

SAMPLE QUESTION PAPER

CLASS-XII (MATHS)

Time allowed: 3 hours

Maximum Marks: 100

General Instructions:

- (i) All questions are compulsory.
- (ii) This question paper contains 29 questions.
- (iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
- (iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
- (v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
- (vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

SECTION-A

Questions from 1 to 4 are of 1 mark each.

- 1. What is the principal value of $\tan^{-1}(\tan \frac{\pi}{4})$?
- 2. A and B are square matrices of order 3 each, $|A| = 2$ and $|B| = 3$. Find $|AB|$
- 3. What is the distance of the point (p, q, r) from the x-axis?
- 4. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 3x^2 - 5$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $g(x) = \frac{1}{x}$. Find $g \circ f$

SECTION-B

Questions from 5 to 12 are of 2 marks each.

- 5. How many equivalence relations on the set $\{1,2,3\}$ containing (1,2) and (2,1) are there in all? Justify your answer.
- 6. Let l_i, m_i, n_i ; $i = 1, 2, 3$ be the direction cosines of three mutually perpendicular vectors in space. Show that $AA' = I_3$, where $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$.
- 7. If $e^y (x+1) = 1$, show that $\frac{dy}{dx} + \frac{1}{x+1} = 0$
- 8. Find the sum of the order and the degree of the following differential equations:

$$\frac{dy}{dx} + \frac{1}{x+1} = 0$$

9. Find the Cartesian and Vector equations of the line which passes through the point $(-2, 4, -5)$ and parallel to the line given by — — —
10. Solve the following Linear Programming Problem graphically:
 Maximize $Z = 3x + 4y$
 subject to
 $x + y$
11. A couple has 2 children. Find the probability that both are boys, if it is known that (i) one of them is a boy (ii) the older child is a boy.
12. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which its area increases, when side is 10 cm long.

SECTION-C

Questions from 13 to 23 are of 4 marks each.

13. If $A + B + C = \pi$, then find the value of

$$\begin{vmatrix} \sin(A) & \cos(B) & \cos(C) \\ \cos(A) & \sin(B) & \sin(C) \\ \sin(A) & \sin(B) & \sin(C) \end{vmatrix}$$

OR

Using properties of determinant, prove that

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

14. It is given that for the function $f(x) = x^3 - 6x^2 + ax + b$ Rolle's theorem holds in $[1, 3]$ with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of 'a' and 'b'
15. Determine for what values of x, the function $f(x) = x^3 + \frac{1}{x}$ ($x \neq 0$) is strictly increasing or strictly decreasing

OR

Find the point on the curve $y = x^2$ at which the tangent is $y = x - 11$

16. Evaluate $\int_0^2 (x^2 + 3) dx$ as limit of sums.
17. Find the area of the region bounded by the y-axis, $y = \cos x$ and $y = \sin x$, $0 \leq x \leq \frac{\pi}{2}$
18. Can $y = ax + b$ be a solution of the following differential equation?
 $y = x^2 + \frac{1}{x} + \dots (*)$
 If no, find the solution of the D.E. (*).

OR

Check whether the following differential equation is homogeneous or not

$$xy' - y^2 = 1 + \cos\left(\frac{y}{x}\right), x \neq 0$$

Find the general solution of the differential equation using substitution $y=vx$.

19. If the vectors $\vec{p} = a\hat{i} + \hat{j} + \hat{k}$, $\vec{q} = \hat{i} + b\hat{j} + \hat{k}$, $\vec{r} = \hat{i} + \hat{j} + c\hat{k}$ are coplanar, then for a, b, c $\neq 1$ show that

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$$

20. A plane meets the coordinate axes in A, B and C such that the centroid of ΔABC is the point (α, β, γ) . Show that the equation of the plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$.
21. If a 20 year old girl drives her car at 25 km/h, she has to spend Rs 4/km on petrol. If she drives her car at 40 km/h, the petrol cost increases to Rs 5/km. She has Rs 200 to spend on petrol and wishes to find the maximum distance she can travel within one hour. Express the above problem as a Linear Programming Problem. Write any one value reflected in the problem.
22. The random variable X has a probability distribution P(X) of the following form, where k is some number:
- $$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$
- (i) Find the value of k (ii) Find $P(X < 2)$ (iii) Find $P(X = 2)$ (iv) Find $P(X \geq 2)$
23. A bag contains $(2n + 1)$ coins. It is known that 'n' of these coins have a head on both its sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{1}{3}$, find the value of 'n'.

SECTION-D

Questions from 24 to 29 are of 6 marks each

24. Using properties of integral, evaluate $\int_0^1 x^2 dx$

OR

Find: $\int_0^1 x^3 dx$

25. Does the following trigonometric equation have any solutions? If Yes, obtain the solution(s):

$$\left(\frac{\pi}{4}\right) + \left(\frac{\pi}{4}\right) = \frac{\pi}{2}$$

OR

Determine whether the operation * define below on \mathbb{Q} is binary operation or not.

$$a * b = ab + 1$$

If yes, check the commutative and the associative properties. Also check the existence of identity element and the inverse of all elements in \mathbb{Q} .

26. Find the value of x, y and z, if $A = \begin{bmatrix} x & \\ & z \end{bmatrix}$ satisfies $A' = A^{-1}$

OR

Verify: $A(\text{adj } A) = (\text{adj } A)A = |A| I$ for matrix $A = \begin{bmatrix} 3 & \\ & 2 \end{bmatrix}$

27. Find — if $y = \left\{ 2 \tan^{-1} \sqrt{\quad} \right\}$
28. Find the shortest distance between the line $x - y + 1 = 0$ and the curve $y^2 = x$
29. Define skew lines. Using only vector approach, find the shortest distance between the following two skew lines:
- $$= (8 + 3\lambda) \hat{i} - (9 + 16\lambda) \hat{j} + (10 + 7\lambda) \hat{k}$$
- $$= 15 \hat{i} + 29 \hat{j} + 5 \hat{k} + \mu (3 \hat{i} + 8 \hat{j} - 5 \hat{k})$$