CBSE SOLVED PAPERS

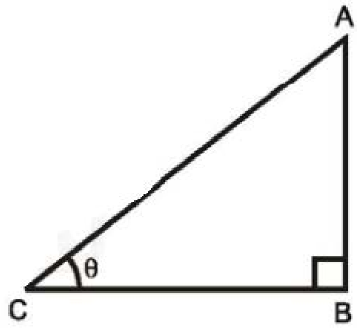
CLASS-X

MATHS

Section - A

Q1. The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3}:1$. What is the angle of elevation of the sun?

Sol: Let AB be the height of the tower and BC be the length of the shadow. Let θ be the angle of elevation of the sun.



In right angled ΔABC, $\frac{AB}{BC} = \tan\theta$

$\left(\because \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}\right)$

$\because \frac{AB}{BC} = \frac{\sqrt{3}}{1}$ (Given)

$\therefore \tan\theta = \sqrt{3}$

But $\tan\theta = \sqrt{3} = \tan 60°$

Q2. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?

Sol: Volume of a solid hemisphere = Surface area of hemisphere.

Let r be the radius of the solid hemisphere.

Volume = $\frac{2}{3}\pi r^3$

Surface area $= 3\pi r^2$

$\therefore \frac{2}{3}\pi r^3 = 3\pi r^2$

$r = \frac{3 \times 3}{2} = \frac{9}{2}$

$\therefore$ Diameter = $2r = 2 \times \frac{9}{2} = 9$ units

Q3. A number is chosen at random from the numbers -3, -2, -1, 0, 1, 2, 3. What will be the probability that square of this number is less than or equal to 1?

Sol: The squares of the numbers chosen at random are 9, 4, 1 and 0.

Out of these numbers, only 0 and **1** are less than or equal to 1.

Correspondingly, the numbers out of the given numbers whose square is less than or equal to 1 are -1, 0 and 1.

So, number of favourable cases = 3

And total number of possible outcomes = 7.

$\therefore$ Required Probability = $\frac{\text{Number of favourable cases}}{\text{Total number of possible outcomes}} = -$

Q4. If the distance between the points (4, k) and (1, 0) is 5, then what can be the possible values of k?

Sol: Distance between any two points (x_1, y_1) and (x_2, y_2) is given by

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here, $(x_1, y_1) = (4, k)$ and $(x_2, y_2) = (1, 0)$

Thus, $\sqrt{(1-4)^2 + (0-k)^2} = 5$

$\Rightarrow \sqrt{(-3)^2 + (-k)^2} = 5$

$\Rightarrow \sqrt{9 + k^2} = 5$

Squaring both sides, we get

$9 + k^2 = 25$

$\Rightarrow k^2 = 25 - 9$

$\Rightarrow k^2 = 16$

$\Rightarrow k = \pm 4$

The possible values of k are 4, -4.

Section - B

Q5. Find the roots of the quadratic equation $\sqrt{2}x^2 + 7x + 5\ \ 2 = 0$

Sol: Comparing the equation $2x^2 + 7x + 5\sqrt{2} = 0$ with $ax^2 + bx + c = 0$.

We get $a = \sqrt{2}, b = 7, c = 5\sqrt{2}$

$D = b^2 - 4ac$

$= (7)^2 - 4 \times \sqrt{2} \times 5\sqrt{2}$

$= 49 - 40 = 9$

So it has real roots.

$$x = \frac{-b \pm \sqrt{D}}{2ac}$$

Q6. Find how many integers between 200 and 500 are divisible by 8.

Sol: The integers between 200 and 500 which are divisible by 8 are 208, 216........496.

This forms an A.P. with first term a = 208 and common difference d = 8.

General term, $T_n = a + (n-1)d$

$496 = 208 + (n-1) \times 8$

$\Rightarrow 496 - 208 = 8(n-1)$

$\Rightarrow 288 = 8(n-1)$

$\Rightarrow n - 1 = \frac{288}{\ }$

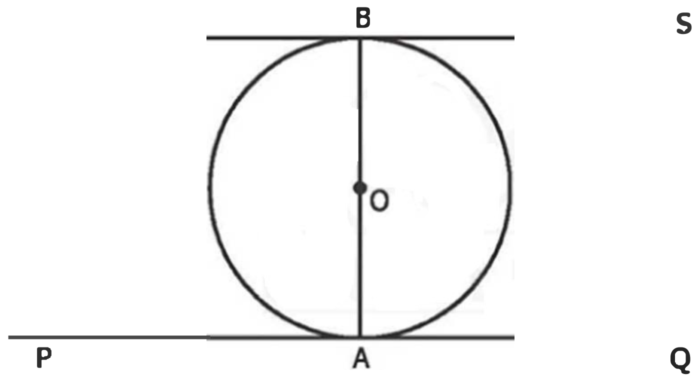
$\Rightarrow n - 1 = 36$

$\Rightarrow n = 36 + 1 = 37$

Here, there are 37 integers lying between 200 and 500 which are divisible by 8.

Q7. Prove that tangents drawn at the ends of a diameter of a circle are parallel to each other.

Sol: Let AB be the diameter of the given circle, and let PQ and RS be the tangent lines drawn to the circle at points A and B respectively. Since tangent at a point to a circle is perpendicular to the radius through the point.



$\therefore AB \perp PQ$ and $AB \perp RS$

$\Rightarrow \angle PAB = 90^{o}$ and $\angle ABS = 90^{o}$

$\Rightarrow \angle PAB = \angle ABS$

$\Rightarrow$ PQ RS [$\because \angle PAB$ and $\angle ABS$ are alternate angles]

Q8. Find the value of k for which the equation $x^2 + k(2x + k - 1) + 2 = 0$ **has real and equal roots.**

Sol: Simplifying the equation $x^2 + k(2x + k - 1) + 2 = 0$

$\Rightarrow x^2 + 2kx + k^2 - k + 2 = 0$

Comparing this equation with $ax^2 + bx + c = 0$

We get $a = 1$, $b = 2k$, $c = k^2 - k + 2$

It will have real and equal roots if $D = b^2 - 4ac = 0$

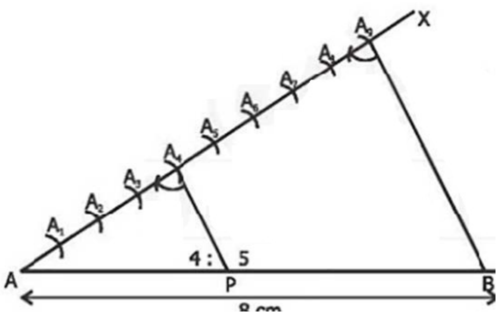
$\Rightarrow (2k)^2 - 4(1)(k^2 - k + 2) = 0$

$\Rightarrow 4k^2 - 4k^2 + 4k - 8 = 0$

$\Rightarrow 4k = 8 \Rightarrow k = 2$

Q9. Draw a line segment of length 8 cm and divide it internally in the ratio 4: 5.

Sol:



Steps of construction:

Step 1: Draw a line segment AB = 8 cm by using a ruler.

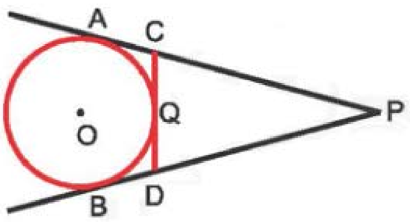
Step 2: Draw a ray making an acute angle $\angle BAX$ with AB.

Step 3: Along AX, mark off $9(= 4 + 5)$ points $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8$ and A_9 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7 = A_7A_8 = A_8A_9$

Step 4: Join BA_9

Step 5: Through A_4 draw a line PA_4 parallel to BA_9 by making an angle equal to $\angle AA_9B$ at A_4 intersecting AB at a point P. The point P so obtained is the required point, which divides AB internally in the ratio 4 : 5.

Q10. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If PA = 12 cm, QC = QD = 3 cm, then find PC + PD



Sol: Since PA and PB are tangents to the circle from external point P. Hence PA = PB

$\because$ CD is another tangent touching the circle at Q.

$\because$ CQ and AC act as tangents from external point C.

$\therefore$ CQ = AC and DQ and DB are tangents from external point D.

$\therefore$ DQ = DB.

Section - C

Q11. If m^{th} term of an A.P is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, then find the sum of its first mn terms.

Sol: Let a be the first term and d be the common difference.

To find the k^{th} term of an A.P., we use the formula $T_k = a + (k-1)d$

Given, $T_m = \frac{1}{n}$

$\Rightarrow a + (m-1)d = \frac{1}{n}$ ....(1)

And $T_n = \frac{1}{m}$

$\Rightarrow a + (n-1)d = \frac{1}{m}$ ....(2)

Subtracting equation (2) from equation (1), we get

$(m-1-n+1)d = \frac{1}{n} - \frac{1}{m}$

$(m-n)d = \frac{m-n}{mn}$

$\Rightarrow d = \frac{1}{mn}$

Substituting the value of d in equation (1), we get

$a + \frac{(m-1)}{mn} = \frac{1}{n}$

$\Rightarrow a + \frac{m}{mn} - \frac{1}{mn} = \frac{1}{n}$

$\Rightarrow a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n}$

$$\Rightarrow a = \frac{1}{mn}$$

To find the sum of first k terms of given A.P., we use the formula

$$S_k = \frac{k}{2}[2a + (k-1)d]$$

$$\therefore S_{mn} = \frac{mn}{2}\left[\frac{2}{mn} + \frac{(mn-1)}{mn}\right]$$

$$= \frac{mn}{2}\left[\frac{2}{mn} + \frac{mn}{mn} - \frac{1}{mn}\right]$$

$$= \frac{mn}{2}\left[\frac{1}{mn} + 1\right] = \frac{1+mn}{2}$$

Q12. Find the sum of n terms of the series

$$\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+\ldots\ldots\ldots.$$

Sol: $d_1 = \left(4-\frac{2}{n}\right)-\left(4-\frac{1}{n}\right)$

$$= 4-\frac{2}{n}-4+\frac{1}{n} = -\frac{1}{n}$$

$$d_2 = \left(4-\frac{3}{n}\right)-\left(4-\frac{2}{n}\right)$$

$$= 4-\frac{3}{n}-4+\frac{2}{n} = -\frac{1}{n}$$

Thus, the given series form an A.P. with first term $\left(4-\frac{1}{n}\right)$ and common difference $-\frac{1}{n}$

Sum of n terms $= \frac{n}{2}[2a + (n-1)d]$

$$= \frac{n}{2}\left[2\left(4 - \frac{1}{n}\right) + (n-1)\left(-\frac{1}{n}\right)\right]$$

$$= \frac{n}{2}\left[8 - \frac{2}{n} - 1 + \frac{1}{n}\right]$$

$$= \frac{n}{2}\left[7 - \frac{1}{n}\right]$$

$$= \frac{7n}{2} - \frac{1}{2}$$

Hence $S_n = \frac{7n}{2} - \frac{1}{2}$

Q13. If the equation $(1+m^2)x^2+2mcx+c^2-a^2=0$ has equal roots then show that $c^2=a^2 1+m^2$

Sol: Comparing the equation with $Ax^2+Bx+C=0$

We get $A=(1+m^2), B=2mc, C=c^2-a^2$

The equation has equal roots if $D=B^2-4AC=0$

$\Rightarrow (2mc)^2-4(1+m^2)(c^2-a^2)=0$

$\Rightarrow 4m^2c^2-4c^2+4a^2-4m^2c^2+4m^2a^2=0$

$\Rightarrow c^2=a^2+a^2m^2$

$\Rightarrow c^2=a^2(1+m^2)$

Hence proved.

Q14. The $\frac{3}{4}$th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel.

Sol: Radius of conical vessel, r = 5 cm

Height of conical vessel, h = 24 cm

Volume of conical vessel $=\frac{1}{3}\pi r^2 h$

$$=\frac{1}{3}\times\frac{22}{7}\times 5\times 5\times 24 \text{ cm}^3$$

Volume of water contained in conical vessel

$=\frac{3}{4}\times\frac{1}{3}\times\frac{22}{7}\times 5\times 5\times 24=\frac{22}{7}\times 5\times 5\times 6 \text{ cm}^3$

Radius of cylindrical vessel, R = 10 cm

Let height of cylindrical vessel = H cm

Volume of cylindrical vessel $= \pi R^2 H$

According to the given equation, Volume of water in conical vessel = Volume of water in cylindrical vessel.

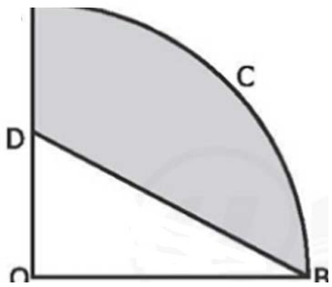
$$\therefore \frac{22}{7} \times 5 \times 5 \times 6 = \frac{22}{7} \times 10 \times 10 \times H$$

$$\therefore H = \frac{\frac{22}{7} \times 5 \times 5 \times 6}{\frac{22}{7} \times 10 \times 10}$$

$$= \frac{3}{2} = 1.5\,\text{cm}$$

Therefore the height of water in cylindrical vessel=1.5 cm

Q15. In the given figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm.
If OD = 2 cm, find the area of the shaded region.



Sol: From the figure, Area of shaded region = Area of the quadrant OACB - Area of ΔDOB

Area of quadrant OACB $= \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5$ (Radius, r of circle = 3.5 cm)

$$= \frac{1}{4} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10}$$

$$= \frac{77}{8} \text{cm}^2$$

Area of $\Delta DOB = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times OB \times OD$$

$$= \frac{1}{2} \times 3.5 \times 2$$

$$= \frac{1}{2} \times \frac{35}{10} \times 2 = \frac{7}{2}$$

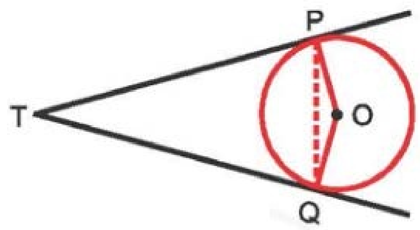
Area of the shaded region $= \frac{77}{8} - \frac{7}{2}$

$$= \frac{77 - 28}{8} = \frac{49}{8}$$

$$= 6.12 \text{ cm}^2$$

Thus, area of the shaded region = 6.12 cm^2

Q16. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



Sol: The lengths of tangents drawn from an external point to a circle are equal.

$\therefore TP = TQ$

$\Rightarrow \Delta TPQ$ is an isosceles triangle.

$\Rightarrow \angle TPQ = \angle TQP$

In ΔTPQ, we have

$\angle TPQ + \angle TQP + \angle PTQ = 180^\circ$ (ASP)

$$\Rightarrow \angle TPQ + \angle TPQ + \angle PTQ = 180^o$$

$$\Rightarrow 2\angle TPQ = 180 - \angle PTQ$$

$$\Rightarrow \angle TPQ = 90^o - \frac{1}{2}\angle PTQ \text{ ....(i)}$$

Since, $OP \perp TP$

$$\therefore \angle OPT = 90^o$$

$$\angle OPQ + \angle TPQ = 90^o$$

$$\angle OPQ = 90^o - \angle TPQ \text{ ....(ii)}$$

From (i) and (ii), we get

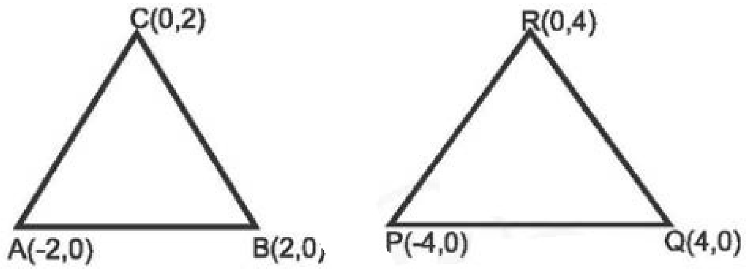
$$\angle OPQ = 90^o - \left(90^o - \frac{1}{2}\angle PTQ\right)$$

$$\angle OPQ = \frac{1}{2}\angle PTQ$$

$$\Rightarrow \angle PTQ = 2\angle OPQ$$

Hence proved.

Q17. Show that ΔABC, where $A(-2,0)$, $B(2,0)$, $C(0,2)$ and ΔPQR where $P(-4,0)$, $Q(4,0)$, $R(0,4)$ are similar triangles.



Sol: By distance formula, in ΔABC,

$$AB = \sqrt{(2+2)^2 + (0-0)^2} = \sqrt{4^2 + 0} = \sqrt{16} = 4$$

$$BC = \sqrt{(0-2)^2 + (2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$CA = \sqrt{(-2-0)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

Also, in ΔPQR,

$$PQ = \sqrt{(4+4)^2 + (0-0)^2} = \sqrt{8^2 + 0} = \sqrt{64} = 8$$

$$QR = \sqrt{(0-4)^2 + (4-0)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$RP = \sqrt{(-4-0)^2 + (0-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$\frac{AB}{PQ} = \frac{4}{8} = \frac{1}{2}$$

$$\frac{BC}{QR} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$$

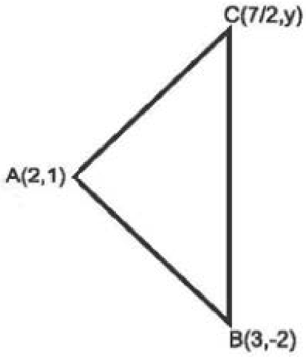
$$\frac{CA}{RP} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

$\Delta ABC \sim \Delta PQR$ (If the corresponding sides of two triangles are proportional, then they are similar.)

Q18. The area of a triangle is 5 square units. Two of its vertices are (2,1) and (3,-2). If the third vertex is $\left(\frac{7}{2}, y\right)$. Find the value of y.

Sol: Area of $\Delta ABC = 5$ square units



The area of triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by:

Area =

$$\frac{1}{2}\left|(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)\right|$$

Let A(2,1) , B(3,-2) and $C\left(\frac{7}{2}, y\right)$

$$\therefore 5 = \frac{1}{2}\left|(2\times -2 - 3\times 1) + \left(3\times y - \frac{7}{2}\times -2\right) + \left(\frac{7}{2}\times 1 - 2\times y\right)\right|$$

$$\Rightarrow 10 = \left|-4 - 3 + 3y + 7 + \frac{7}{2} - 2y\right|$$

$$\Rightarrow 10 = \left|y + \frac{7}{2}\right|$$

$$\Rightarrow \pm 10 = y + \frac{7}{2}$$

$$\Rightarrow y + \frac{7}{2} = 10 \text{ or } y + \frac{7}{2} = -10$$

$$\therefore y = 10 - \frac{7}{2} \text{ or } y = -10 - \frac{7}{2}$$

$$\Rightarrow y = \frac{20 - 7}{2} \text{ or } \frac{-20 - 7}{2}$$

Thus, $y = \frac{13}{2}$ or $\frac{-27}{2}$

Q19. Two different dice are thrown together. Find the probability that the numbers obtained

(i) Have a sum less than 7

(ii) Have a product less than 16.

(iii) is a doublet of odd numbers

Sol: If two dice are thrown together, the sample space is

$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$

$(2,1),(2,2),(2,3),(2,4)(2,5),(2,6)$

$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$

$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$

$(5,1),(52),(5,3),(5,4),(5,5),(5,6)$

$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$\therefore$ Total number of outcomes = ⌣

(i) Have a sum less than 7

The favourable cases are $\{(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4),$
$(3,1),(3,2),(3,3),(4,1),(4,2),(5,1)\}$

So, number of favourable cases = 15

P(Have a sum less than 7) $= \frac{15}{36} = \frac{5}{12}$

(ii) Have a product less than 16.

The favourable cases are $(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$
$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\ (3,1),(3,2),(3,3),(3,4),(3,5),(4,1)(4,2),(4,3),$
$(5,1),(5,2),(5,3)$

So, number of favorable cases = 23

P(Have a product less than 16) $= \frac{23}{36}$

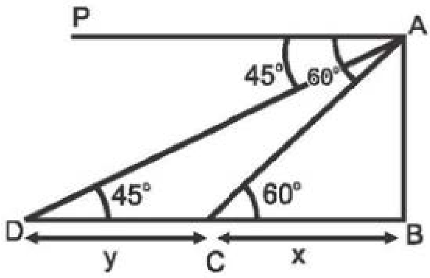
(iii) is a doublet of odd numbers

The favourable cases are $(1,1),(3,3),(5,5)$

So, number of favourable cases = 3

P(is a doublet of odd numbers) $= \frac{3}{36} = \frac{1}{12}$

Q20. A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/h.



Sol: Let AB be the height of the cliff and it is given AB = 150 m.

Let C and D be two positions of the boat such that $\angle PAD = 45^\circ$ and $\angle PAC = 60^\circ$

Now $\angle PAD = \angle ADB = 45^\circ$ (Alternate angles)

And $\angle PAC = \angle ACB = 60^\circ$ (Alternate angles)

Let BC = x and CD = y

In ΔABC, $\frac{AB}{BC} = \tan 60^o$

$$\Rightarrow \frac{150}{x} = \tan 60^o$$

$$\Rightarrow \frac{x}{150} = \cot 60^o = \frac{1}{\sqrt{3}}$$

$$\therefore x = \frac{150}{\sqrt{3}} = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{150\sqrt{3}}{3} = 50\sqrt{3}\,m \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

In ΔABD, $\frac{AB}{BD} = \tan 45^o$

$$\frac{150}{x+y} = 1$$

$$\Rightarrow x + y = 150 \ldots\ldots\ldots\ldots\ldots(2)$$

Substituting value of x from equation (1) in equation (2) we get,

$$50\sqrt{3} + y = 150$$

$$\therefore y = 150 - 50\sqrt{3}$$

$\therefore$ Distance travelled from point C to D is $\left(150 - 50\sqrt{3}\right)m$

Time required = 2 min $= \frac{2}{60}h = \frac{1}{30}h$

$$\therefore \text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{150 - 50\sqrt{3}}{1/30}$$

$= 30(150 - 50\sqrt{3})$

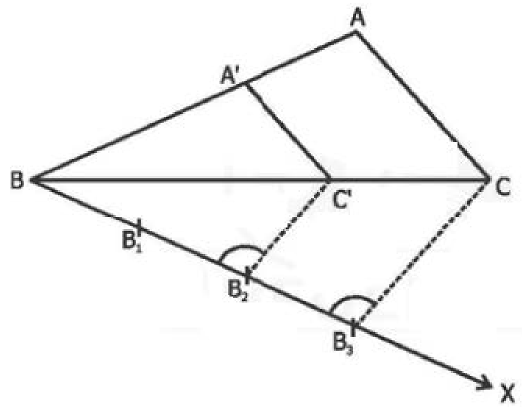
$= 30(150 - 86.6)$

$= 30 \times 63.4 = 1902 \text{ m/h}$

Section - D

Q21. Construct an isosceles triangle with base 8 cm and altitude 4 cm. Construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of the isosceles triangle.

Sol:



Steps of construction:

Step 1: Draw BC = 8 cm, with B as centre and radius 4 cm mark and arc. With C as centre and radius 4 cm, cut the previous arc at A. Then AB= AC = 4 cm.

Step 2: ΔABC is the required isosceles triangle.

Step 3: Below, BC make an acute angle $\angle CBX$

Step 4: Along BX, mark three points B_1, B_2 and B_3 such that

$$BB_1 = B_1B_2 = B_2B_3$$

Step 5: Join B_3C

Step 6: Since we have to construct a triangle each of whose sides is two third of the corresponding sides of ΔABC, so take two parts out of three equal parts on BX i.e. from B_2, draw $B_2C' \parallel B_3C$, and meeting BC at C'.

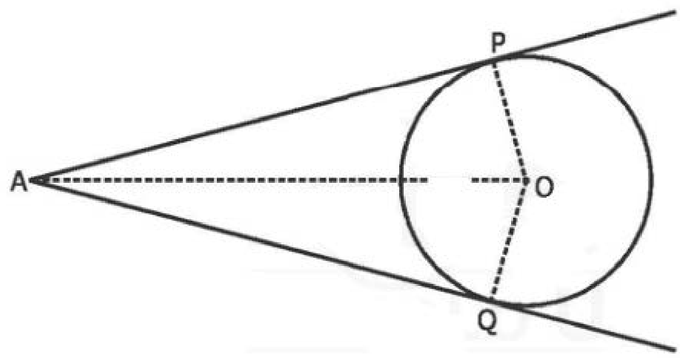
Step 7: From C', draw $A'C' \parallel AC$, meeting AB at A'.

$A'BC'$ is the required triangle whose each side is two third of the corresponding side of ΔABC.

Q22. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Sol: Given: AP and AQ are two tangents drawn from an external point A to a circle $C(O,r)$

To prove: AP =AQ



Construction: Join OP, OQ and OA.

Proof: In order to prove that AP = AQ, we shall prove that $\Delta OPA \cong \Delta OQA$

Since a tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore OP \perp AP$ and $OQ \perp AQ$

$\Rightarrow \angle OPA = \angle OQA = 90^{o}$ ....(i)

Now in right triangles OPA and OQA, we have

$\angle OPA = \angle OQA = 90^{o}$ [From (i)]

$OP = OQ$ (radii of circle)

$OA = OA$ (common)

$\therefore \Delta OPA \cong \Delta OQA$ (R.H.S. rule)

$\Rightarrow AP = AQ$ (Corresponding parts of congruent triangles are equal)

Hence the lengths of tangents drawn from an external point to a circle are equal.

Q23. The ratio of the sums of first m and first n terms of an A.P. is $m^2 : n^2$. Show that the ratio of its m^{th} and n^{th} terms is $(2m-1):(2n-1)$

Sol: The sum of k terms of an AP is given by $S_k = \frac{k}{2}[2a + (k-1)d]$

where a is the first term and d is the common difference.

Given $\frac{S_m}{S_n} = \frac{\frac{m}{2}[2a + (m-1)d]}{\frac{n}{2}[2a + (n-1)d]} = \frac{m^2}{n^2}$

i.e. $\frac{m[2a + (m-1)d]}{n[2a + (n-1)d]} = \frac{m^2}{n^2}$

$\Rightarrow \frac{2a + (m-1)d}{2a + (n-1)d} = \frac{m}{n}$

$\Rightarrow \{2a + (m-1)d\}n = \{2a + (n-1)d\}m$

$\Rightarrow 2an + n(m-1)d = 2am + m(n-1)d$

$\Rightarrow 2a(n-m) = d[mn - m - mn + n]$

$\Rightarrow 2a(n-m) = d(n-m)$

$\Rightarrow d = 2a$

$$\therefore \frac{T_m}{T_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+(m-1)2a}{a+(n-1)2a} = \frac{a(1+2m-2)}{a(1+2n-2)} = \frac{2m-1}{2n-1}$$

Hence proved!

Q24. Speed of a boat in still water is 15 km/h. It goes 30 km upstream and returns back at the same point in 4 hours 30 minutes. Find the speed of the stream.

Sol: Let the speed of the stream be x km/hr.

Speed in downstream $= (15+x)\text{km/hr}$

Speed in upstream $= (15-x)\text{km/hr}$

Time taken by the boat to go 30 km upstream $= \frac{30}{15-x}$ hours

Time taken by the boat to return 30 km downstream $= \frac{30}{15+x}$ hours

It is given that the boat returns to the same point in 4 hours 30 minutes.

$$\therefore \frac{30}{15-x} + \frac{30}{15+x} = \frac{9}{2}$$

$$\Rightarrow \quad \frac{30(15+x)+30(15-x)}{(15-x)(15+x)}=\frac{9}{2}$$

$$\Rightarrow \quad \frac{450+30x+450-30x}{225-x^2}=\frac{9}{2}$$

$$\Rightarrow \quad \frac{900}{225-x^2}=\frac{9}{2}$$

$$\Rightarrow 1800=2025-9x^2$$

$$\Rightarrow 9x^2=225$$

$$\Rightarrow x^2=\frac{225}{9}=25$$

$$\Rightarrow x=\pm 5$$

But the speed of the stream can never be negative. Hence, the speed of the stream is 5 km/hr.

Q25. If $a \neq b \neq 0$, prove that the points $(a,a^2),(b,b^2),(0,0)$ will not be collinear.

Sol: Let Δ be the area of the triangle formed by the points $(a,a^2),(b,b^2)$ and $(0,0)$

The area of triangle with vertices $(x_1,y_1),(x_2,y_2)$ and (x_3,y_3) is given by

$$\text{Area} = \frac{1}{2}\left|(x_1y_2-x_2y_1)+(x_2y_3-x_3y_2)+(x_3y_1-x_1y_3)\right|$$

$$\Delta=\frac{1}{2}\left|\left(a\times b^2-b\times a^2\right)+\left(b\times 0-0\times b^2\right)+\left(0\times a^2-a\times 0\right)\right|$$

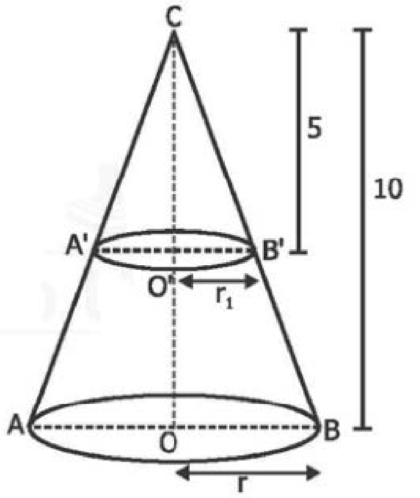
$$=\frac{1}{2}\left|ab^2-a^2b\right| \qquad \because a\neq 0, b\neq 0$$

$$\therefore \Delta=\frac{1}{2}\left|ab^2-a^2b\right|\neq 0$$

Hence the points $(a,a^2),(b,b^2),(0,0)$ are not collinear.

Q26. The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the volumes of the two parts.

Sol: Let CAB be a cone of height 10 cm and base radius r. Suppose it is cut by a plane parallel to the base of the cone at point O', Let O'A' = r_1 and CO' = 5 cm.



Clearly, $\Delta CO'A' \sim \Delta COA$

$$\therefore \frac{CO}{CO'} = \frac{OA}{O'A'} \Rightarrow \frac{10}{5} = \frac{r}{r_1}$$

$$\Rightarrow \frac{r}{r_1} = 2 \Rightarrow r = 2r_1$$

Volume of cone $CA'B' = \frac{1}{3}\pi r_1^2 \times 5 = \frac{5}{3}\pi r_1^2$

Volume of frustum $A'B'BA = \frac{1}{3}\pi\left(r^2 + r_1^2 + rr_1\right)(OO')$

$$= \frac{1}{3}\pi\left[(2r_1)^2 + r_1^2 + 2r_1.r_1\right] \times 5$$

$$= \frac{5}{3}\pi\left(4r_1^2 + r_1^2 + 2r_1^2\right)$$

$$= \frac{35\pi r_1^2}{3}$$

$$\frac{\text{Volume of cone}}{\text{Volume of frustrum}} = \frac{\frac{5}{3}\pi r_1^2}{\frac{35}{3}\pi r_1^2} = \frac{5}{3}\pi r_1^2 \times \frac{3}{35}\pi r_1^2$$

Thus, the ratio of volumes of two parts is 1 : 7.

Q27. Peter throws two different dice together and finds the product of the two numbers obtained. Rina throws a die and squares the number obtained. Who has the better chance to get the number 25.

Sol: Sample space for two dice=

$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$

$(2,1),(2,2),(2,3),(2,4)(2,5),(2,6)$

$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$

$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$

$(5,1),(52),(5,3),(5,4),(5,5),(5,6)$

$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

Total possible outcomes = 36

The only favorable case for Peter to get 25 is pair (5,5)
[No other pair gives product 25]

Hence probability of Peter getting 25 $= \frac{1}{36}$

Rina throws a die, so possible outcomes are {1,2,3,4,5,6}

Then, she squares the numbers obtained.

So the sample space is {1,4,9,16,25,36}

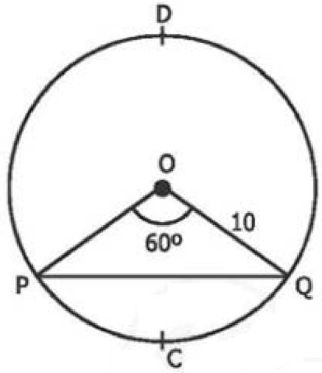
There is only one way of getting 25.

Hence probability of Rina getting 25 = $\frac{1}{6}$

As $\frac{1}{6} > \frac{1}{36}$

So, Rina has a better chance to get 25.

Q28. A chord PQ of a circle of radius 10 cm subtends an angle of $60°$ at the centre of circle. Find the area of major and minor segments of the circle.



Sol: Area of $\Delta OPQ = r^2 \sin\frac{\theta}{2}\cos\frac{\theta}{2}$, where r = 10 m, $\theta = 60^{\circ}$

$$= (10)^2 \sin 30^{\circ} \cos 30^{\circ}$$

$$= 100 \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= 25\sqrt{3}$$

Area of sector OPCQO $= \frac{\theta}{360}\pi r^2$

$$= \frac{60}{360} \times \frac{22}{7} \times 10 \times 10$$

$$= \frac{1100}{21} cm^2$$

Area of minor segment PCQP = Area of sector OPCQO - Area of ΔOPQ

$$= \frac{1100}{21} - 25\sqrt{3}$$

$$= 52.38 - 43.3 = 9.08\ cm^2$$

Area of circle $= \pi r^2$

$$= \frac{22}{7} \times 10 \times 10 = \frac{2200}{7} = 314.28\ cm^2$$

Area of major segment PQDP = Area of circle - Area of minor segment

$$= 314.28 - 9.08 = 305.20\ cm^2$$

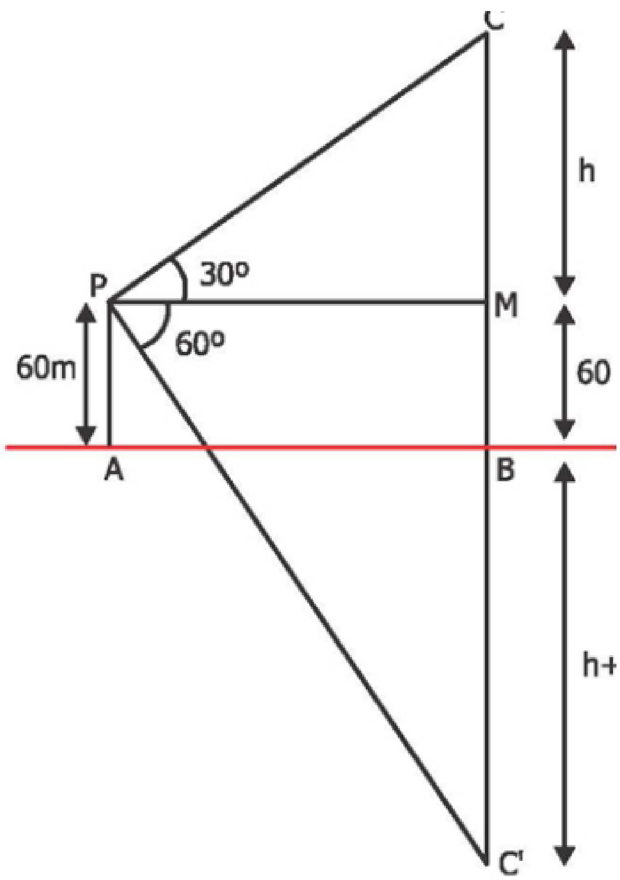
Q29. The angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30^{o} and the angle of depression of its shadow in water of lake is $60°$. Find the height of the cloud from the surface of water.

Sol: Let AB be the surface of the lake and P be the point of observation such that AP = 60 m. Let C be the position of the cloud and C'be its reflection in the lake. Then $CB = C'B$

Let PM be perpendicular from P on CB. Then $\angle CPM = 30°$ and $\angle C'PM = 60°$. Let CM = h. then $CB = h + 60^{o}$

Consequently, $C'B = h + 60^{o}$

In ΔCMP, we have $\tan 30^{o} = \frac{CM}{PM}$



$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{PM}$

$\Rightarrow PM = \sqrt{3}h$ ....(1)

In $\Delta PMC'$, we have

$\tan 60^{o} = \frac{C'M}{PM}$

$\tan 60^{o} = \frac{C'B + BM}{PM}$

$\sqrt{3} = \frac{h + 60 + 60}{PM}$

$\sqrt{3} = \frac{h + 60 + 60}{PM}$

$$\Rightarrow PM = \frac{h+120}{\sqrt{3}} \text{..................(2)}$$

From equations (1) and (2) we get

$$\sqrt{3}h = \frac{h+120}{\sqrt{3}}$$

$$\Rightarrow 3h = h + 120$$

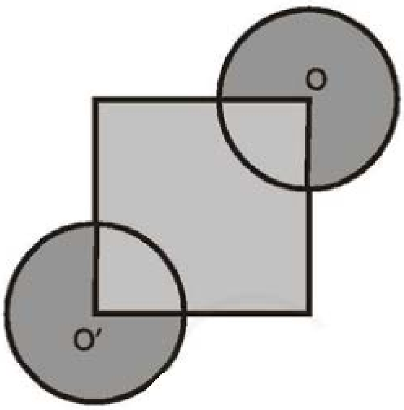
$$\Rightarrow 2h = 120$$

$$\Rightarrow h = 60$$

Now, $CB = CM + MB = h + 60 = 60 + 60 = 120$

Hence, the height of the cloud from the surface of the lake is 120 m.

Q30. In the given figure, the side of square is 28 cm and radius of each circle is half of the length of the side of the square where O and O' are centres of the circles. Find the area of shaded region.

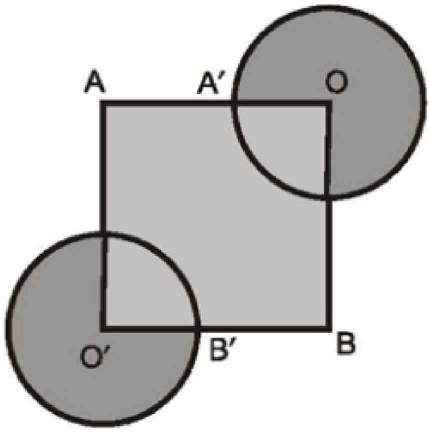


Sol: Given AO = 28 cm

Radius of circles = OA' = O'B' = 14 cm

Area of the square AOBO' $= (\text{side})^2$

$= 28 \times 28 = 784\ cm^2$



Area of the circle with centre O $= \pi r^2$

$= \frac{22}{7} \times 14 \times 14 = 616\ cm^2$

Similarly, Area of circle with centre $O' = 616\ cm^2$

Area of quadrant $= \frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times 616 = 154\ cm^2$

$\therefore$ Area of shaded region = Area of square + 2 (Area of circle) - 2 (area of quadrant)

$= 784 + 2(616) - 2(154)$

$= 784 + 1232 - 308 = 1708\ cm^2$

Q31. In a hospital, used water is collected in a cylindrical tank of diameter 2 m and height 5 m. After recycling, this water is used to irrigate a park of hospital whose length is 25 m and breadth is 20 m. If tank is filled completely then what will be the height of standing water used for irrigating the park. Write your views on recycling of water.

Sol: Diameter of cylindrical tank = 2m

$\therefore$ Radius of cylindrical tank = 1 m

Height of cylindrical tank = 5m

Volume of water in cylindrical tank = $\pi r^2 h = \frac{22}{7} \times 1 \times 1 \times 5 = \frac{110}{7} m^3$

Length of the park = 25 m

Breadth of the park = 20 m

Let height of standing water in the park = h m

Volume of water in the park $= 25 \times 20 \times h$

[Park is of cuboidal shape.]

$\therefore$ Volume of water in cylindrical tank = Volume of water in park

$\frac{110}{7} = 25 \times 20 \times h$

$\therefore h = \frac{110}{7 \times 20 \times 25}$

$= \frac{11}{350} = 0.0314 \text{ m}$

Thus, the height of water used for irrigating the park = 0.0314 m

Recycling of Water: It is the need of hour. Our water sources are becoming increasingly polluted because of failure to impose regulations on huge industries and most individuals' casual approach to disposing of their wastewater. The need to recycle our wastewater is becoming critical as water shortages spread through the world – and not just the developing parts.