CBSE CLASS-XII
MATHS
QUESTIONS WITH SOLUTIONS

Time : 3 Hrs. **Max. Marks : 100**

General Instruction :

(i) All questions are compulsory.

(ii) The question paper consists of **29** questions divided into four section A, B, C and D. Sections A comprises of **4** questions of **one mark** each, Section B comprises of **8** questions of **two marks** each, Section C comprises of **11** questions of **four marks** each and Section D comprises of **6** questions of **six marks** each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 3 questions of four marks each and 3 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is **not** permitted. You may ask for logarithmic tables, if required.

SECTION - A

Question numbers 1 to 4 carry 1 mark each

1. If for any 2 × 2 square matrix A, A(adj A) = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of |A|.

Sol. $A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

by using property

$A(\text{adj } A) = |A|\, I_n$

$\Rightarrow |A|\, I_n = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$\Rightarrow |A|\, I_n = 8\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \Rightarrow |A| = 8$

2. Determine the value of 'k' for which the following function is continuous at x = 3 :

$$f(x) = \begin{cases} \dfrac{(x+3)^2 - 36}{x-3} & , x \neq 3 \\ k & , x = 3 \end{cases}$$

Sol. $\lim\limits_{x \to 3} f(x) = \lim\limits_{x \to 3} \dfrac{(x+3)^2 - 36}{x-3}$

$= \lim\limits_{x \to 3} \dfrac{(x+3-6)(x+3+6)}{(x-3)}$

$= 12$

given that f(x) is continuous at x = 3

$\therefore \lim\limits_{x \to 3} f(x) = f(3)$

$\Rightarrow k = 12$

3. Find : $\displaystyle\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x}\,dx$

Sol. $\displaystyle\int \frac{\sin^2 x - \cos^2 x}{\sin x \cdot \cos x}\,dx$

$= 2\displaystyle\int \frac{-\cos 2x}{\sin 2x}\,dx$

$= -2\displaystyle\int \cot 2x\,dx$

$= \dfrac{-2\log|\sin 2x|}{2} + C$

$= -\log|\sin 2x| + C$

4. Find the distance between the planes $2x - y + 2z = 5$ and $5x - 2{\cdot}5y + 5z = 20$.

Sol. $2x - y + 2z = 5$ …(1)

$5x - 2{\cdot}5\,y + 5z = 20$

or $2x - y + 2z = 8$ …(2)

Distance between plane (1) & (2)

$= \left|\dfrac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}}\right| = \left|\dfrac{3}{\sqrt{9}}\right| = 1$

SECTION - B

Question numbers 5 to 12 carry 2 marks each

5. If A is a skew-symmetric matrix of order 3, then prove that det A = 0.

Sol. Let $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ be a skew symmetric matrix of order 3

$\therefore\ |A| = \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$

$|A| = -a(0 + bc) + b(ac - 0)$

$= -abc + abc = 0$ Proved

6. Find the value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in $[-\sqrt{3}, 0]$.

Sol. $f(x) = x^3 - 3x$

(i) f(x) being a polynomial is continuous on $[-\sqrt{3}, 0]$

(ii) $f(-\ 3) = f(0) = 0$

(iii) $f'(x) = 3x^2 - 3$ and this exist uniquely on $[-\sqrt{3}, 0]$

$\therefore$ f(x) is derivable on $(-\sqrt{3}, 0)$

$\therefore$ f(x) satisfies all condition of Rolle's theorem

$\therefore$ There exist atleast one $c \in (-\sqrt{3}, 0)$ where $f'(c) = 0$

$\Rightarrow\ 3c^2 - 3 = 0$

$\Rightarrow\ c = \pm 1 \ \Rightarrow\ c = -1$

7. The volume of a cube is increasing at the rate of 9 cm^3/s. How fast is its surface area increasing when the length of an edge is 10 cm ?

Sol. Assumed volume of cube = V

Given that, $\frac{dV}{dt} = 9\ cm^3/sec$

$$\frac{dA}{dt} = ?$$

$$l = 10\ cm$$

$$\frac{dV}{dt} = \frac{d}{dt}(l)^3 = 9 \Rightarrow 3l^2 \frac{dl}{dt} = 9$$

$$\frac{dl}{dt} = \frac{3}{l^2} \qquad \ldots\ldots\ldots(1)$$

Now $\frac{dA}{dt} = \frac{d}{dt}(6\,l^2) = 12\,l\,\frac{dl}{dt} = 12\,l \times \frac{3}{l^2}$ (form (1))

$$= \frac{36}{l} = \frac{36}{10} = 3.6\ cm^2/\ sec$$

8. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on **R**.

Sol. $f(x) = x^3 - 3x^2 + 6x - 100$

$f'(x) = 3x^2 - 6x + 6$

$f'(x) = 3\ (x^2 - 2x + 2)$

$f'(x) = 3\ [(x - 1)^2 + 1]$

$f'(x) > 0$ for all $x \in R$

So, f(x) is increasing on R.

9. The x-coordinate of a point on the line joining the points P(2, 2, 1) and Q(5, 1, –2) is 4. Find its z-coordinate.

Sol.

k 1

P R Q

(2, 2, 1) (5, 1, –2)

Let R divides PQ in the ratio k : 1

$$R\left(\frac{5k+2}{k+1}, \frac{k+2}{k+1}, \frac{-2k+1}{k+1}\right)$$

given x co-ordinate of R = 4

$$\therefore \quad \frac{5k+2}{k+1} = 4$$

$$\Rightarrow \quad k = 2$$

$$\therefore \quad \text{z co-ordinate} = \frac{-2(2)+1}{2+1} = -1$$

10. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events.

Sol. $A = \{2, 4, 6\}$ $\quad P(A) = \frac{3}{6} = \frac{1}{2}$

$B = \{1, 2, 3\}$

$A \cap B = \{2\}$ $\quad P(B) = \frac{3}{6} = \frac{1}{2}$

$P(A \cap B) = \frac{1}{6}$

Here, $P(A)\,P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Since, $P(A \cap B) \neq P(A)\,P(B)$, so events A and B are not independent events.

11. Two tailors, A and B, earn ₹ 300 and ₹ 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.

Sol.

	Tailor A	Tailor B	Minimum Total No.
No. of shirts	6	10	60
No. of trousers	4	4	32
Wage	Rs 300/day	Rs 400/day	

Let tailor A and tailor B works for x days and y days respectively

$\therefore\ x \geq 0, \quad y \geq 0$

minimum number of shirts = 60

$\therefore\ 6x + 10y \geq 60$

$3x + 5y \geq 30$

minimum no. of trousers = 32

$\therefore\ 4x + 4y \geq 32$

$\Rightarrow\ x + y \geq 8$

Let z be the total labour cost

$\therefore\ z = 300x + 400y$

$\therefore$ The given L.P. Problem reduces to : $z = 300x + 400y$

$x \geq 0,\ y \geq 0,\ 3x + 5y \geq 30$ and $x + y \geq 8$

12. Find : $\int \frac{dx}{5 - 8x - x^2}$

Sol. $\int \frac{dx}{5 - 8x - x^2}$

$= -\int \frac{dx}{\{(x+4)^2 - 21\}}$

$= \int \frac{dx}{(\sqrt{21})^2 - (x+4)^2}$

$= \frac{1}{2\sqrt{21}} \log \left| \frac{\sqrt{21} + (x+4)}{\sqrt{21} - (x+4)} \right| + C$

SECTION - C

Question numbers 13 to 23 carry 4 marks each

13. If $\tan^{-1}\frac{x-3}{x-4}+\tan^{-1}\frac{x+3}{x+4}=\frac{\pi}{4}$, then find the value of x.

Sol. $\tan^{-1}\left[\dfrac{\frac{x-3}{x-4}+\frac{x+3}{x+4}}{1-\left(\frac{x^2-9}{x^2-16}\right)}\right]=\frac{\pi}{4}$

$$\frac{(x+4)(x-3)+(x+3)(x-4)}{(x^2-16)-(x^2-9)}=1$$

$2x^2-24=-7$

$2x^2=-7+24$

$x^2=\frac{17}{2}$

$x=\pm\sqrt{\frac{17}{2}}$

14. Using properties of determinants, prove that

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}=(a-1)^3$$

OR

Find matrix A such that

$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix}A=\begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$$

Sol. Use $R_1=R_1-R_2$; $R_2=R_2-R_3$; $R_3=R_3$

L.H.S.

$$=\begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

$$=\begin{vmatrix} (a-1)(a+1) & (a-1) & 0 \\ 2(a-1) & (a-1) & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

Taking common $(a-1)^2$

$$=(a-1)^2\begin{vmatrix} (a+1) & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

$=(a-1)^2[(a+1)(1-0)-1(2-0)]$

$=(a-1)^2[(a+1)-2]$

$=(a-1)^3$

$=$ R.H.S.

OR

Let matrix A is

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$$

$$\begin{bmatrix} 2a - c & 2b - d \\ a & b \\ -3a + 4c & -3b + 4d \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$$

Comparing both the sides

$2a - c = -1$,

$2b - d = -8$

And $a = 1$, $b = -2$

After solving we get

$c = 3$, $d = -4$

So, $A = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$

15. If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$.

OR

If $e^y(x + 1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

Sol. We have $x^y + y^x = a^b$.

Differentiating w.r.t. x, we get $\frac{d}{dx}(x^y) + \frac{d}{dx}(y^x) = 0$. …(1)

Let $u = x^y$ $\therefore$ $\log u = y \log x$

$\Rightarrow \frac{1}{u}\frac{du}{dx} = y.\frac{1}{x} + \log x.\frac{dy}{dx}$; $\Rightarrow \frac{du}{dx} = u\left(\frac{y}{x} + \log x \frac{dy}{dx}\right)$

or $\frac{d}{dx}(x^y) = x^y\left(\frac{y}{x} + \log x \frac{dy}{dx}\right)$ …(2)

Let $v = y^x$ $\therefore$ $\log v = x \log y$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = x.\frac{1}{y}\frac{dy}{dx} + \log y \,.\, 1$; $\Rightarrow \frac{dv}{dx} = v\left(\frac{x}{y}\frac{dy}{dx} + \log y\right)$

or $\frac{d}{dx}(y^x) = y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right)$ …(3)

Using (2) and (3) in (1),

we get $x^y\left(\frac{y}{x} + \log x \frac{dy}{dx}\right) + y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0$. …(4)

$\Rightarrow (x^y \log x + xy^{x-1})\frac{dy}{dx} = -(y^x \log y + yx^{y-1})$ or $\frac{dy}{dx} = -\frac{y^x \log y + yx^{y-1}}{x^y \log x + xy^{x-1}}$

OR

Let $e^y (x + 1) = 1$

$$e^y(1) + (x + 1)\, e^y \frac{dy}{dx} = 0$$

$$\Rightarrow \quad (x + 1) \frac{dy}{dx} + 1 = 0 \qquad \ldots(1)$$

Again differentiating w.r.t. x

$$\therefore \quad (x + 1) \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right).1 = 0$$

$$\frac{d^2y}{dx^2} = -\frac{\frac{dy}{dx}}{(x+1)}$$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} \cdot \frac{dy}{dx} \qquad \text{[equation (1)]}$$

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

16. Find : $\displaystyle\int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)}\, d\theta$

Sol. $\displaystyle\int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)}\, d\theta$

$$= \int \frac{\cos\theta \; d\theta}{(4+\sin^2\theta)(5-4(1-\sin^2\theta))}$$

$$\int \frac{\cos\theta \; d\theta}{(\sin^2\theta+4)(4\sin^2\theta+1)}$$

Put $\sin\theta = t$

$\cos\theta \, d\theta = dt$

$$\therefore \; I = \int \frac{1}{(4+t^2)(1+4t^2)} dt$$

Consider

$$\frac{1}{(4+t^2)(1+4t^2)} = \frac{At+B}{4+t^2} + \frac{Ct+D}{1+4t^2}$$

$1 = (At + B)(1 + 4t^2) + (Ct + D)(4 + t^2)$

$= At + B + 4At^3 + 4Bt^2 + 4Ct + Ct^3 + 4D + Dt^2$

$= (4A + C)t^3 + (4B + D)t^2 + (A + 4C)t + (B + 4D)$

$4A + C = 0 \Rightarrow C = -4A$

$4B + D = 0 \Rightarrow D = -4B$

$A + 4C = 0 \Rightarrow A = -4C$

$B + 4D = 1$

By solving we get $A = 0,\ B = -\frac{1}{15},\ C = 0,\ D = \frac{4}{15}$

$$\therefore \ \frac{1}{(4+t^2)(1+4t^2)} = \frac{-1/15}{4+t^2} + \frac{4/15}{1+4t^2}$$

$$\therefore I = -\frac{1}{15}\int \frac{1}{4+t^2}dt + \frac{4}{15}\times\frac{1}{4}\int \frac{1}{\frac{1}{4}+t^2}dt$$

$$= - -\frac{1}{15}\times\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right) + \frac{1}{15}\times\frac{1}{\frac{1}{2}}\tan^{-1}\left(\frac{t}{1/2}\right) + C$$

$$= -\frac{1}{30}\tan^{-1}\left(\frac{t}{2}\right) + \frac{2}{15}\tan^{-1}(2t) + C$$

$$= \frac{2}{15}\tan^{-1}(2\sin\theta) - \frac{1}{30}\tan^{-1}\left(\frac{\sin\theta}{2}\right) + C$$

17. Evaluate : $\displaystyle\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}dx$

OR

Evaluate : $\displaystyle\int_1^{4} \{|x-1|+|x-2|+|x-4|\}dx$

Sol. $I = \displaystyle\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}dx$ …(1)

$$I = \int_0^{\pi} \frac{(\pi - x)(-\tan x)}{-\sec x - \tan x}dx$$

$I = \displaystyle\int_0^{\pi} \frac{(\pi - x)\tan x}{\sec x + \tan x}dx$ …(2)

Adding (1) & (2)

$$2I = \int_0^{\pi} \frac{\pi\tan x}{\sec x + \tan x}dx$$

$$\Rightarrow \ 2I = 2\pi\int_0^{\pi/2} \frac{\tan x}{\sec x + \tan x}dx$$

$$\left\{\because \int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx \ \ \text{whenever} f(2a-x) = f(x)\right\}$$

$$I = \pi\int_0^{\pi/2} \frac{\tan x}{\sec x + \tan x}dx$$

$$I = \pi\int_0^{\pi/2} \frac{\tan x(\sec x - \tan x)}{\sec^2 x - \tan^2 x}dx$$

$$I = \pi\int_0^{\pi/2} (\sec x\tan x - \tan^2 x)\,dx$$

$$= \pi\int_0^{\pi/2} (\sec x\tan x - \sec^2 x + 1)\,dx$$

$I = \pi\left[\sec x - \tan x + x\right]_0^{\pi/2}$

$= \pi\left[\lim_{x\to\frac{\pi}{2}^-}(\sec x - \tan x) + \frac{\pi}{2} - \sec 0\right]$

$= \pi \lim_{x\to\frac{\pi}{2}^-} \frac{1-\sin x}{\cos x} + \frac{\pi^2}{2} - \pi$

$= \pi \lim_{x\to\left(\frac{\pi}{2}\right)^-} \frac{1-\sin^2 x}{\cos x(1+\sin x)} + \frac{\pi^2}{2} - \pi$

$= \frac{\pi^2}{2} - \pi$

OR

Let f (x) = | x – 1| +| x – 2 | + |x – 4|

We have three critical points x = 1, 2, 4

(i) when $x < 1$

(ii) when $1 \le x < 2$

(iii) when $2 \le x < 4$

(iv) when $x \ge 4$

$f(x) = -(x-1)-(x-2)-(x-4)$	if	$x < 1$
$= (x-1)-(x-2)-(x-4)$	if	$1 \le x < 2$
$= (x-1)+(x-2)-(x-4)$	if	$2 \le x < 4$
$= (x-1)+(x-2)+(x-4)$	if	$x \ge 4$

$\therefore f(x) = -3x + 7$	if	$x < 1$
$= -x + 5$	if	$1 \le x < 2$
$= x + 1$	if	$2 \le x < 4$
$= 3x - 7$	if	$x \ge 4$

$\therefore\ I = \int_1^4 f(x)\,dx$

$\therefore\ I = \int_1^2 f(x)dx + \int_2^4 f(x)dx$

$\therefore I = \int_1^2 (-x+5)dx + \int_2^4 (x+1)dx$

$= \left[-\frac{x^2}{2} + 5x\right]_1^2 + \left[\frac{x^2}{2} + x\right]_2^4$

$= \left(-\frac{4}{2}+10\right) - \left(-\frac{1}{2}+5\right) + \left(\frac{16}{2}+4\right) - \left(\frac{4}{2}+2\right)$

$= 8 - \frac{9}{2} + 12 - 4 = \frac{23}{2}$

18. Solve the differential equation $(\tan^{-1} x - y)dx = (1 + x^2)\, dy$.

Sol. We have

$$\frac{dy}{dx} = \frac{\tan^{-1} x - y}{1+x^2}$$

$$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{\tan^{-1} x}{1+x^2}$$

$$\text{I.F} = e^{\int \frac{1}{1+x^2}dx} = e^{\tan^{-1} x}$$

$$y.e^{\tan^{-1} x} = \int \frac{\tan^{-1} x}{1+x^2} \times e^{\tan^{-1}} dx$$

Put $t = \tan^{-1} x$

$$dt = \frac{1.dx}{1+x^2}$$

$$= \int \underset{\mathrm{I}}{t} \;.\; \underset{\mathrm{II}}{e^t}\, dt$$

$$= t.e^t - \int 1.e^t\, dt$$

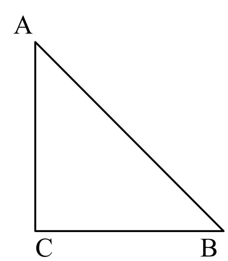
$$y.e^{\tan^{-1} x} = t\, e^t - e^t + c$$

$$y.e^{\tan^{-1} x} = (\tan^{-1} x - 1)\, e^{\tan^{-1} x} + c$$

$$y = \tan^{-1} x - 1 + ce^{\tan^{-1} x}$$

19. Show that the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively, are the vertices of a right-angled triangle, Hence find the area of the triangle.

Sol. $\overrightarrow{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}$



$\overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}$

$\overrightarrow{CA} = \hat{i} - 3\hat{j} - 5\hat{k}$

$\overrightarrow{BC} \cdot \overrightarrow{CA} = 0$

$\overrightarrow{BC} \perp \overrightarrow{CA}$

$\therefore$ ΔABC is a right angled triangle

$$\Delta = \frac{1}{2} |\overrightarrow{BC}||\overrightarrow{AC}|$$

$$\Delta = \frac{1}{2}\sqrt{4+1+1}\,\sqrt{1+9+25}$$

$$= \frac{1}{2}\sqrt{6}\,\sqrt{35}$$

$$= \frac{1}{2}\sqrt{210}$$

20. Find the value of λ, if four points with position vectors $3\hat{i}+6\hat{j}+9\hat{k}$, $\hat{i}+2\hat{j}+3\hat{k}$, $2\hat{i}+3\hat{j}+\hat{k}$ and $4\hat{i}+6\hat{j}+\lambda\hat{k}$, are coplanar.

Sol. We have

P.V. of A = $3\hat{i}+6\hat{j}+9\hat{k}$

P.V. of B = $\hat{i}+2\hat{j}+3\hat{k}$

$\overrightarrow{AB} = -2\hat{i} - 4\hat{j} - 6\hat{k}$

$\overrightarrow{AC} = -\hat{i} - 3\hat{j} - 8\hat{k}$

$\overrightarrow{AD} = \hat{i} + (\lambda - 9)\hat{k}$

Now $\quad \overrightarrow{AB}.(\overrightarrow{AC} \times \overrightarrow{AD}) \Rightarrow \begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & (\lambda - 9) \end{vmatrix} = 0$

$\Rightarrow \; -2(-3\lambda + 27) + 4(-\lambda + 9 + 8) - 6(0 + 3) = 0$

$\Rightarrow \; 6\lambda - 54 - 4\lambda + 68 - 18 = 0$

$2\lambda - 4 = 0$

$\lambda = 2$

$\because \; \overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are coplanar and so the points A, B, C and D are coplanar.

21. There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean and variance of X.

Sol. X denote sum of the numbers so, X can be 4, 6, 8, 10, 12

X	Number on card	P(x)	X P(x)	X^2 P(x)
4	(1, 3)	$\frac{1}{4} \times \frac{1}{3} \times 2 = \frac{1}{6}$	2/3	8/3
6	(1, 6)	$\frac{1}{4} \times \frac{1}{3} \times 2 = \frac{1}{6}$	1	6
8	(3, 5) or (1, 7)	$\frac{1}{4} \times \frac{1}{3} \times 2 + \frac{1}{4} \times \frac{1}{3} \times 2 = \frac{1}{3}$	8/3	64/3
10	(3, 7)	$\frac{1}{4} \times \frac{1}{3} \times 2 = \frac{1}{6}$	5/3	50/3
12	(5, 7)	$\frac{1}{4} \times \frac{1}{3} \times 2 = \frac{1}{6}$	2	24

Mean $= \sum X\, P(x) = 8$

Variance $= \sum X^2\, P(x) - (\sum X\, P(x))^2 = \frac{212}{3} - 64 = \frac{20}{3}$

22. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance? Is regularity required only in school? Justify your answer.

Sol. Let E_1 be students having 100% attendance

E_2 be students having irregular attendance

E be students having A grade

$P(E_1) = \frac{30}{100}$ $\quad P(E_2) = \frac{70}{100}$

$P\left(\frac{E}{E_1}\right) = \frac{70}{100} \times \frac{30}{100} = 21\%$

$P\left(\frac{E}{E_2}\right) = \frac{10}{100} \times \frac{70}{100} = 7\ \%$

By Baye's theorem,

$$\text{So, } P\left(\frac{E_1}{E}\right) = \frac{P(E_1)\,P\left(\frac{E}{E_1}\right)}{P(E_1)\ P\left(\frac{E}{E_1}\right) + P(E_2)\ P\left(\frac{E}{E_2}\right)} = \frac{\frac{30}{100} \times \frac{21}{100}}{\frac{30}{100} \times \frac{21}{100} + \frac{70}{100} \times \frac{7}{100}} = \frac{63}{63+49} = \frac{63}{112}$$

23. Maximize $Z = x + 2y$

Subject to the constraints

$x + 2y \geq 100$

$2x - y \leq 0$

$2x + y \leq 200$

$x, y \geq 0$

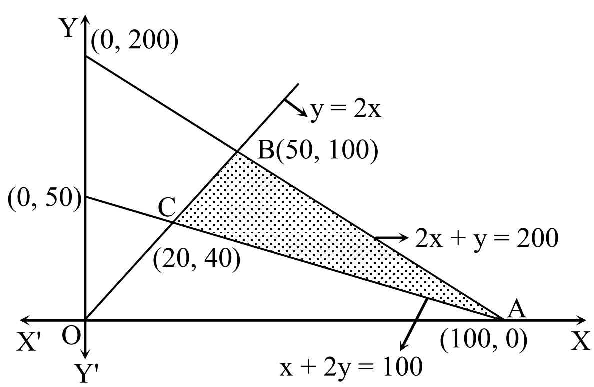
Solve the above LPP graphically.

Sol. $x + 2y = 100$

$2x - y = 0$ ……(1)

$2x + y = 200$ ……(2)

$x = 0, y = 0$ ……(3)



Corner points are A (100, 0), B(50, 100), C(20, 40)

Corner points	$Z = x + 2y$	
A(100, 0)	100	← minimum
B(50, 100)	250	← maximum
C(20, 40)	100	← minimum

Maximum at point B and maximum value 250

SECTION - D

Question numbers 24 to 29 carry 6 marks each

24. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations

$x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

Sol. Product of the matrices

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8\, I_3$$

Hence $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}^{-1} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$

Now, given system of equations can be written in matrix form, as follows

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix}$$

$x = \frac{24}{8}$, $y = \frac{-16}{8}$, $z = \frac{-8}{8}$

$x = 3$, $y = -2$, $z = -1$

25. Consider $f : R - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.

OR

Let A = Q × Q and let * be a binary operation on A defined by (a, b) * (c, d) = (ac, b + ad) for (a, b), (c, d) ∈ A. Determine, whether * is commutative and associative.Then, with respect to * on A

(i) Find the identity element in A.

(ii) Find the invertible elements of A.

Sol. $f(x) = \frac{4x+3}{3x+4}$, $x \in R - \left\{-\frac{4}{3}\right\}$

f is one – one →

Let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$ and $f(x_1) = f(x_2)$

$\Rightarrow \frac{4x_1+3}{3x_1+4} = \frac{4x_2+3}{3x_2+4}$

$\Rightarrow 12x_1 x_2 + 16 x_1 + 9 x_2 + 12 = 12x_1 x_2 + 9 x_1 + 16 x_2 + 12$

$\Rightarrow 7x_1 = 7x_2 \quad \Rightarrow x_1 = x_2$

$\therefore$ f is one – one

f is onto →

Let $k \in R - \left\{\frac{4}{3}\right\}$ be any number

$f(x) = k \Rightarrow \frac{4x+3}{3x+4}$

$\Rightarrow 4x + 3 = 3kx + 4k$

$\Rightarrow x = \frac{4k-3}{4-3k}$

Also $\frac{4k-3}{4-3k} = -\frac{4}{3}$

implies – 9 = – 16 (which is impossible)

$\therefore f\left(\frac{4k-3}{4-3k}\right) = k$ i.e. f is onto

$\therefore$ The function f is invertible i.e. f^{-1} exist inverse of f

Let $f^{-1}(x) = k$

$f(k) = x$

$\Rightarrow \frac{4k+3}{3k+4} = x$

$\Rightarrow k = \frac{4x-3}{4-3x}$

$\therefore f^{-1}(x) = \frac{4x+3}{4-3x}$, $x \in R - \left\{-\frac{4}{3}\right\}$

$f^{-1}(0) = -\frac{3}{4}$

and when

$f^{-1}(x) = 2$

$\Rightarrow \frac{4x-3}{4-3x} = 2$

$\Rightarrow 4x - 3 = 8 - 6x$

$\Rightarrow 10x = 11$

$\Rightarrow x = \frac{11}{10}$

OR

(i) Let (e, f) be the identify element for *

$\therefore$ for $(a, b) \in Q \times Q$, we have

$(a, b) * (e, f) = (a, b) = (e, f) * (a, b)$

$\Rightarrow$ $(ae, af + b) = (a, b) = (ea, eb + f)$

$\Rightarrow$ $ae = a, \quad af + b = b, \quad a = ea, \quad b = eb + f$

$\Rightarrow$ $e = 1, \quad af = 0, \quad e = 1, \quad b = (1)\, b + f$

($\because$ a need not be '0')

$\Rightarrow$ $e = 1, \quad f = 0, \quad e = 1, \quad f = 0$

$\therefore$ $(e, f) = (1, 0) \in Q \times Q$

$\therefore$ (1, 0) is the identify element of A

(ii) Let $(a, b) \in Q \times Q$

Let $(c, d) \in Q \times Q$

such that

$(a, b) * (c, d) = (1, 0) = (c, d) * (a, b)$

$\Rightarrow$ $(ac, ad + b) = (1, 0) = (ca, cb + d)$

$\Rightarrow$ $ac = 1, ad + b = 0, ca = 1, cb + d = 0$

$\Rightarrow$ $c = \frac{1}{a}, d = -\frac{b}{a}, \left(\frac{1}{a}\right) b + d = 0 \ (a \neq 0)$

$\therefore$ $(c, d) = \left(\frac{1}{a}, \frac{-b}{a}\right) \ (a \neq 0)$

$\therefore$ for $a \neq 0$, $(a, b)^{-1} = \left(\frac{1}{a}, \frac{-b}{a}\right)$

26. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

Sol. If each side of square base is x and height is h then volume

$V = x^2h \quad \Rightarrow h = \frac{V}{x^2}$

S is surface area then

$S = 4hx + 2x^2 = 4\left(\frac{V}{x^2}\right)x + 2x^2$

$\Rightarrow S = \frac{4V}{x} + 2x^2$

Diff. w. r. to x

$\frac{dS}{dx} = -\frac{4V}{x^2} + 4x$ and $\frac{d^2S}{dx^2} = +\frac{8V}{x^3} + 4$

Now $\frac{dS}{dx} = 0 \Rightarrow 4x = \frac{4V}{x^2}$

$\Rightarrow x^3 = V \Rightarrow x = V^{1/3}$

at $x = V^{1/3}$, $\frac{d^2S}{dx^2} > 0$

$\Rightarrow$ S is minimum when $x = V^{1/3}$

and $h = \frac{V}{x^2} = \frac{V}{V^{2/3}} = V^{1/3} \Rightarrow x = h$

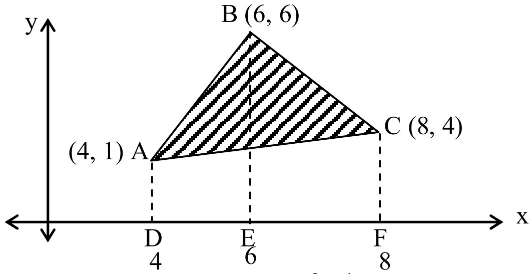
$\Rightarrow x = h$ means it is a cube

27. Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are A (4, 1), B (6, 6) and C (8, 4).

OR

Find the area enclosed between the parabola $4y = 3x^2$ and the straight line $3x - 2y + 12 = 0$.

Sol.



Equation of AB is $y - 1 = \frac{6-1}{6-4}(x-4)$

$\Rightarrow\ 2y - 2 = 5x - 20$

$\Rightarrow\ y = \frac{5x}{2} - 9$

Equation of BC is

$\Rightarrow\ y - 6 = \frac{4-6}{8-6}(x-6)$

$\Rightarrow\ y = -x + 12$

Equation of AC is

$\Rightarrow\ y - 1 = \frac{4-1}{8-4}(x-4)$

$\Rightarrow\ 4y - 4 = 3x - 12$

$\Rightarrow\ y = \frac{3x}{4} - 2$

Area of ΔABC = area ABED + area BEFC – area ADFC

$$= \int_4^6 \left(\frac{5x}{2} - 9\right) dx + \int_6^8 (-x + 12)\, dx - \int_4^6 \left(\frac{3x}{4} - 2\right) dx$$

$$= \left|\left(\frac{5x^2}{4} - 9x\right)_4^6\right| + \left|\left(\frac{-x^2}{2} + 12x\right)_6^8\right| - \left|\left(\frac{3x^2}{8} - 2x\right)_4^6\right| = 7 \text{ sq units}$$

OR

Parabola $4y = 3x^2$ …(1)

line $3x - 2y + 12 = 0$ …(2)

from (2) $y = \frac{3x + 12}{2}$

putting this value of y is (1) we get

$6x + 24 = 3x^2$

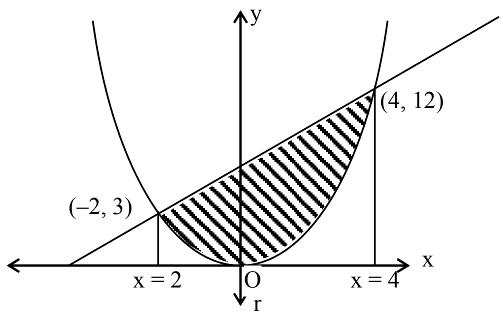
$\Rightarrow\ x = 4, -2$

when $x = 4$ then $y = 12$

$x = -2$ then $y = 3$

Required area

$$= \int_{-2}^{4} (\text{y of line})\, dx - \int_{-2}^{4} (\text{y of parabola})\, dx$$



$$= \int_{-2}^{4}\left(\frac{3x+12}{2}-\frac{3x^2}{4}\right)dx$$

$$= \frac{3}{4}\int_{-2}^{4}(8+2x-x^2)\,dx$$

$$= \frac{3}{4}\left[8x+x^2-\frac{x^3}{3}\right]_{-2}^{4} = 27 \text{ sq. units}$$

28. Find the particular solution of the differential equation $(x-y)\frac{dy}{dx} = (x+2y)$, given that y = 0 when x = 1.

Sol. $(x-y)\frac{dy}{dx} = (x+2y)$

$$\frac{dy}{dx} = \frac{x+2y}{x-y}$$

Let $y = Vx$

$$\frac{dy}{dx} = V + x\frac{dV}{dx}$$

$$\Rightarrow V + x\frac{dV}{dx} = \frac{x+2(Vx)}{x-Vx}$$

$$\Rightarrow V + x\frac{dV}{dx} = \frac{1+2V}{1-V}$$

$$\Rightarrow x\frac{dV}{dx} = \frac{1+2V-V+V^2}{1-V}$$

$$\Rightarrow \int\frac{1-V}{1+V+V^2}dV = \int\frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2}\int\left\{\frac{(2V+1)-3}{1+V+V^2}\right\}dV = \int\frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2}\left[\int\frac{2V+1}{1+V+V^2}dV - 3\int\frac{dV}{1+V+V^2}\right] = \int\frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2}\log|1+V+V^2| + \frac{3}{2}\int\frac{dV}{\left(V+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \log|x| + C$$

$$\Rightarrow -\frac{1}{2}\log|1+V+V^2| + \frac{3}{2}\,\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{V+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = \log|x| + C$$

$$\Rightarrow -\frac{1}{2}\log\left|1+\frac{y}{x}+\frac{y^2}{x^2}\right| + \sqrt{3}\tan^{-1}\left(\frac{2\frac{y}{x}+1}{\sqrt{3}}\right) = \log|x| + C$$

we have y = 0 when x = 1

$$\Rightarrow 0 + \sqrt{3}\ \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 0 + C$$

$$\Rightarrow C = \sqrt{3}\ \tan^{-1}\frac{1}{\sqrt{3}}$$

$\therefore$ Solution

$$\Rightarrow -\frac{1}{2}\log\left|1+\frac{y}{x}+\frac{y^2}{x^2}\right| + \sqrt{3}\ \tan^{-1}\left(\frac{2\frac{y}{x}+1}{\sqrt{3}}\right) = \log|x| + \sqrt{3}\ \tan^{-1}\frac{1}{\sqrt{3}}$$

29. Find the coordinates of the point where the line through the points (3, –4, –5) and (2, –3, 1), crosses the plane determined by the points (1, 2, 3), (4, 2, –3) and (0, 4, 3).

OR

A variable plane which remains at a constant distance 3p from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of triangle ABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$.

Sol. Equation of line passing through (3, – 4, – 5) and (2, – 3, 1)

$$\frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6} \qquad \ldots(1)$$

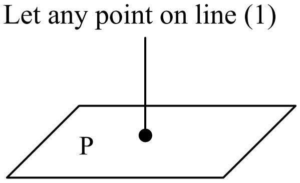
Equation of plane passing through (1, 2, 3) (4, 2, – 3) and (0, 4, 3)

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 3 & 0 & -6 \\ -1 & 2 & 0 \end{vmatrix} = 0$$

$\Rightarrow$ (x – 1) (12) – (y – 2) (– 6) + (z – 3) (6) = 0

$\Rightarrow$ 2x + y + z – 7 = 0 $\qquad$ …(2)

Let any point on line (1)



is P (– k + 3, k – 4, 6k – 5)

it lies on plane

$\therefore$ 2(– k + 3) + k – 4 + 6k – 5 – 7 = 0

5k = 10

$\Rightarrow$ k = 2

$\therefore$ P (1, –2, 7)

OR

Let the equation of plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad \ldots(1)$$

It cut the co-ordinate axes at A, B and C

$\therefore$ A (a, 0, 0), B (0, b, 0), C (0, 0, c)

Let the centroid of ΔABC be (x, y, z)

$$\therefore \quad \left(x = \frac{a}{3}, y = \frac{b}{3}, z = \frac{c}{3}\right) \qquad \ldots(2)$$

given that distance of plane (1) from origin is 3p

$$\therefore \left|\frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}\right| = 3p$$

$$\Rightarrow \quad \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{9p^2}$$

from (2)

$$\Rightarrow \quad \frac{1}{9x^2} + \frac{1}{9y^2} + \frac{1}{9z^2} = \frac{1}{9p^2}$$

$$\Rightarrow \quad \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2} \quad \text{Proved}$$