CBSE Sample Papers for Class 10 SA2 Maths Solved 2016 Set 3

Answers:

Section A

1.Determine the value of k for which the indicated value of x is a solution: $x^2 + kx - 4 = 0$; $x = -4$.

Ans.

$$x = -4 \text{ is a solution}$$
$$x^2 + kx - 4 = 0$$
$$(-4)^2 + k \times (-4) - 4 = 0$$
$$16 - 4k - 4 = 0 \quad \Rightarrow -4k = -12$$
$$k = 3$$

2.Find the sum of the following AP: 2, 7, 12, ... upto 10 terms.

Ans.

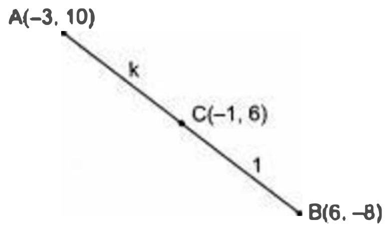
Here,

$$a = 2,\ d = 7 - 2 = 5,\ n = 10$$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{10} = \frac{10}{2}[2 \times 2 + (10-1)5]$$
$$= 5[4 + 45] = 5 \times 49 = 245$$

3.Find the ratio in which the joining of points (- 3,10) and (6, - 8) is divided by point (- 1, 6).

Ans.

Let point C(− 1, 6) divides the joining of A(− 3, 10) and B(6, − 8) in the ratio $k : 1$.



$$x \text{ coordinate of C} = \frac{k \times 6 \quad 1 \times (-3)}{k+1}$$
$$-1 = \frac{6k - 3}{k + 1}$$
$$-k - 1 = 6k - 3$$
$$7k = 2$$
$$k = 2$$

Ratio is $k : 1 = \frac{2}{7} : 1 = 2 : 7$

4.Find the area of a quadrant of a circle whose circumference is 22 cm.
Ans.

Circumference of the circle = 22 cm

$$2\pi r = 22 \text{ cm}$$

$$r = \frac{22 \times 7}{2 \times 22} = \text{—} \text{ cm}$$

$$\text{Area of the quadrant} = \frac{1}{4}\pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{8} \text{ cm}^2$$

Section B

5.Find discriminant of the following quadratic equation and examine the nature of real roots (if they exist): $7y^2 + 4y + 5 = 0$.
Ans.

Given quadratic equation:

$$7y^2 + 4y + 5 = 0$$

Here,

$$a = 7,\ b = 4 \text{ and } c = 5$$

$$D = b^2 - 4ac$$

$$= (4)^2 - 4 \times 7 \times 5$$

$$= 16 - 140 = -124$$

$$D = -124$$

D is negative.

Equation has no real roots.

6.Find the sum of the first 17 terms of the AP whose nth term is given by $t_n = 7 - 4n$.

Ans.

$$t_n = 7 - 4n$$

$$t_1 = 7 - 4 \times 1 = 3$$

$\Rightarrow$ $$t_2 = 7 - 4 \times 2 = -1$$

$$d = t_2 - t_1 = -1 - 3 = -4$$

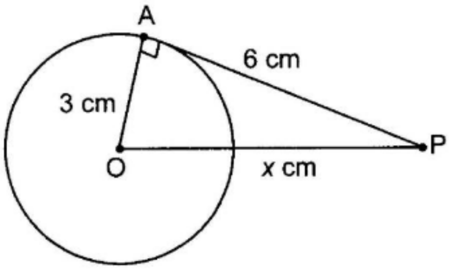
So, sum of 17 terms,

$$S_{17} = \frac{17}{2}[2 \times 3 + (17 - 1) \times (-4)]$$

$$= \frac{17}{2}(6 - 64) = \frac{17}{2} \times (-58)$$

$$= 17 \times (-29) = -493$$

7.In figure, O is the centre of the circle, radius of the circle is 3 cm and PA is a tangent drawn to the circle from point P. If OP = x cm and AP = 6 cm, then find the value of x.



Ans.

In right ΔOAP,

$$OP^2 = OA^2 + AP^2$$

$\Rightarrow$ $$x^2 = (3)^2 + (6)^2$$

$\Rightarrow$ $$x^2 = 9 + 36$$

$\Rightarrow$ $$x^2 = 45$$

$\Rightarrow$ $$x = 3\sqrt{5} \text{ cm}$$

8.2000 tickets of a lottery were sold and there are 8 prizes on these tickets. Your friend has purchased one lottery ticket. What is the probability that your friend wins a prize?

Ans.

Number of lottery tickets = 2000

Total number of prizes = 8

Probability to win a prize $= \frac{8}{2000} = \frac{1}{250}$

9.The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Ans.

Circumference of 1st circle $= 2\pi \times 19 = 38\pi$ cm

Circumference of 2nd circle $= 2\pi \times 9 = 18\pi$ cm

Let the radius of required circle be x cm

A.T.Q.,

$$2\pi x = 38\pi + 18\pi$$

$$2\pi x = 56\pi$$

$$x = \frac{56\pi}{2\pi} = 28 \text{ cm}$$

10.The diameter of a solid metallic sphere is 16 cm. The sphere is melted and recast into 8 equal solid spherical balls. Determine the radius of the balls.

Ans.

$$\text{Radius of metallic sphere} = \frac{16}{2} = 8 \text{ cm}$$

$$\text{Volume of the sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (8)^3 \text{ cm}^3$$

Let the radius of each shperical ball be x cm

$$\text{Volume of one spherical ball} = \frac{4}{3}\pi(x)^3 \text{ cm}^3$$

$$\text{Volume of 8 spherical balls} = 8 \times \frac{4}{3}\pi(x)^3 \text{ cm}^3$$

$$= \frac{32}{3}\pi(x)^3 \text{ cm}^3$$

A.T.Q.,

$$\frac{32}{3}\pi(x)^3 = \frac{4}{3}\pi(8)^3$$

$$\frac{32}{3}(x)^3 = \frac{4}{3}(8)^3$$

$$x^3 = \frac{4}{3} \times 8^3 \times \frac{3}{32}$$

$$x^3 = 64 \quad \Rightarrow \quad x = 4 \text{ cm}$$

Section C

11.The sum of an integer and its reciprocal is 145/12 find the integer.

Ans.

Let the integer be x

A.T.Q.,

$$x + \frac{1}{x} = \frac{145}{12}$$

$$\frac{x^2+1}{x} = \frac{145}{12}$$

$$12x^2 + 12 = 145x$$

$$\Rightarrow \quad 12x^2 - 145x + 12 = 0$$

$$\Rightarrow \quad 12x^2 - 144x - x + 12 = 0$$

$$\Rightarrow \quad 12x(x - 12) - 1(x - 12) = 0$$

$$\Rightarrow \quad (x - 12)(12x - 1) = 0$$

$$x = 12 \text{ or } x = \frac{1}{12}$$

Rejecting $x = \frac{1}{12}$ $\because$ x is an integer

$$x = 12$$

12.Find the 12th term from the end in the AP 56, 63, 70, ... ,329.

Ans.

12th term from the end of AP 56, 63, 70, ... 322, 329 is 12th term of the AP 329, 322, ... 56.

Here, $a = 329,\ d = -7$

$$a_{12} = 329 + 11 \times (-7) = 329 - 77 = 252$$

13.Solve for x : x - 1/x = 3, x not equal to zero

Ans.

$$x - \frac{1}{x} = 3$$

$$\frac{x^2 - 1}{x} = 3$$

$$x^2 - 1 = 3x$$

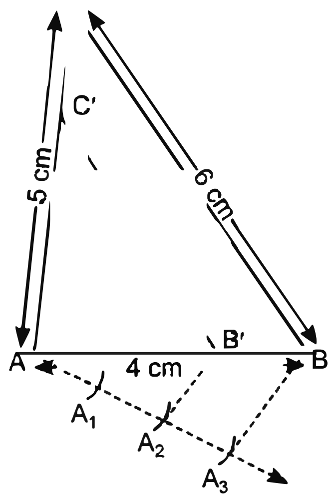
$$x^2 - 3x - 1 = 0$$

$$D = b^2 - 4ac = (-3)^2 - 4 \times 1 \times (-1) = 13$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-3) \pm \sqrt{13}}{2 \times 1} = \frac{3 \pm \sqrt{13}}{2}$$

14.Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it, whose sides are — of the corresponding sides of the first triangle.

Ans.



C′AB′ is the required triangle.

15.A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. Find the probability that the marble taken out will be (i)red (ii) white (iii)not green.

Ans.

Total number of marbles = 5 + 8 + 4 = 17

(*i*) Number of red marbles = 5

Required probability = $\frac{5}{17}$

(*ii*) Number of white marbles = 8

Probability of getting white marbles = $\frac{8}{17}$

(*iii*) Number of green marbles = 4

Probability of getting green marbles = $\frac{4}{17}$

Probability of not getting a green marble = $1 - \frac{4}{17} = \frac{13}{17}$

16.A piggy bank contains hundred 50 p coins, fifty Rs 1 coins, twenty Rs 2 coins and ten Rs 5 coins. If it is equally likely that one of the coins will fall out when the piggy bank is turned upside down, find the probability that the coin (i)will be a 50 p coin (ii)will not be a Rs 5 coin.

Ans.

Total number of coins = 100 + 50 + 20 + 10 = 180

(*i*) Number of 50p coins = 100

$\therefore$ Probability of getting a 50p coin = $\frac{100}{180} = \frac{5}{9}$

(*ii*) No. of ₹5 coins = 10

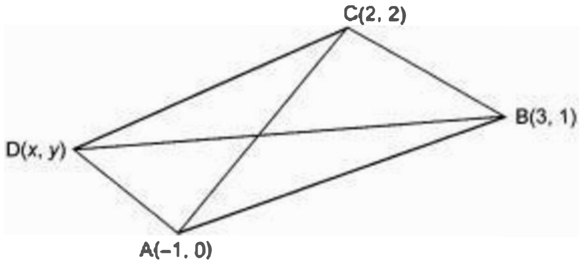
No. of coins other than ₹5 = 180 − 10 = 170

P(will not be ₹5) = $\frac{170}{180}$

17.The three vertices of a parallelogram taken in order are (-1, 0), (3, 1) and (2, 2) respectively. Find the coordinates of the fourth vertex.

Ans.

Let the coordinates of fourth vertex be D(x, y)



$$\text{Mid point of AC} = \left(\frac{-1+2}{2}, \frac{0 \quad 2}{2}\right)$$

$$= \left(\frac{1}{2}, 1\right)$$

$$\text{Mid point of BD} = \left(\frac{3+x}{2}, \frac{1+y}{2}\right)$$

Diagonals of a parallelogram bisect each other.

Coordinates of mid point of BD = coordinates of mid point of AC

$$\left(\frac{3+x}{2}, \frac{1+y}{2}\right) = \frac{1}{2}, 1)$$

$\Rightarrow$ $$\frac{3+x}{2} = \frac{1}{2} \text{ and } \frac{1+y}{2} = 1$$

$\Rightarrow$ $$3 + x = 1 \text{ and } 1 + y = 2$$

$\Rightarrow$ $x = -2$ and $y = 1$

Coordinates of fourth vertex are (−2, 1).

18.Using distance formula, show that the points A, B and C are collinear: A(2, 3), B(3, 4), C(6, 7)

Ans.

$AB = \sqrt{(3-2)^2 + (4-3)^2} = \sqrt{2}$ units

$BC = \sqrt{(6-3)^2 + (7-4)^2} = \sqrt{18} = 3\sqrt{2}$ units

$AC = \sqrt{(6-2)^2 + (7-3)^2} = \sqrt{32} = 4\sqrt{2}$ units

Now , $AB + BC = \sqrt{2} + 3\sqrt{2}$

$= 4\sqrt{2} = AC$

A, B and C are collinear.

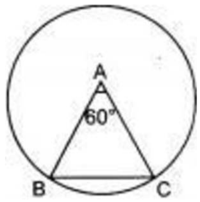
19.Find the area of the segment of a circle of radius 12 cm whose corresponding sector has a central angle of 60°.

Ans.

Consider the figure,

$\angle BAC = 60°$ and AB = AC [radius of a semicircle]

BAC is an equilaterals triangle



$$\text{Area of } \Delta ABC = \frac{\sqrt{3}}{4} \times (12)^2 = 36\sqrt{3} \text{ cm}^2$$

$$\text{Area of sector BAC} = \frac{60}{360} \times \pi r^2 = 24\pi \text{ cm}^2$$

$$\text{Area of segment} = \text{Area of sector BAC} - \text{Area of triangle BAC}$$

$$= (24\pi - 36\sqrt{3}) \text{ cm}^2$$

$$= 12(2\pi - 3\sqrt{3}) \text{ cm}^2$$

20.A drinking glass is in the shape of a frustum of a cone of height 14 cm. The radii of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass. (Take pi = 22/7)

Ans.

Radii of the frustum of a cone: R_1 = 4 cm, R_2 = 2 cm and h = 14 cm

$$\text{Volume of the frustum} = \frac{\pi h}{3}(R_1^2 + R_2^2 + R_1R_2)$$

$$= \frac{22}{7} \times \frac{14}{3} \times (4^2 + 2^2 + 4 \times 2) \text{ cm}^3$$

$$= \frac{44}{3}(16 + 4 + 8) = 410.67 \text{ cm}^3$$

Section D

21.A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Ans.

Let speed of the train be x km/h

$$\text{Distance} = 360 \text{ km}$$

$$\text{Time taken} = \frac{360}{x} \text{ hrs}$$

If speed of the train becomes $(x + 5)$ km/h

$$\text{Distance} = 360 \text{ km}$$

$$\text{Time taken} = \frac{360}{x+5} \text{ hrs}$$

A.T.Q.,

$$\frac{360}{x} - \frac{360}{x+5} = 1$$

$$\frac{360(x+5) - 360x}{x(x+5)} = 1$$

$$1800 = x^2 + 5x$$

$$x^2 \quad 5x - 1800 = 0$$

$$(x + 45)(x - 40) = 0$$

$$x = -45 \text{ or } x = 40 \qquad [\text{Rejecting } x = -45]$$

∴ Speed of train = 40 km/h

22.Three positive integers a_1 a_2, a_3 are in AP such that $a_1 + a_2 + a_3 = 33$ and a_1 x a_2x a_3 = 1155. Find the integers a_1, a_2 and a_3.

Ans.

Let $a_1 = a - d$, $a_2 = a$ and $a_3 = a + d$

$$a_1 + a_2 + a_3 = 33$$

$$a - d + a + a + d = 33$$

⇒ $$3a = 33$$

⇒ $$a = 11 \qquad \ldots(i)$$

Also, $$a_1 \times a_2 \times a_3 = 1155$$

⇒ $$(a - d) \times (a) \times (a + d) = 1155$$

$$(11 - d) \quad (11) \times (11 + d) = 1155 \qquad [\text{Using } (i)]$$

$$(11 - d)(11 + d) = \frac{1155}{11}$$

⇒ $$121 - d^2 = 105$$

⇒ $$d^2 = 16 \Rightarrow d \pm 4$$

When $a = 11$ and $d = 4$

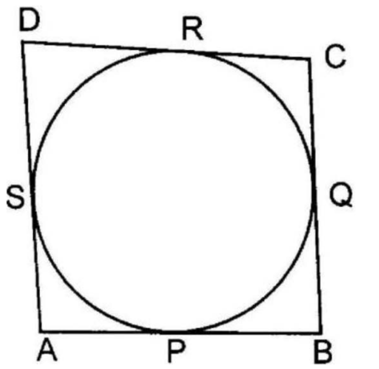
$$a_1 = 11 - 4 = 7,\ a_2 = a = 11 \text{ and } a_3 = 11 + 4 = 15$$

When $a = 11$ and $d = -4$

$$a_1 = 11 + 4 = 15,\ a_2 = 11,\ a_3 = 11 - 4 = 7$$

23. A village Panchayat constructed a circular tank to serve as a bird bath. A fencing was made in the shape of quadrilateral ABCD to circumscribe the circle. Prove that AB + CD = AD + BC.

What values does the village Panchayat depict through this action?



Ans.

Here, AP = AS (tangents from an external point are equal) ...(*i*)

Similarly, BP = BQ ...(*ii*)

CR = CQ ...(*iii*)

DR = DS ...(*iv*)

Adding, (*i*), (*ii*), (*iii*) and (*iv*), we ge

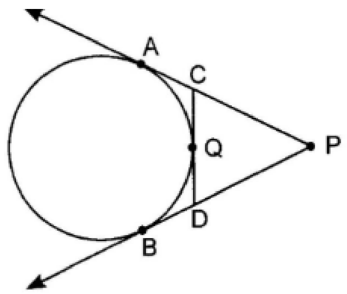
AP + BP + CR + DR = AS + BQ + CQ + DS

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

⇒ AB CD = AD + BC Hence proved.

Values reflected are : Care towards nature and love for creatures.

24.In figure, PA and PB are tangents to circle drawn from an external point P. CD is a third tangent touching the circle at Q. If PB = 7 cm and CQ = 2.5 cm, find the length of CP.



Ans.

Here, PB = PA (tangents from an external point are equal)

PB = 7 cm

PA = 7 cm

Now, PA = CP + AC

Also, AC = CQ (tangents from an external point are equal)

PA = CP + CQ

7 = CP 2.5

CP = 7 − 2.5 = 4.5 cm

25.The lengths of tangents drawn from an external point (point outside the circle) to a circle are equal. Prove it.

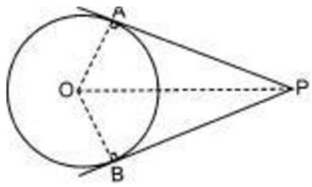
Ans.

Given: A circle C(O, *r*). P is a point outside the circle and PA and PB are tangents to a circle.

To Prove: PA = PB

Construction: Join OA, OB and OP.

Proof: Consider triangles OAP and OBP.



$\angle OAP = \angle OBP = 90°$ ...(*i*)

[Radius is perpendicular to the tangent at the point of contact]

OA = OB (radii) ...(*ii*)

OP is common ...(*iii*)

OAP $\cong$ ΔOBP [from (*i*), (*ii*), (*iii*)]

AP = BP (cpct)

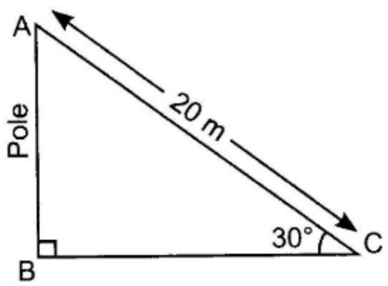
Hence proved.

26.A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole if the angle made by the rope with the ground level is 30°.

Ans.

Let AB be pole and AC be rope

AC = 20 m and $\angle ACB = 30°$



In right ΔABC,

$$\frac{AB}{AC} = \sin 30°$$

$$\frac{AB}{20} = \frac{1}{2} \Rightarrow AB = 10 \text{ m}$$

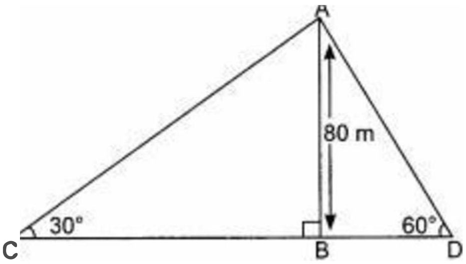
27.Two men standing on either side of a cliff 80 m high, observe the angles of elevation of the top of the cliff to be 30° and 60° respectively. Find the distance between the two men.

Ans.

Let AB be the cliff and two men are standing at C and D.

Now, $AB = 80$ m

$\angle ACB = 30°$ and $\angle ADB = 60°$



In right ΔABC,

$$\frac{AB}{BC} = \tan 30°$$

$$\frac{80}{BC} = \frac{1}{3} \Rightarrow BC = 80\sqrt{3} \text{ m}$$

In righ ΔABD,

$$\frac{AB}{BD} = \tan 60° \Rightarrow \frac{80}{BD} = 3$$

$\Rightarrow$ $$BD = 80 \text{ m}$$

Now, $$CD = BC \quad BD$$

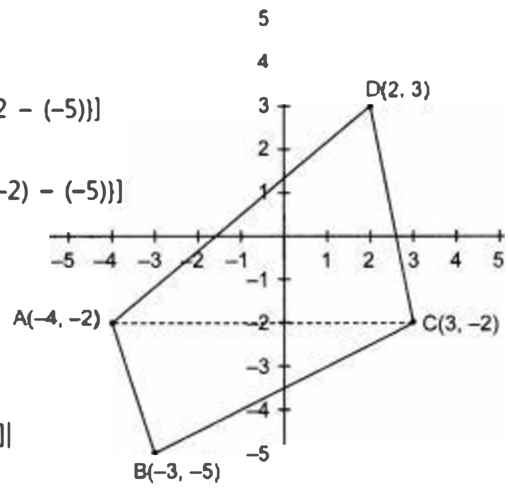
$$= 80\sqrt{3} + \frac{80}{\sqrt{3}}$$

$$= \frac{320}{3} \text{ m} = \frac{320 \; 3}{3} \text{ m}$$

28. Find the area of the quadrilateral formed by joining the points: A(-4, -2), B(-3, -5), C(3, -2) and D(2, 3).

Ans.

Consider the figure,



Area of $\Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}[(-4)(-5 - (-2)\} + (-3)\ \{-2 - (-2)\} \quad 3\{-2 - (-5)\}]$

$= \frac{1}{2}[(-4)(-5 - (-2)\} + (-3)\{(-2) - (-2)\} + 3\{(-2) - (-5)\}]$

$= \frac{1}{2}|-4 \times -3 - 3 \times 0 + 3 \times 3|$

$= \frac{1}{2}|12 + 0 + 9| = \frac{21}{2}$ sq. units

Area $\Delta ACD = \frac{1}{2}|-4(-2 - 3) + 3\ [3 - (-2)] + 2[-2 - (-2)]|$

$= \frac{1}{2}[(-4)(-2 - 3) + 3\{3 - (-2)\} + 2\{(-2) - (-2)\}]$

$= \frac{1}{2}|20 + 15 + 0| = \frac{35}{2}$ sq. units

Area of quadrilateral ABCD

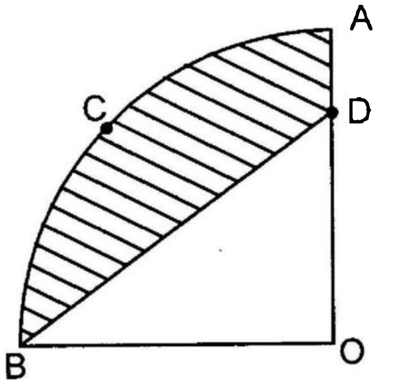
= area of ΔABC + area of ΔACD

$= \frac{21}{2} + \frac{35}{2} = \frac{56}{2}$ sq. units = 28 sq. units.

29.In figure, OACB is quadrant of a circle with centre O and radius 8 cm. If OD = 5 cm, find

(i) the area of the quadrant OACB.

(ii) the area of the shaded region. (Take pi = 22/7)



Ans.

(*i*) Area of quadrant OACB $= \frac{1}{4} \times \pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 8 \times 8 = \frac{352}{\ } \text{ cm}^2$$

(*ii*) Area of ΔBOD $= \frac{1}{2} \times OB \times OD$

$$= \frac{1}{2} \times 8 \times 5 = 20 \text{ cm}^2$$

Area of shaded region $= \frac{352}{7} - 20 = \frac{352 - 140}{7}$

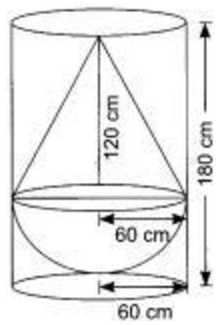
$$= \frac{212}{7} \text{ cm}^2 = 30.28 \text{ cm}^2$$

30.A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.(Take pi = 22/7)

Ans.

Radius of the conical part = 60 cm

Height of the conical part = 120 cm



Volume of conical part $= \frac{1}{3}\pi \times \quad h$

$= \frac{1}{3}\pi \times (60)^2 \times 120$

$= 144000\pi \text{ cm}^3$

Radius of hemisphere = 60 cm

Volume of hemisphere $= \frac{2}{3}\pi \times (60)^3 = 144000\pi \text{ cm}^3$

Volume of solid $= 144000\pi + 144000\pi = 288000\pi \text{ cm}^3$

Volume of cylinder $= \pi \times (60)^2 \times 180$

$= 648000\pi \text{ cm}^3$

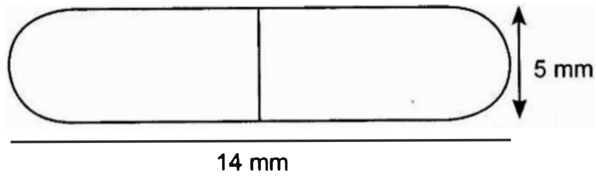
Volume of water left $= 648000\pi - 288000\pi$

$= 360000\pi \text{ cm}^3$

$= 360000 \times \frac{22}{7} \text{ cm}^3$

$= 1131428.57 \text{ cm}^3$

31.A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area. (Take pi = 22/7)



Ans.

Radius of hemisphere = $\frac{5}{2}$ mm

Curved surface area of hemispheres = $2\pi r^2 = 2\pi\left(\frac{5}{2}\right)^2$

$= \frac{25}{2}\pi \text{ mm}^2$

Surface area of two hemispheres = $2 \times \frac{25}{2}\pi = 25\pi \text{ mm}^2$

Length of cylindrical part = $(14 - 5) = 9$ mm

Radius of cylindrical part = $\frac{5}{2}$ mm

Curved surface area of cylindrical part = $2\pi rh = 2 \times \pi \times \frac{5}{2} \times 9 \text{ mm}^2$

$= 45\pi \text{ mm}^2$

Total surface area of the capsule = $25\pi + 45\pi = 70\pi \text{ mm}^2$

$= 70 \times \frac{22}{7} = 220 \text{ mm}^2$