ICSE Board
Class IX Mathematics
Paper 4 – Solution

SECTION – A (40 Marks)

Q. 1.

(a) Consider

$$\frac{3\sqrt{2}+2\sqrt{3}}{5\sqrt{2}-4\sqrt{3}} = x - y\sqrt{6}$$

$$\Rightarrow \frac{3\sqrt{2}+2\sqrt{3}}{5\sqrt{2}-4\sqrt{3}} \times \frac{5\sqrt{2}+4\sqrt{3}}{5\sqrt{2}+4\sqrt{3}} = x - y\sqrt{6}$$

$$\Rightarrow \frac{30+12\sqrt{6}+10\sqrt{6}+24}{50-48} = x - y\sqrt{6}$$

$$\Rightarrow \frac{54+22\sqrt{6}}{2} = \frac{2\left(27+11\sqrt{6}\right)}{2} = x - y\sqrt{6}$$

$$\Rightarrow 27+11\sqrt{6} = x - y\sqrt{6}$$

$$\Rightarrow x = 27, y = -11$$

(b) Dimensions of the brick:

Length (l) = 20 cm, breadth (b) = 5 cm and height (h) = 5 cm

Volume of brick (V) = lbh = 500 cm^3

Dimensions of the wall:

Length (l) = 2.5 m = 250 cm, breadth (b) = 0.5 m = 50 cm and

height (h) = 5 m = 500 cm

Volume of the wall (V_1) = LBH= 6250000 cm^3

Let N be the number of bricks required to make the wall.

Then, N × V = V_1

$$\Rightarrow N = \frac{6250000}{500} = 12500$$

Thus, 12500 bricks are required to make the wall.

(c) Let P = Rs. x, rate = r%

Then, $\frac{x \times r \times 1}{100} = 350 \Rightarrow rx = 35000$ ....(1)

Now, principal for second year = Rs. (x + 350)

$\Rightarrow$ rx + 350r = 42000

$\Rightarrow$ 35000 + 350r = 42000 [From (1)]

$\Rightarrow$ 350r = 42000 – 35000

$\Rightarrow 350r = 7000$

$\Rightarrow r = \frac{7000}{350} = 20\%$

Q. 2.

(a) $a^2 - 3a + 1 = 0$ ....(1)

On dividing equation (1) by a, we get, $a - 3 + \frac{1}{a} = 0$

(i) $a + \frac{1}{a} = 3$ ....(2)

On squaring equation (2), we have

$$\left(a + \frac{1}{a}\right)^2 = 3^2$$

$$a^2 + \frac{1}{a^2} + 2 \times a \times \frac{1}{a} = 9$$

$$a^2 + \frac{1}{a^2} + 2 = 9$$

$$a^2 + \frac{1}{a^2} = 9 - 2 = 7$$

(ii) Cubing equation (2), we have

$$\left(a + \frac{1}{a}\right)^3 = 3^3$$

$$a^3 + \frac{1}{a^3} + 3 \times a \times \frac{1}{a}\left(a + \frac{1}{a}\right) = 27$$

$$a^3 + \frac{1}{a^3} + 3(3) = 27$$

$$a^3 + \frac{1}{a^3} = 27 - 9 = 18$$

(b)

$$20-45(m+n)^2=5\left[4-9(m+n)^2\right]$$
$$=5\left[(2)^2-\{3(m+n)\}^2\right]$$
$$=5\left[2+3(m+n)\right]\left[2-3(m+n)\right]$$
$$=5(2+3m+3n)(2-3m-3n)$$

(c) Here

$$\frac{5\sin 62^\circ}{\cos 28^\circ}=\frac{5\sin(90^\circ\ -28^\circ\ )}{\cos 28^\circ}=\frac{5\cos 28^\circ}{\cos 28^\circ}=5$$

And $\dfrac{2\sec 34^\circ}{\text{cosec}56^\circ}=\dfrac{2\sec(90^\circ-56^\circ)}{\text{cosec}56^\circ}=\dfrac{2\text{cosec}56^\circ}{\text{cosec}56^\circ}=2$

$$\therefore \frac{5\sin 62^\circ}{\cos 28^\circ}-\frac{2\sec 34^\circ}{\text{cosec}56^\circ}=5-2=3$$

Q. 3

(a) Let the side of square be x cm

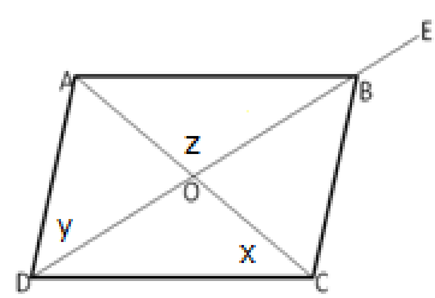
$\therefore$ Its perimeter is 4x cm

Given, $4x = 4(p + 3q)$

$$\Rightarrow x=\frac{4(p+3q)}{4}=(p+3q)\text{cm}$$
$$\therefore \text{Area}=(\text{side})^2=(p+3q)^2$$
$$=(p^2+9q^2+6pq)\text{cm}^2$$

(b) m∠AOB = z = 90° [Diagonals of a rhombus bisect each other at right angle]

m∠ABO = 180 – 160 = 20°



In ΔAOB, m∠BAO + m∠AOB + m∠ABO = 180°

m∠BAO + 90° + 20° = 180°

m∠BAO = 70°

Also, x = m∠BAO = 70° [Alternate interior angles are equal as AB || DC]

In ΔADB, AD = AB

∠ABD = ∠ADB (Angles opposites to equal sides are equal)

Therefore, y = 20°

(c) $x = 2^{\frac{1}{3}} + 2^{\frac{-1}{3}}$

On cubing both sides

$$x^3 = \left(2^{\frac{1}{3}}\right)^3 + \left(2^{\frac{-1}{3}}\right)^3 + 3 \times 2^{\frac{1}{3}} \times 2^{\frac{-1}{3}}\left(2^{\frac{1}{3}} + 2^{\frac{-1}{3}}\right)$$

[By using $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$]

$$\Rightarrow x^3 = 2 + 2^{-1} + 3 \times 2^{\circ} \times x$$

$$x^3 = 2 + \frac{1}{2} + 3x$$

$$\Rightarrow 2x^3 = 5 + 6x \text{ or } 2x^3 = 6x + 5$$

Q. 4.

(a) We have $\frac{\log 4}{\log \frac{1}{2}} = \frac{\log 2^2}{\log 2^{-1}} = \frac{2\log 2}{-1\log 2} = -2$

Now $\frac{\log(a-b)}{\log 5} = -2$

$$\Rightarrow \log(a-b) = -2\log 5$$

$$\Rightarrow \log(a-b) = \log 5^{-2}$$

$$\Rightarrow a - b = \frac{1}{5^2}$$

$$\Rightarrow a - b = \frac{1}{25} \quad \text{....(1)}$$

Again, $\frac{\log(a+b)}{\log 2} = -2$

$$\Rightarrow \log(a+b) = -2\log 2$$

$$\Rightarrow \log(a+b) = \log 2^{-2}$$

$$\Rightarrow a + b = \frac{1}{4} \quad \text{....(2)}$$

From (1) and (2), we get

$$a = \frac{29}{200},\ b = \frac{21}{200}$$

(b) Given: In the equilateral ΔABC, AD is perpendicular to BC.

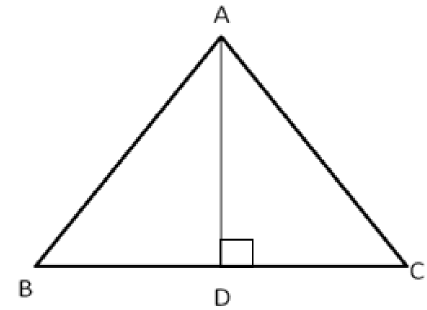
To prove: $4AD^2 = 3AB^2$

Proof: $AD \perp BC$ [Given]

BD = DC

[In an equilateral triangle ⊥ from the vertex bisects the base]

In right ΔADB, $AD^2 + BD^2 = AB^2$ [Pythagoras theorem]



$$\Rightarrow AD^2 + \left[\frac{1}{2}BC\right]^2 = AB^2 \quad \left[\because BD = \frac{1}{2}BC\right]$$

$$\Rightarrow AD^2 + \frac{BC^2}{4} = AB^2$$

$$\Rightarrow AD^2 + \frac{AB^2}{4} = AB^2 \left[\because AB = BC\right]$$

$$\Rightarrow 4AD^2 + AB^2 = 4AB^2$$

$$\Rightarrow 4AD^2 = 3AB^2$$

(c)

Sum of exterior angles = $\frac{1}{6}^{\text{th}}$ of the sum of interior angles

$$360 = \frac{1}{6} \times (2n - 4) \times 90$$

$$\Rightarrow 4 \times 6 = 2n - 4$$

$$\Rightarrow 24 = 2n - 4$$

$$\Rightarrow 2n = 28 \Rightarrow n = 14$$

SECTION - B

Q. 5.

(a)

$$3x - 7 = \frac{1}{y} \quad ....(1)$$

$$x + \frac{1}{y} = 1 \quad ....(2)$$

Substituting $\frac{1}{y} = a$

3x – a = 7 ....(3)

x + a = 1 ....(4)

Applying 1(3) – 3(4), we get

−4a = 4

$$\Rightarrow a = \frac{-4}{4} = -1$$

$$\therefore \frac{1}{y} = -1$$

$$\Rightarrow y = -1$$

Substituting value of y in equation (2) we get

x = 2

Hence, x = 2 and y = −1.

(b) Given: ∠R = ∠S and ∠RPQ = ∠PQS

To prove: PS = QR

Proof: In ΔPQS and ΔPQR, we have

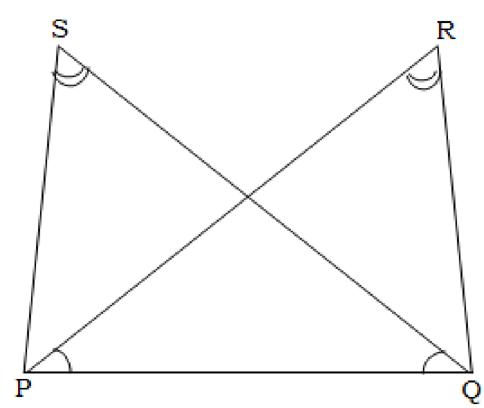
PQ = PQ [Common]

∠PSQ = ∠PRQ [Given]

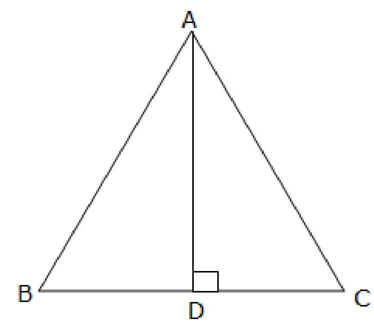
∠RPQ = ∠PQS [Given]

Hence, ΔPQS ≅ ΔPQR [AAS]

∴ PS = PR [CPCT]



(c) Let ABC be an equilateral triangle whose sides measure ‘a’ units each. Draw AD $\perp$ BC. Then, D is the mid-point of BC.



$\Rightarrow \quad AB = a,\ BD = \frac{1}{2}BC = \frac{a}{2}$

Since ΔABD is a right triangle right – angled at D.

$\therefore\ AB^2 = AD^2 + BD^2$

$\Rightarrow a^2 = AD^2 + \left(\frac{a}{2}\right)^2$

$\Rightarrow AD^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$

$\Rightarrow AD = \frac{\sqrt{3}a}{2}$

$\therefore\quad \text{Altitude} = \frac{\sqrt{3}}{2}a$

Now,

Area of $\Delta ABC = \frac{1}{2}(\text{Base} \times \text{Height})$

Area of $\Delta ABC = \frac{1}{2}(BC \times AD)$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2} \times a \times \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{4}a^2$

Q. 6.

(a) The given points are A(4, −1), B(6, 0), C(7, 2) and D(5, 1).

Using distance formula,

Diagonal AC = $\sqrt{(4-7)^2 + (-1-2)^2} = \sqrt{9+9} = \sqrt{18}$

Diagonal BD = $\sqrt{(6-5)^2 + (0-1)^2} = \sqrt{1+1} = \sqrt{2}$

Since, AC ≠ BD, ABCD is not a square.

(b)

$(e - y)^3 + (y - g)^3 + (g - e)^3$

Here, e – y + y – g + g – e = 0

Here, by the result, if a + b + c = 0, then, $a^3 + b^3 + c^3 = 3abc$

$(e - y)^3 + (y - g)^3 + (g - e)^3 = 3 \times (e - y)\ (y - g)\ (g - e)$

(c)

$(\text{Per.})^2 + (\text{Base})^2 = (\text{Hyp.})^2$ [By Pythagoras theorem]

$\Rightarrow 5^2 + (\text{Base})^2 = 13^2$

$\Rightarrow (\text{Base})^2 = 13^2 - 5^2$

$\Rightarrow \text{Base} = \sqrt{169-25} = \sqrt{144} = 12$

$\therefore \tan\theta = \frac{5}{12}, \frac{1}{\cos\theta} = \sec\theta = \frac{13}{12}$

$\Rightarrow \tan\theta + \frac{1}{\cos\theta} = \frac{5}{12} + \frac{13}{12} = \frac{5+13}{12} = \frac{18}{12} = \frac{3}{2}$

Q. 7.

(a) Given: ΔABC in which AD is the median.

To prove: $ar(\Delta ABC) = ar(\Delta ADC)$

Construction: Draw $AL \perp BC$.

Proof: Since D is the mid-point of BC, we have $BD = DC$

$\Rightarrow \frac{1}{2} BD \times AL = \frac{1}{2} DC \times AL$

$\Rightarrow ar(\Delta ABD) = ar(\Delta ADC)$

Hence, a median of a triangle divides in into two triangles of equal area.

(b) Let the width of concrete wall be x m

In ΔPQR, $\angle Q = 90°$, $\angle P = \theta°$ and $\angle R = \phi°$

By Pythagoras theorem, we have

$PQ^2 = PR^2 - QR^2$

$\Rightarrow PQ^2 = (x+2)^2 - x^2$

$\Rightarrow PQ^2 = x^2 + 4x + 4 - x^2$

$\Rightarrow PQ^2 = 4(x+1)$

$\Rightarrow PQ = 2\sqrt{x+1}$

Now, $\cot\phi = \frac{QR}{PQ} = \frac{x}{2\sqrt{x+1}}$ and $\tan\theta = \frac{QR}{PQ} = \frac{x}{2\sqrt{x+1}}$

(i) $\left(\sqrt{x+1}\right)\cot\phi = \left(\sqrt{x+1}\right) \times \frac{x}{2\sqrt{x+1}} = \frac{x}{2}$

(ii) $\left(\sqrt{x^3+x^2}\right)\tan\theta = \left(\sqrt{x^2(x+1)}\right)\tan\theta = x\left(\sqrt{x+1}\right) \times \frac{x}{2\sqrt{x+1}} = \frac{x^2}{2}$

(iii) $\cos\theta = \frac{PQ}{PR} = \frac{2\sqrt{x+1}}{x+2}$

(c) Given, circumference = 704 m

$\therefore\ 2\pi r = 704$

$\Rightarrow\ 2\times\frac{22}{7}\times r = 704$

$\Rightarrow r = \frac{7\times 704}{2\times 22} = 112\text{ m}$

and $R = 112 + 14 = 126\text{ m}$

$\therefore$ Surface area of road $= \pi R^2 - \pi r^2$

$$= \pi(R+r)(R-r)$$

$$= \frac{22}{7}(126+112)(126-112)$$

$$= \frac{22}{7}\times 238\times 14 = 10472\text{ m}^2$$

Given Rate of paving = Rs. 100 per m^2

$\therefore$ Total cost $= 10472\times 100 =$ Rs. 1047200

Q. 8.

(a) Given: A ΔABC and D, E, F are the mid-points of BC, CA and DF

AB = 5.8 cm, EF = 6 cm and DF = 5 cm

To find: BC and CA

EF || BC [As E and F are mid-points of AC and AB]

Also, $EF = \frac{1}{2}BC$

BC = 2 × EF = 2 × 6 = 12 cm

Thus, BC = 12 cm

DF || AC [As D and F are mid-points of AB and BC respectively]

And $DF = \frac{1}{2}AC \Rightarrow 5 = \frac{1}{2}AC \Rightarrow AC = 10\text{cm}$

(b)

$$x^3+y^3+z^3-3xyz$$

$$=(x+y)^3-3xy(x+y)+z^3-3xyz$$

$$=(x+y)^3+z^3-3xy(x+y)-3xyz$$

$$=(x+y+z)\left[(x+y)^2-(x+y)z+z^2\right]-3xy(x+y+z)$$

$$=(x+y+z)\left(x^2+2xy+y^2-xz-yz+z^2-3xy\right)$$

$$=(x+y+z)\left(x^2+y^2+z^2-yz-xz-xy\right)$$

(c) P = Rs. 30,000, A = Rs. 39,930, T = 3 half years $\Rightarrow$ n = 3

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$39{,}930 = 30{,}000\left(1+\frac{r}{100}\right)^3$$

$$\Rightarrow \frac{39930}{30000} = \left(1+\frac{r}{100}\right)^3$$

$$\Rightarrow \frac{1331}{1000} = \left(1+\frac{r}{100}\right)^3$$

$$\Rightarrow \left(\frac{11}{10}\right)^3 = \left(1+\frac{r}{100}\right)^3$$

$$\Rightarrow \frac{r}{100} = \frac{11}{10} - 1 = \frac{1}{10}$$

$$\Rightarrow \quad r = 10\%$$

So, rate of interest per annum $= 20\%$

Q. 9.

(a) Given: AD = AB, AE bisects $\angle A$

Construction: Join DE

To prove: BE = ED

Proof: In ΔABE and ΔADE

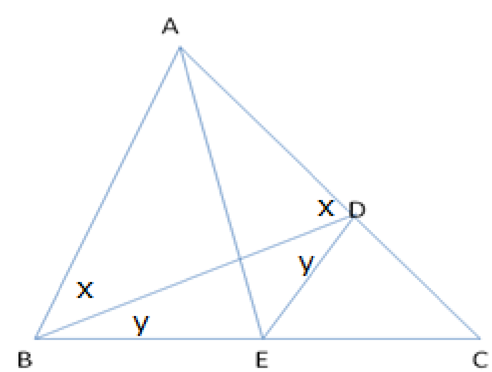
AE = AE [Common]

AD = AB [Given]

And $\angle BAE = \angle DAE$ [AE bisects $\angle A$]

$\Rightarrow \Delta ABE \cong \Delta ADE$ [S.A.S. Congruency]

So, BE = ED [CPCT]



(b)

$$\text{Given: } \frac{x-b-c}{a} + \frac{x-c-a}{b} + \frac{x-a-b}{c} = 3$$

$$\Rightarrow \frac{x-b-c}{a} - 1 + \frac{x-c-a}{b} - 1 + \frac{x-a-b}{c} - 1 = 3 - 3$$

$$\Rightarrow \frac{x-a-b-c}{a} + \frac{x-c-a-b}{b} + \frac{x-a-b-c}{c} = 0$$

$$\Rightarrow (x-a-b-c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = 0$$

$$\Rightarrow x-a-b-c = 0 \qquad \left(\text{as } \frac{1}{a}+\frac{1}{b}+\frac{1}{c} \neq 0\right)$$

$$\Rightarrow x = a+b+c$$

(c) Given that AB and CD are two chords of a circle with centre O, intersecting at a point E. PQ is the diameter through E, such that ∠AEQ = ∠DEQ.

To prove that AB = CD.

Draw perpendiculars OL and OM on chords AB and CD respectively.

Now, m∠LOE = 180° – 90° – m∠LEO ... [Angle sum property of a triangle]

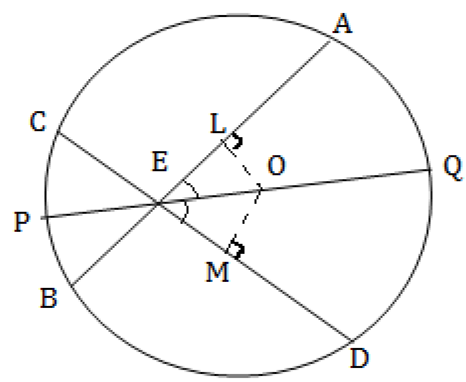
= 90° – m∠LEO

⇒ m∠LOE = 90° – m∠AEQ

⇒ m∠LOE = 90° – m∠DEQ

⇒ m∠LOE = 90° – m∠MEQ

⇒ ∠LOE = ∠MOE



In ΔOLE and ΔOME,

∠LEO = ∠MEO

∠LOE = ∠MOE

EO = EO

ΔOLE ≅ ΔOME

OL = OM

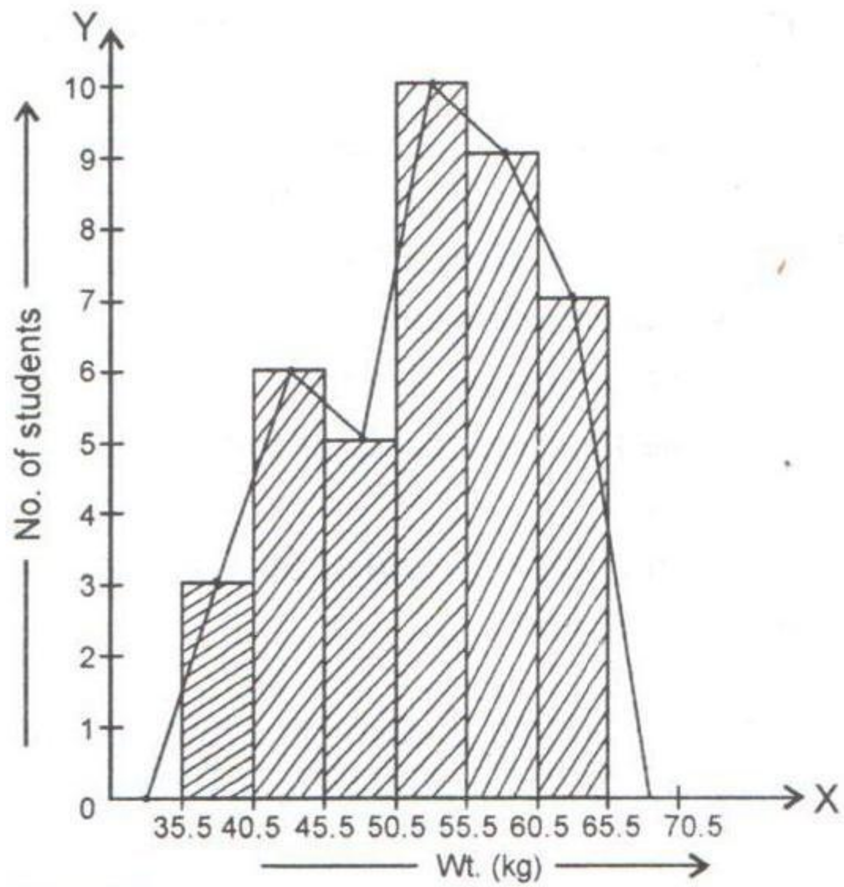
Therefore, cords AB and CD are equidistant from the centre.

Hence AB = CD

Q. 10.

(a) Adjustment factor $= \frac{41-40}{2} = 0.5$

C.I	**C.I after Adjustment**	**Frequency**
36-40	35.5 – 40.5	3
41-45	40.5 – 45.5	6
46-50	45.5 – 50.5	5
51-55	50.5 – 55.5	10
56-60	55.5 – 60.5	9
61-65	60.5 – 65.5	7



(b) Let the numerator be x and denominator be y

Then, the required fraction is $\frac{x}{y}$

According to the given conditions

$\frac{x+2}{y+1} = \frac{5}{8}$

$\Rightarrow 8x + 16 = 5y + 5$

$\Rightarrow 8x - 5y = -11$ ....(1)

And $\frac{x+1}{y+1} = \frac{1}{2}$

$\Rightarrow 2x + 2 = y + 1$

$\Rightarrow 2x - y = -1$ ....(2)

On Solving (1) and (2), we get

y = 7 and x = 3

Hence, the required fraction is $\frac{3}{7}$

Q. 11.

(a)

(i) Draw AB = 5.2 cm

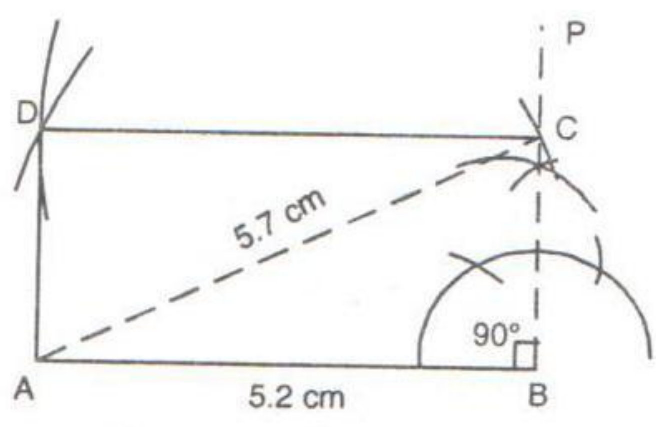
(ii) At B construct $m\angle ABP = 90^\circ$

(iii) With A as the centre and radius 5.7 cm, draw an arc to cut BP at C.

(iv) With C as centre and radius equal to 5.2 cm draw an arc.

(v) With B as centre and radius equal to 5.7 cm, cut the previous arc at D

(vi) Join AD and DC



(b) $4x - y = 13 \Rightarrow 4x = 13 + y \Rightarrow x = \frac{13+y}{4}$

Taking convenient values of y, we get

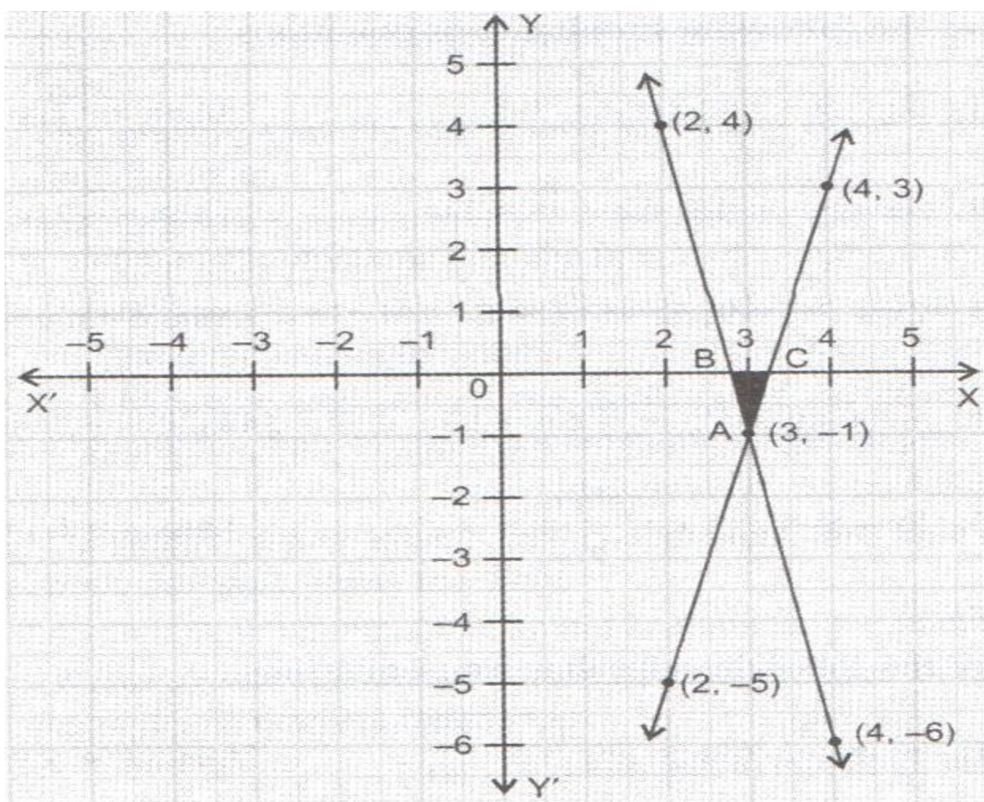
x	3	4	2
y	-1	3	−5

And $5x + y = 14 \Rightarrow 5x = 14 - y \Rightarrow x = \frac{14-y}{5}$

Taking convenient values of y, we get

x	3	2	4
y	−1	4	−6

Now plot these points on the graph paper,



i. From graph, the coordinates of the point of intersection of two lines are (3, −1).

ii. In ΔABC, BC = 0.6 cm, AD = 1 cm

$\therefore$ Area (ΔABC) = $\frac{1}{2} \times BC \times AD$ = $\frac{1}{2} \times 0.6 \times 1 = 0.3 \text{ cm}^2$

(c) Radius of the drum $= \frac{70}{2} = 35\text{cm}$

$\therefore$ No. of revolution = $\frac{\text{Distance by which the bucket is raised}}{\text{Circumference of the drum}}$

$$= \frac{11 \times 100}{2\pi \times 35} = \frac{11 \times 100 \times 7}{2 \times 35 \times 22} = 5$$

No. of revolutions = 5