ICSE Board Class IX Mathematics Paper 2 – Solution

SECTION - A (40 Marks)

Q. 1.

(a)
$$\frac{\sin 30^{\circ} - \sin 90^{\circ} + 2\cos 0^{\circ}}{\tan 30^{\circ} \times \tan 60^{\circ}} = \frac{\frac{1}{2} - 1 + 2}{\frac{1}{\sqrt{3}} \times \sqrt{3}} = \frac{1}{2} + 1 = \frac{3}{2} = 1\frac{1}{2}$$

(b)
$$\frac{3 \times 27^{n+1} + 9 \times 3^{n-1}}{8 \times 3^{3n} - 5 \times 27^{n}} = \frac{3 \times 3^{3n+3} + 9 \times 3^{3n-1}}{8 \times 3^{3n} - 5 \times 3^{3n}}$$
$$= \frac{3^{3n} \left(3 \times 3^{3} + 9 \times 3^{-1}\right)}{3^{3n} \left(8 - 5\right)}$$
$$= \frac{3 \times 27 + 9 \times \frac{1}{3}}{3}$$
$$= \frac{81 + 3}{3}$$
$$= \frac{81 + 3}{3}$$
$$= 28$$

(c) Given, $\frac{2+\sqrt{3}}{2-\sqrt{3}} = x + y\sqrt{3}$

Rationalize the denominator

$$\frac{\left(2+\sqrt{3}\right)\left(2+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)} = x + y\sqrt{3}$$
$$\Rightarrow \frac{4+3+4\sqrt{3}}{4-3} = x + y\sqrt{3}$$
$$\Rightarrow 7 + 4\sqrt{3} = x + y\sqrt{3}$$

Comparing the real and irrational parts on both sides, we get x = 7, y = 4

Q. 2.

(a)

Given that, arc AXB = $\frac{1}{2}$ arc BYC $\Rightarrow \angle AOB = \frac{1}{2} \angle BOC$ Since AOC is a straight line, $\angle AOB + \angle BOC = 180^{\circ}$ $\therefore \frac{1}{2} \angle BOC + \angle BOC = 180^{\circ}$ $\therefore \frac{3}{2} \angle BOC = 180^{\circ}$ $\therefore \angle BOC = 180^{\circ} \times \frac{2}{3} = 120^{\circ}$



(b) Given, x + y = 6; x - y = 4We know that $(x + y)^2 = (x - y)^2 + 4xy$ $(6)^2 = (4)^2 + 4xy$ $\Rightarrow 36 - 16 = 4xy$ $\Rightarrow 20 = 4xy$ $\Rightarrow xy = 5$

(c) Let,
$$\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b} = k$$

 $\Rightarrow \log a = k(b-c), \log b = k(c-a) \text{ and } \log c = k(a-b)$
Now, $A = a^a.b^b.c^c$
Taking log both sides
 $\log A = a \log a + b \log b + c \log c$
 $= a.k(b-c) + b.k(c-a) + c.k(a-b)$
 $= k[ab - ac + bc - ab + ac - bc]$
 $= k \times 0 = 0$
 $\Rightarrow \log A = 0$
 $\Rightarrow \log A = \log 1 \Rightarrow A = 1$
 $\therefore a^a.b^b.c^c = 1$
Hence Proved.

Q. 3.

(a) In \triangle ADC,

 $x + 2x + 90^\circ = 180^\circ$ (sum of all angles in a \triangle is 180°)

 \Rightarrow 3x = 90°

 \Rightarrow x = 30°

 \Rightarrow m \angle D = 30°

 \therefore m∠B = m∠D = 30° [Opposite angles of ||gm are equal]

And m $\angle A$ = m $\angle C$ = 180° - 30°

[sum of co- interior angles = 180° in a ||gm]

 \Rightarrow m \angle A = m \angle C = 150°

Thus, the angles of a parallelogram are 150° , 30° , 150° and 30° .

(b) Let x = $5.3\overline{47} = 5.34747....$ (i) Multiplying (i) by 10, we get 10x = 53.4747....(ii) Multiplying (ii) by 100, we get 1000x = 5347.47....(iii) Subtracting (ii) from (iii), we get 1000x - 10x = 5347.47.... - 53.47... $\Rightarrow 990x = 5294$ $\Rightarrow x = \frac{5294}{990}$ $\Rightarrow x = \frac{2647}{495}$ $\therefore 5.3\overline{47} = \frac{2647}{495}$



Classes	Class Marks	Days	
Rain (mm)			
0 - 10	5	0	
10 - 20	15	8	
20 - 30	25	10	
30 - 40	35	14	
40 - 50	45	20	
50 - 60	55	15	
60 - 70	65	8	
70 - 80	75	7	
80 - 90	85	6	
90 - 100	95	4	
100 - 110	105	0	

(c) While drawing frequency polygon, we represent each class by its mid-value.

The frequency polygon is as follows:



(d)

Q. 4.

(a) Each exterior angle of first polygon = $\frac{360^{\circ}}{n-1}$

Each exterior angle of second polygon = $\frac{360^{\circ}}{n+2}$

$$\frac{360^{\circ}}{n-1} - \frac{360^{\circ}}{n+2} = 6$$

$$\Rightarrow \frac{1}{n-1} - \frac{1}{n+2} = \frac{1}{60}$$

$$\Rightarrow \frac{n+2-n+1}{(n-1)(n+2)} = \frac{1}{60}$$

$$\Rightarrow \frac{3}{n^2+2n-n-2} = \frac{1}{60}$$

$$\Rightarrow n^2 + n - 2 = 180$$

$$\Rightarrow n^2 + n - 182 = 0$$

$$\Rightarrow n^2 + 14n - 13n - 182 = 0$$

$$\Rightarrow n(n+14) - 13(n+14) = 0$$

$$\Rightarrow (n+14)(n-13) = 0$$

$$n = -14 \text{ is not applicable}$$

$$\therefore n - 13 = 0$$

$$\therefore n = 13$$

(b)

Given: A ||gm ABCD, E and F are the midpoints of AB and DC. Line PQ meets AD, EF and BC at P, G and Q respectively. To prove: PG = GQ Proof: $AE = \frac{1}{2}AB$ and $DF = \frac{1}{2}DC$ [E and F are the mid-points] $\Rightarrow AE = DF$ Also, AE || DF [As AB || DC] AE and DF are the parts of AB and DC. \therefore AEFD is a parallelogram [\because Opposite sides of parallelogram are equal and parallel] \therefore AD || EF || BC \therefore PG = GQ [By intercept theorem AD, EF and BC are parallel, PQ cuts them] Hence proved.



(c) P = Rs. 5000, r = 12%, T = 1 year Amount at the end of first year

$$= 5000 + \left(\frac{5000 \times 12 \times 1}{100}\right)$$

Amount at the end of first year after payment of Rs. 2000

Amount at the end of second year

$$= 3600 + \left(\frac{3600 \times 12 \times 1}{100}\right)$$
$$= \text{Rs.}3600 + 432 = \text{Rs.}4032$$

Amount at the end of second year after payment of Rs. $2000 = \text{Rs} \cdot 4032 - \text{Rs} \cdot 2000 = \text{Rs} \cdot 2032$

Amount at the end of third year

$$= 2032 + \left(\frac{2032 \times 12 \times 1}{100}\right)$$

= Rs. 2032 + Rs. 243.84
= Rs. 2275.84

SECTION - B (40 Marks)

Q. 5.

(a) Area of a square formed = 484 m^2

 \Rightarrow (Side)² = 484

 \Rightarrow Side = 22 m

Thus, perimeter of a square = $4 \times \text{Side} = 4 \times 22 = 88 \text{ m}$

Let r be the radius of the circle formed.

Now,

Circumference of a circle = Perimeter of a square

$$\Rightarrow 2 \times \frac{22}{7} \times r = 88$$

$$\Rightarrow r = \frac{88 \times 7}{2 \times 22}$$

$$\Rightarrow r = 14 \text{ m}$$

$$\therefore \text{ Area of a circle} = \pi r^2 = \frac{22}{7} \times 14 \times 14 = 616 \text{ m}^2$$

(b)
$$\sin\theta = \frac{p}{q} = \frac{perpendicular}{hypotenuse} = \frac{AB}{AC}$$

By Pythagoras theorem,
 $BC^2 = AC^2 - AB^2 = q^2 - p^2$
 $BC = \sqrt{q^2 - p^2}$
 $\cos\theta = \frac{BC}{AC} = \frac{\sqrt{q^2 - p^2}}{q}$
Now,
 $\log q = \frac{p}{AC} = \sqrt{q^2 - p^2} = \sqrt{q^2 - p^2} + R$



$$\sin\theta + \cos\theta = \frac{p}{q} + \frac{\sqrt{q^2 - p^2}}{q} = \frac{\sqrt{q^2 - p^2} + p}{q}$$

(c) Points (a, 0), (0, b) and (1, 1) are collinear. So, area of triangle formed by these points will be 0.

$$\Rightarrow \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow [a(b - 1) + 0(1 - 0) + 1(0 - b)] = 0$$

$$\Rightarrow ab - a - b = 0$$

$$\Rightarrow ab = a + b$$

On dividing throughout by ab, we get

$$1 = \frac{1}{a} + \frac{1}{b}$$

Q. 6.

(a)

$$\begin{aligned} & \left(x^2 + y^2 - z^2\right)^2 - \left(2xy\right)^2 \\ &= \left(x^2 + y^2 - z^2 + 2xy\right) \left(x^2 + y^2 - z^2 - 2xy\right) \\ &= \left(x^2 + y^2 + 2xy - z^2\right) \left(x^2 + y^2 - 2xy - z^2\right) \\ &= \left\{\left(x + y\right)^2 - \left(z\right)^2\right\} \left\{\left(x - y\right)^2 - \left(z\right)^2\right\} \\ &= \left(x + y + z\right) \left(x + y - z\right) \left(x - y - z\right) \left(x - y + z\right) \end{aligned}$$

(b) Given: A parallelogram ABCD, such that $BD \perp AC$. To prove: ABCD is a rhombus Proof: In $\triangle OAB$ and $\triangle OBC$ OA = OC [Diagonals of a ||gm bisect each] $\angle AOB = \angle BOC$ [Each 90°] OB = OB $\therefore \triangle OAB \cong \triangle OBC$ [SAS axioms of congruency] AB = BC [C.P.C.T] BC = AD and AB = DC [Given] $\Rightarrow AB = DC = BC = AD$ Hence ABCD is a rhombus.



(c)
$$3a = p\left(\frac{x}{2} - y\right)$$

 $\Rightarrow \frac{x}{2} - y = \frac{3a}{p}$
 $\Rightarrow y = \left(\frac{x}{2} - \frac{3a}{p}\right)$
 $\Rightarrow y = \left(\frac{px - 6a}{2p}\right)$
Given $a = 32, x = 4, p = 5$
 $\Rightarrow y = \frac{5 \times 4 - 6 \times 32}{2 \times 5}$
 $\Rightarrow y = \frac{20 - 192}{10}$
 $\Rightarrow y = -17.2$

Q.7. (a) $2x - 3y = 7 \Rightarrow 2x = 7 + 3y$ $\Rightarrow x = \frac{7 + 3y}{2}$

Taking convenient values of y, we get

Х	3.5	5	8
У	0	1	3

And $x + 6y = 11 \Rightarrow x = 11 - 6y$

Taking convenient values of y, we get

	-		
Х	5	-1	-7
У	1	2	3

Now we plot these points on graph paper as follows:



The point of intersection of the two lines is (5, 1). Hence the solution set is x = 5, y = 1. (b) (i) Parallelograms BFED and AFEC are on the same base FE and between the same parallels AD || EF. Thus, they are equal in area.

 \therefore ar (||gm BFED) = ar (||gm AFEC) = 140 cm²

(ii) Δ BFD and ||gm BFED are on the same base BD and between the same parallels BD and FE.

$$\therefore A(\Delta BFD) = \frac{1}{2} \times A(||gm BFED) = \frac{1}{2} \times 140 = 70 \text{ cm}^2$$

Q. 8.

(a) Given: PQRS is a quadrilateral, and PR and QS are its diagonals.

To Prove: (PQ + QR + RS + SP) > (PR + QS) Proof:

- 1. In $\triangle PQS$, SP + PQ > QS
- 2. In \triangle PQR, PQ + QR > PR
- 3. In \triangle QRS, (QR + RS) > QS
- 4. In \triangle RSP, (RS + SP) > PR

[Sum of the two sides of a Δ is greater than the third side]

Adding 1, 2, 3 and 4

2(PQ + QR + RS + SP) > 2(PR + QS)

 \Rightarrow PQ + QR + RS + SP > PR + QS [Hence Proved]



(b) Let the two places P_1 and P_2 be 30 km apart and let A start from P_1 and B from P_2 .

Let A's speed of walking be x km/ hr and B's speed be y km/ hr. According to the question, when they walk in the same direction 10(x - y) = 30(i) When they walk in opposite directions 2(x + y) = 30(ii) On dividing (i) by (ii), we get $\frac{5(x-y)}{x+y} = 1 \Longrightarrow 5x - 5y = x + y$ \Rightarrow 4x - 6y = 0(iii) On multiplying (ii) by 2, we get 4x + 4y = 60....(iv) Subtracting (iii) from (iv), we get $10y = 60 \Rightarrow y = 6$ Substituting value of y in (iv), 4x + 4(6) = 60 \Rightarrow 4x + 24 = 60 \Rightarrow 4x = 36 $\Rightarrow x = \frac{36}{4} = 9$ \therefore A's speed = 9 km/ hr and B's speed = 6 km/ hr 10

Q. 9.

(a) Mean height of the 10 girls = 1.38 m Sum of heights of 10 girls = $1.38 \times 10 = 13.8$ m Mean height of 40 boys = 1.44 m Sum of heights of 40 boys = $1.44 \times 40 = 57.6$ m Sum of heights of 50 students (10 girls + 40 boys) = 13.8 + 57.6 = 71.4 m : Mean height of 50 students = $\frac{71.4}{50}$ = 1.428 m (b) Given: $m \angle D = 90^\circ$, AB = 8 cm, BC = 6 cm and CA = 3 cm To find: length of CD Let CD = x cmIn $\triangle ADC$, $AD^2 + CD^2 = AC^2$ [By Pythagoras theorem] $AD^{2} = AC^{2} - DC^{2} = 3^{2} - x^{2}$ $AD^2 = 9 - x^2$ In $\triangle ADB$, $AD^2 + BD^2 = AB^2$ [By Pythagoras theorem] $9-x^2+(6+x)^2=8^2$ [: $AD^2=9-x^2$ and BD=6+x] $\rightarrow 9 - x^2 + 36 + x^2 + 12x - 64$

$$\Rightarrow 12x = 64 - 45 = 19$$

$$\therefore x = \frac{19}{12}$$

$$CD = \frac{19}{12} cm = 1\frac{7}{12}$$

(c) Volume of rectangular tank = 80 × 60 × 60 cm³ = 288000 cm³ One liter = 1000 cm³ Volume of water flowing in per sec

$$= 1.5 \text{ cm}^2 \times 3.2 \frac{\text{m}}{\text{s}}$$
$$= 15. \text{ cm}^2 \times \frac{(3.2 \times 100) \text{ cm}}{\text{s}}$$
$$= 480 \frac{\text{cm}^3}{\text{s}}$$

Volume of water flowing in 1 min = $480 \times 60 = 28800 \text{ cm}^3$ Hence,

 $28800\ \mbox{cm}^3$ of water can be filled in 1 min

 $\Rightarrow \text{Time required to fill 288000 cm}^3 \text{ of water } = \left(\frac{1}{28800} \times 288000\right) \text{ min} = 10 \text{ min}$

Q. 10.

(a) Using sec $(90^{\circ} - \theta) = \csc \theta$, $\tan (90^{\circ} - \theta) = \cot \theta$ And $\cos (90^{\circ} - \theta) = \sin \theta$ $\frac{\sec(90^{\circ} - \theta).\cos \sec \theta - \tan(90^{\circ} - \theta)\cot \theta + \cos^{2} 25^{\circ} + \cos^{2} 65^{\circ}}{3\tan 27^{\circ} \tan 63^{\circ}}$ $= \frac{\csc \theta.\cos \sec \theta - \cot \theta.\cot \theta + \cos^{2} (90^{\circ} - 65^{\circ}) + \cos^{2} 65^{\circ}}{3\tan (90^{\circ} - 63^{\circ})\tan 63^{\circ}}$ $= \frac{\csc^{2} \theta - \cot^{2} \theta + \sin^{2} 65^{\circ} + \cos^{2} 65^{\circ}}{3\cot 63^{\circ} \tan 63^{\circ}}$ $= \frac{1+1}{3}$ $= \frac{2}{3}$

(b) Steps of construction:

- 1. We draw AB = 4.5 cm
- 2. Now we construct m \angle BAE = 60°
- 3. Then we cut off AD = 4.5 cm from AE
- 4. We draw two arcs of radius 4.5 cm, one with centre at D and other at B.
- 5. Let them cut at C.
- 6. Join DC and BC.

ABCD is the required rhombus.



(c) Construction: Draw $OM \perp AB$ and $ON \perp CD$. Join OB and OD.



 $BM = \frac{AB}{2} = \frac{5}{2}$ and $ND = \frac{CD}{2} = \frac{11}{2}$ (Perpendicular from centre bisects the chord) Let ON be x. Then, OM will be 6 - x. In ΔMOB, $OM^2 + MB^2 = OB^2$ $\therefore (6-x)^2 + \left(\frac{5}{2}\right)^2 = 0B^2$ $\therefore 36 + x^2 - 12x + \frac{25}{4} = 0B^2$ (1) In \land NOD, $ON^2 + ND^2 = OD^2$ $\therefore OD^{2} = x^{2} + \left(\frac{11}{2}\right)^{2} = x^{2} + \frac{121}{4} \qquad \dots (2)$(radii of same circle) We have OB = OD $36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$ [From (1) and (2)] $\therefore 12x = 36 + \frac{25}{4} - \frac{121}{4} = \frac{144 + 25 - 121}{4} = \frac{48}{4} = 12$ $\therefore 12x = 12$ \Rightarrow x = 1 From equation (2), $OD^{2} = (1)^{2} + \left(\frac{121}{4}\right) = 1 + \frac{121}{4} = \frac{125}{4}$ \Rightarrow OD = $\frac{5}{2}\sqrt{5}$ Hence, the radius of the circle is $\frac{5}{2}\sqrt{5}$ cm.

Q. 11.

(a) Given :
$$a + \frac{1}{a} = p$$
,

Cubing on both the sides, we get

$$\left(a + \frac{1}{a}\right)^3 = p^3$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right) = p^3$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3p = p^3$$

$$\Rightarrow a^3 + \frac{1}{a^3} = p^3 - 3p = p\left(p^2 - 3\right)$$

Hence Proved.

(b) Let
$$\frac{1}{x+y} = a$$
, $\frac{1}{x-y} = b$
Then, we have
 $5a + 3b = 4$ (i)
 $2a + 5b = \frac{27}{5}$ (ii)

Multiplying equation (i) by 2 and equation (ii) by 5, we get 10a + 6b = 8(iii) 10a + 25b = 27(iv)

Subtracting (iv) from (iii), we get -19b = -19

$$b = \frac{-19}{-19} = 1$$

Substituting value of b in equation (i), we get 5a + 3(1) = 4

$$\Rightarrow 5a + 3 = 4 \Rightarrow 5a = 1 \Rightarrow a = \frac{1}{5}$$

Now
$$\frac{1}{x+y} = a$$
, $\frac{1}{x-y} = b$

$$\therefore \frac{1}{x+y} = \frac{1}{5}, \quad \frac{1}{x-y} = 1$$
$$\Rightarrow x+y=5 \quad \dots(v)$$

$$x - y = 1$$
(vi)

Adding (v) and (vi), we get $2y = 4 \Rightarrow y = 2$ Substituting the value of y in equation (vi), we get $x - 2 = 1 \Rightarrow x = 3$ Hence, x = 3, y = 2. (c)

(c)								
	х	25	35	45	55	65	75	Total
	f	10	6	8	12	5	9	50
	fx	250	210	360	660	325	675	2,480

Here,
$$\sum f = 50, \sum f x = 2480$$

 \therefore Mean = $\frac{\sum f x}{\sum f} = \frac{2480}{50} = 49.6$