MATHEMATICS

(Maximum Marks: 100)

(Time allowed: Three hours)

*(Candidates are allowed additional 15 minutes for **only** reading the paper.*

They must NOT start writing during this time.)

Section A - *Answer* **Question 1** *(compulsory) and* **five** *other questions.*

Section B and Section C - *Answer* **two** *questions from* ***either*** *Section B* ***or*** *Section C.*

All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

Slide rule may be used.

SECTION A (80 Marks)

Question 1

(i) If the matrix $\begin{pmatrix} 6 & -x^2 \\ 2x-15 & 10 \end{pmatrix}$ is symmetrix, find the value of x.

(ii) If $y - 2x - k = 0$ touches the conic $3x^2 - 5y^2 = 15$, find the value of k.

(iii) Prove that $\frac{1}{2}cos^{-1}\left(\frac{1-x}{1+x}\right) = tan^{-1}\sqrt{x}$

(iv) Using L'Hospital's Rule, evaluate:

$$\lim_{x \to \pi/2} (x\ tanx - \frac{\pi}{4}.\sec x)$$

(v) Evaluate: $\int \frac{1}{x^2} sin^2\left(\frac{1}{x}\right) dx$

(vi) Evaluate:

$$\int_0^{\pi/4} \log(1 + tan\theta)\, d\theta$$

(vii) By using the data $\bar{x} = 25$, $\bar{y} = 30$, $b_{yx} = 1{\cdot}6$ and $b_{xy} = 0{\cdot}4$, find:

(a) The regression equation y on x.

(b) What is the most likely value of y when $x = 60$?

(c) What is the coefficient of correlation between x and y?

This Paper consists of 5 printed pages and 1 blank page.

(viii) A problem is given to three students whose chances of solving it are $\frac{1}{4}, \frac{1}{5}$ and $\frac{1}{3}$ respectively. Find the probability that the problem is solved.

(ix) If $a + ib = \frac{x+iy}{x-iy}$, prove that $a^2 + b^2 = 1$ and $\frac{b}{a} = \frac{2xy}{x^2-y^2}$

(x) Solve: $\frac{dy}{dx} = 1 - xy + y - x$

Question 2

(a) Using properties of determinants, prove that: [5]

$$\begin{vmatrix} a & b & b+c \\ c & a & c+a \\ b & c & a+b \end{vmatrix} = (a+b+c)(a-c)^2$$

(b) Given that: $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$, find AB. [5]

Using this result, solve the following system of equation:

$x - y = 3, 2x + 3y + 4z = 17$ and $y + 2z = 7$

Question 3

(a) Solve the equation for x: [5]

$$\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x, x \neq 0$$

(b) If A, B and C are the elements of Boolean algebra, simplify the expression $(A'+B')(A + C') + B'(B + C)$. Draw the simplified circuit. [5]

Question 4

(a) Verify Langrange's mean value theorem for the function: [5]

$f(x) = x(1 - \log x)$ and find the value of 'c' in the interval $[1, 2]$

(b) Find the coordinates of the centre, foci and equation of directrix of the hyperbola $x^2 - 3y^2 - 4x = 8$. [5]

Question 5

(a) If $y = \cos(\sin x)$, show that: [5]

$$\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$$

(b) Show that the surface area of a closed cuboid with square base and given volume is minimum when it is a cube. [5]

Question 6

(a) Evaluate: [5]

$$\int \frac{\sin 2x}{(1+\sin x)(2+\sin x)} dx$$

(b) Draw a rough sketch of the curve $y^2 = 4x$ and find the area of the region enclosed by the curve and the line $y = x$. [5]

Question 7

(a) Calculate the Spearman's rank correlation coefficient for the following data and interpret the result: [5]

X	35	54	80	95	73	73	35	91	83	81
Y	40	60	75	90	70	75	38	95	75	70

(b) Find the line of best fit for the following data, treating x as dependent variable (Regression equation x on y): [5]

X	14	12	13	14	16	10	13	12
Y	14	23	17	24	18	25	23	24

Hence, estimate the value of x when $y = 16$.

Question 8

(a) In a class of 60 students, 30 opted for Mathematics, 32 opted for Biology and 24 opted for both Mathematics and Biology. If one of these students is selected at random, find the probability that: [5]

(i) The student opted for Mathematics or Biology.

(ii) The student has opted neither Mathematics nor Biology.

(iii) The student has opted Mathematics but not Biology.

(b) Bag A contains 1 white, 2 blue and 3 red balls. Bag B contains 3 white, 3 blue and 2 red balls. Bag C contains 2 white, 3 blue and 4 red balls. One bag is selected at random and then two balls are drawn from the selected bag. Find the probability that the balls drawn are white and red. [5]

Question 9

(a) Prove that locus of z is circle and find its centre and radius if $\frac{z-i}{z-1}$ is purely imaginary. [5]

(b) Solve: $(x^2 - yx^2)dy + (y^2 + xy^2)\,dx = 0$ [5]

SECTION B (20 Marks)

Question 10

(a) If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors of equal magnitude, prove that $(\vec{a} + \vec{b} + \vec{c})$ is equally inclined with vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$. [5]

(b) Find the value of λ for which the four points with position vectors $6\hat{i} - 7\hat{j}$, $16\hat{i} - 19\hat{j} - 4\hat{k}$, $\lambda\hat{j} - 6\hat{k}$ and $2\hat{i} - 5\hat{j} + 10\hat{k}$ are coplanar. [5]

Question 11

(a) Show that the lines $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ *and* $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ intersect. Find the coordinates of their point of intersection. [5]

(b) Find the equation of the plane passing through the point $(1, -2, 1)$ and perpendicular to the line joining the points A(3, 2, 1) and B(1, 4, 2). [5]

Question 12

(a) A fair die is rolled. If face 1 turns up, a ball is drawn from Bag A. If face 2 or 3 turns up, a ball is drawn from Bag B. If face 4 or 5 or 6 turns up, a ball is drawn from Bag C. Bag A contains 3 red and 2 white balls, Bag B contains 3 red and 4 white balls and Bag C contains 4 red and 5 white balls. The die is rolled, a Bag is picked up and a ball is drawn. If the drawn ball is red, what is the probability that it is drawn from Bag B? [5]

(b) An urn contains 25 balls of which 10 balls are red and the remaining green. A ball is drawn at random from the urn, the colour is noted and the ball is replaced. If 6 balls are drawn in this way, find the probability that: [5]

(i) All the balls are red.

(ii) Not more than 2 balls are green.

(iii) Number of red balls and green balls are equal.

SECTION C (20 Marks)

Question 13

(a) A machine costs ₹ 60,000 and its effective life is estimated to be 25 years. A sinking fund is to be created for replacing the machine at the end of its life time when its scrap value is estimated as ₹ 5,000. The price of the new machine is estimated to be 100% more than the price of the present one. Find the amount that should be set aside at the end of each year, out of the profits, for the sinking fund if it accumulates at an interest of 6% per annum compounded annually. [5]

(b) A farmer has a supply of chemical fertilizer of type A which contains 10% nitrogen and 6% phosphoric acid and of type B which contains 5% nitrogen and 10% phosphoric acid. After soil test, it is found that at least 7 kg of nitrogen and same quantity of phosphoric acid is required for a good crop. The fertilizer of type A costs ₹ 5.00 per kg and the type B costs ₹ 8.00 per kg. Using Linear programming, find how many kilograms of each type of the fertilizer should be bought to meet the requirement and for the cost to be minimum. Find the feasible region in the graph. [5]

Question 14

(a) The demand for a certain product is represented by the equation $p = 500 + 25x - \frac{x^2}{3}$ in rupees where x is the number of units and p is the price per unit. Find: [5]

(i) Marginal revenue function.

(ii) The marginal revenue when 10 units are sold.

(b) A bill of ₹ 60,000 payable 10 months after date was discounted for ₹ 57,300 on 30th June, 2007. If the rate of interest was 11¼% per annum, on what date was the bill drawn? [5]

Question 15

(a) The price relatives and weights of a set of commodities are given below: [5]

Commodity	A	B	C	D
Price relatives	125	120	127	119
Weights	x	$2x$	y	$y+3$

If the sum of the weights is 40 and weighted average of price relatives index number is 122, find the numerical values of x and y.

(b) Construct 3 yearly moving averages from the following data and show on a graph against the original data: [5]

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Annual sales in lakhs	18	22	20	26	30	22	24	28	32	35