MATHEMATICS

(Maximum Marks: 100)

(Time allowed: Three hours)

(*Candidates are allowed additional 15 minutes for* ***only*** *reading the paper.*

They must NOT start writing during this time.)

The Question Paper consists of three sections A, B and C.

Candidates are required to attempt all questions from ***Section A*** *and all questions*

EITHER *from* ***Section B*** ***OR*** ***Section C***

Section A: *Internal choice has been provided in three questions of four marks each and two questions of six marks each.*

Section B: *Internal choice has been provided in two questions of four marks each.*

Section C: *Internal choice has been provided in two questions of four marks each.*

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A (80 Marks)

Question 1 **[10×2]**

(i) A binary operation * defined on Q-{1} is given by a * b = a+b – ab. Find the identity element.

(ii) Without expanding at any stage, find the value of the determinant:

$$\Delta = \begin{vmatrix} 2 & x & y+z \\ 2 & y & z+x \\ 2 & z & x+y \end{vmatrix}$$

(iii) Solve: $\sin^{-1} \cos (\sin^{-1} x) = \frac{\pi}{3}$

(iv) Find the value of k if $M = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and M 0.

(v) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$

(vi) Find —, if at.

(vii) Find the differential equation of the family of curves , where A and B are arbitrary constants.

(viii) Find the intervals in which the function $f(x)$ is strictly increasing where,
$(x) = 10 - 6x - 2x^2$.

(ix) A family has two children. What is the probability that both children are boys, given that at least one of them is a boy?

(x) Given that the events A and B are such that P(A) = $\frac{1}{\ }$, P($B)$ $-$ and P (B) = k. Find k if:

(a) A and B are mutually exclusive.

(b) A and B are independent.

Question 2 **[4]**

Let R^+ be the set of all positive real numbers and f: $R^+ \longrightarrow [4, \infty)$: f(x) = $x^2 + 4$. Show that inverse of f exists and find f^{-1}.

Question 3 **[4]**

Using properties of determinants, prove:

$$\begin{vmatrix} y \end{vmatrix} = (1 \quad pxyz)(x \quad y)(y \quad z)(z \quad x), \text{ where } p \text{ is any scalar.}$$

Question 4 **[4]**

Prove that $\quad - \quad - \quad - \qquad (-)$.

Question 5 **[4]**

(a) Prove that the function $f(x) = | \quad 1|, \quad R,$ is continuous at x = 1 but not differentiable.

OR

(b) Verify Rolle's Theorem for the following function:

$$f(x) \qquad\qquad [0 \ \pi]$$

Question 6 **[4]**

If $\qquad\qquad$ then show that:

$$(1 \qquad)y$$

Question 7 **[4]**

(a) Evaluate : $\int \frac{\quad}{\sqrt{(\quad 5)(x \quad 4)}}$

OR

(b) Evaluate: $\int_1^3 (\qquad x)dx$ expressing as a limit of sum.

Question 8 **[4]**

(a) Find the equations of the normals to the curve which are parallel to the line $x + 14y + 4 = 0$.

OR

(b) A circular disc of radius 3 cm. is heated. Due to expansion its radius increases at the rate of 0·05 cm/s. Find the rate at which its area increases when the radius is 3·2 cm.

Question 9 **[4]**

Solve the following differential equation:

—

Question 10 **[4]**

Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values 'x' has the following form, where 'k' is some unknown constant.

$$P(X=x) = \begin{cases} 0.1, & \text{if } x = 0 \\ kx, & \text{if } x = 1 \text{ or } 2 \\ k(5-x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the value of 'k'.

(b) What is the probability that you study:

(i) at least two hours?

(ii) exactly two hours?

(iii) at most 2 hours?

Question 11 **[6]**

(a) Evaluate $\begin{bmatrix} 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} \quad \end{bmatrix}$

Hence , Solve the system of equations,

OR

(b) Using elementary transformations, find the inverse of the matrix:

$$\begin{bmatrix} - & & 1 \end{bmatrix}$$

Question 12 **[6]**

(a) Show that the altitude of a right circular cone of maximum volume that can be inscribed in a sphere of radius r is —.

OR

(b) An open topped box is to be made by removing equal squares from each corner of a 3 m by 8 m rectangle sheet of aluminium and by folding up the sides. Find the volume of the largest such box.

Question 13 **[6]**

Evaluate: $\int$ ————

Question 14 **[6]**

A, B and C throw a die one after the other in the same order till one of them gets a ‘6’ and wins the game. Find their respective probability of winning, if A starts the game.

SECTION B (20 Marks)

Question 15 **[6]**

(a) Find the area of the parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

(b) Find the angle between the line —— – —— and the plane = 4.

(c) Find the Cartesian equation of the line passing through the points $(-1,0,2)$ and$(3,4,6)$.

Question 16 **[4]**

(a) Show that:

$$(\vec{a} \times \vec{b}) \qquad \begin{vmatrix} \vec{a}.\vec{a} & \overrightarrow{a.b} \\ \overrightarrow{a.b} & \overrightarrow{b.b} \end{vmatrix}$$

OR

(b) Show that:

$$\vec{a}.\left(\vec{b} + \vec{c}\right) \times \left(\vec{a} \quad 2\vec{b} \quad 3\vec{c}\right) = [\vec{a} \quad \vec{b} \quad \vec{c}]$$

Question 17 **[4]**

(a) Find the shortest distance between the lines —— —— —— and

—— —— ——.

OR

(b) Find the cartesian equation of the plane passing through the intersection of the planes

$\vec{r}\,.(\,2\,\hat{\imath} + \,6\,\hat{\jmath}\,) + \,12 = 0$ and $3x - \,y + 4z = 0$ and at a unit distance from origin.

Question 18

Using integration, find the area of the following region: **[6]**

$$\left\{(x\ \ y): — \quad — \qquad - \quad -\right\}$$

SECTION C (20 Marks)

Question 19 **[6]**

(a) Find the cost of increasing from 100 to 200 units if the marginal cost in Rupees per unit is given by the function MC = $0{\cdot}003\ x^2 - 0{\cdot}01\ x + 2{\cdot}5$.

(b) If two lines of regression are $4x + 2y - 3 = 0$ and $3x + 6y + 5 = 0$, find the correlation coefficient between x and y.

(c) The total variable cost of manufacturing x units in a firm is ₹ ($3x$ + —). Show that average variable cost increases with output x.

Question 20 **[4]**

(a) Given that the observations are (9,-4), (10, -3), (11,-1), (13,1), (14,3), (15,5), (16,8), find the two lines of regression. Estimate the value of y when x = 13.5.

OR

(b) Find the regression coefficient b_{yx} and b_{xy} and the two lines of regression for the following data.

X	2	6	4	7	5
Y	8	8	5	6	2

Also, compute the correlation coefficient.

Question 21 **[4]**

(a) If the demand function is given by x = ———, where the price is ₹ p per unit and the manufacturer produces x unit per week at the total cost of ₹ $x^2 + 78x + 2500$, find the value of x for which the profit is maximum.

OR

(b) The fixed cost of new product is ₹ 35000 and the variable cost per unit is ₹ 500. If the demand function : $p = 5000 - 100\,x$, find the break-even value(s)?

Question 22 **[6]**

A toy company manufactures two types of dolls A and B. Market test and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demands for the dolls of type B is atmost half of that for dolls of type A. Further, the production level of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of ₹ 12 and ₹ 16 per doll respectively on dolls A and B, how many of each type of dolls should be produced weekly, in order to maximise the profit?