QUANT TECHNIQUES STRAIGHT FROM SERIAL CAT TOPPER BYJU

INDEX

1)Power Cycle

The last digit of a number of the form a^b falls in a particular sequence or order depending on the unit digit of the number (a) and the power the number is raised to (b). The power cycle of a number thus depends on its' unit digit.

Consider the power cycle of 2

$2^1=2$, $2^5=32$
$2^2=4$ $2^6=64$
$2^3=8$ $2^7=128$
$2^4=16$ $2^8=256$

As it can be observed, the unit digit gets repeated after every 4th power of 2. Hence, we can say that 2 has a power cycle of 2,4,8,6 with frequency 4.

This means that, a number of the form

2^{4k+1} will have the last digit as 2 2^{4k+2} will have the last digit as 4
2^{4k+3} will have the last digit as 8 2^{4k+4} will have the last digit as 6 (where k=0, 1, 2, 3…)

This is applicable not just for 2, but for all numbers ending in 2.

Therefore to find the last digit of a number raised to any power, we just need to know the power cycle of digits from 0 to 9, which are given below

Unit digit	Power cycle	Frequency
0	0	1
1	1	1
2	2,4,8,6	4
3	3,9,7,1	4
4	4,6	2
5	5	1
6	6	1
7	7,9,3,1	4
8	8,4,2,6	4
9	9,1	2

For example

1) Find the remainder when 3^{75} is divided by 5.

1) Express the power in the form, 4k+x where x=1, 2, 3, 4. In this case 75 = 4k+3.
2) Take the power cycle of 3 which is 3,9,7,1. Since the form is 4k+3, take the third digit in the cycle, which is 7
Any number divided by 5, the remainder will be that of the unit digit divided by 5. Hence the remainder is 2.
Sometimes, you may get a question in the term of variables, where you need to substitute values to get the answer in the fastest way possible.
For example,

2) Find the unit digit of

Put n=1, the problem reduces to $7^{3^{\wedge}4}$, which is $7^{81.}$
Since 81=4k+1, take the first digit in the power cycle of 7, which is 7.

3) What is the first non zero integer from the right in $8330^{1957} + 8370^{1982}$?

a) 3 b) 1 c) 9 d) none of these

8370^{1982} will end with more number of zeroes so we need to consider only the first part. Rightmost non-zero integer of the expression will be = unit digit of 833^{1957}
= *unit digit of $3^{1957.}$* Since 1957=4k+1, take the first digit in the power cycle of 3, which is 3.

4) If N = $(13)^{1!+2!+3!+..+13!} + (28)^{1!+2!+3!..+28!} + (32)^{1!+2!+3!+...+32!} + (67)^{1!+2!+3!+......+67!}$, then the unit digit of N is

(a) 4 (b) 8 (c) 2 (d) none of these

Based on Power Cycle
After 4! Every number is of the form 4k+4 , here we need to check the nature of the power till 4!
Every term's power is of the form 4k+1.
So taking the first digit from the power cycles of 3,8,2, and 7 we will get the unit digit as (3+8+2+7 = ..0).
Ans = 0

2) Useful technique to find the last 2 digits of any expression of the form a^b

Depending on the last digit of the number in question, we can find the last two digits of that number.
We can classify the technique to be applied into 4 categories

TYPE	METHOD	EXAMPLES
1) **Numbers ending in 1**	The last digit is always 1. The 2^{nd} last digit = product of tens digit of base * unit digit of the power. In 21^{67}; 2 is the tens digit of base and 7 is the unit digit of power	1) 21^{67} =__41(2 * 7=__4_) 2) 41^{87} =__ 81(4 *7= __8_) 3) 1261^{167} =__21 (6 * 7= __2_) 4) 31^{124} =__ 21(3 * 4=__2_)
2) **Numbers ending with 5**	Last two digits: always 25 or 75	e.g.) 1555^{34} = __ 25

3) **Numbers ending in 3, 7, 9**	Change the power so that the base ends with 1 and then use the same technique as for those numbers ending with 1. eg) 3^4, 7^4&9^2 all will end in1.	e.g.) $17^{288} = (17^4)^{72}$ (taking the power 4 as 7^4 will end in 1. $(17^2 * 17^2)^{72}$ $=(\_89*\_89)^{72}$(as last 2 digits of 17^2=89) $= (\_21)^{72}$ (as last 2 digits of 89*89=21) Answer =__41(as 2*2=4)
4) **For even numbers (2,4,6,8)**	Use the pattern of the number 1024 =2^{10} i.e. *2^{10} raised to even power ends with 76 and * 2^{10}raised to odd power ends with 24.	e.g.) $2^{788} = (2^{10})^{78} * 2^8$ = __76 * __56 = __56.

It is also important to note that,
1. 76 multiplied by the last 2 digits of any power of 2 ($>2^1$)will end in the same last 2 digits as of that power of 2
E.g. 76*04 = 04, 76*08 = 08, 76*16 = 16, 76*32 = 32

2. The last two digits of x^2, $(50-x)^2$, $(50+x)^2$, $(100-x)^2$ will always be the same. For example last 2 digits of 12^2,38^2,62^2,88^2,112^2.... will all be the same (..44).
Also, last two digits of $11^2=39^2=61^2=89^2=111^2=139^2=161^2=189^2$ and so on

3.*To find the squares of numbers from 30-70 we can use the following method*

5) To find 41^2
Step1 : Difference from 25 will be first 2 digits = **16**
Step 2 : Square of the difference from 50 will be last 2 digits = **81** Answer = **1681**.

6) To find 43^2
Step1 : Difference from 25 will be first 2 digits = **18**
Step 2 :Square of the difference from 50 will be last 2 digits = **49** Answer = **1849**

4. Combining all these techniques we can find the last 2 digits for any number because every even number can be written as 2* an odd number

3) "Minimum of all" regions in Venn Diagrams

7) ***In a survey conducted among 100 men in a company, 100 men use brand A, 75 use brand B, 80 use brand C, 90 use brand D & 60 use brand E of the same product. What is the minimum possible number of men using all the 5 brands, if all the 100 men use at least one of these brands?***

Sum of the difference from 100 = (100-100) + (100-75)+(100-80)+(100-90)+(100-60) = 95
Again take the difference from 100 = **5 (answer)**

4) Similar to different Grouping in Permutation & Combination

All questions in Permutation and Combination fall into 4 categories, and if you master these 4 categories, you can understand all concepts in P&C easily.

1) Similar to Different

2) Different to Similar

3) Similar to Similar

4) Different to Different

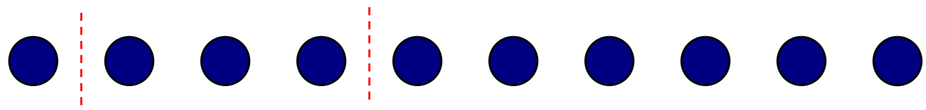
In this booklet, we will look at the first category; i.e. Similar to Different, where I will give a unique approach to the number of ways of dividing 'n' **identical** (similar) things into 'r' **distinct** (different) groups

a) NO LIMIT QUESTIONS

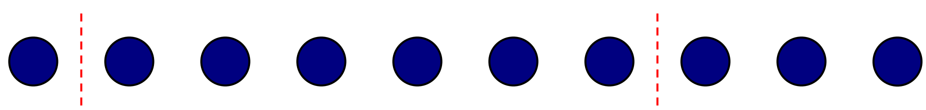
Let me explain this with an example. Suppose I have 10 identical chocolates to be divided among 3 people. The 10 chocolates need to be distributed into 3 parts where a part can have **zero or more** chocolates.

So let us represent chocolates by zeroes. The straight red lines (Call then "ones") are used to divide them into parts. So you can see that for dividing into 3 parts, you need only two lines.

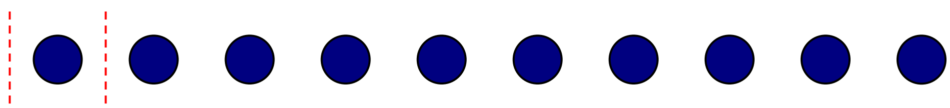
Suppose you want to give 1^{st} person 1 chocolate, 2^{nd} 3 chocolates and 3^{rd} 6 chocolates. Then you can show it as:



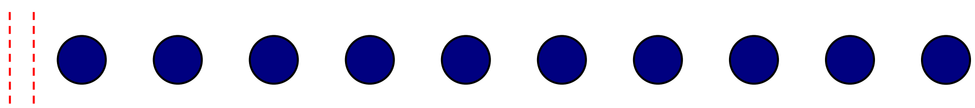
Suppose you want to give one person 1 chocolate, another person 6 chocolates and another one 3, then it can be represented as:



Now if first person gets 0, second gets 1 and third gets 9 chocolates then it can be represented as:



Now suppose you want to give first person 0, second also 0 and third all of 10 then you can show it like:



➔**So, for dividing 10 identical chocolates among 3 persons you can assume to have 12 (10 zeroes +2 ones) things among which ten are identical and rest 2 are same and of one kind.** So the number of ways in which you can distribute ten chocolates among 3 people is the same in which you can arrange 12 things, among which 10 are identical and of one kind while 2 are identical and of one kind which can be done in ——

The above situation is same as finding the number of positive integral solutions of a + b + c = 10. a, b, c is the number of chocolates given to different persons.

b) LOWER LIMIT QUESTIONS

➔Now suppose we have a restriction that the groups cannot be empty i.e. in the above example all 3 persons should get at least 1.

You have to divide ten chocolates among 3 persons so that each gets at least one. Start by giving them one each initially and take care of this condition. You can do this in just 1 way as all the chocolates are identical. Now, you are left with 7 chocolates and you have to divide them among 3 people in such that way that each gets 0 or more. You can do this easily as explained above using the zeroes and ones. Number of ways = ——

The above situation is same as finding the number of natural number solutions of a + b + c = 7. (a, b, c are the number of chocolates given to different persons)

➔ Now suppose I change the question and say that you have to divide 10 chocolates among 3 persons in such a way that the first person gets at least 1, the second at least 2 and the third at least 3.
It's as simple as the last one. First fullfill the required condition.
Give the 1^{st} person "1", second person "2" and the third person"3" chocolates and then divide the remaining 4 (10–1–2-3) chocolates among those 3
This is same as arranging 4 zeroes and 2 ones which can be done in $^6C_2$ ways.

The above situation is same as finding the number of positive integral solutions of a + b + c = 10 such that

$a \geq 1, b \geq 2, c \geq 3$. a, b, c is the number of chocolates given to different persons.In this case the answer is $^6C_2$.

8) Rajesh went to the market to buy 18 fruits in all. If there were mangoes, bananas, apples and oranges for sale then in how many ways can Rajesh buy at least one fruit of each kind?
a) $^{17}C_3$ b) $^{18}C_4$ c) $^{21}C_3$ d) $^{21}C_4$

This is a Grouping type 1 Similar to Different question, with a lower limit condition. M+B+A+O=18
Remove one from each group, therefore 4 is subtracted from both sides.
The problem changes toM+B+A+O=14. Using the logical shortcut you just learnt, the answer is based on the arrangement of 14 zeroes and 3 ones (i.e. $^{17}C_3$)

9) The number of non negative integral solutions of $x_1+x_2+x_3 \leq 10$
a) 84 b) 286 c) 220 d) none of these

By non-negative integral solutions, the conditions imply that we can have 0 and natural number values for x_1, x_2, x_3, and x_4. To remove the sign ≤ add another dummy variable x_4. The problem changes to$x_1+x_2+x_3+x_4=10$
This is an example of grouping type1 (Similar to Distinct). It is the arrangement of 10 zeroes and 3 ones.
Using the shortcut of zeroes and ones, Therefore the answer is $^{13}C_3=286$

5) APPLICATION OF FACTORIALS

A thorough understanding of Factorials is important because they play a pivotal role not only in understanding concepts in Numbers but also other important topics like Permutation and Combination

Definition of Factorial→ N! = 1x2x3x…(n-1)xn

Eg 1) 5!= 1x2x3x4x5=120 eg 2) 3!=1x2x3=6

Let us now look at the application of Factorials

I) Highest power in a factorial or in a product

Questions based on highest power in a factorial are seen year after year in CAT. Questions based on this can be categorized based on the nature of the number (prime or composite) whose highest power we are finding in the factorial, i.e

a) Highest power of a prime number in a factorial:

To find the highest power of a prime number (x) in a factorial (N!), continuously divide N by x and add all the quotients.

10) The highest power of 5 in 100!

—=20; — =4; Adding the quotients, its 20+4=24. So highest power of 5 in 100! = 24

ALTERNATIVE METHOD

— — =20+4=24 (We take upto 5^2 as it is the highest power of 5 which is less than 100)

b) Highest power of a composite number in factorial

Factorize the number into primes. Find the highest power of all the prime numbers in that factorial using the previous method. Take the least power.

11) To find the highest power of 10 in 100!

Solution: Factorize 10=5*2.

1. Highest power of 5 in 100! =24 2. Highest power of 2 in 100! =97

Therefore, the answer will be 24, because to get a 10, you need a pair of 2 and 5, and only 24 such pairs are available. So take the lesser number i.e. 24 & this is the answer.

12) Highest power of 12 in 100!

Solution: $12=2^2*3$. Find the highest power of 2^2 and 3 in 100!

First find out the highest power of 2.

Listing out the quotients: $= 50;$ $= 25;$ $= 12;$ $= 6;$ $= 3;$ $=$

Highest power of 2 = 50 + 25 + 12 + 6 + 3 + 1 = 97. So highest power of 2^2 = 48 (out of 97 2's, only 48 are 2^2)

Now for the highest power of 3. $= 33;$ $= 11;$ $= 3;$ $= 1;$

Highest power of 3 = 48. Thus, the highest power of 12 = 48

II) Number of zeros in the end of a factorial or a product

Finding the number of zeroes forms the base concept for a number of application questions. In base 10, number of zeros in the end depends on the number of 10s; i.e. effectively, on the number of 5s

In base N, number of zeroes in the end is the highest power of N in that product

13) Find the number of zeroes in 13! In base 10

Solution: We need to effectively find the highest power of 10 in 13! = Highest power of 5 in 13! As this power will be lesser. —

14) Find the number of zeroes at the end of 15! in base 12.

Solution: Highest power of 12 in 15! =highest power of 2^2 *3 in 15! =Highest power of 3 in 15!= 5

III) Number of factors of any factorial

Let us look at an example to understand how to find the number of factors in a factorial

15) Find the factors of 12!

STEP 1: Prime factorize 12! i.e. find out the highest power of all prime factors till 12 (i.e. 2,3,5,7,11).

12! = $2^{10}*3^5*5^2*7*11$

STEP2: Use the formula N=a^m*b^n(a, b are the prime factors). Then number of factors= (m+1)(n+1)

The number of factors= (10+1)(5+1)(2+1)(1+1)(1+1) =792. Answer=792

APPLICATION QUESTION BASED ON FACTORIAL

16) How many natural numbers are there such that their factorials are ending with 5 zeroes?

10! is 1*2*3*4*(5)*6*7*8*9*(2*5). From this we can see that highest power of 5 till 10! is 2. Continuing like this, 10!-14!, highest power of 5 will be 2. The next 5 will be obtained at 15 = (5*3).

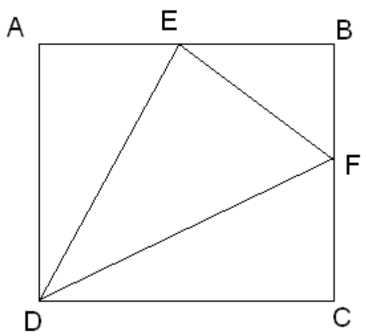
Therefore, from 15! To 19! - The highest power of 5 will be 3.

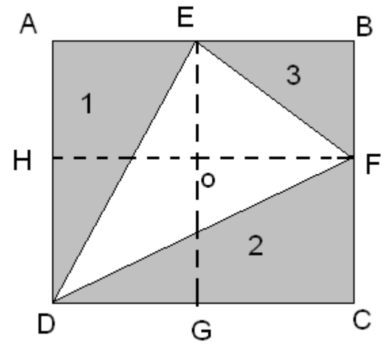
20!-24! – Highest Power = 4, In 25, we are getting one extra five, as 25=5*5. Therefore, 25! to 29!, we will get highest power of 5 as 6. The answer to the question is therefore, 0. There are no natural numbers whose factorials end with 5 zeroes.

6) USE OF GRAPHICAL DIVISION IN GEOMETRY

Let's look at a technique which will help you solve a geometry question in no time

17) ABCD is a square and E and F are the midpoints of AB and BC respectively. Find the ratio of Area(ABCD): Area(DEF)?





Lets divide the figure using dotted lines as shown in Figure B. Area of ABCD=100%. Area AEGD=50%. Then Area in shaded region 1(AED)= 25%. Similarly, DCFH=50%. Area in shaded region 2(DCF)=25%. Now Area of EOFB= 25%. .Area of shaded region 3(BEF)=12.5%. Total area outside triangle= 62.5%. Area inside triangle= 100-62.5=37.5%. required ratio = 100/37.5 = 8:3. To learn this directly from BYJU, refer the video in the CD given

7) ASSUMPTION METHOD

This involves assuming simple values for the variables in the questions, and substituting in answer options based on those values. Assumption helps to tremendously speedup the process of evaluating the answer as shown below.

18) k & $2k^2$ are the two roots of the equation $x^2 - px + q$. Find $q + 4q^2 + 6pq$ =

a) q^2 b) p^3 c) 0 d) $2p^3$

Solution: Assume an equation with roots 1&2 (k=1)
=>p (sum of roots)= 3 and q(product of roots)=2.
Substitute in $q + 4q^2 + 6pq = 54$. Look in the answer options for 54 on substituting values of p=3 and q=2. we get $2p^3 = 54$.=>Ans = $2p^3$.

19) Let 'x' be the arithmetic mean and y,z be the two geometric means between any two positive numbers. The value of —— is

a) 2 b) 3 c) – d) –

Assume a GP 1 2 4 8 implies 2 GMs between 1 and 8, i.e. y=2 and z=4.

Arithmetic Mean, —— .Substitute in ——. Answer =—— $(2^3+4^3)/(2x4x4.5) = 2$.

NOTE:- Assume a GP 1 1 1 1 then x=1, y=1 z=1. Answer on substitution=2, which will make the calculation even faster, half of the problems in Algebra can be solved using assumption. This is not direct substitution. In the next eg: see how you can use the same technique in a different question.

20) Consider the set S={2,3,4……2n+1), where n is a positive integer larger than 2007. Define X as the average of odd integers in S and Y as the average of the even integers in S. What is the value of X-Y?

a) 1 b) – c) —— d) 2008 e) 0

The question is independent of n, which is shown below.

Take n=2. Then S= {2,3,4,5). X= 4 and Y=3. X-Y =1, Take n=3. then S={2,3,4,5,6,7}. X=5 and Y=4. X-Y=1
Hence you can directly mark the answer option (a) .You can solve the question in less than 60 seconds.

There were more questions which could be solved using similar strategies. The methods given above clearly show that for someone with good conceptual knowledge and right strategies the quant section is a cakewalk.