ICSE Board

## SECTION - A (40 Marks)

Q. 1.
(a) $\mathrm{P}=$ Rs. $6000, \mathrm{R}=10 \%$ p.a., $\mathrm{n}=1 \frac{1}{2}$ years $=\frac{3}{2}$ years
$\mathrm{A}=\mathrm{P}\left(1+\frac{\mathrm{R}}{2 \times 100}\right)^{2 \mathrm{n}}(\because$ Interest is compounded half yearly $)$
$=6000\left(1+\frac{10}{2 \times 100}\right)^{3}$
$=6000\left(1+\frac{5}{100}\right)^{3}$
$=6000 \times(1.05)^{3}$
$=$ Rs. 6945.75
Amount $=$ Rs. 6945.75
C.I. $=6945.75-6000=$ Rs. 945.75
(b) We have
$\frac{(\sqrt{11}-\sqrt{7})(\sqrt{11}-\sqrt{7})}{(\sqrt{11}+\sqrt{7})(\sqrt{11}-\sqrt{7})}=x-y \sqrt{77}$
$\Rightarrow \frac{11-\sqrt{77}-\sqrt{77}+7}{11-7}=x-y \sqrt{77}$
$\Rightarrow \frac{18-2 \sqrt{77}}{4}=x-y \sqrt{77}$
$\Rightarrow \frac{9}{2}-\frac{1}{2} \sqrt{77}=\mathrm{x}-\mathrm{y} \sqrt{77}$
$\Rightarrow \mathrm{x}=\frac{9}{2}, \mathrm{y}=\frac{1}{2}$
(c)

Sonu and Monu's field together form a quadrilateral $A B C D$.


Sonu's field is $\triangle A B D$,
$s=\frac{a+b+c}{2}=\frac{52+25+63}{2}=70$
$s-a=70-52=18, s-b=70-25=45$ and $s-c=70-63=7$
Area of $\triangle A B D=$

$$
\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{70.18 .45 .7}=630 \mathrm{sq} \mathrm{~m}
$$

Monu's field is $\triangle B C D$,
$s=\frac{a+b+c}{2}=\frac{114+25+101}{2}=120$
$s-a=120-114=6, s-b=120-25=95$ and $s-c=120-101=19$
Area of $\triangle B C D=$

$$
\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{120.6 .95 .19}=1140 \mathrm{sq} \mathrm{~m}
$$

Total area is $=630+1140=1770 \mathrm{sq} \mathrm{m}$
The cost of fertilization is Rs 20 per sq m .
Therefore the total cost is $=1770 \times 20=$ Rs 35,400 .

## Q. 2.

(a) In $\triangle \mathrm{DFC}, \mathrm{DC}^{2}=\mathrm{DF}^{2}+\mathrm{FC}^{2} \quad$ [Pythagoras Theorem]
$\Rightarrow 5^{2}=\mathrm{DF}^{2}+4^{2}$
$\Rightarrow 5^{2}-4^{2}=\mathrm{DF}^{2} \Rightarrow \mathrm{DF}^{2}=25-16 \Rightarrow \mathrm{DF}=3$
$\therefore$ Area of $\triangle \mathrm{DEC}=\frac{1}{2} \times(4+4) \times 3=\frac{1}{2} \times 8 \times 3=12 \mathrm{~cm}^{2}$
$\mathrm{FX}=\mathrm{DX}-\mathrm{DF}=9-3=6 \mathrm{~cm}$


Area of trapezium CEBA $=\frac{1}{2} \times(4+4+6+6) \times 6=\frac{1}{2} \times 20 \times 6=60 \mathrm{~cm}^{2}$
$\therefore$ Area of figure $\mathrm{ABCDE}=$ area of $\triangle \mathrm{DEC}+$ area of trapezium $\mathrm{ECBA}=12+60=72 \mathrm{~cm}^{2}$
(b) Given that AB and CD are two chords of a circle with centre 0 , intersecting at a point $E . P Q$ is the diameter through E , such that $\angle \mathrm{AEQ}=\angle \mathrm{DEQ}$.

To prove that $\mathrm{AB}=\mathrm{CD}$.
Draw perpendiculars OL and OM on chords AB and CD respectively.
Now, $\mathrm{m} \angle \mathrm{LOE}=180^{\circ}-90^{\circ}-\mathrm{m} \angle \mathrm{LEO} . .$. [Angle sum property of a triangle]

$$
=90^{\circ}-\mathrm{m} \angle \mathrm{LEO}
$$

$\Rightarrow \mathrm{m} \angle \mathrm{LOE}=90^{\circ}-\mathrm{m} \angle \mathrm{AEQ}$
$\Rightarrow \mathrm{m} \angle \mathrm{LOE}=90^{\circ}-\mathrm{m} \angle \mathrm{DEQ}$
$\Rightarrow \mathrm{m} \angle \mathrm{LOE}=90^{\circ}-\mathrm{m} \angle \mathrm{MEQ}$
$\Rightarrow \angle \mathrm{LOE}=\angle \mathrm{MOE}$
In $\triangle$ OLE and $\triangle O M E$,

$\angle$ LEO $=\angle$ MEO
$\angle \mathrm{LOE}=\angle \mathrm{MOE}$
EO = EO
$\Delta \mathrm{OLE} \cong \triangle \mathrm{OME}$
$\mathrm{OL}=\mathrm{OM}$
Therefore, cords AB and CD are equidistant from the centre.
(c)

$$
\begin{aligned}
& (\sqrt[3]{4})^{2 x+\frac{1}{2}}=(\sqrt[3]{8})^{5} \\
& \Rightarrow\left[(4)^{1 / 3}\right]^{2 x+\frac{1}{2}}=\left[8^{1 / 3}\right]^{5} \\
& \Rightarrow\left(2^{2 / 3}\right)^{\left(2 x+\frac{1}{2}\right)}=\left[8^{1 / 3}\right]^{5} \\
& \Rightarrow(2)^{\frac{2}{3}\left(2 x+\frac{1}{2}\right)}=(2)^{5} \\
& \Rightarrow \frac{2}{3}\left(2 x+\frac{1}{2}\right)=5 \\
& \Rightarrow 4 x+1=15 \\
& \Rightarrow x=\frac{7}{2}
\end{aligned}
$$

## Q. 3.

(a) Given, $\log x=a+b$ and $\log y=a-b$

$$
\begin{aligned}
\log \frac{10 \mathrm{x}}{\mathrm{y}^{2}} & \left.=\log 10 \mathrm{x}-\log \mathrm{y}^{2} \quad \text { [Using quotient law }\right] \\
& =\log 10+\log \mathrm{x}-2 \log \mathrm{y} \\
& =1+(\mathrm{a}+\mathrm{b})-2(\mathrm{a}-\mathrm{b}) \\
& =1+\mathrm{a}+\mathrm{b}-2 \mathrm{a}+2 \mathrm{~b} \\
& =1-\mathrm{a}+3 \mathrm{~b}
\end{aligned}
$$

(b) Given : In $\triangle \mathrm{ABC}, \mathrm{AD}$ is the bisector of $\angle \mathrm{BAC}$ and BC is produced to E


To Prove: $\angle \mathrm{ABC}+\angle \mathrm{ACE}=2 \angle \mathrm{ADC}$
Proof:
Let $\angle \mathrm{BAD}=\angle \mathrm{DAC}=\mathrm{x}$ and $\angle \mathrm{ABC}=\mathrm{y}$
Now, $\angle \mathrm{ACE}=\angle \mathrm{ABC}+\angle \mathrm{BAC} \quad \ldots .[$ Exterior angle $=$ Sum of interior opposite $\angle \mathrm{s}$ ]
$\Rightarrow \angle \mathrm{ACE}=\mathrm{y}+2 \mathrm{x}$
In $\triangle \mathrm{ABD}, \angle \mathrm{ADC}=\mathrm{x}+\mathrm{y} \quad \ldots$. [Exterior angle $=$ Sum of interior opposite $\angle \mathrm{s}$ ]
$\therefore \angle \mathrm{ABC}+\angle \mathrm{ACE}=\mathrm{y}+\mathrm{y}+2 \mathrm{x}=2(\mathrm{x}+\mathrm{y})$
$\Rightarrow \angle \mathrm{ABC}+\angle \mathrm{ACE}=2 \angle \mathrm{ADC}$
(c) Steps of Construction:

1. Draw $\mathrm{AB}=4.5 \mathrm{~cm}$.
2. Draw $\angle \mathrm{BAS}=120^{\circ}$ and draw $\mathrm{EA} \perp \mathrm{AB}$.
3. From A , cut an arc of measure 3.3 cm on EA such that $\mathrm{AX}=3.3 \mathrm{~cm}$.
4. Through $X$, draw a line QP which is parallel to AB which cuts AS at D.
5. Through B draw an arc taking radius 3.6 cm at C on PQ .
6. Join CB.

Thus, ABCD is the required trapezium.

Q. 4.
(a) We can see that $\triangle \mathrm{ABC}$ is a right-angles triangle.
$\Rightarrow \mathrm{AB}^{2}+\mathrm{BC}^{2}=\mathrm{AC}^{2} \quad \ldots . .[\mathrm{By}$ Pythagoras theorem]
$\Rightarrow 15^{2}+\mathrm{BC}^{2}=25^{2}$
$\Rightarrow \mathrm{BC}^{2}=400$
$\Rightarrow \mathrm{BC}=20 \mathrm{~cm}$
Now $B C=D B+C D$
$\Rightarrow 20=\mathrm{DB}+7$
$\Rightarrow \mathrm{DB}=13 \mathrm{~cm}$
Again ADB is a right angled triangle.
$\Rightarrow \mathrm{AB}^{2}+\mathrm{DB}^{2}=\mathrm{AD}^{2} \ldots . .[$ By Pythagoras theorem $]$
$\Rightarrow 15^{2}+13^{2}=390$
$\Rightarrow \mathrm{BC}=19.8 \mathrm{~cm}$
In the right-angled $\triangle \mathrm{CDE}$
$\Rightarrow \mathrm{ED}^{2}+\mathrm{CE}^{2}=\mathrm{CD}^{2} \quad \ldots$..[By Pythagoras theorem $]$
$\Rightarrow \mathrm{ED}^{2}=\mathrm{CD}^{2}-\mathrm{CE}^{2}=7^{2}-\mathrm{x}^{2}$

In the right-angled $\triangle \mathrm{AED}$
$\Rightarrow \mathrm{ED}^{2}+\mathrm{AE}^{2}=\mathrm{AD}^{2} \quad \ldots . .[$ By Pythagoras theorem $]$
$\Rightarrow \mathrm{ED}^{2}=\mathrm{AD}^{2}-\mathrm{AE}^{2}=19.8^{2}-(25-\mathrm{x})^{2}$
Since in both the cases length of ED is same
and hence $E D^{2}$ is also same in both the cases.
$\Rightarrow 7^{2}-x^{2}=19.8^{2}-(25-x)^{2}$
$\Rightarrow 7^{2}-x^{2}=19.8^{2}-625-x^{2}+50 \mathrm{x}$
$\Rightarrow 7^{2}-19.8^{2}+625=50 \mathrm{x}$
$\Rightarrow 281.96=50 \mathrm{x}$
$\Rightarrow \mathrm{x}=5.63 \mathrm{~cm}$
So,
$E D^{2}=7^{2}-5.6^{2}=17.64$
$\Rightarrow \mathrm{ED}=4.2 \mathrm{~cm}=\mathrm{DE}$
(b)

Each interior angle of a regular pentagon $=\frac{(2 \times 5-4) \times 90}{5} \quad[\mathrm{n}=5]$

$$
\begin{aligned}
& =\frac{6 \times 90}{5} \\
& =108^{\circ}
\end{aligned}
$$

Each exterior angle of a regular decagon $=\frac{360}{10}=36^{\circ} \quad[\mathrm{n}=10]$
$\therefore$ Each interior angle of a regular pentagon $=3$ (Exterior angle of a regular decagon)
(c) Given $\tan \theta+\cot \theta=3$,

Squaring both sides,

$$
\begin{aligned}
& (\tan \theta+\cot \theta)^{2}=3^{2} \\
& \Rightarrow \tan ^{2} \theta+\cot ^{2} \theta+2 \tan \theta \cot \theta=9 \\
& \Rightarrow \tan ^{2} \theta+\cot ^{2} \theta+2 \tan \theta \times \frac{1}{\tan \theta}=9 \quad\left[\because \cot \theta=\frac{\cos \theta}{\sin \theta}\right] \\
& \Rightarrow \tan ^{2} \theta+\cot ^{2} \theta+2=9 \\
& \Rightarrow \tan ^{2} \theta+\cot ^{2} \theta=7
\end{aligned}
$$

## Section - B (40 Marks)

Q. 5.
(a) Consider equation, $x-2 y=1$
$\Rightarrow y=\frac{x-1}{2}$

| x | 1 | 3 | 5 |
| :---: | ---: | ---: | ---: |
| y | 0 | 1 | 2 |

$\therefore$ Points are $(1,0),(3,1)$ and $(5,2)$.
Now consider equation $x+y=4$

| x | 0 | 2 | 4 |
| :---: | :--- | :--- | :--- |
| y | 4 | 2 | 0 |

$\therefore$ Points are $(0,4),(2,2)$ and $(4,0)$.

Now plotting these points on the graph paper, we get


Since the lines intersect at $(3,1)$, therefore the solution is $\mathrm{x}=3$ and $\mathrm{y}=1$.
(b) In 15 days A and B together can do a piece of work.

Therefore, in 1 day they do $\frac{1}{15}$ work
Let us assume that A takes x days and B takes y days to do the work alone.
So A's one day's work $=\frac{1}{x}$
B's one day's work $=\frac{1}{y}$
$\frac{1}{x}=\frac{3}{2} \cdot \frac{1}{y}$
$\Rightarrow 3 \mathrm{x}-2 \mathrm{y}=0$
$\Rightarrow 2 \mathrm{y}=3 \mathrm{x}$
$\Rightarrow y=\frac{3 x}{2}$
Also, $\frac{1}{\mathrm{x}}+\frac{1}{\mathrm{y}}=\frac{1}{15}$
$\Rightarrow \frac{1}{\mathrm{x}}+\frac{2}{3 \mathrm{x}}=\frac{1}{15}$
$\Rightarrow \frac{3+2}{3 \mathrm{x}}=\frac{1}{15}$
$\Rightarrow 3 \mathrm{x}=75$
$\Rightarrow \mathrm{x}=25$
$\Rightarrow \mathrm{y}=\frac{3 \times 25}{2}=37.5$
Hence, A will do the work alone in 25 days and B will do it alone 37 and half days.
(c) Let there be $n$ sides of the polygon. Then, each interior angle is of measure

$$
\begin{aligned}
& \left(\frac{2 \mathrm{n}-4}{\mathrm{n}} \times 90^{\circ}\right) \\
& \therefore \frac{2 \mathrm{n}-4}{\mathrm{n}} \times 90=108 \\
& \Rightarrow(2 \mathrm{n}-4) \times 90=108 \mathrm{n} \\
& \Rightarrow 180 \mathrm{n}-360=108 \mathrm{n} \\
& \Rightarrow 180 \mathrm{n}-108 \mathrm{n}=360 \\
& \Rightarrow 72 \mathrm{n}=360 \\
& \Rightarrow \mathrm{n}=5
\end{aligned}
$$

Hence the given polygon has 5 sides.

## Q. 6.

(a) (i) Interest for first year $=\frac{5600 \times 14 \times 1}{100}=$ Rs. 784
(ii) Amount at the end of the first year $=5600+784=$ Rs. 6384
(iii) Interest for the second year $=$
$\frac{6384 \times 14 \times 1}{100}=$ Rs. $893.76=$ Rs. 894 (to the nearest rupee)
(b)
(i) Since, the point P lies on the x -axis, its ordinate is 0 .
(ii) Since, the point $Q$ lies on the $y$-axis, its abscissa is 0 .
(iii) The co-ordinates of $P$ and $Q$ are $(-12,0)$ and $(0,-16)$ respectively.

$$
\begin{aligned}
\mathrm{PQ} & =\sqrt{(-12-0)^{2}+(0+16)^{2}} \\
& =\sqrt{144+256} \\
& =\sqrt{400} \\
& =20
\end{aligned}
$$

(c) Here $\mathrm{m} \angle \mathrm{A}+\mathrm{m} \angle \mathrm{C}=90^{\circ}$ as $\mathrm{m} \angle \mathrm{B}=90^{\circ}$

$$
\Rightarrow 30^{\circ}+\mathrm{m} \angle \mathrm{C}=90^{\circ}
$$

$$
\Rightarrow \mathrm{m} \angle \mathrm{C}=60^{\circ}
$$

In right-angled $\triangle \mathrm{ABC}$,

$$
\tan 30^{\circ}=\frac{\mathrm{BC}}{\mathrm{AB}}
$$

$$
\Rightarrow \frac{1}{\sqrt{3}}=\frac{8}{\mathrm{AB}}
$$

$$
\Rightarrow \mathrm{AB}=8 \sqrt{3} \mathrm{~cm}
$$

$$
\sin 30^{\circ}=\frac{\mathrm{BC}}{\mathrm{AC}}
$$

$$
\Rightarrow \frac{1}{2}=\frac{8}{\mathrm{AC}}
$$



$$
\Rightarrow \mathrm{AC}=16 \mathrm{~cm}
$$

Q. 7.
(a)

$$
\begin{aligned}
& \frac{3^{n+1}}{3^{n(n-1)}} \div \frac{9^{n+1}}{\left(3^{n+1}\right)^{n-1}} \\
& =\frac{3^{n+1}}{3^{n(n-1)}} \times \frac{\left(3^{n+1}\right)^{n-1}}{9^{n+1}} \\
& =\frac{3^{n+1}}{3^{n(n-1)}} \times \frac{3^{(n+1)(n-1)}}{(3 \times 3)^{n+1}} \\
& =\frac{3^{n+1}}{3^{n^{2}-n}} \times \frac{3^{\left(n^{2}-1\right)}}{\left(3^{2}\right)^{n+1}} \\
& =\frac{3^{n+1}}{3^{n^{2}-n}} \times \frac{3^{\left(n^{2}-1\right)}}{3^{2 n+2}} \\
& =3^{n+1+n^{2}-1-\left(n^{2}-n\right)-(2 n+2)} \\
& =3^{n+1+n^{2}-1-n^{2}+n-2 n-2} \\
& =3^{-2} \\
& =\frac{1}{3^{2}} \\
& =\frac{1}{9}
\end{aligned}
$$

(b)

Area of an isosceles $\Delta=\frac{1}{4} \mathrm{~b} \sqrt{4 \mathrm{a}^{2}-\mathrm{b}^{2}}$
(where $b$ is the base and $a$ is the length of equal sides)
Given, $\mathrm{b}=8 \mathrm{~cm}$ and area $=12 \mathrm{~cm}^{2}$
$\Rightarrow \frac{1}{4} \times 8 \times \sqrt{4 \mathrm{a}^{2}-8^{2}}=12$
$\Rightarrow \sqrt{4 \mathrm{a}^{2}-8^{2}}=6$
$\Rightarrow 4 \mathrm{a}^{2}-8^{2}=36$
$\Rightarrow 4 \mathrm{a}^{2}=100$
$\Rightarrow \mathrm{a}^{2}=25$
$\Rightarrow \mathrm{a}=5 \mathrm{~cm}$
$\therefore$ Perimeter $=2 \mathrm{a}+\mathrm{b}=2 \times 5+8=18 \mathrm{~cm}$
(c) Given $\mathrm{AC}=\mathrm{CD}$

To prove: $\mathrm{BC}<\mathrm{CD}$
Proof: In $\triangle \mathrm{ACD}$,
$\mathrm{m} \angle \mathrm{ACD}=180^{\circ}-70^{\circ}=110^{\circ} \quad$ [Linear pair]
$\angle \mathrm{CAD}=\angle \mathrm{ADC}=\frac{70^{\circ}}{2}=35^{\circ} \quad$ [Angles opposite to equal sides are equal]
In $\triangle \mathrm{ABC}$,

$$
\begin{aligned}
& \mathrm{m} \angle \mathrm{BAC}=70^{\circ}-35^{\circ}=35^{\circ} \\
& \mathrm{m} \angle \mathrm{ABC}=180^{\circ}-\left(70^{\circ}+35^{\circ}\right) \\
& \quad\left[\angle \mathrm{BAC}=\angle \mathrm{BAD}-\angle \mathrm{Cum} \text { of all } \angle \mathrm{s} \text { of a } \triangle \text { is } 180^{\circ}\right] \\
& \therefore \angle \mathrm{BAC}<\angle \mathrm{ABC} \\
& \therefore \mathrm{BC}<\mathrm{AC}
\end{aligned}
$$

So, $\mathrm{BC}<\mathrm{CD} \quad$ [Since $\mathrm{AC}=\mathrm{CD}$ ]

Q. 8.
(a)
$\cos \theta=\frac{2 \sqrt{m n}}{m+n}$
Now,

$$
\begin{aligned}
& \sin ^{2} \theta+\cos ^{2} \theta=1 \\
& \Rightarrow \sin ^{2} \theta=1-\cos ^{2} \theta \\
&=1-\left(\frac{2 \sqrt{m n}}{m+n}\right)^{2} \\
&=1-\left(\frac{4 m n}{(m+n)^{2}}\right) \\
&=\frac{(m+n)^{2}-4 m n}{(m+n)^{2}} \\
&=\frac{m^{2}+n^{2}+2 m n-4 m n}{(m+n)^{2}} \\
&=\frac{m^{2}+n^{2}-2 m n}{(m+n)^{2}} \\
&=\frac{(m-n)^{2}}{(m+n)^{2}} \\
&=\left(\frac{m-n}{m+n}\right)^{2} \\
& \Rightarrow \sin \theta=\frac{m-n}{m+n}
\end{aligned}
$$

(b) Mean $=20$

Number of terms $=5$
$\therefore$ Total sum $=20 \times 5=100$
Let the excluded number be $x$.
Then, $\frac{(100-x)}{4}=23$
$\Rightarrow 100-\mathrm{x}=23 \times 4=92$
$\Rightarrow \mathrm{x}=8$
Hence, the excluded number is 8 .
(c) Let the side of each of the three equal cubes be 'a' cm.

Surface area of one cube $=6 \mathrm{a}^{2} \mathrm{~cm}^{2}$
Therefore, sum of surface areas of the three cubes $=3 \times 6 \mathrm{a}^{2}=18 \mathrm{a}^{2} \mathrm{~cm}^{2}$
Now,
Length of the new cuboid $=3 \mathrm{a} \mathrm{cm}$
Breadth of the new cuboid $=\mathrm{acm}$
Height of the new cuboid = a cm
Total surface area of the new cuboid $=2[(3 a \times a)+(a \times a)+(a \times 3 a)]$

$$
\begin{aligned}
& =2\left[3 a^{2}+a^{2}+3 a^{2}\right] \\
& =2\left[7 a^{2}\right] \\
& =14 a^{2} \mathrm{~cm}^{2}
\end{aligned}
$$

Thus, the required ratio of T.S.A. of the new cuboid to that of the sum of the S.A. of the 3 cubes $=14 \mathrm{a}^{2}: 18 \mathrm{a}^{2}=7: 9$.

## Q. 9.

(a) Given: In quadrilateral $A B C D ; A D=B C . P, Q, R, S$ are the mid-points of $A B, B D, C D$ and AC respectively.
To Prove: PQRS is a rhombus.
Proof: In $\triangle \mathrm{ACD}, \mathrm{RS} \| \mathrm{AD}$ and $\mathrm{RS}=\frac{1}{2} \mathrm{AD}$
[Line joining the mid-points of the two sides of triangle is parallel and half of the third side.]
Similarly,
In $\triangle A B D, P Q \| A D$ and $P Q=\frac{1}{2} A D$


In $\triangle \mathrm{BCD}, \mathrm{QR} \| \mathrm{BC}$ and $\mathrm{QR}=\frac{1}{2} \mathrm{BC}$
In $\triangle \mathrm{ABC}, \mathrm{SP}| | \mathrm{BC}$ and $\mathrm{SP}=\frac{1}{2} \mathrm{BC}$
As AD = BC [Given]
$\mathrm{RS}=\mathrm{PQ}=\mathrm{QR}=\mathrm{SP}$ and $\mathrm{RS}|\mid \mathrm{PQ}$ and QR$| \mid \mathrm{SP} \quad$ [From (i), (ii), (iii) and (iv)]
Hence PQRS is a rhombus.
(b) Given that we have to construct a grouped frequency distribution table of class size 5 . So, the class intervals will be as $0-5,5-10,10-15,15-20$, and so on. Required grouped frequency distribution table is as follows:

| Distance (in km) | Tally marks | Number of engineers |
| :---: | :---: | :---: |
| 0-5 | NN | 5 |
| 5-10 | NNMNI | 11 |
| 10-15 | NNMNI | 11 |
| 15-20 | NN\||| | 9 |
| 20-25 | \| | 1 |
| 25-30 | \| | 1 |
| 30-35 | \\| | 2 |
| Total |  | 40 |

Only 4 engineers have homes at a distance of more than or equal to 20 km from their work place.

Most of the engineers have their workplace at a distance of upto 15 km from their homes.
(c)
$\log _{x}(8 x-3)-\log _{x} 4=2$
$\Rightarrow \log _{x}\left(\frac{8 x-3}{4}\right)=2$
$\Rightarrow \frac{8 \mathrm{x}-3}{4}=\mathrm{x}^{2}$
$\Rightarrow 8 \mathrm{x}-3=4 \mathrm{x}^{2}$
$\Rightarrow 4 \mathrm{x}^{2}-8 \mathrm{x}+3=0$
$\Rightarrow 4 \mathrm{x}^{2}-6 \mathrm{x}-2 \mathrm{x}+3=0$
$\Rightarrow 2 \mathrm{x}(2 \mathrm{x}-3)-1(2 \mathrm{x}-3)=0$
$\Rightarrow(2 \mathrm{x}-3)(2 \mathrm{x}-1)=0$
$\Rightarrow 2 \mathrm{x}-3=0$ or $2 \mathrm{x}-1=0$
$\Rightarrow \mathrm{x}=\frac{3}{2}$ or $\mathrm{x}=\frac{1}{2}$
Q. 10.
(a)

Let us assume, on the contrary that $\sqrt{5}$ is a rational number.
Therefore, we can find two integers $\mathrm{a}, \mathrm{b}(\mathrm{b} \neq 0)$ such that $\sqrt{5}=\frac{\mathrm{a}}{\mathrm{b}}$
Where a and b are co-prime integers.

$$
\begin{aligned}
\sqrt{5} & =\frac{a}{b} \\
\Rightarrow & a=\sqrt{5} b \\
\Rightarrow & a^{2}=5 b^{2}
\end{aligned}
$$

Therefore, $\mathrm{a}^{2}$ is divisible by 5 then a is also divisible by 5 .
So $\mathrm{a}=5 \mathrm{k}$, for some integer k .
Now, $\mathrm{a}^{2}=(5 \mathrm{k})^{2}=5\left(5 \mathrm{k}^{2}\right)=5 \mathrm{~b}^{2}$
$\Rightarrow \mathrm{b}^{2}=5 \mathrm{k}^{2}$
This means that $\mathrm{b}^{2}$ is divisible by 5 and hence, b is divisible by 5 .
This implies that a and b have 5 as a common factor.
And this is a contradiction to the fact that a and b are co-prime.
So our assumption that $\sqrt{5}$ is rational is wrong.
Hence, $\sqrt{5}$ cannot be a rational number. Therefore, $\sqrt{5}$ is irrational.
(b)

$$
\begin{aligned}
& \tan \left(\theta_{1}+\theta_{2}\right)=\frac{\tan \theta_{1}+\tan \theta_{2}}{1-\tan \theta_{1} \tan \theta_{2}}=\frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2} \times \frac{1}{3}\right)} \\
& \Rightarrow \tan \left(\theta_{1}+\theta_{2}\right)=\frac{\frac{3+2}{6}}{1-\frac{1}{6}}=\frac{\frac{5}{6}}{\frac{6-1}{6}}=\frac{5}{6} \times \frac{6}{5}=1 \\
& \Rightarrow \tan \left(\theta_{1}+\theta_{2}\right)=1=\tan 45^{\circ} \\
& \Rightarrow\left(\theta_{1}+\theta_{2}\right)=45^{\circ}
\end{aligned}
$$

(c) $\frac{x^{2}+1}{x}=4$

$$
\begin{align*}
& \Rightarrow x^{2}+1=4 x \\
& \Rightarrow x^{2}-4 x+1=0 \tag{i}
\end{align*}
$$

On dividing equation (i) by $x$, we have

$$
\begin{align*}
& x-4+\frac{1}{x}=0 \\
& \Rightarrow x+\frac{1}{x}=4 \tag{ii}
\end{align*}
$$

On cubing equation (ii) both sides, we have

$$
\begin{aligned}
& \left(\mathrm{x}+\frac{1}{\mathrm{x}}\right)^{3}=(4)^{3} \\
& \Rightarrow \mathrm{x}^{3}+\frac{1}{\mathrm{x}^{3}}+3 \times \mathrm{x} \times \frac{1}{\mathrm{x}}\left(\mathrm{x}+\frac{1}{\mathrm{x}}\right)=64 \\
& \Rightarrow \mathrm{x}^{3}+\frac{1}{\mathrm{x}^{3}}+3 \times 4=64 \\
& \Rightarrow \mathrm{x}^{3}+\frac{1}{\mathrm{x}^{3}}=64-12 \\
& \Rightarrow \mathrm{x}^{3}+\frac{1}{\mathrm{x}^{3}}=52 \\
& \therefore\left(2 \mathrm{x}^{3}+\frac{2}{\mathrm{x}^{3}}\right)=2\left(\mathrm{x}^{3}+\frac{1}{\mathrm{x}^{3}}\right)=2 \times 52=104
\end{aligned}
$$

Q. 11.
(a) $4 a^{3} b-44 a^{2} b+112 b$

$$
\begin{aligned}
& =4 a b\left[a^{2}-11 a+28\right] \\
& =4 a b\left[a^{2}-7 a-4 a+28\right] \\
& =4 a b[a(a-7)-4(a-7)] \\
& =4 a b(a-7)(a-4)
\end{aligned}
$$

(b) Construction: Draw TM $\perp$ QS

Area of $\triangle R Q S=\frac{1}{2} \times Q S \times R N=\frac{1}{2} \times 35 \times 20=350 \mathrm{~cm}^{2}$
Now, QS = QM + MS
$\Rightarrow 35=25+\mathrm{MS}$
$\Rightarrow \mathrm{MS}=10 \mathrm{~cm}$
In $\triangle$ STM,
$\mathrm{MS}^{2}+\mathrm{TM}^{2}=\mathrm{ST}^{2}$

$\Rightarrow \mathrm{TM}^{2}=\mathrm{ST}^{2}-\mathrm{MS}^{2}=(26)^{2}-(10)^{2}=676-100=576$
$\Rightarrow \mathrm{TM}=24 \mathrm{~cm}=\mathrm{PQ}$
$\therefore$ Area of trapezium PQST $=\frac{1}{2} \times(\mathrm{PT}+\mathrm{QS}) \times \mathrm{PQ}=\frac{1}{2} \times(25+35) \times 24=720 \mathrm{~cm}^{2}$
Thus, area of given figure $=$ Area of $\triangle R Q S+$ Area of trapezium PQST

$$
\begin{aligned}
& =350 \mathrm{~cm}^{2}+720 \mathrm{~cm}^{2} \\
& =1070 \mathrm{~cm}^{2}
\end{aligned}
$$

(c) Given: In $\triangle \mathrm{ABC}, \mathrm{AB}=\mathrm{AC}=\mathrm{x}, \mathrm{BC}=20 \mathrm{~cm}$, Area of $\triangle \mathrm{ABC}=250 \mathrm{~cm}^{2}$

To find: x
Construction: Draw AD $\perp$ BC
Since $\triangle A B C$ is an isosceles triangle. $A D$ bisects $B C$.
$\mathrm{BD}=\mathrm{DC}=20 / 2=10 \mathrm{~cm}$
Area of $\triangle \mathrm{ABC}=\frac{1}{2} \times \mathrm{BC} \times \mathrm{AD}=250 \mathrm{~cm}^{2}$ [Given]

$$
\frac{1}{2} \times 20 \times \mathrm{AD}=250 \Rightarrow \mathrm{AD}=25 \mathrm{~cm}
$$

In rt. $\triangle \mathrm{ADC}$,

$$
\mathrm{AD}^{2}+\mathrm{DC}^{2}=\mathrm{AC}^{2}
$$



20 cm
$25^{2}+10^{2}=x^{2}$
$x^{2}=625+100=725$
[Pythagoras Theorem]
$\Rightarrow \mathrm{x}=5 \sqrt{29} \mathrm{~cm}$

