

Conservation of K.E.

$$\frac{1}{2}m\nu^2 = \frac{1}{2}m\nu_1^2 + \frac{1}{2}m\nu_2^2 \quad ....(ii)$$

$m\nu^2 = m\nu_1{}^2 + mv_2{}^2$ From (ii)

$$m\left(\nu^2 - \nu_1{}^2\right) = m\nu_2^2 \quad (iii)$$

From (i)

$m(v - v_1) = mv_2$ (iv)

Dividing respecting sides of (iii) by (iv)

$$\frac{m\left(v + v_1\right)\left(v - v_1\right)}{m\left(v - v_1\right)} = \frac{mv_2^2}{mv_2}$$

$$\Rightarrow \qquad v_2 = v + v_1$$

Substituting in (i)

$$mv = mv_1 + m(v + v_1)$$

$\Rightarrow \quad v_1 = 0$

Similarly solving,

$$v_2 = v$$

i.e., Ball A comes to rest and ball B starts moving with velocity v i.e, they exchange their velocities on collision.

21. Consider an incompressible non-viscous liquid entering the cross-section A_1 at A with a velocity v_1 and coming out at a height h_2 at B with velocity v_2.

The P.E. and K.E. increases since h_2 and v_2 are more than h_1 and v_1 respectively. This is done by the pressure doing work on the liquid. If P_1 and P_2 are the pressure at A and B, for a small displacement at A and B,

The work done on the liquid A = (P_1A_1)

$$\Delta x_1 = P_1A_1v_1\Delta t$$

The work done by the liquid at B = $-(P_2A_2)$

$$\Delta x_2 = P_2A_2v_2\Delta t$$

(Considering a small time Δt so that area may be same)

Net work done by pressure = $(P_1 - P_2)$ Av Δt since $A_1v_1 = A_2v_2$

From conservation of energy,

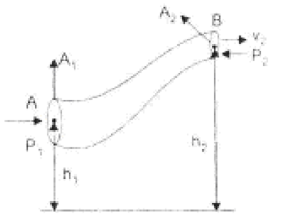
$$(P_1 - P_2)Av\ \Delta t = \text{change in (K.E. + P.E.)}$$

$$(P_1 - P_2)\,A\nu\,\Delta t = A\nu\rho\Delta tg\,(h_2 - h_1) + \frac{1}{2}A\nu\Delta t\rho\left(\nu_2^2 - \nu_1^2\right)$$

$\therefore \quad P_1 - P_2 = \rho g\,(h_2 - h_1) + \frac{\rho}{2}\left(\nu_2^2 - \nu_1^2\right)$

(i.e.) $\quad P_1 + \rho gh_1 + \frac{\rho}{2}\nu_1^2 = P_2 + \rho hg_2 + \frac{\rho}{2}\nu_2^2$

$$\frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant}$$



22. (i) Fog particles are formed due to condensation of water vapour as they rise up. Due to condensation, they become heavy and appear suspended.

    (ii) When two boats move in parallel directions close to each other the stream of water between boat is set into vigorous motion. As a result pressure exerted by water in between the boats becomes less than pressure of water beyond the boats. Due to this difference in pressure the boats attract each other.

    (iii) A bridge undergoes alternating stress and strain for a large number of times during its use. When bridge is used for long time, it loses its elastic strength. Therefore, the amount of strain in the bridge for a given stress will become large and ultimately, the bridge will collapse. So, they are declared unsafe after long use.

23. (i) All the planets move around in elliptical orbits with the sun at its focus.

    (ii) The line joining the sun and the planet sweeps out equal areas in equal intervals of time.

    (iii) The square of the time period of revolution of the planet is directly proportional to the cube of the semi-major axis of the elliptical orbit $T^2 \propto a^3$.

    Areal velocity and angular momentum remain constant.

24. According to law of equipartition of energy for any dynamical system in thermal equilibrium the total energy is distributed equally amongst all the degrees of freedom and the energy associated with **each molecule per**

degree of freedom is $\frac{1}{2}k_BT$, where k_B is Boltzmann constant and T is temperature of the system.

Diatomic gas N_2 has 5 degrees of freedom. Using law of equipartition of energy total internal energy of one mole of gas is

$$u = 5 \times \left(\frac{1}{2}k_BT\right) \times N_A = \frac{5}{2}RT$$

$$C_v = \left(\frac{du}{dt}\right)$$

$$C_v = \frac{d}{dt}\left(\frac{1}{2}RT\right) = \frac{5}{2}R$$

$$C_p = C_v + R = \frac{5}{2}R + R = \frac{7}{9}R$$

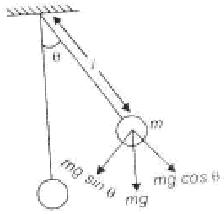
$$\gamma = \frac{C_p}{C_u} = \frac{(7/2)R}{(5/2)R} = 1.4$$

25. Restoring force is provided by the portion mg sin θ of gravitational force, Since, it acts perpendicular to length l, the restoring torque = –mg sin θ.l

Also, $\tau = I\alpha = ml_2\alpha$

$\therefore \quad ml^2\alpha = -mg \sin\theta.l$

$$\alpha = -\frac{g \sin\theta}{l}$$



For small angles of oscillation, $\sin\theta = \theta$

$$\alpha = -\frac{g}{l}.\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}.\theta \quad \text{i.e.} \quad \frac{d^2\theta}{dt^2} + \omega^2\theta = 0$$

giving $\omega = \sqrt{\frac{g}{l}}$ and $T = 2\pi\sqrt{\frac{l}{g}}$

Time period doesn't depend on mass of bob.

OR

(i) Let the particle be at R when its velocity $V = \frac{V_{max}}{2}$ and its displacement from the mean position O be y.

As $\quad \nu = \omega\sqrt{A^2 - y^2}$

So $\quad y = \sqrt{A^2 - \nu^2/\omega^2}$

Given $\quad \nu = A\ \omega/2,$

then $\quad y = \sqrt{A^2 - \frac{A^2\omega^2}{4\omega^2}} = \frac{\sqrt{3}}{2}A$

(ii) $P.E. = U_t = \frac{1}{2}m\omega^2[A^2 \sin^2 \omega t]$

$K.E. = \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t$

$u = K.E.$

$\sin^2\omega t = \cos^2\omega t \Rightarrow \sin \omega t = \cos \omega t$

$\therefore \quad \omega t = \frac{\pi}{4} \quad \Rightarrow \quad \frac{2\pi}{T}t = \frac{\pi}{4}$

$$\Rightarrow \quad t = \frac{T}{8}$$

26. Mass of sphere = m, Radius of sphere = r

Moment of Inertia $I = \frac{2}{5}mr^2$

Total energy = $K_R + K_T$

$$K_{tot} = \frac{1}{2}I\omega^2 + \frac{1}{2}m\nu^2$$

$$= \frac{1}{2}\cdot\frac{2}{5}mr^2\left(\frac{\nu^2}{r^2}\right) + \frac{1}{2}m\nu^2 \qquad (\nu = r\omega)$$

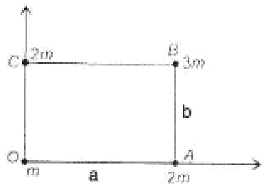
$$K_{tot} = \frac{1}{2}\left(\frac{7}{5}\right)m\nu^2$$

$$\text{Fraction of K.E. of rotation} = \frac{K_R}{K_{tot}} = \frac{\frac{1}{2}\left(\frac{2}{5}\right)m\nu^2}{\frac{1}{2}\left(\frac{7}{5}\right)m\nu^2} = \frac{2}{7}$$

$$\therefore \quad \text{Fraction of K.E. of translation} = \frac{K_T}{K_{tot}} = \frac{5}{7}$$

27. Let $m_1 = m, m_2 = 2\ m,\ m_4 = 2m$

Let mass m_1 be at origin



$\therefore$ For $m_1; x_1 = 0,\ y_1 = 0$

For $m_2;\ x_2 = a\hat{i},\ y_2 = 0$

For $m_3;\ x_3 = a\hat{i},\ y_3 = b\hat{j}$

For m_4; $x_4 = 0$, $y_4 = b\hat{j}$

Coordinates of COM of the system are

$$x = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{m \times 0 + 2m \times a\hat{i} + 3m \times a\hat{i} + 2m \times 0}{m + 2m + 3m + 2m}$$

$$x = \frac{5ma\hat{i}}{8m} = \frac{5a\hat{i}}{8}$$

$$y = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{m \times 0 + 2m \times 0 + 3m \times b\hat{j} + 2m \times b\hat{j}}{m + 2m + 3m + 2m}$$

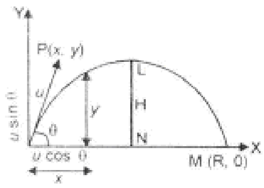
$$= \frac{5mb\hat{j}}{8m} = \frac{5}{8}b\hat{j}$$

$\therefore$ Centre of mass of system is $= \frac{5}{8}\left(a\hat{j} + b\hat{j}\right)$

Students may note that choice of the mass at the origin, may lead to varying result.

28. A body thrown up in space and allowed to proceed with effect of gravity alone is called projectile.

Suppose a body is projected with velocity *u* at an angle θ with the horizontal, P(x, y) is any point on its trajectory at time *t*. Horizontal component of velocity is unaffected by gravity, but the vertical component ($u \sin\theta$) changes due to gravity



$x = (u \cos\theta)t$

$$y = (u \sin\theta)\, t - \frac{1}{2} g t^2$$

$$= u \sin q \times \frac{x}{u \cos\theta} - \frac{1}{2} g \left(\frac{x}{u cis\theta}\right)^2$$

$$y = x \tan\theta - \frac{g x^2}{2u^2 \cos^2\theta} \quad ....(i)$$

It represents the equation of a parabola, hence the path followed by a projectile is a parabola.

(i) When the body returns to the same horizontal level y = 0

$$\therefore \quad 0 = x \tan\theta - \frac{g x^2}{2u^2 \cos^2\theta} \quad \text{[From (i)]}$$

$$\text{or} \quad x \tan\theta = \frac{g x^2}{2u^2 \cos^2\theta}$$

But coordinates of M are (r, 0). Putting x = R,

$$\text{we have} \quad R = \frac{u^2 \sin 2\theta}{g}$$

(ii) The greatest vertical distance attained by the projectile above the horizontal plane from the point of projection is called maximum height.

Maximum height, LN = H

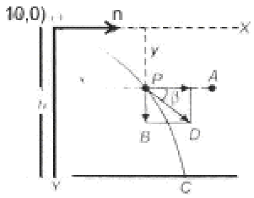
At maximum, height $\nu = 0$

$$\therefore \quad \nu^2 - \nu_y^2 = -2gH,$$

where $u_y = u \sin\theta$

or $(u \sin \theta)^2 = 2gH$

or $H = \frac{u^2 \sin^2 \theta}{2g}$



(i) Time taken to reach the ground (Time of flight)

Let it be *T*

h = vertical height of point of projection *O* from *C*

Taking motion of object along *OY* direction

$y_0 = 0,\ y = h,\ u_y = 0,\ a_y = g,\ t = T$

$$y = y_0 + u_y t + \frac{1}{2} a_y t^2$$

Putting values of y_0, y and, u_y, ay, we have

$$h = \frac{1}{2} g T^2$$

$$\Rightarrow T = \sqrt{\frac{2h}{g}}$$

(ii) Horizontal distance from foot of building where it will strike the ground (Horizontal range R). Taking motion of object along *OX*-direction; we have $x_0 = 0$, $x = R$, $u_x = u$, $a_x = 0$

$$t = T = \sqrt{\frac{2h}{g}}$$

As $x = x_0 + u_x t + \frac{1}{2} a_x t^2$

Putting the values we have

$$R = u\sqrt{\frac{2h}{g}}$$

(iii) Velocity with which body reach the ground. At any instant to the object posses two perpendicular velocities.

Horizontal velocity $\nu_x = u$ represented by PA

Vertical velocity ν_x represented by PB

$$\nu_x = u_x + a_x t$$

$$\nu_y = 0,\ a_y = g, \text{ we have}$$

$$\nu_y = gt$$

Resultant velocity ν^1at ν_s^1 and ν_s^1 is given by

$$\nu = \sqrt{\nu_x^2 + \nu_y^2}$$

$$\nu = \sqrt{\nu^2 + g^2 t^2}$$

Let $\vec{\nu}$ makes an angle β with horizontal direction then

$$\tan\beta = \frac{\nu_y}{\nu_x} = \frac{gt}{u} \quad \text{or} \quad \beta = \tan^{-1}\left(\frac{gt}{u}\right)$$

29. From the forces acting on the vehicle in a banked curve (θ).

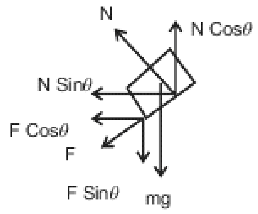
$$N\cos\theta - F_f \sin\theta = mg$$

$$N\sin\theta + F_f\cos\theta = mv^2/r.\ F_f = \mu N$$

Dividing the equation, we have,

$$\frac{\nu^2}{rg} = \frac{N\sin\theta + \mu N\cos\theta}{N\cos\theta + \mu N\sin\theta}$$

$$\nu^2 = rg\left[\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right]$$ [dividing each term of right side by N cosθ]



$$\nu = \sqrt{rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)}$$

If $\mu = 0$ i.e., banked road is perfectly smooth. Then from above

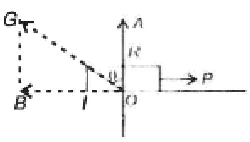
$$\nu_0 = (rg \tan \theta)^{1/2}$$

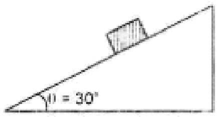
$$\nu_0 = rg \tan\theta$$

or $\tan\theta = \frac{\nu_0^2}{rg}$

OR

Angle which the resultant of force of limiting friction *F* and normal reaction *R* makes with direction of normal reaction *R*.



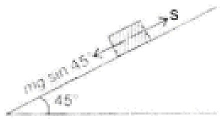


$$\theta = \tan^{-1}(\mu) = 30°$$

$\Rightarrow$ $\mu = \tan 30° = \frac{1}{\sqrt{3}}$

Max friction $f_{max} = \mu mg \cos 45°$

$$= \frac{1}{\sqrt{3}} mg \frac{1}{\sqrt{2}}$$



Net force

$$F = (\text{mg sin } 45^\circ - \mu \text{ mg cos } 45^\circ)$$

$$\text{ma} = \left(\frac{\text{mg}}{\sqrt{2}} - \frac{1}{\sqrt{3}}\frac{\text{mg}}{\sqrt{2}}\right)$$

$$\therefore \quad a = \frac{g}{\sqrt{2}}\left[1 - \frac{1}{\sqrt{3}}\right]$$

$$= \frac{10}{\sqrt{2}}\left[\frac{\sqrt{3}-1}{\sqrt{3}}\right] = 10\left[\frac{1.73-1}{\sqrt{6}}\right]$$

$$a = \frac{7.3}{\sqrt{6}} = 2.99 \text{ m/s}^2$$

30. $y(x,t) = 8 \cos(300t - 0.15x)$

On comparing with $y = a \cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

(i) Direction of propogation is + x-axis.

(ii) Wavelength $\frac{2\pi}{\lambda} = 0.15$

$$\lambda = \frac{2\pi}{0.15} = 41.87 \text{ m}$$

(iii) $\frac{2\pi}{T} = 300$

$$2\pi\upsilon = 300$$

$$\gamma = \frac{300}{2\pi} = 47.7 \text{ H}_z$$

(iv) $v = \lambda v = \frac{2\pi}{0.15} \times \frac{300}{2\pi} = 3000 \text{ m/s}$

(v) $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$$= \frac{2\pi}{\lambda} \times 0.2 = \frac{2\pi \times 0.2 \times 0.15}{2\pi} = 0.03 \text{ radian}$$

OR

Progressive Wave		***Stationary Wave***	
(i)	All particles have same phase and amplitude.	(i)	Amplitude varies with position.
(ii)	Speed of motion is same.	(ii)	Speed varies with position.
(iii)	Energy is transported.	(iii)	Energy is not transported.
(iv)	Same change in pressure and density is with every point.	(iv)	Pressure and density varies with point.

$$y = 12 \sin 300t \cos 2x$$

Comparing with equation of stationary wave

$$y = 2A \sin \omega t \cos Kx$$

$$K = 2$$

Distance between two consecutive nodes $= \frac{\lambda}{2}$

Where λ is wavelength

$$K = \frac{2\pi}{\lambda}$$

$$\Rightarrow \quad \frac{\pi}{(\lambda/2)} = 2$$

$$\therefore \quad \frac{\lambda}{2} = \frac{\pi}{2}$$

So, the distance between two nearest nodes is $\frac{\pi}{2}$.