ICSE Board
Class IX Mathematics
Paper 5 – Solution

Time: 2½ hrs **Total Marks: 80**

SECTION – A

Q. 1.

(a) Let x= $0.\overline{001}$

Then, x = 0.001001001 ….(i)

Therefore, 1000x = 1.001001001 ….(ii)

Subtracting (i) from (ii), we get 999x = 1 ⇒ x = $\frac{1}{999}$

Hence, $0.\overline{001} = \frac{1}{999}$

(b) On arranging the numbers in ascending order, we get

3, 4, 9, 10, 12, 15, 18, 27, 47, 48, 75, 81

n = 12 (even)

$$\text{Median} = \frac{\frac{n}{2}^{th}\text{ term} + \left(\frac{n}{2}+1\right)^{th}\text{ term}}{2}$$

$$= \frac{\frac{12}{2}^{th}\text{ term} + \left(\frac{12}{2}+1\right)^{th}\text{ term}}{2}$$

$$= \frac{6^{th}\text{ term} + 7^{th}\text{ term}}{2}$$

$$= \frac{15+18}{2}$$

$$= 16.5$$

(c) Given, side of the square = $\left(\frac{x+1}{2}\right)$units

And diagonal = $\frac{3-x}{\sqrt{2}}$ units = = $\sqrt{2}$side

$$\Rightarrow \frac{3-x}{\sqrt{2}} = \sqrt{2}\left(\frac{x+1}{2}\right)$$

$$\Rightarrow 3-x = x+1$$

$$\Rightarrow 2x = 2 \Rightarrow x = 1$$

$$\therefore \text{length of side} = \frac{x+1}{2} = \frac{1+1}{2} = 1\text{unit}$$

Q. 2.

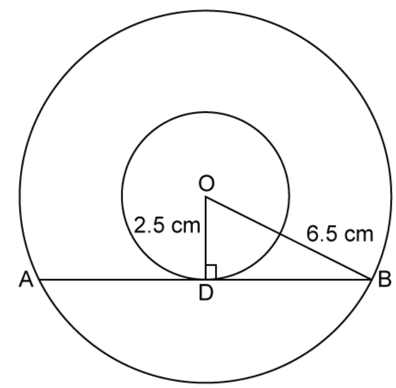
(a)

Since AB is a tangent to the inner circle.

$\angle ODB = 90°$ ....(tangent is $\perp$ to the radius of a circle)

AB is a chord of the outer circle.

We know that, the perpendicular drawn from the centre to a chord of a circle, bisects the chord.



$\Rightarrow AB = 2DB$

In ΔODB,

By Pythagoras theorem,

$OB^2 = OD^2 + DB^2$

$\Rightarrow 6.5^2 = 2.5^2 + DB^2$

$\Rightarrow DB^2 = 6.5^2 - 2.5^2$

$\Rightarrow DB^2 = 42.25 - 6.25$

$\Rightarrow DB^2 = 36$ cm

$\Rightarrow DB = 6$ cm

$AB = 2DB = 2(6) = 12$ cm

(b) Given points are A(8, 2), B(5, −3) and C(0, 0).

Using the distance formula, we get,

$AC = \sqrt{(8-0)^2 + (2-0)^2} = \sqrt{68}$

$BC = \sqrt{(5-0)^2 + (-3-0)^2} = \sqrt{34}$

$AB = \sqrt{(5-8)^2 + (-3-2)^2} = \sqrt{34}$

Since, BC = AB, ΔABC is an isosceles triangle.

(c) By Pythagoras theorem

$y^2 = x^2 + (\text{Base})^2$

$\Rightarrow (\text{Base})^2 = y^2 - x^2$

$\Rightarrow \text{Base} = \sqrt{y^2 - x^2}$

$\therefore \cos\theta = \frac{\sqrt{y^2 - x^2}}{y}, \tan\theta = \frac{x}{\sqrt{y^2 - x^2}}$

$\cos\theta \times \tan\theta = \frac{\sqrt{y^2 - x^2}}{y} \times \frac{x}{\sqrt{y^2 - x^2}} = \frac{x}{y}$

Q. 3

(a) $a^2 + b^2 - c^2 - 2ab = a^2 + b^2 - 2ab - c^2$

$$= (a-b)^2 - (c)^2$$

$$= (a-b+c)(a-b-c)$$

(b)

Let $x = 9^{\log 4}, y = 16^{\log 3}$

$\log x = \log 9^{(\log 4)}$

$\log x = \log 4.\log 9$ ....(1)

$\log y = 16^{\log 3}$

$\Rightarrow \log y = \log 3.\log 16 = \log 3.\log 4^2$

$\Rightarrow$ log y = 2 log 3. Log 4

$\Rightarrow$ log y = log 9. Log 4 ....(2)

$\Rightarrow$ log x = log y [From (1) and (2)]

Hence x = y

(c)

$$\left(\frac{81}{16}\right)^{-3/4} \times \left[\left(\frac{25}{9}\right)^{-3/2} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{3^4}{2^4}\right)^{-3/4} \times \left[\left(\frac{5^2}{3^2}\right)^{-3/2} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{3}{2}\right)^{4\times -3/4} \times \left[\left(\frac{5}{3}\right)^{2\times -3/2} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{3}{2}\right)^{-3} \times \left[\left(\frac{5}{3}\right)^{-3} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{3}{2}\right)^{-3} \times \left(\frac{5}{3} \div \frac{5}{2}\right)^{-3}$$

$$= \left(\frac{3}{2}\right)^{-3} \times \left(\frac{5}{3} \times \frac{2}{5}\right)^{-3}$$

$$= \left(\frac{3}{2}\right)^{-3} \times \left(\frac{2}{3}\right)^{-3}$$

$$= \left(\frac{3}{2} \times \frac{2}{3}\right)^{-3}$$

$$= (1)^{-3}$$

$$= 1$$

Q. 4.

(a) In ΔABC and ΔCDE

$\angle BAC = \angle CED$ [Given]

$AC = EC$ [Given]

$\angle ACB = \angle DCE$ [Vertically opposite $\angle$s]

Hence $\Delta ACB \cong \Delta ECD$ [$\because$ ASS – condition of congruency is satisfied]

$\therefore$ AB = ED [CPCT]

Then, $2x + 4 = 3y + 8$

$2x - 3y = 4$ ....(1)

Also, BC = CD

$x = 2y$

$x - 2y = 0$ ....(2)

Solving (1) and (2), we get

$x = 8$ and $y = 4$

(b) Amount at the end of first year = Principal for second year

P = Rs. 1250, A = Rs. 1375, n = 1, rate = r%

$$1375 = 1250\left(1 + \frac{r}{100}\right)^1$$

$$\frac{1375}{1250} = \frac{100 + r}{100}$$

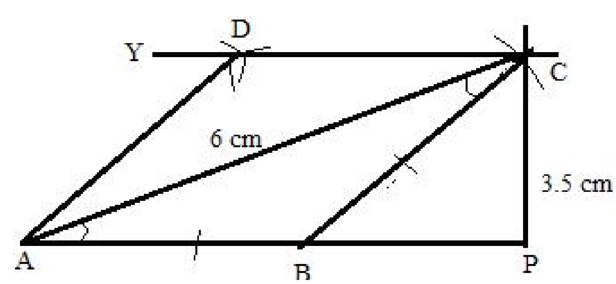
$$\Rightarrow 125000 + 1250r = 137500$$

$$\Rightarrow 1250r = 137500 - 125000$$

$$\Rightarrow 1250r = 12500 \Rightarrow r = \frac{12500}{1250} = 10\%$$

(c) Steps of construction:

1) Draw a line AP.
2) Now draw AC = 6 cm and CP = 3.5 cm
3) Draw a line BC such that AB = BC.
4) Now at C draw a line CY parallel to AP.



5) At point C and A, taking radius same as AB draw arcs cutting each other at D.
6) Now join AD.

ABCD is the required rhombus.

SECTION – B

Q. 5.

(a) (i) $3x - y - 2 = 0$

$\Rightarrow y = 3x - 2$

Taking convenient value of x

x	0	2	3
y	−2	4	7

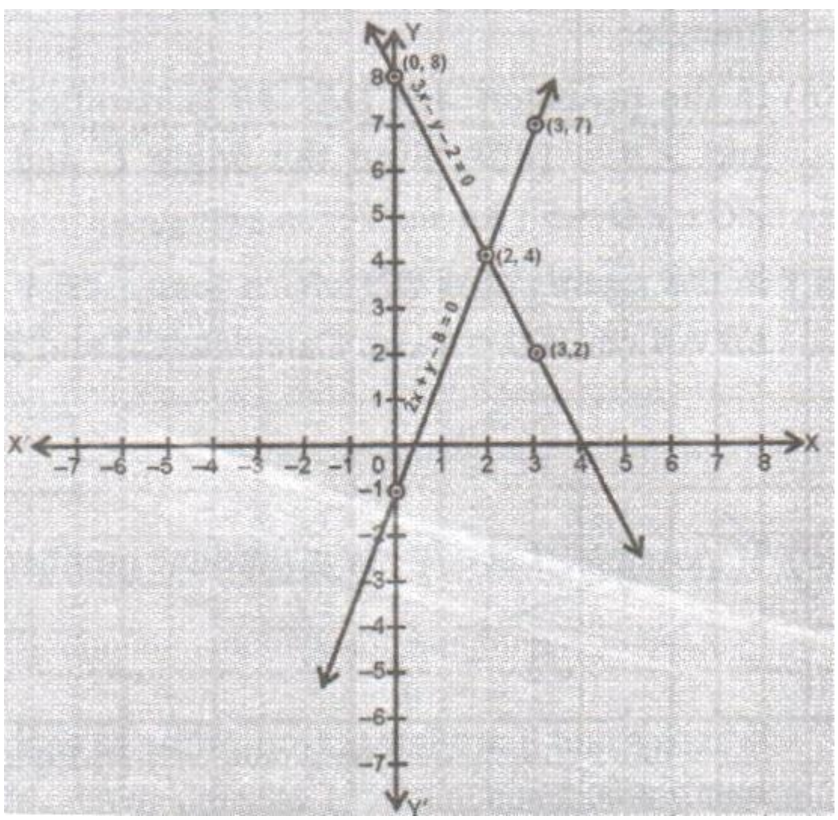
$2x + y - 8 = 0$

$y = 8 - 2x$

Taking convenient value of x

x	0	2	3
y	8	4	2

Now plot these points on the graph paper,

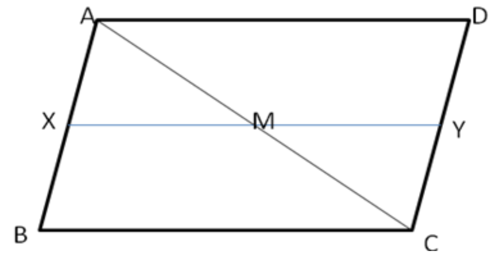


(ii) The coordinates of the point of intersection are (2, 4).

(b) Given: ABCD is a parallelogram, M is the midpoint of AC, X and Y are points on AB and DC respectively such that AX = CY.

To prove: (a)$\Delta AXM \cong \Delta CYM$ (b) XMY is a straight line

Construction: Join XM and MY



Proof:

(a) In Δs AMX and CMY

AM = MC [Given]

AX = CY [Given]

$\angle XAM = \angle YCM$ [Alternate angles]

So, $\Delta AXM \cong \Delta CYM$ [SAS]

(b) $\angle AMX = \angle CMY$ [Vertically opposite angles]

$\therefore$ XMY is a straight line.

Q. 6.

(a) Let the speed of boat in still water be = x kmph

And speed of the stream = y kmph

Speed of boat upstream = (x − y) kmph

Speed of boat downstream = (x + y) kmph

Time taken for upstream journey = $\frac{8}{x-y}$

Time taken for downstream journey = $\frac{8}{x+y}$

As per the problem, $\frac{8}{x-y}$ = 1 hr

x − y = 8 ....(1)

Also,

$$\frac{8}{x+y} = \frac{40}{60} = \frac{2}{3}$$

x + y = 12 ....(2)

Solving (1) and (2) we get

x = 10 kmph; y = 2 kmph

(b) Edge of the cubical tank = 1.5 m = 150 cm

Surface area of the tank = $5 \times 150 \times 150$ cm^2

Area of each square tile = side × side = 25×25 cm^2

$$\therefore \text{Number of tiles required} = \frac{\text{Surface area of the tank}}{\text{area of each tile}} = \frac{5 \times 150 \times 150}{25 \times 25} = 180$$

Cost of 1 dozen tiles, i.e. cost of 12 tiles = Rs. 360

Cost of one tile = Rs. $\frac{360}{12}$ = Rs. 30

Thus, the cost of 180 tiles = 180 × 30 = Rs. 5400

(c) $3p - 2q = 5$ ....(1)

$q - 1 = 3p$ ....(2)

From equation (2),

$$p = \frac{q-1}{3}$$

Substituting the value of p in equation (1), we get

$$3\left(\frac{q-1}{3}\right) - 2q = 5$$

$$\Rightarrow q - 1 - 2q = 5$$

$$\Rightarrow -q = 5 + 1$$

$$\Rightarrow q = -6$$

Substituting the value of q in equation (2) we get,

$$\Rightarrow q - 1 = 3p$$

$$\Rightarrow -6 - 1 = 3p$$

$$\Rightarrow -7 = 3p$$

$$\Rightarrow p = -\frac{7}{3}$$

$$\Rightarrow p = -\frac{7}{3}, q = -6$$

Q. 7.

(a)

$A = 60° \text{ and } B = 30°$

$\Rightarrow A - B = 60° - 30° = 30°$

$\therefore \tan(A - B) = \tan 30° = \frac{1}{\sqrt{3}}$

$$\text{And, } \frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\tan 60° - \tan 30°}{1 + \tan 60° \tan 30°}$$

$$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{2}$$

$$= \frac{1}{\sqrt{3}}$$

$$\therefore \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

(b) Length of garden = 120 – 2 × 5 and breadth = 70 – 2 × 5

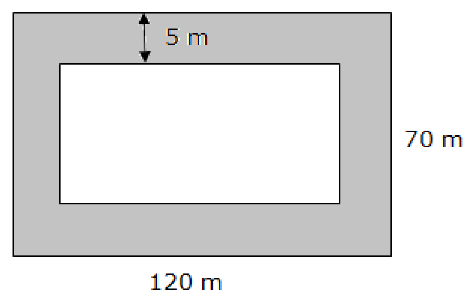
$\Rightarrow$ l = 110 m, b = 60 m

Area of garden = l × b = 110 × 60 = 600m^2

Given, rate = Rs. 10m^2

$\therefore$ Cost = Area × rate

Cost = Rs. 66000



(c) Given: A rectangle PQRS

To prove: $PR^2 + QS^2 = PQ^2 + QR^2 + RS^2 + SP^2$

Proof: In ΔPSR

$PR^2 = PS^2 + SR^2$ ....(1) [Pythagoras theorem]

In ΔQRS,

$QS^2 = QR^2 + RS^2$ ....(2)

Adding (1) and (2), we get

$PR^2 + QS^2 = PS^2 + SR^2 + QR^2 + RS^2$

$= RS^2 + QR^2 + PS^2 + PQ^2 [\because RS = PQ]$

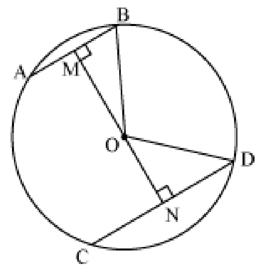
$PR^2 + QS^2 = PQ^2 + QR^2 + RS^2 + SP^2$

$= RS^2 + QR^2 + PS^2 + PQ^2 [\because RS = PQ]$

$\therefore PR^2 + QS^2 = PQ^2 + QR^2 + RS^2 + SP^2$

Q. 8.

(a) Construction: Draw $OM \perp AB$ and $ON \perp CD$. Join OB and OD.



$BM = \frac{AB}{2} = \frac{5}{2}$ and $ND = \frac{CD}{2} = \frac{11}{2}$ (Perpendicular from centre bisects the chord)

Let ON be x, so OM will be $6 - x$.

In ΔMOB, $OM^2 + MB^2 = OB^2$

$\therefore (6-x)^2 + \left(\frac{5}{2}\right)^2 = OB^2$

$\therefore 36 + x^2 - 12x + \frac{25}{4} = OB^2$ ....(1)

In ΔNOD, $ON^2 + ND^2 = OD^2$

$\therefore OD^2 = x^2 + \left(\frac{11}{2}\right)^2 = x^2 + \frac{121}{4}$ ....(2)

We have $OB = OD$ ....(radii of same circle)

$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$ [From (1) and (2)]

$\therefore 12x = 36 + \frac{25}{4} - \frac{121}{4} = \frac{144 + 25 - 121}{4} = \frac{48}{4} = 12$

$\therefore 12x = 12 \Rightarrow x = 1$

From equation (2),

$OD^2 = (1)^2 + \left(\frac{121}{4}\right) = 1 + \frac{121}{4} = \frac{125}{4} \Rightarrow OD = \frac{5}{2}\sqrt{5}$

Hence, the radius of the circle is $\frac{5}{2}\sqrt{5}$ cm.

(b) We know,

$p^3 + q^3 = (p+q)^3 - 3pq(p+q)$

$= (1+pq)^3 - 3pq(1+pq)$

$= (1+pq)^3 - 3pq(1+pq)$

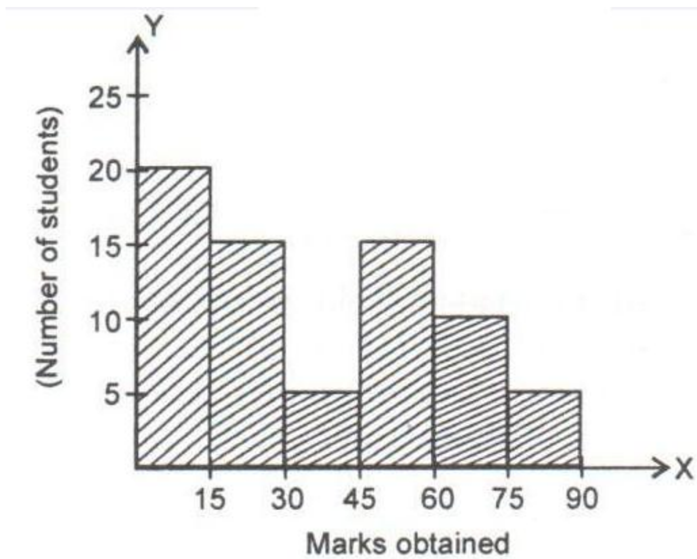
$= 1 + p^3q^3 + 3pq(1+pq) - 3pq(1+pq)$

$= 1 + p^3q^3$

Hence, $p^3 + q^3 = 1 + p^3q^3$

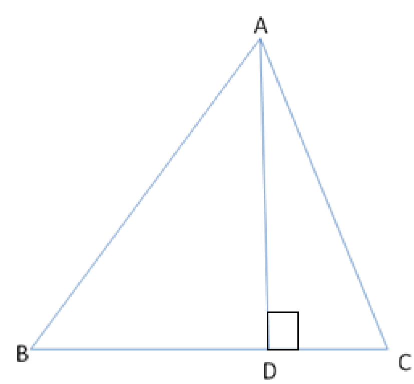
(c) Rewriting we get the continuous frequency distribution as following:

C.I	Frequency (No. of students)
Below 15	20
15 – 30	35 – 20 = 15
30 – 45	40 – 35 = 5
45 – 60	55 – 40 = 15
60 – 75	65 – 55 = 10
75 - 90	70 – 65 = 5



Q. 9.

(a) Given: AD ⊥ BC



To prove:

AB > BD

AC > CD

AB + AC > BC

Proof: In ΔABD, ∠ADB is the greatest angle

[There can be only one right angle]

i. So, the side opposite to ∠ADB in ΔABD is greatest

i.e., AB > BD ....(1)

ii. Similarly, ∠ADC is the greatest angle in ΔADC

So, AC > CD [∠ADC = 90°] ....(2)

iii. On adding (1) and (2), we get AB + AC > BD + CD

AB + AC > BC

(b) We have,

$$\frac{9^n \times 3^2 \times \left(3^{-n/2}\right)^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$$

$$\Rightarrow \frac{\left(3^2\right)^n \times 3^2 \times 3^{2n/2} - \left(3^3\right)^n}{3^{3m} \times 2^3} = \frac{1}{27}$$

$$\Rightarrow \frac{3^{2n} \times 3^2 \times 3^n - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27}$$

$$\Rightarrow \frac{3^{2n+2+n} - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27}$$

$$\Rightarrow \frac{3^{3n+2} - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27}$$

$$\Rightarrow \frac{3^{3n}\left(3^2 - 1\right)}{3^{3m} \times 2^3} = \frac{1}{27}$$

$$\Rightarrow \frac{3^{3n} \times 8}{3^{3m} \times 8} = \frac{1}{27}$$

$$\Rightarrow 3^{3n-3m} = 3^{-3} \qquad \text{[on equating the exponents]}$$

$$\Rightarrow 3n - 3m = -3 \Rightarrow n - m = -1 \Rightarrow m - n = 1$$

(c) We have,

$$x = 30^o \Rightarrow 2x = 60^o$$

$$\therefore \tan 2x = \tan 60^o = \sqrt{3}$$

And, $\frac{2\tan x}{1-\tan^2 x} = \frac{2\tan 30^o}{1-\tan^2 30^o}$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{2/\sqrt{3}}{1 - \frac{1}{3}} = \frac{2/\sqrt{3}}{2/3}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{2}$$

$$= \sqrt{3}$$

$$\therefore \tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Q. 10.

(a) In $\Delta ABC,\ \tan 30^\circ = \frac{BC}{AB}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BC}{15}$$

$$\Rightarrow BC = \frac{AB}{\sqrt{3}} = \frac{15}{\sqrt{3}} = \frac{15\sqrt{3}}{\sqrt{3}} = 5\sqrt{3}\ \text{cm}$$

(b)

(i) Given that ABCD is a parallelogram.

So, AB||DE. That is, AB||FE.

Since the parallelograms have the same base AB, and the height on base AB is equal, the areas of ||gm ABCD and ||gm ABEF will be equal.

Hence, $ar(||gm\ ABEF) = ar(||gm\ ABCD) = 80\ cm^2$

(ii) We know that the diagonal of a parallelogram, divides the parallelogram into two triangles with equal areas.

So, $ar(\Delta ABD) = \frac{1}{2} ar(||gm\ ABCD) = \frac{1}{2}(80) = 40\ cm^2$

(iii) Similarly,

$$ar(\Delta BEF) = \frac{1}{2} ar(||gm\ ABEF) = \frac{1}{2}(80) = 40\ cm^2$$

(c) ABCD be a regular polygon

BC and ED when produced meet at P such that $\angle CPD = 90^\circ$

$\angle CPD = 90^\circ$

Let $\angle BCD = x^\circ$

So, $\angle CDE = x^\circ$

$\angle PCD = 180 - x$

$\angle PDC = 180 - x$

In ΔCPD,

$180^\circ - x^\circ + 180^\circ - x^\circ + 90^\circ = 180^\circ$ [Sum of all $\angle$s of a Δ]

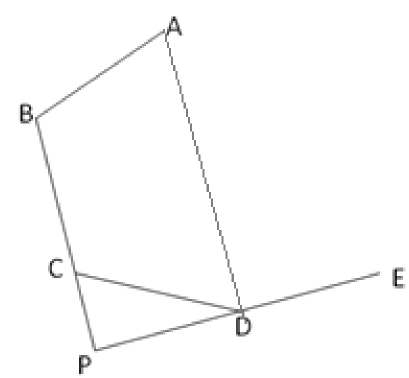
$270^\circ - 2x^\circ = 0$

$2x^\circ = 270^\circ$

$x^\circ = 135^\circ$

Each external angle = $180^\circ - x^\circ = 180 - 135 = 45^\circ$

No. of sides = $\frac{360^\circ}{45^\circ} = 8^\circ$



Q. 11.

(a)

$$\sqrt[3]{\frac{p}{q}} = \left(\frac{p}{q}\right)^{3-4x} = \left(\frac{p}{q}\right)^{4x-3}$$

$$\Rightarrow \left(\frac{p}{q}\right)^{1/3} = \left(\frac{p}{q}\right)^{-3+4x}$$

$$\Rightarrow \frac{1}{3} = -3 + 4x$$

$$\Rightarrow 4x = 3 + \frac{1}{3}$$

$$\Rightarrow 4x = \frac{10}{3}$$

$$\Rightarrow x = \frac{10}{12}$$

$$\Rightarrow x = \frac{5}{6}$$

(b) a + b = 1, a – b = 7

$$(a+b)^2 - (a-b)^2 = 4ab$$

$$\Rightarrow 1^2 - 7^2 = 4ab$$

$$\Rightarrow 1 - 49 = 4ab$$

$$\Rightarrow 4ab = -48$$

$$\Rightarrow ab = -12 \quad ....(1)$$

Now, we know that

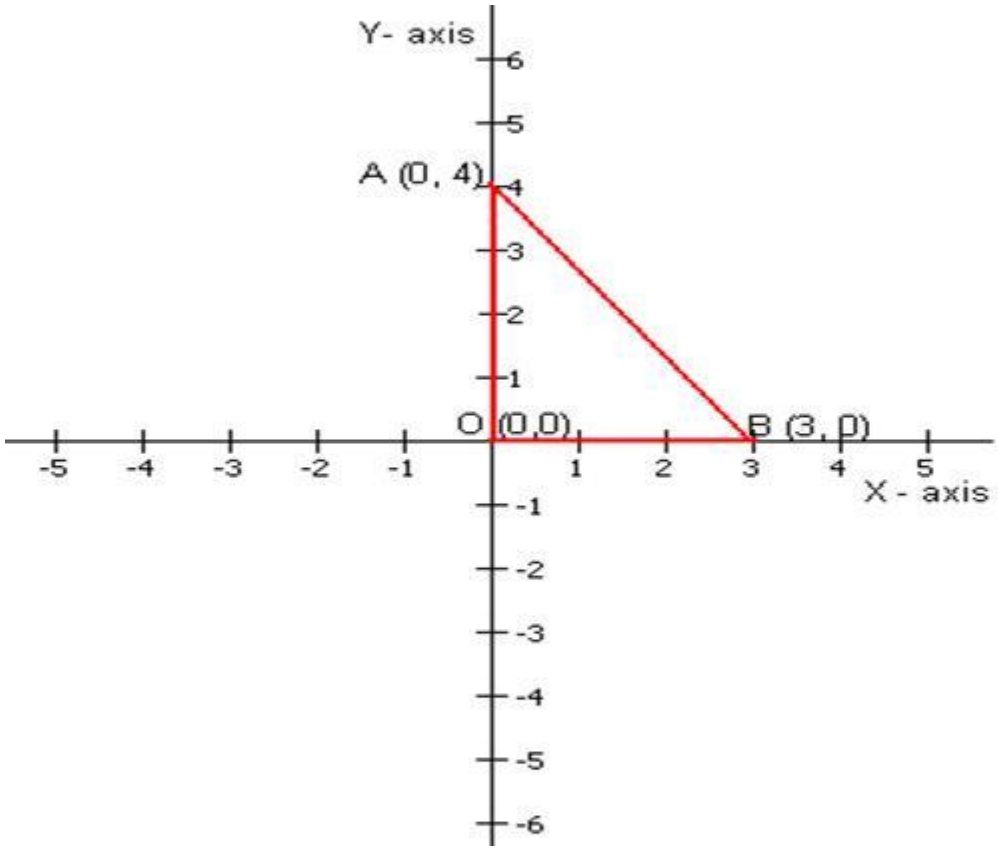
$$a^2 + b^2 = (a+b)^2 - 2ab = 1^2 - 2\times(-12)$$

$$\Rightarrow a^2 + b^2 = 1 + 24 = 25$$

(1) $5\left(a^2 + b^2\right) = 25 \times 5 = 125$

(2) ab = -12 [using equation (1)]

(c) The given points A(0, 4), O(0, 0), B(3, 0) can be plotted as follows:



Clearly, AOB is a right-angled triangle.

OA = 4 units, OB = 3 units.

Area of $\Delta AOB = \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times 3 \times 4$

= 6 square units