ICSE Board Class IX Mathematics Paper 1 – Solution

SECTION - A (40 Marks)

Q. 1.

(a) Here, C.I. = Rs. 287, r = 5% p.a., n = 2 years C.I. = P $\left[\left(1 + \frac{r}{100} \right)^n - 1 \right] = 287$ $\Rightarrow P \left[\left(1 + \frac{5}{100} \right)^2 - 1 \right] = 287$ $\Rightarrow P \left[\left(\frac{105}{100} \right)^2 - 1 \right] = 287$ $\Rightarrow P \left[\left(\frac{21}{20} \right)^2 - 1 \right] = 287$ $\Rightarrow P \left[\left(\frac{441}{400} - 1 \right] = 287$ $\Rightarrow P \left[\left(\frac{441 - 400}{400} \right) \right] = 287$ $\Rightarrow P \times \frac{41}{400} = 287$ $\Rightarrow P \times \frac{41}{400} = 287$ $\Rightarrow P = \frac{287 \times 400}{41} = 2800$ Thus P = Rs. 2,800

(b) Let us assume that $\sqrt{2}$ is a rational number.

Then,
$$\sqrt{2} = \frac{p}{q}$$
(1)

Where p and q are integers, co-prime to each other and $q \neq 0$. On squaring both sides, we get

$$2 = \frac{p^2}{q^2} \Longrightarrow p^2 = 2q^2 \quad \dots (2)$$

By equation (2), we can say that p^2 is an even integer.

 \therefore p is also an even integer (\because the square of an even integer is always even) Let p = 2k, where k is an integer. From (2), $p^2 = 2q^2$ $(2k)^2 = 2q^2$ $4k^2 = 2q^2$ $\Rightarrow q^2 = 2k^2$

 $q^2 \mbox{ is an even integer, } q \mbox{ is also an even integer.}$

Thus, p and q have a common factor 2 which contradicts the hypothesis that p, q are co-prime to each other.

 $\therefore \sqrt{2}$ is an irrational number.

(c)

$$\frac{\cos 37^{\circ} \cdot \csc 53^{\circ}}{\tan 5^{\circ} \cdot \tan 25^{\circ} \cdot \tan 45^{\circ} \cdot \tan 65^{\circ} \cdot \tan 85^{\circ}}$$

$$= \frac{\cos 37^{\circ} \cdot \csc (90^{\circ} - 37^{\circ})}{\tan 5^{\circ} \cdot \tan 25^{\circ} \cdot \tan 45^{\circ} \cdot \tan (90^{\circ} - 25^{\circ}) \cdot \tan (90^{\circ} - 5^{\circ})}$$

$$= \frac{\cos 37^{\circ} \cdot \sec 37^{\circ}}{\tan 5^{\circ} \cdot \tan 25^{\circ} \cdot \tan 45^{\circ} \cdot \cot 25^{\circ} \cdot \cot 5^{\circ}} \qquad \begin{bmatrix} \because \csc (90^{\circ} - \theta) = \sec \theta \\ \tan (90^{\circ} - \theta) = \cot \theta \end{bmatrix}$$

$$= \frac{1}{\tan 5^{\circ} \cdot \cot 5^{\circ} \cdot \tan 25^{\circ} \cdot \cot 25^{\circ} \cdot 1} \qquad \begin{bmatrix} \because \sec \theta = \frac{1}{\cos \theta} \\ \tan 45^{\circ} = 1 \end{bmatrix}$$

$$= \frac{1}{1} \qquad \qquad \begin{bmatrix} \because \cot \theta = \frac{1}{\tan \theta} \end{bmatrix}$$

$$= 1$$

Q. 2

(a) In $\triangle ABD$ and $\triangle BCD$ $m \angle A = m \angle C = 90^{\circ}$ AB = DC [Given] BD = BD [Common] $\triangle ABD \cong \triangle CDB$ [R.H.S.] $\Rightarrow \angle CBD = \angle ADB$ [C.P.C.T.] $\therefore \angle CBD = x^{\circ} = 50^{\circ}$ Also, BC = AD [C.P.C.T.] $\Rightarrow 3y - 20 = y - 10$ $\Rightarrow 3y - y = 20 - 10$ $\Rightarrow 2y = 10$ $\Rightarrow y = 5 \text{ cm}$

(b)
$$2 \log 3 - \frac{1}{2} \log 16 + \log 12 = \log 3^2 - \log(16)^{\frac{1}{2}} + \log 12$$

= $\log 9 - \log 4 + \log 12$
= $\log \frac{9 \times 12}{4}$
= $\log 27$

- (c) Steps of construction for constructing parallelogram:
 - 1) Draw a line AB of measure 6 cm.
 - 2) Draw an angle of measure 45° at point A such that \angle DAB = 45° and AD = 5 cm.
 - 3) Now draw a line CD parallel to line AB of measure 6 cm.
 - 4) Join BC. Join diagonals AC and BD. Let them intersect at 0.

Thus, ABCD is the required parallelogram.



(b) Given,
$$(2a + b, a - 2b) = (7, 6)$$

 $\Rightarrow 2a + b = 7$ (1)
 $a - 2b = 6$ (2)
Multiplying equation (1) by 2, we get
 $4a + 2b = 14$ (3)
Adding equations (2) and (3), we get
 $5a = 20$
 $\Rightarrow a = 4$
Substituting $a = 4$ in equation (1), we get
 $2(4) + b = 7$
 $\Rightarrow b = -1$
 $\therefore a = 4$ and $b = -1$

(c) Let A = (0, 0), B = (5, 0), C = (8, 4) and D = (3, 4)
AB =
$$\sqrt{(5-0)^2 + (0-0)^2} = \sqrt{25+0} = 5$$

BC = $\sqrt{(8-5)^2 + (4-0)^2} = \sqrt{9+16} = \sqrt{25} = 5$
CD = $\sqrt{(8-3)^2 + (4-4)^2} = \sqrt{25+0} = 5$
DA = $\sqrt{(0-3)^2 + (0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$
AC = $\sqrt{(8-0)^2 + (4-0)^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$
BD = $\sqrt{(3-5)^2 + (4-0)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$
Now, AB = BC = CD = DA and AC \neq BD.
Hence, ABCD is a rhombus.
 \therefore Area of rhombus = $\frac{1}{2} \times AC \times BD = \frac{1}{2} \times 4\sqrt{5} \times 2\sqrt{5} = 20$ sq. units

Q. 4.

(a) Given side of an equilateral triangle = a

Let,
$$AD \perp DC \therefore BD = DC = \frac{a}{2}$$

In right $\triangle ABD$, $AB^2 = AD^2 + BD^2$ [Using Pythagoras theorem]



$$a^{2} = AD^{2} + \left(\frac{a}{2}\right)^{2} \Rightarrow AD^{2} = a^{2} - \frac{a^{2}}{4} = \frac{3a^{2}}{4} \Rightarrow AD = \frac{a\sqrt{3}}{2}$$

Now, Area of $\triangle ABC = \frac{1}{2} \times Base \times height = \frac{1}{2} \times BC \times AD = \frac{1}{2} \times a \times \frac{a\sqrt{3}}{2} = \frac{\sqrt{3}}{4}a^{2}$

(b) We know, each exterior angle = $\frac{360^{\circ}}{n}$

$$\therefore \frac{360^{\circ}}{n} - \frac{360^{\circ}}{n+2} = 6$$

$$\Rightarrow 360^{\circ} \left[\frac{n+2-n}{n(n+2)} \right] = 6$$

$$\Rightarrow \frac{360^{\circ} \times 2}{n^{2}+2n} = 6$$

$$\Rightarrow n^{2} + 2n = \frac{360^{\circ} \times 2}{6}$$

$$\Rightarrow n^{2} + 2n = 120$$

$$\Rightarrow n^{2} + 2n - 120 = 0$$

$$\Rightarrow n^{2} + 12n - 10n - 120 = 0$$

$$\Rightarrow n(n+12) - 10(n+12) = 0$$

$$\Rightarrow (n+12)(n-10) = 0$$

$$\Rightarrow n = -12 \text{ or } n = 10$$

Neglecting n = -12 as number of sides cannot be negative. \therefore n = 10 Thus, number of sides are 10.

(c)
$$\frac{4}{\tan^2 60^\circ} + \frac{1}{\cos^2 30^\circ} - \tan^2 45^\circ$$
$$= \frac{4}{\left(\sqrt{3}\right)^2} + \frac{1}{\left(\sqrt{3}/2\right)^2} - (1)^2$$
$$= \frac{4}{3} + \frac{4}{3} - 1$$
$$= \frac{5}{3}$$



From the graph, we find that the two lines intersect at the point (-2, -1)Therefore, the solution is x = -2, y = -1

(b) Let his starting salary be Rs. x and the fixed annual increment be Rs. y Salary after 4 years = x + 4yAccording to question, x + 4y = 1500(i) And salary after 10 years = x + 10y $\Rightarrow x + 10y = 1800$ (ii) Subtracting (i) from (ii), we get 6y = 300 $\Rightarrow y = 50$ Substituting the value of y in (i), we get x + 4y = 1500 $\Rightarrow x + 4 \times 50 = 1500$ $\Rightarrow x = 1300$ \therefore Starting salary = Rs. 1300 and Annual increment = Rs. 50 (c)

$$x = \frac{1}{\sqrt{2} - 1} = \frac{1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} = \frac{\sqrt{2} + 1}{2 - 1} = \sqrt{2} + 1$$

$$\therefore x^{2} = (\sqrt{2} + 1)^{2} = 2 + 1 + 2\sqrt{2} = 3 + 2\sqrt{2}$$

Therefore,

$$x^{2} - 6 + \frac{1}{x^{2}} = (\sqrt{2} + 1)^{2} - 6 + \frac{1}{(\sqrt{2} + 1)^{2}}$$

$$= 3 + 2\sqrt{2} - 6 + \frac{1}{3 + 2\sqrt{2}}$$

$$= 3 + 2\sqrt{2} - 6 + 3 - 2\sqrt{2}$$

$$= 0$$

Q. 6.

(a) Let the sum be Rs. P, A = Rs. 3630, r = 10%, n = 2 years We know that,

$$A = P \left(1 + \frac{r}{100} \right)^{n}$$

$$\Rightarrow 3630 = P \left(1 + \frac{10}{100} \right)^{2}$$

$$\Rightarrow 3630 = P \left(\frac{110}{100} \right)^{2}$$

$$\Rightarrow P = \frac{3630 \times 10 \times 10}{11 \times 11}$$

$$\Rightarrow P = \text{Rs. } 3000$$

(b) In right $\triangle PQS$,

PS² + QS² = PQ² [By Pythagoras theorem] ⇒ PS² = $(10)^2 - (6)^2 = 100 - 36 = 64$ ⇒ PS = 8 cm ∴ RS = RQ + QS = 9 + 16 = 15 cm Now in Δ PSR, ⇒ PR² = PS² + RS² ⇒ PR² = $(8)^2 + (15)^2 = 64 + 225 = 289$ ⇒ PR = 17 cm (c) Consider the following figure:



Distance of smaller chord AB from centre of circle = 4 cm

i.e.,
$$OM = 4 \text{ cm}$$

 $MB = \frac{AB}{2} = \frac{6}{2} = 3 \text{ cm}$
In ΔOMB ,
 $OM^2 + MB^2 = OB^2$
 $\therefore 4^2 + 3^2 = OB^2$
 $\therefore 16 + 9 = OB^2$
 $\therefore 0B^2 = 25$
 $\therefore OB = \sqrt{25} = 5 \text{ cm}$
In ΔOND ,
 $OD = OB = 5 \text{ cm}$ (radii of the same circle)
 $ND = \frac{CD}{2} = \frac{8}{2} = 4 \text{ cm}$
 $ON^2 + ND^2 = OD^2$
 $\therefore ON^2 = OD^2 - ND^2$
 $\therefore ON^2 = 5^2 + 4^2 = 25 - 16 = 9$
 $\therefore ON = \sqrt{9} = 3 \text{ cm}$

Q. 7.

(a) Arranging the numbers in ascending order: 1, 2, 3, 3, 3, 4, 5, 5, 6, 7 Number of terms = n = 10 (even) ∴ Mean = $\frac{1+2+3+3+3+4+5+5+6+7}{10} = \frac{39}{10} = 3.9$ And median = $\frac{5^{\text{th}} \text{term} + 6^{\text{th}} \text{term}}{2} = \frac{3+4}{2} = \frac{7}{2} = 3.5$ (b) Area of room = (8×5) m² = 40 m² Let the length of carpet be 'x' m.

Area of carpet = $l \times b = \left(x \times \frac{80}{100}\right) = 0.80 \text{ x m}^2$ Area of carpet = Area of floor $\Rightarrow 0.80 \text{ x} = 40$ $\Rightarrow x = \frac{40}{80} \times 100 = 50 \text{ m}$ $\therefore \text{ Cost of carpet = } 50 \times \text{Rs. } 22.50 = \text{Rs. } 1125$

(c) PR = PQ [Given]

 $\Rightarrow \angle 1 = \angle 2$ [angles opposite to equal sides] : Exterior angle of a triangle is greater than any one of the interior opposite angles.

$$\therefore \angle 1 > \angle 3$$

$$\Rightarrow \angle 2 > \angle 3 \qquad [\because \angle 1 = \angle 2]$$

In $\triangle PSR, \angle R > \angle S$

$$\Rightarrow PS > PR \qquad [sides opposite to greater angle is longer]$$

$$\Rightarrow PS > PQ \qquad [\because PR = PQ]$$



Q. 8.

(a) Length (l) of the greenhouse = 30 cm

Breadth (b) of the greenhouse = 25 cm

Height (h) of the greenhouse = 25 cm

i. Total surface area of the greenhouse = 2[lb + lh + bh]

$$= [2(30 \times 25 + 30 \times 25 + 25 \times 25)]$$
$$= [2(750 + 750 + 625)]$$
$$= (2 \times 2125)$$
$$= 4250 \text{ cm}^2$$

Thus, the area of the glass is 4250 cm^2 .

ii. Total length of tape = 4(l + b + h)

= [4(30 + 25 + 25)] cm

= 320 cm

Thus, 320 cm of tape is required for all the 12 edges.

(b) Given, 1. AOC is the diameter 2. Arc AXB = $\frac{1}{2}$ Arc BYC From Arc AXB = $\frac{1}{2}$ Arc BYC we can see that Arc AXB : Arc BYC = 1: 2 $\Rightarrow \angle BOA : \angle BOC = 1 : 2$ Since AOC is the diameter of the circle, hence, $\angle AOC = 180^{\circ}$ Now, Assume that $\angle BOA = x^{\circ}$ and $\angle BOC = 2x^{\circ}$ $\angle AOC = \angle BOA + \angle BOC = 180^{\circ}$ $\Rightarrow x + 2x = 180$ $\Rightarrow x = 60$ Hence $\angle BOA = 60^{\circ}$ and $\angle BOC = 120^{\circ}$



(c)

Age in years	10-20	20-30	30-40	40-50	50-60	60-70
Number of patients	90	40	60	20	120	30

Take class intervals i.e. age in years along x-axis and number of patients of width equal to the size of the class intervals and height equal to the corresponding frequencies to get the required histogram.

In order to draw frequency polygon, we take imaginary intervals 0-10 at the beginning and 70-80 at the end each with frequency zero and join the mid-points of top of the rectangles. Thus, we obtain a complete frequency polygon, shown below:



Q. 9. (a) $2\cos^{2}\theta + \sin\theta - 2 = 0$ $\Rightarrow 2\cos^{2}\theta + \sin\theta = 2$ $\Rightarrow \sin\theta = 2 - 2\cos^{2}\theta$ $\Rightarrow \sin\theta = 2(1 - \cos^{2}\theta)$ $\Rightarrow \sin\theta = 2 \times \sin^{2}\theta$ $\Rightarrow \frac{\sin^{2}\theta}{\sin\theta} = \frac{1}{2}$ $\Rightarrow \sin\theta = \frac{1}{2}$

$$\Rightarrow \sin \theta = \frac{1}{2}$$
$$\Rightarrow \sin \theta = \sin 30^{\circ}$$
$$\Rightarrow \theta = 30^{\circ}$$

(b) Assume
$$p^{\frac{1}{x}} = p^{\frac{1}{y}} = p^{\frac{1}{z}} = k$$

 $\Rightarrow p^{\frac{1}{x}} = k, p^{\frac{1}{y}} = k, p^{\frac{1}{z}} = k$
 $\Rightarrow p = k^{x}, q = k^{y}, r = k^{z}$
Also, pqr = 1
 $\Rightarrow k^{x} \times k^{y} \times k^{z} = 1$
 $\Rightarrow k^{x+y+z} = k^{0}$
 $\Rightarrow x+y+z=0$

(c) From the given figure,

 $XD = \frac{1}{2}AD$ (:: X is the midpoint of AD) And BY = $\frac{1}{2}$ BC (:: Y is the midpoint of BC) \therefore XD = BY (as AD = BC opposite sides of $\|gm$) Δ Also, XD || BY (:: AD || BC opp. Sides of ||gm) : XBYD is a parallelogram (Opposite sides are equal and parallel) In $\triangle AFD$, X is the midpoint of AD and XE || DF : E is the midpoint of AF ---- (i) $\therefore AE = EF$ Now in $\triangle CEB$, Y is the midpoint of BC and YF || BE. ∴ F is the midpoint of CE \therefore EF = FC ---- (ii) From (i) and (ii) AE = EF = FE [Hence proved] 11



Q. 10. (a) $\frac{2\sqrt{7} + 3\sqrt{5}}{\sqrt{7} + \sqrt{5}}$ $= \frac{2\sqrt{7} + 3\sqrt{5}}{\sqrt{7} + \sqrt{5}} \times \frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} - \sqrt{5}}$ = $\frac{14 - 2\sqrt{35} + 3\sqrt{35} - 15}{(\sqrt{7})^2 - (\sqrt{5})^2}$ [:: $(a + b)(a - b) = a^2 - b^2$] $=\frac{\sqrt{35}-1}{7-5}$ $=\frac{\sqrt{35}-1}{2}$ $=\frac{1}{2}\sqrt{35}-\frac{1}{2}$

On comparing this with $P\sqrt{35} + Q$, $P = \frac{1}{2}$ and $Q = \frac{-1}{2}$

$$\therefore 2P + Q = 2 \times \frac{1}{2} + \left(\frac{-1}{2}\right) = 1 - \frac{1}{2} = \frac{1}{2}$$

(c)
$$3 \cos A - 4 \sin A = 0$$

 $\Rightarrow 3 \cos A = 4 \sin A$
 $\Rightarrow \frac{\sin A}{\cos A} = \frac{3}{4}$
 $\Rightarrow \tan A = \frac{3}{4}$
By Pythagoras theorem,
 $AC^2 = AB^2 + BC^2$
 $= 4^2 + 3^2$
 $= 16 + 9 = 25$
 $\Rightarrow AC = \sqrt{25} = 5$
 $\sin A = \frac{BC}{AC} = \frac{3}{5}$
 $\cos A = \frac{AB}{AC} = \frac{4}{5}$
 $\therefore \frac{\sin A + 2\cos A}{3\cos A - \sin A} = \frac{\frac{3}{5} + 2 \times \frac{4}{5}}{3 \times \frac{4}{5} - \frac{3}{5}} = \frac{\frac{3}{5} + \frac{8}{5}}{\frac{12}{5} - \frac{3}{5}} = \frac{\frac{11}{5}}{\frac{11}{9}} = \frac{9}{5}$

(c) i.
$$a + \frac{1}{a} = 4$$
,
On squaring, we get
 $a^{2} + \frac{1}{a^{2}} + 2 = 16$
 $\Rightarrow a^{2} + \frac{1}{a^{2}} = 16 - 2 = 14$
ii. By part (i), we have $a^{2} + \frac{1}{a^{2}} = 14$
On squaring, we get
 $\Rightarrow a^{4} + \frac{1}{a^{4}} + 2 = 196$
 $\Rightarrow a^{4} + \frac{1}{a^{4}} = 196 - 2 = 194$

Q. 11.

(a) Given: In $\triangle ABC$, AD is the median To prove: Area of $\triangle ABD = Area$ of $\triangle ADC$ Construction: We draw $AE \perp BC$ Proof: Area of $\triangle ABD = \frac{1}{2} \times BD \times AE$ [:: Area $= \frac{1}{2} \times base \times height$] Similarly, area of $\triangle ADC = \frac{1}{2} \times DC \times AE$ Here, we have BD = DC \therefore Area of $\triangle ABD = Area$ of $\triangle ADC$ Hence proved.

(b) Given, Area of $\triangle PQR = 44.8 \text{ cm}^2$ Since QL is the median and Median divides triangle into two triangles of equal areas Area of $\triangle LQR = \text{Area of } \triangle PQL = 22.4 \text{ cm}^2$ In $\triangle QLR$, LM is the median, Area of $\triangle LMR = \frac{1}{2} \times 22.4 = 11.2 \text{ cm}^2$

Area of
$$\Delta LMR = \frac{1}{2} \times 22.4 = 11.2 \text{ cm}^2$$

(c)
$$x^3 - 3x^2 - x + 3$$

= $x^2 (x - 3) - 1(x - 3)$
= $(x - 3)(x^2 - 1)$
= $(x - 3)(x - 1)(x + 1)$

