Sample Paper-01
Mathematics Class – XI

ANSWERS

Section A

1. **Solution**

Number of subsets

$^{10}C_0 +^{10}C_1 +^{10}C_2 +^{10}C_3 +^{10}C_4 +^{10}C_5 +^{10}C_6 +^{10}C_7 +^{10}C_8 +^{10}C_9 +^{10}C_{10} = 2^{10}$

2. **Solution**

$^3C_1 +^3C_2 +^3C_3 + = 2^3 - 1 \ = 7$

3. **Solution**

1. Each card can be drawn in 52 ways and so the total number of ways =

   $52 \times 52 \times 52 = 52^3$

2. If there is no replacement the first card can be drawn in 52 ways, the second by 51 ways and the third by 50 ways. Hence the total number of ways is

   $52 \times 51 \times 50 = 132600$

4. **Solution:**

1. None of the factors are zero
2. Factors must be of the form $(a+ib); k(b+ia)$ where k is a real number

5. **Solution**

Length of arc = $r\theta$

Hence length of arc==2units

6. **Solution**

1 Full rotation is $2\pi radians$

500 radians = $\frac{500}{2\pi}$ rotations

$$\frac{500}{2\pi} = 79.57 rotations$$

79 full rotations and 0.57 of a rotation

$$0.5 < 0.57 < 0.75$$

The incomplete rotation is between $\frac{1}{2}$ *and* $\frac{3}{4}$ of a rotation . Hence 500 radians is in third quadrant. So $\cos\theta$ is negative

Section B

7. **Solution**

Let

$n = 1$

Then $n(n+1)(2n+1)$ = 6 and divisible by 6

Let it be divisible by 6 for

$n = m$

Then

$m(m+1)(2m+1) = 6k$ Where k is an integer

For $n = m+1$ the expression is

$$(m+1)(m+2)(2m+2+1) = (m+2)(m+1)(2m+1) + 2(m+1)(m+2)$$
$$= m(m+1)(2m+1) + 2(m+1)(2m+1) + 2(m+1)(m+2)$$
$$= m(m+1)(2m+1) + 2(m+1)(3m+3)$$
$$= m(m+1)(2m+1) + 6(m+1)^2$$
$$= 6k + 6(m+1)^2, This \quad is \quad divisible \quad by \quad 6$$

8. **Solution**

$1 - 2\sin^2 x - 5\sin x - 3 = 0$

$2sin^2 x + 5\sin x + 2 = 0$

Let $sinx = t$

Then, $2t^2 + 5t + 2 = 0$

Solving this quadratic

$2t(t+2) + (t+2) = 0$

$(2t+1)(t+2) = 0$

$t = -2, t = -\frac{1}{2}$

$\sin x = \frac{-1}{2}$

First value of t is rejected as $\sin x$ should lie between $(-1 \quad and \quad 1)$

General solution is $x = (-1)^{n+1}\frac{\pi}{6} + n\pi$

9. **Solution**

When

$m = 0$

The given equation reduces to a first degree and it will have only one solution

Also when the discriminant is zero it will have only one solution

Discriminant is

$4\left(m+1\right)^2 - 4m^2.4 = 0$

$4\left(m^2 + 1 + 2m\right) - 16m^2 = 0$

On simplifying and solving,

$\left(m-1\right)\left(3m+1\right) = 0$

$m = 1, m = -\frac{1}{3}$

Hence the three values of m for which the equation will have only one solution is

$m = 0, m = 1, m = -\frac{1}{3}$

10. **Solution**

$A.P \qquad a-d, a.a+d$

$GP \qquad \frac{b}{g}, b, bg$

$a - d + a + a + d = 3a$

$3a = 126$

$a = 42$

$a + b = 76$

$b = 34$

$a - d + \frac{b}{g} = 85 \ldots (1)$

$a + d + bg = 84 \ldots (2)$

$2a + \frac{b}{g} + bg = 169$

$34g^2 - 85g + 34 = 0$

$g = \frac{85 \pm \sqrt{85^2 - 4 \times 34 \times 34}}{2 \times 34}$

$g=2 \quad or \quad \frac{1}{2}$

$When \quad g=2$

$42-d+\frac{34}{2}=85$

$d=-26$

$a=42, \quad d=-26, \quad g=2, \quad b=34$

AP

$68, \quad 42, \quad 16$

GP

$17, \quad 34, \quad 68$

$m=1, m=-\frac{1}{3}$

11. **Solution**

$f(x+1)=4^{x+1}$

$f(x)=4^{x}$

$f(x+1)-f(x)= 4^{x+1}-4^{x}$

$=4^{x}.4-4^{x}$

$=4^{x}(3)$

$=3f(x)$

12. **Solution**

$$\log\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}$$

$$=\log\frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}$$

$$=\log\frac{(1+x)^3}{(1-x)^3}$$

$$=3\log\frac{(1+x)}{(1-x)}$$

$$=3f(x)$$

13. **Solution**

$$\sin(45+30)=\sin 45\cos 30+\cos 45\sin 30$$
$$=\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2}$$
$$=\frac{\sqrt{3}+1}{2\sqrt{2}}$$
$$=\frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\cos(45+30)=\cos 45\cos 30-\sin 45\sin 30$$
$$=\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\times\frac{1}{2}$$
$$=\frac{\sqrt{3}-1}{2\sqrt{2}}$$
$$=\frac{\sqrt{6}-\sqrt{2}}{4}$$

14. **Solution**

$$\frac{sin3\theta}{\sin\theta}-\frac{\cos 3\theta}{\cos\theta}=\frac{\sin 3\theta\cos\theta-\cos 3\theta\sin\theta}{\sin\theta\cos\theta}$$
$$=\frac{\sin(3\theta-\theta)}{\sin\theta\cos\theta}$$
$$=\frac{2\sin 2\theta}{2\sin\theta\cos\theta}$$
$$=\frac{2\sin 2\theta}{\sin 2\theta}=2$$

15. **Solution**

$$x^2+4(mx+1)^2=1$$
$$x^2+4(m^2x^2+2mx+1)=1$$
$$x^2+4m^2x^2+8mx+4=1$$
$$x^2(1+4m^2)+8mx+3=0$$

The line being a tangent ,it touches the ellipse at two coincident points, and so Discriminant must be zero,

$$(8m)^2-4(3)(1+4m^2)=0$$
$$64m^2-12-48m^2=0$$
$$16m^2=12$$
$$m^2=\frac{12}{16}$$
$$m^2=\frac{3}{4}$$

16. **Solution**

Divide the equation by

$$-\sqrt{3^2+-4^2}=-5$$

Hence, $-\frac{3}{5}x+\frac{4}{5}y-4=0$

Where, $\cos\alpha=\frac{-3}{5}$ *and* $\sin\alpha=\frac{4}{5}$ *and* $p=4$

17. **Solution**

Multiply both numerator and denominator with $x-7$.Then denominator becomes a perfect square and it is always positive

Now

$$(x+3)(x-7)\le 0$$

Critical points are

$$(-3,7)$$

Hence, $-3\le x<7$

18. **Solution**

$$\lim_{x\to\infty}\frac{x^2-ax+4}{3x^2-bx+7}=\lim_{x\to\infty}\frac{x^2(1-\frac{a}{x}+\frac{4}{x^2})}{x^2(3-\frac{b}{x}+\frac{7}{x^2})}$$

$$=\frac{1}{3}$$

19. **Solution**

$$\lim_{x\to 0}\frac{\tan x}{\sin 3x}=\lim_{x\to 0}\frac{\sin x}{x}\times\frac{1}{cosx}\times\frac{1}{\frac{3sin3x}{3x}}$$

$$=1\times 1\times\frac{1}{3}=\frac{1}{3}$$

Section C

20. **Solution**

Form a quadratic equation whose roots are

$1+i$ *and* $1-i$

The equation is

$$x^2-2x+2=0$$

The given expression

$$x^3+x^2-4x+13=x(x^2-2x+2)+3(x^2-2x+2)+7$$

$$x^3+x^2-4x+13=x(0)+(0)+7$$

$$x^3+x^2-4x+13=7$$

21. **Solution**

$$x^2-(\alpha+\beta)x+\alpha\beta-k^2=0$$

Discriminant of the above quadratic is

$\{(\alpha+\beta)\}^2-4(\alpha\beta-k^2)=(\alpha-\beta)^2+k^2$ is always positive and hence the roots are real.

22. **Solution**

Let the roots be

$p\alpha$ *and* $q\beta$

Then

$$p\alpha+q\alpha=-\frac{n}{l}\ldots(1)$$

$$pq\alpha^2=\frac{n}{l}$$

$$\alpha=\frac{\sqrt{n}}{\sqrt{l}}\times\frac{1}{\sqrt{pq}}\ldots(2)$$

Hence substituting equation 2 in equation 1

$$(p+q)\frac{\sqrt{n}}{\sqrt{l}}\times\frac{1}{\sqrt{pq}}+\frac{n}{l}=0$$

On simplifying ,

$$\frac{\sqrt{p}}{\sqrt{q}}+\frac{\sqrt{q}}{\sqrt{p}}+\frac{\sqrt{n}}{\sqrt{l}}=0$$

23. **Solution**

$$\lim_{x\to\pi}(\pi-x)\tan\frac{x}{2}=\lim_{x\to\pi}\frac{2(\pi-x)}{2}\cot\frac{\pi-x}{2}$$

$$=\lim_{x\to\pi}\frac{2(\pi-x)}{2}\frac{\cos\frac{\pi-x}{2}}{\sin\frac{\pi-x}{2}}$$

$$=\lim_{x\to\pi} 2\frac{\cos\frac{\pi-x}{2}}{\frac{\sin\frac{\pi-x}{2}}{\frac{\pi-x}{2}}}$$

$$=\lim_{\frac{x-\pi}{2}\to 0} 2\frac{\cos\frac{\pi-x}{2}}{\frac{\sin\frac{\pi-x}{2}}{\frac{\pi-x}{2}}}$$

$$=\lim_{\frac{x-\pi}{2}\to 0} 2\frac{\cos\frac{x-\pi}{2}}{\frac{\sin\frac{x-\pi}{2}}{\frac{x-\pi}{2}}}=2$$ **since the limit of** $\boldsymbol{\frac{\sin\frac{x-\pi}{2}}{\frac{x-\pi}{2}}}$ **= 1**

24. **Solution**

Let

$a=x-1$

$b=x$

$c=x+1$

Then

$(x-1-i)((x-1+i)(x+1+i)(x+1-i)=\{(x-1)^2-i^2\}\{(x+1)^2-i^2\}$

$=\{(x-1)^2+1\}\{(x+1)^2+1\}$

$=\{(x-1)(x+1)\}^2+(x-1)^2+(x+1)^2+1$

$=(x^2-1)^2+(x-1)^2+(x+1)^2+1$

$=x^4+1$

$=b^4+1$

25. **Solution**

Multiply both Numerator and denominator with $(1-i)^2$ Then

$$\frac{\left(1+i\right)^n}{\left(1-i\right)^{n-2}}=\frac{\left(1+i\right)^n\left(1-i\right)^2}{\left(1-i\right)^n}$$

multiplying both Numerator & denominator with $\left(1+i\right)^n$

$$=\frac{\left(1+i\right)^n\left(-2i\right)\left(1+i\right)^n}{\left(1-i\right)^n\left(1+i\right)^n}$$

Simplifying

$$=\frac{\left\{\left(1+i\right)^2\right\}^n\left(-2i\right)}{\left(1-i^2\right)^n}$$

On expanding and simplifying

$$=\frac{2^n i^n\left(-2\right)i}{2^n}$$

$$=-2i^{n+1}$$

$$=\frac{2\left(i\right)^{n+1}}{i^2}=2i^{n-1}$$

26. **Solution**

Let the point be $A(1,2) and B(3,4)$

The mid-point of the line joining A and B is $C(2,3)$

Slope of line AB = $\frac{4-2}{3-1}=1$

Let the required point be $D(\alpha,\beta)$

Then D must be a point on the line perpendicular to the line AB and passing through point C

$\therefore$ *Slope of* $CD=-1$

Equation of CD

$y-3=-1(x-2)$

$x+y=5$

Equation of AB

$y-2=1(x-1)$

$x-y+1=0$

The point $D(\alpha,\beta)$ must satisfy the equation

$x+y=5$

$\therefore \alpha+\beta=5\ldots(1)$

The perpendicular distance from (α,β) to AB is

$$\frac{\alpha-\beta+1}{\sqrt{2}}=\sqrt{2}$$

$\alpha-\beta=1\ldots(2)$

Solving equations 1 and 2

$\alpha=3,\beta=2$