(b) Area of room = (8×5) m^2 = 40 m^2

Let the length of carpet be 'x' m.

Area of carpet = $l \times b = \left(x \times \frac{80}{100}\right) = 0.80x$ m^2

Area of carpet = Area of floor

$\Rightarrow 0.80x = 40$

$\Rightarrow x = \frac{40}{80} \times 100 = 50$ m

$\therefore$ Cost of carpet = 50 × Rs. 22.50 = Rs. 1125

(c) PR = PQ [Given]

$\Rightarrow \angle 1 = \angle 2$ [angles opposite to equal sides]

$\because$ Exterior angle of a triangle is greater than any one of the interior opposite angles.

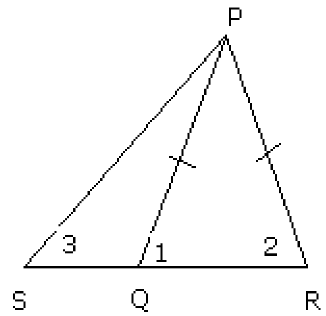
$\therefore \angle 1 > \angle 3$

$\Rightarrow \angle 2 > \angle 3$ [$\because \angle 1 = \angle 2$]

In ΔPSR, $\angle R > \angle S$

$\Rightarrow$ PS > PR [sides opposite to greater angle is longer]

$\Rightarrow$ PS > PQ [$\because$ PR = PQ]



Q. 8.

(a) Length (l) of the greenhouse = 30 cm

Breadth (b) of the greenhouse = 25 cm

Height (h) of the greenhouse = 25 cm

i. Total surface area of the greenhouse = 2[lb + lh + bh]

= [2(30 × 25 + 30 × 25 + 25 × 25)]

= [2(750 + 750 + 625)]

= (2 × 2125)

= 4250 cm^2

Thus, the area of the glass is 4250 cm^2.

ii. Total length of tape = 4(l + b + h)

= [4(30 + 25 + 25)] cm

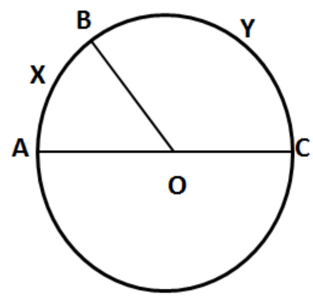
= 320 cm

Thus, 320 cm of tape is required for all the 12 edges.

(b) Given,

1. AOC is the diameter

2. Arc AXB = $\frac{1}{2}$ Arc BYC



From Arc AXB = $\frac{1}{2}$ Arc BYC we can see that

Arc AXB : Arc BYC = 1: 2

$\Rightarrow \angle BOA : \angle BOC = 1 : 2$

Since AOC is the diameter of the circle,

hence, $\angle AOC = 180^o$

Now,

Assume that $\angle BOA = x^o$ and $\angle BOC = 2x^o$

$\angle AOC = \angle BOA + \angle BOC = 180^o$

$\Rightarrow x + 2x = 180$

$\Rightarrow 3x = 180$

$\Rightarrow x = 60$

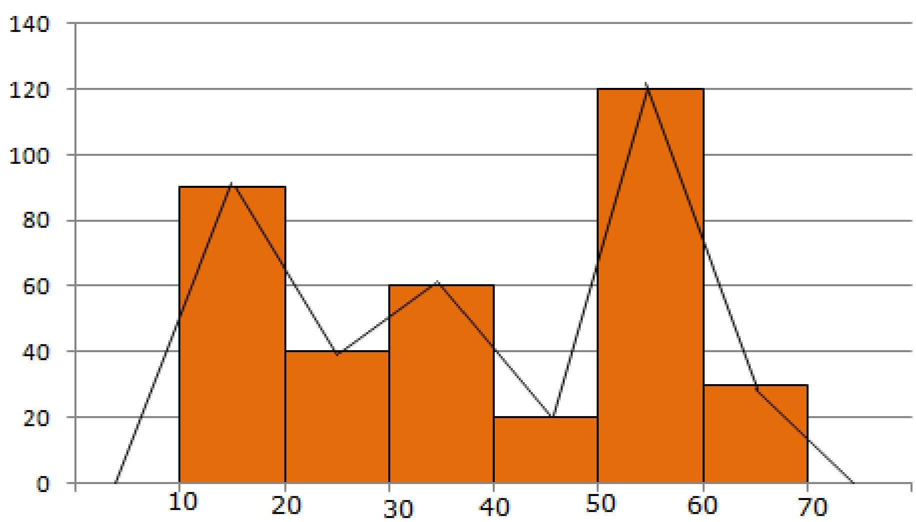
Hence $\angle BOA = 60^o$ and $\angle BOC = 120^o$

(c)

Age in years	10-20	20-30	30-40	40-50	50-60	60-70
Number of patients	90	40	60	20	120	30

Take class intervals i.e. age in years along x-axis and number of patients of width equal to the size of the class intervals and height equal to the corresponding frequencies to get the required histogram.

In order to draw frequency polygon, we take imaginary intervals 0-10 at the beginning and 70-80 at the end each with frequency zero and join the mid-points of top of the rectangles. Thus, we obtain a complete frequency polygon, shown below:



Q. 9.

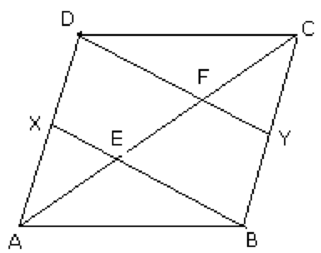
(a)

$$2\cos^2\theta + \sin\theta - 2 = 0$$
$$\Rightarrow 2\cos^2\theta + \sin\theta = 2$$
$$\Rightarrow \sin\theta = 2 - 2\cos^2\theta$$
$$\Rightarrow \sin\theta = 2(1 - \cos^2\theta)$$
$$\Rightarrow \sin\theta = 2 \times \sin^2\theta$$
$$\Rightarrow \frac{\sin^2\theta}{\sin\theta} = \frac{1}{2}$$
$$\Rightarrow \sin\theta = \frac{1}{2}$$
$$\Rightarrow \sin\theta = \sin 30°$$
$$\Rightarrow \theta = 30°$$

(b) Assume $p^{\frac{1}{x}} = p^{\frac{1}{y}} = p^{\frac{1}{z}} = k$

$$\Rightarrow p^{\frac{1}{x}} = k, p^{\frac{1}{y}} = k, p^{\frac{1}{z}} = k$$
$$\Rightarrow p = k^x, q = k^y, r = k^z$$
Also, $pqr = 1$
$$\Rightarrow k^x \times k^y \times k^z = 1$$
$$\Rightarrow k^{x+y+z} = k^0$$
$$\Rightarrow x + y + z = 0$$

(c) From the given figure,



$XD = \frac{1}{2}AD$ ($\because$ X is the midpoint of AD)

And $BY = \frac{1}{2}BC$ ($\because$ Y is the midpoint of BC)

$\therefore$ XD = BY (as AD = BC opposite sides of ||gm)

Also, XD || BY ($\because$ AD || BC opp. Sides of ||gm)

$\therefore$ XBYD is a parallelogram (Opposite sides are equal and parallel)

In ΔAFD, X is the midpoint of AD and XE || DF

$\therefore$ E is the midpoint of AF

$\therefore$ AE = EF ---- (i)

Now in ΔCEB, Y is the midpoint of BC and YF || BE.

$\therefore$ F is the midpoint of CE

$\therefore$ EF = FC ---- (ii)

From (i) and (ii)

AE = EF = FE [Hence proved]

Q. 10.

(a)

$$\frac{2\sqrt{7}+3\sqrt{5}}{\sqrt{7}+\sqrt{5}}$$

$$=\frac{2\sqrt{7}+3\sqrt{5}}{\sqrt{7}+\sqrt{5}}\times\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$$

$$=\frac{14-2\sqrt{35}+3\sqrt{35}-15}{\left(\sqrt{7}\right)^2-\left(\sqrt{5}\right)^2} \qquad \left[\because (a+b)(a-b)=a^2-b^2\right]$$

$$=\frac{\sqrt{35}-1}{7-5}$$

$$=\frac{\sqrt{35}-1}{2}$$

$$=\frac{1}{2}\sqrt{35}-\frac{1}{2}$$

On comparing this with $P\sqrt{35}+Q$, $P=\frac{1}{2}$ and $Q=\frac{-1}{2}$

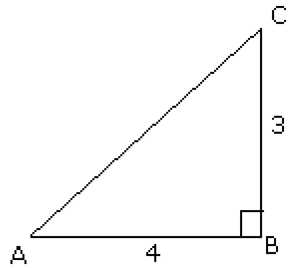
$$\therefore 2P+Q=2\times\frac{1}{2}+\left(\frac{-1}{2}\right)=1-\frac{1}{2}=\frac{1}{2}$$

(c) 3 cos A – 4 sin A = 0

$\Rightarrow 3\cos A = 4\sin A$

$\Rightarrow \frac{\sin A}{\cos A}=\frac{3}{4}$

$\Rightarrow \tan A=\frac{3}{4}$



By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= 4^2 + 3^2$

$= 16 + 9 = 25$

$\Rightarrow AC=\sqrt{25}=5$

$\sin A=\frac{BC}{AC}=\frac{3}{5}$

$\cos A=\frac{AB}{AC}=\frac{4}{5}$

$$\therefore \frac{\sin A+2\cos A}{3\cos A-\sin A}=\frac{\frac{3}{5}+2\times\frac{4}{5}}{3\times\frac{4}{5}-\frac{3}{5}}=\frac{\frac{3}{5}+\frac{8}{5}}{\frac{12}{5}-\frac{3}{5}}=\frac{\frac{11}{5}}{\frac{11}{9}}=\frac{9}{5}$$

(c) i. $a + \frac{1}{a} = 4,$

On squaring, we get

$a^2 + \frac{1}{a^2} + 2 = 16$

$\Rightarrow a^2 + \frac{1}{a^2} = 16 - 2 = 14$

ii. By part (i), we have $a^2 + \frac{1}{a^2} = 14$

On squaring, we get

$\Rightarrow a^4 + \frac{1}{a^4} + 2 = 196$

$\Rightarrow a^4 + \frac{1}{a^4} = 196 - 2 = 194$

Q. 11.

(a) Given: In ΔABC, AD is the median

To prove: Area of ΔABD = Area of ΔADC

Construction: We draw AE ⊥ BC

Proof:

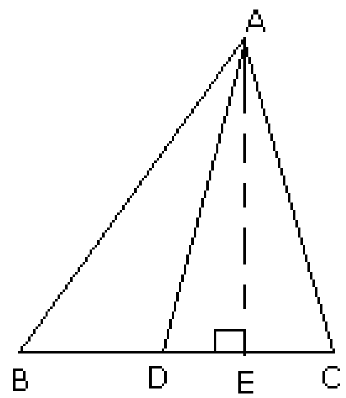
Area of ΔABD = $\frac{1}{2} \times BD \times AE$ $\quad$ [$\because$ Area $= \frac{1}{2} \times \text{base} \times \text{height}$]

Similarly, area of ΔADC = $\frac{1}{2} \times DC \times AE$

Here, we have BD = DC

$\therefore$ Area of ΔABD = Area of ΔADC

Hence proved.



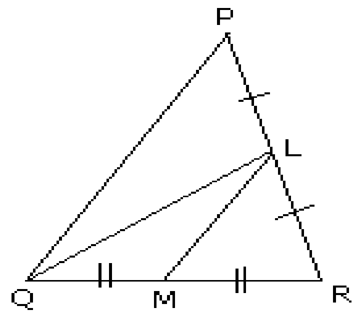
(b) Given, Area of ΔPQR = 44.8 cm^2

Since QL is the median and Median divides triangle into two triangles of equal areas

Area of ΔLQR = Area of ΔPQL = 22.4 cm^2

In ΔQLR, LM is the median,

Area of ΔLMR = $\frac{1}{2} \times 22.4 = 11.2 cm^2$



(c) $x^3 - 3x^2 - x + 3$

$= x^2 (x - 3) - 1(x - 3)$

$= (x - 3)(x^2 - 1)$

$= (x - 3)(x - 1)(x + 1)$