General Instructions:

(i) All questions are compulsory.

(ii) Please check that this question paper contains 26 questions.

(iii) Questions 1 – 6 in Section A are very short-answer type questions carrying 1 mark each.

(iv) Questions 7 – 19 in Section B are long-answer I type questions carrying 4 marks each.

(v) Questions 20 – 26 in Section C are long-answer II type questions carrying 6 marks each.

(vi) Please write down the serial number of the question before attempting it.
SECTION A

प्रश्न संख्या 1 से 6 तक प्रश्न का 1 अंक है।
Question numbers 1 to 6 carry 1 mark each.

1. वह अवकल समीकरण ज्ञात कीजिए जो वक्र \( y = cx + c^2 \) को निरूपित करता है।
   Find the differential equation representing the curve \( y = cx + c^2 \).

2. निन्हे अवकल समीकरण का समाकलन गुणक लिखिए।
   \[ (1 + y^2) \, dx - (\tan^{-1} y - x) \, dy = 0 \]
   Write the integrating factor of the following differential equation
   \[ (1 + y^2) \, dx - (\tan^{-1} y - x) \, dy = 0 \]

3. यदि \( A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} \) है, तो \( A^{-1} \) लिखिए।
   If \( A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} \), then write \( A^{-1} \).

4. यदि \( \vec{a} = \hat{i} + 2\hat{j} - \hat{k} \), \( \vec{b} = 2\hat{i} + \hat{j} + \hat{k} \) तथा \( \vec{c} = 5\hat{i} - 4\hat{j} + 3\hat{k} \) है, तो \( (\vec{a} + \vec{b}) \cdot \vec{c} \) का मान ज्ञात कीजिए।
   If \( \vec{a} = \hat{i} + 2\hat{j} - \hat{k} \), \( \vec{b} = 2\hat{i} + \hat{j} + \hat{k} \) and \( \vec{c} = 5\hat{i} - 4\hat{j} + 3\hat{k} \), then find the value of \( (\vec{a} + \vec{b}) \cdot \vec{c} \).

5. निम्न रेखा के दिक्-अनुपातों को लिखिए:
   \[ x = -3, \quad \frac{y-4}{3} = \frac{2-z}{1}. \]
   Write the direction ratios of the following line:
   \[ x = -3, \quad \frac{y-4}{3} = \frac{2-z}{1}. \]
Write the value of \( \vec{a} \cdot (\vec{b} \times \vec{a}) \).

 SECTION B

Question numbers 7 to 19 carry 4 marks each.

**Question 7:**

Evaluate:

\[ \int (\sqrt{\cot x} + \sqrt{\tan x}) \, dx \]

**Solution:**

\[ \int (\sqrt{\cot x} + \sqrt{\tan x}) \, dx \]

**Question 8:**

Find:

\[ \int \frac{x^3 - 1}{x^3 + x} \, dx \]

**Solution:**

\[ \int \frac{x^3 - 1}{x^3 + x} \, dx \]

Show that four points A, B, C and D whose position vectors are \( 4\hat{i} + 5\hat{j} + \hat{k} \), \( -\hat{j} - \hat{k} \), \( 3\hat{i} + 9\hat{j} + 4\hat{k} \) and \( 4(-\hat{i} + \hat{j} + \hat{k}) \) respectively are coplanar.
\[
\frac{x - a + d}{\alpha} \times \frac{y - a}{\alpha} = \frac{z - a - d}{\alpha + \delta} \quad \text{and} \quad \frac{x - b + c}{\beta - \gamma} \times \frac{y - b}{\beta} = \frac{z - b - c}{\beta + \gamma}
\]

Show that the following two lines are coplanar

\[
\frac{x - a + d}{\alpha - \delta} = \frac{y - a}{\alpha} = \frac{z - a - d}{\alpha + \delta} \quad \text{and} \quad \frac{x - b + c}{\beta - \gamma} - \frac{y - b}{\beta} = \frac{z - b - c}{\beta + \gamma}
\]

**OR**

Find the acute angle between the plane \(5x - 4y + 7z - 13 = 0\) and the \(y\)-axis.

11. A and B बारी-बारी से एक पासे को उछालते हैं जब तक कि उनमें से कोई एक पासे पर चार से बड़ी संख्या प्राप्त कर खेल को जीत नहीं लेता। यदि A खेल को शुरू करे, तो B के जीतने की प्रायिकता क्या है?

**अध्ययन**

एक पासे को तीन बार उछालने के परीक्षण में घटना A तथा B को निम्न प्रकार से परिभाषित किया गया है:
A: पहली उछाल पर संख्या 5 और दूसरी उछाल पर संख्या 6 प्रकट होना।
B: तीसरी उछाल पर संख्या 3 या 4 प्रकट होना।
यदि A का घटना होना दिया गया है, तो घटना B की प्रायिकता ज्ञात कीजिए।

A and B throw a die alternatively till one of them gets a number greater than four and wins the game. If A starts the game, what is the probability of B winning?

**OR**

A die is thrown three times. Events A and B are defined as below:
A: 5 on the first and 6 on the second throw.
B: 3 or 4 on the third throw.
Find the probability of B, given that A has already occurred.
Using the properties of determinants, prove the following:

\[
\begin{vmatrix}
1 & x & x+1 \\
2x & x(x-1) & x(x+1) \\
3x(1-x) & x(x-1)(x-2) & x(x+1)(x-1)
\end{vmatrix} = 6x^2 (1-x^2)
\]

If \( x = \alpha \sin 2t \) (1 + \cos 2t) and \( y = \beta \cos 2t \) (1 - \cos 2t), show that
\[
\frac{dy}{dx} = \frac{\beta}{\alpha} \tan t.
\]

Find:
\[
\frac{d}{dx} \cos^{-1} \left( \frac{x - x^{-1}}{x + x^{-1}} \right)
\]

Find the derivative of the following function \( f(x) \) w.r.t. \( x \), at \( x = 1 \)
\[
\cos^{-1} \left( \sin \sqrt{\frac{1+x}{2}} \right) + x^x
\]
मान ज्ञात कीजिएः

\[
\int_{0}^{\frac{\pi}{2}} \frac{2\sin x}{2\sin x + 2\cos x} \, dx
\]

मान ज्ञात कीजिएः

\[
\int_{0}^{3/2} |x \cdot \cos(\pi x)| \, dx
\]

Evaluate:

\[
\int_{0}^{\frac{\pi}{2}} \frac{2\sin x}{2\sin x + 2\cos x} \, dx
\]

OR

Evaluate:

\[
\int_{0}^{3/2} |x \cdot \cos(\pi x)| \, dx
\]

एक अन्य क्रम के लिए धन एकजित करने हेतु, तीन विद्यालयों A, B तथा C के विद्यार्थियों ने एक प्रदर्शनी में तीन मोहरे में आयोजित की। उन्होंने इस प्रदर्शनी में पुनः चक्रित कागज से बने कागज के ढेरे, स्क्रीन-पुस्तकें एवं हलका रंगीन पेस्टल कागज क्रमशः ₹ 20, ₹ 15 और ₹ 5 प्रति इकाई से बेचा है। विद्यालय A ने 25 कागज के ढेरे, 12 स्क्रीन-पुस्तकें एवं 34 हलके रंगीन पेस्टल कागज बेचे, विद्यालय B ने 22 कागज के ढेरे, 15 स्क्रीन-पुस्तकें एवं 28 हलके रंगीन पेस्टल कागज और विद्यालय C ने 26 कागज के ढेरे, 18 स्क्रीन-पुस्तकें एवं 36 हलके रंगीन पेस्टल कागज बेचे। आयोजकों का प्रयोग करके, यह ज्ञात कीजिए कि इन विद्यार्थियों ने प्रति विद्यालय कितना धन अर्जित किया?

इस प्रकार की प्रदर्शनी के आयोजन से विद्यार्थियों में किन पूँजीय का ज्ञान होता है?
To raise money for an orphanage, students of three schools A, B and C organised an exhibition in their locality, where they sold paper bags, scrap-books and pastel sheets made by them using recycled paper, at the rate of ₹ 20, ₹ 15 and ₹ 5 per unit respectively. School A sold 25 paper bags, 12 scrap-books and 34 pastel sheets. School B sold 22 paper bags, 15 scrap-books and 28 pastel sheets while School C sold 26 paper bags, 18 scrap-books and 36 pastel sheets. Using matrices, find the total amount raised by each school.

By such exhibition, which values are generated in the students?

\[ \begin{pmatrix} a & b \\ 1 & 2 \end{pmatrix} \]

Prove that:

\[ 2 \tan^{-1}\left( \frac{a - b}{a + b} \tan \frac{x}{2} \right) = \cos^{-1}\left( \frac{a \cos x + b}{a + b \cos x} \right) \]

OR

Solve the following for \( x \)

\[ \tan^{-1}\left( \frac{x - 2}{x - 3} \right) + \tan^{-1}\left( \frac{x + 2}{x + 3} \right) = \frac{\pi}{4}, \ |x| < 1. \]
19. यदि $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$ है, तो $A^2 - 5A + 16I$ ज्ञात कीजिए।

If $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$, find $A^2 - 5A + 16I$.

खण्ड स

SECTION C

प्रश्न संख्या 20 से 26 तक प्रत्येक प्रश्न के 6 अंक हैं।

Question numbers 20 to 26 carry 6 marks each.

20. एक महाविद्यालय के 40% विद्यार्थियों छात्रावास में रहते हैं और बाकी के बाहर रहते हैं। वर्ष के अन्त में छात्रावास में रहने वाले 50% छात्र A ग्रेड (श्रेणी) में उत्तीर्ण होते हैं तथा बाहर रहने वालों में से केवल 30% छात्र ही A ग्रेड (श्रेणी) प्राप्त करते हैं। वर्ष के अन्त में एक छात्र न्यायिक चुना जाता है और पाया जाता है कि उसने A ग्रेड (श्रेणी) प्राप्त किया है या उन्होंने नहीं।

40% students of a college reside in hostel and the remaining reside outside. At the end of the year, 50% of the hostlers got A grade while from outside students, only 30% got A grade in the examination. At the end of the year, a student of the college was chosen at random and was found to have gotten A grade. What is the probability that the selected student was a hosteler?
The postmaster of a local post office wishes to hire extra helpers during the Deepawali season, because of a large increase in the volume of mail handling and delivery. Because of the limited office space and the budgetary conditions, the number of temporary helpers must not exceed 10. According to past experience, a man can handle 300 letters and 80 packages per day, on the average, and a woman can handle 400 letters and 50 packets per day. The postmaster believes that the daily volume of mail and packages will be no less than 3400 and 680 respectively. A man receives ₹225 a day and a woman receives ₹200 a day. How many men and women helpers should be hired to keep the pay-roll at a minimum? Formulate an LPP and solve it graphically.

The postmaster of a local post office wishes to hire extra helpers during the Deepawali season, because of a large increase in the volume of mail handling and delivery. Because of the limited office space and the budgetary conditions, the number of temporary helpers must not exceed 10. According to past experience, a man can handle 300 letters and 80 packages per day, on the average, and a woman can handle 400 letters and 50 packets per day. The postmaster believes that the daily volume of mail and packages will be no less than 3400 and 680 respectively. A man receives ₹225 a day and a woman receives ₹200 a day. How many men and women helpers should be hired to keep the pay-roll at a minimum? Formulate an LPP and solve it graphically.

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If the function \( f : \mathbb{R} \to \mathbb{R} \) be defined by \( f(x) = 2x - 3 \) and \( g : \mathbb{R} \to \mathbb{R} \) by \( g(x) = x^3 + 5 \), then find the value of \((f \circ g)^{-1}(x)\).

OR

Let \( A = \mathbb{Q} \times \mathbb{Q} \), where \( \mathbb{Q} \) is the set of all rational numbers, and \( * \) be a binary operation defined on \( A \) by
\[
(a, b) * (c, d) = (ac + bd, ad + bc),
\]
for all \((a, b), (c, d) \in A\).

Find
(i) the identity element in \( A \).
(ii) the invertible element of \( A \).

If the function \( f(x) = 2x^3 - 9mx^2 + 12m^2x + 1 \), where \( m > 0 \) attains its maximum and minimum at \( p \) and \( q \) respectively such that \( p^2 = q \), then find the value of \( m \).

Find the differential equation for all the straight lines, which are at a unit distance from the origin.

OR

Show that the differential equation \( 2xy \frac{dy}{dx} = x^2 + 3y^2 \) is homogeneous and solve it.
Find the direction ratios of the normal to the plane, which passes through the points \((1, 0, 0)\) and \((0, 1, 0)\) and makes angle \(\frac{\pi}{4}\) with the plane \(x + y = 3\). Also find the equation of the plane.

Using integration, find the area of the region bounded by the lines \(y = 2 + x, \ y = 2 - x\) and \(x = 2\).