ICSE QUESTION PAPER

MATHEMATICS

(Two hours and a half)

Answers to this Paper must be written on the paper provided separately.

*You will **not** be allowed to write during the first 15 minutes.*

This time is to be spent in reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answers.

*Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*

All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer. Omission of essential working will result in the loss of marks.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables are provided.

SECTION A [40 Marks]

*(Answer **all** questions from this Section.)*

Question 1.

(a) *Ranbir borrows ₹ 20,000 at 12% per annum compound interest. If he repays ₹ 8400 at the end of the first year and ₹ 9680 at the end of the second year, find the amount of loan outstanding at the beginning of the third year.* **[3]**

(b) *Find the value of x, which satisfy the inequation* $-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \le 2, x \in W$. *Graph the solution set on the number line.* **[3]**

(c) *A die has 6 faces marked by the given numbers as shown below :*

$\boxed{1}$ $\boxed{2}$ $\boxed{3}$ $\boxed{-1}$ $\boxed{-2}$ $\boxed{-3}$

The die is thrown once. What is the probability of getting

(i) *a positive integer.*

(ii) *an integer greater than –3.*

(iii) *the smallest integer.* **[4]**

Solution :

(a) *Given* : Principal for the first year (P) = ₹ 20,000, r = 12%.

We know that
$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$\text{Amount after the first year} = 20,000\left(1 + \frac{12}{100}\right)^1$$
$$= 20,000\left(\frac{112}{100}\right)$$
$$= ₹\ 22,400$$

Money repays at the end of first year = ₹ 8,400 (given)

$$\text{Principal for the second year} = ₹\,22{,}400 - ₹\,8{,}400$$
$$= ₹\,14{,}000$$
$$\text{Amount after second year} = 14{,}000\left(1 + \frac{12}{100}\right)^{1}$$
$$= ₹\,15{,}680$$
$$\text{Money repays at the end of the second year} = ₹\,9{,}680 \qquad \text{(given)}$$

$\therefore$ The loan outstanding at the beginning of the third year

$$= ₹\,15{,}680 - ₹\,9{,}680$$
$$= ₹\,6{,}000.$$

(b) *Given* :

$$-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \le 2$$
$$-\frac{17}{6} < \frac{3-4x}{6} \le 2$$

Multiplying throughout by 6

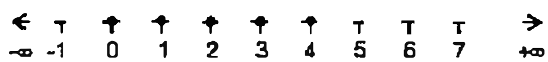
$$-17 < 3 - 4x \le 12$$

$$-17 < 3 - 4x \quad \text{and} \quad 3 - 4x \le 12$$
$$4x < 3 + 17 \qquad\qquad 3 - 12 \le 4x$$
$$4x < 20 \qquad\qquad -9 \le 4x$$
$$x < 5 \qquad \Rightarrow \qquad -\frac{9}{4} < x$$

$$\left\{5 > x \ge \frac{-9}{4}\right\}$$

Hence, the solution set is $\{x : x \in W, -\frac{9}{4} \le x < 5\}$

$\therefore$ {0, 1, 2, 3, 4}



The graph of the solution set is shown by dots on the number line.

(c)
$$\text{No. of sample space } n(S) = 6$$

(i)
$$\text{a positive integer} = \{1, 2, 3\}$$
$$\text{No. of favourables } n(E) = 3$$
$$\text{Probability} = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2} \qquad \textbf{Ans.}$$

(ii)
$$\text{an integer greater than } -3 = \{1, 2, 3, -1, -2\}$$
$$\text{No. of favourables } n(E) = 5$$
$$\text{Probability} = \frac{n(E)}{n(S)} = \frac{5}{6} \qquad \textbf{Ans.}$$

(iii) Smallest integer = – 3

$$\text{Probability of smallest integer } \frac{n(E)}{n(S)} = \frac{1}{6}.$$

Question 2.

(a) *Find x, y if* $\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3\begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2\begin{bmatrix} y \\ 3 \end{bmatrix}$. **[3]**

·ount in a bank and deposited ₹ 800
94 at the time of maturity, find the

[3]

A (–4, 2) and B(3, 6) is divided by

[4]

$\begin{bmatrix} y \\ 3 \end{bmatrix}$

$\begin{matrix} 2y \\ 6 \end{matrix}\Bigg]$

$\begin{matrix} 2y \\ 6 \end{matrix}\Bigg]$

$\begin{matrix} 2y \\ 6 \end{matrix}\Bigg]$

$\begin{bmatrix} 2y \\ 6 \end{bmatrix}$

$2x = 6$

$x = 3$ **Ans.**

: ₹ 800

deposited = $1\frac{1}{2}$ years = 18 months.

ıen

$\times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$\times \frac{18 \times 19}{2 \times 12} \times \frac{r}{100}$

$4r$

800 = ₹ 14,400

ırity value

4

4 – 14,400

Ans.

points A (–4, 2) and B (3, 6) in

B(3,6)
(x_2, y_2)

$\therefore$ Coordinates of P is $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right) = \left(\frac{3k-4}{k+1}, \frac{6k+2}{k+1}\right)$.

But coordinate of P is $(x, 3)$

$$\Rightarrow \qquad \frac{6k+2}{k+1} = 3$$

$$6k + 2 = 3k + 3$$

$$3k = 1 \Rightarrow k = \frac{1}{3}$$

$\therefore$ The required ratio is $\frac{1}{3} : 1$ *i.e.,* $1 : 3$ (internally) **Ans.**

(i) $$x = \frac{3k-4}{k+1}$$

Putting $k = \frac{1}{3}$, we get

$$x = \frac{3 \times \frac{1}{3} - 4}{\frac{1}{3} + 1} = \frac{1 - 4}{\frac{1+3}{3}} = \frac{-3}{4/3} = \frac{-9}{4}$$ **Ans.**

(ii) $\therefore$ Coordinate of P is $\left(\frac{-9}{4}, 3\right)$

$$\text{Length of AP} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left(-\frac{9}{4} + 4\right)^2 + (3 - 2)^2}$$

$$= \sqrt{\left(\frac{-9 + 16}{4}\right)^2 + (1)^2} = \sqrt{\frac{49}{16} + 1}$$

$$= \sqrt{\frac{49 + 16}{16}} = \sqrt{\frac{65}{16}} = \frac{\sqrt{65}}{4}$$ **Ans.**

Question 3.

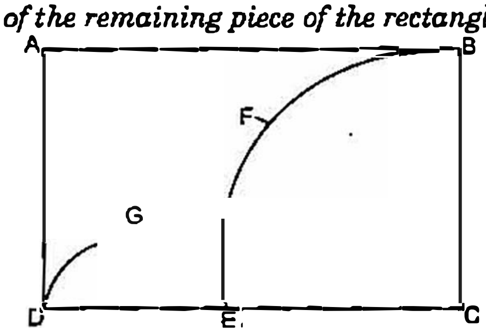
(a) *Without using trigonometric tables, evaluate*

$$\sin^2 34° + \sin^2 56° + 2 \tan 18° \tan 72° - \cot^2 30°$$ **[3]**

(b) *Using the Remainder and Factor Theorem, factorise the following polynomial :*

$$x^3 + 10x^2 - 37x + 26$$ **[3]**

(c) *In the figure given below, ABCD is a rectangle. AB = 14 cm, BC = 7 cm. From the rectangle, a quarter circle BFEC and a semicircle DGE are removed. Calculate the area of the remaining piece of the rectangle. (Take $\pi = 22/7$)* **[4]**



Solution :

(a) *Given* :

$$
\begin{aligned}
& \sin^2 34^\circ + \sin^2 56^\circ + 2\tan 18^\circ \tan 72^\circ - \cot^2 30^\circ \\
= \; & \sin^2 34^\circ + \sin^2 (90^\circ - 34^\circ) + 2\tan 18^\circ \tan (90^\circ - 18^\circ) - \cot^2 30^\circ \\
= \; & \sin^2 34^\circ + \cos^2 34^\circ + 2\tan 18^\circ \cot 18^\circ - (\sqrt{3}) \\
= \; & 1 + 2\tan 18^\circ \times \frac{1}{\tan 18^\circ} \quad 3 \\
= \; & 1 + 2 - 3 \\
= \; & 0
\end{aligned}
$$

(b) Let $f(x) = x^3 + 10x^2 - 37x + 26$

Putting $x = 1$, we get

$$f(1) = 1 + 10 - 37 + 26 = 0$$

$\therefore$ By factor theorem, $x - 1$ is factor of $f(x)$.

$$
\begin{array}{r|l}
 & x^2 + 11x - 26 \\
\hline
x - 1 & x^3 + 10x^2 - 37x + 26 \\
 & x^3 - x^2 \\
 & - \quad + \\
 & \quad 11x^2 - 37x \\
 & \quad 11x^2 - 11x \\
 & \qquad\quad + \\
\hline
 & \qquad - 26x + 26 \\
 & \qquad - 26x + 26 \\
 & \qquad + \\
\hline
 & \qquad\quad \times \\
\hline
\end{array}
$$

On dividing $x^3 + 10x^2 - 37x + 26$ by $x - 1$, we get $x^2 + 11x - 26$ as the quotient and remainder $= 0$.

$\therefore$ The other factor of $f(x)$ are the factor of $x^2 + 11x - 26$

Now,

$$
\begin{aligned}
& x^2 + 11x - 26 \\
= \; & x^2 + 13x - 2x - 26 \\
= \; & x(x + 13) - 2(x + 13) \\
= \; & (x + 13)(x - 2)
\end{aligned}
$$

Hence, $x^3 + 10x^2 - 37x + 26 = (x - 1)(x - 2)(x + 13)$ **Ans.**

(c)

$$
\begin{aligned}
\text{Area of rectangle ABCD} &= 14 \times 7 = 98 \text{ cm}^2 \\
\text{Area of quarter circle BFEC} &= \frac{1}{4}\pi(7)^2 = \frac{49}{4}\pi \\
\text{Area of semi-circle DGE} &= \frac{1}{2}\pi \left.\frac{7}{2}\right)^2 = \frac{1}{2} \times \frac{49}{4}\pi \\
\text{Area of remaining piece of rectangle} &= 98 - \left. \frac{49}{4}\pi + \frac{1}{2} \times \frac{49}{4}\pi \right] \\
&= 98 - \frac{49}{4}\pi\left[1 + \frac{1}{2}\right]
\end{aligned}
$$

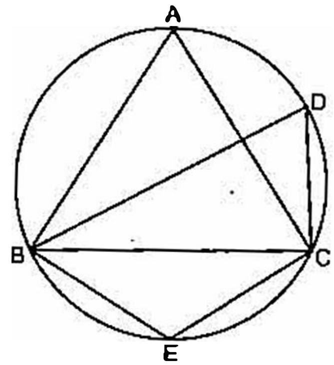
$$= 98 - \frac{49}{4} \times \frac{22}{7} \times \frac{3}{2} = 98 - \frac{231}{4}$$
$$= 98 - 57.75$$
$$= 40{\cdot}25 \text{ cm}^2.$$ **Ans.**

Question 4.

(a) *The numbers 6, 8, 10, 12, 13 and x are arranged in an ascending order. If the mean of the observations is equal to the median, find the value of x.* **[3]**

(b) *In the figure, $\angle DBC = 58°$. BD is a diameter of the circle. Calculate :*

(i) $\angle BDC$ (ii) $\angle BEC$ (iii) $\angle BAC$ **[3]**



(c) *Using graph paper to answer the following questions. (Take 2 cm = 1 unit on both axis)*

(i) *Plot the points A (– 4, 2) and B (2, 4)*

(ii) *A′ is the image of A when reflected in the y-axis. Plot it on the graph paper and write the coordinates of A′.*

(iii) *B′ is the image of B when reflected in the line AA′. Write the coordinates of B′.*

(iv) *Write the geometric name of the figure ABA′B′.*

(v) *Name a line of symmetry of the figure formed.* **[4]**

Solution :

(a) Numbers in ascending order are 6, 8, 10, 12, 13, x.

$$\text{Mean} = \frac{6 + 8 + 10 + 12 + 13 + x}{6} = \frac{49 + x}{6}$$

$$\text{No. of terms } (n) = 6 \text{ (even)}$$

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2} + 1\right)^{th} \text{term}}{2}$$

$$\text{Median} = \frac{\left(\frac{6}{2}\right)^{th} \text{term} + \left(\frac{6}{2} + 1\right)^{th} \text{term}}{2} = \frac{3^{rd} + 4^{th}}{2}$$

$$= \frac{10 + 12}{2} = \frac{22}{2} = 11$$

According to given condition

$$\frac{49 + x}{6} = 11$$

$\Rightarrow \quad 49 + x = 66$

$x = 17$ **Ans.**

(b) In Δ BCD; $\angle DBC = 58°$ (given)

(i) $\angle BCD = 90°$ (Angle in the semicircle as BD is diameter)

$\therefore \angle DBC + \angle BCD + \angle BDC = 180°$

$58° + 90° + \angle BDC = 180°$

$\angle BDC = 180° - (90° + 58°)$

$= 180° - 148°$

$= 32°$ **Ans.**

(ii) $\angle BEC + \angle BDC = 180°$ ($\because$ BECD is a cyclic quadrilateral)

$\angle BEC = 180° - \angle BDC$

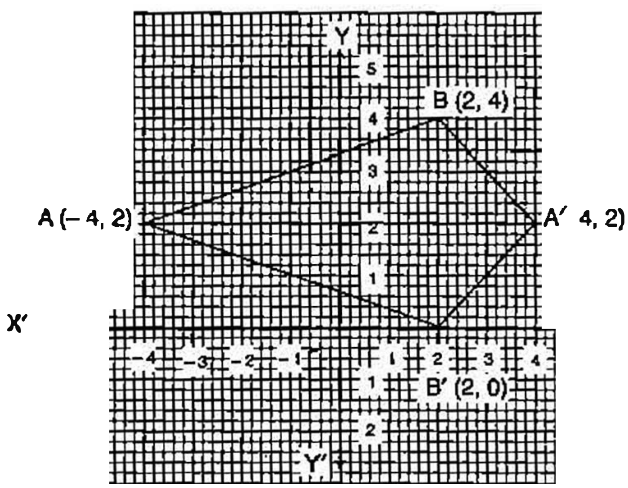
$= 180° - 32°$

$\angle BEC = 148°$ **Ans.**

(iii) $\angle BAC = \angle BDC$ (Angle of same segment are equal)

$\angle BAC = 32°$ **Ans.**

(c) (i) See Graph.



(ii) Coordinate of A′ = (4, 2)

(iii) Coordinate of B′ = (2, 0)

(iv) Geometric name = Kite.

(v) AA′ is the symmetric line.

SECTION B [40 Marks]

*Answer any **four** Questions in this Section.*

Question 5.

(a) *A shopkeeper bought a washing machine at a discount of 20% from a wholesaler, the printed price of the washing machine being ₹ 18,000. The shopkeeper sells it to a consumer at a discount of 10% on the printed price. If the rate of sales tax is 8% find :*

(i) *the VAT paid by the shopkeeper.*

(ii) *the total amount that the consumer pays for the washing machine.* [3]

(b) *If* $\frac{x^2+y^2}{x^2-y^2}=\frac{17}{8}$, *then find the valu*

(i) $x : y$.

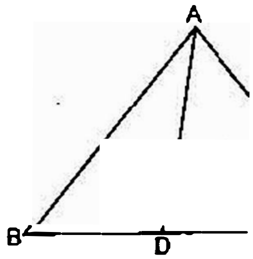
(ii) $\frac{x^3+y^3}{x^3-y^3}$

(c) *In* ΔABC, $\angle ABC = \angle DAC$. $AB = 8$ *cm*.

(i) *Prove that* ΔACD *is similar to* Δ

(ii) *Find BC and CD*

(iii) *Find area of* ΔACD : *area of* ΔABC



Solution :

(a) *Given* : Printed price of washing machine

(i) Amount of discount to shopkeeper =

=

Shopkeeper's price =

=

Sales Tax paid by shopkeeper = 1

Discount for consumer = $\frac{1}{10}$

Price for consumer = ₹ 1

= ₹ 1

Tax charged by the shopkeeper = $\frac{8}{100}$

= ₹ 1,

Since, Tax paid by the shopkeeper = ₹ 1,1

VAT paid by the shopkeeper = Tax

= ₹ 1,2

= ₹ 144

(ii) Total amount paid by the consumer for wash

= ₹ 16,20

= ₹ 17,496.

(b) *Given* : $\frac{x^2+y^2}{x^2-y^2}=\frac{17}{8}$

(i) Applying componendo and dividendo

$$\frac{(x^2+y^2)+(x^2-y^2)}{(x^2-y^2)-(x^2-y^2)}=\frac{17+8}{17-8}$$

$$\frac{2x^2}{2y^2} = \frac{25}{9} \Rightarrow \frac{x^2}{y^2} = \frac{25}{9}$$

$$\frac{x}{y} = \frac{5}{3}$$

$$x : y = 5 : 3.$$ **Ans.**

(ii) As $$\frac{x}{y} = \frac{5}{3}$$

Cubing both sides, we get

$$\frac{x^3}{y^3} = \frac{(5)^3}{(3)^3} = \frac{125}{27}$$

Applying componendo and Dividendo

$$\frac{x^3 + y^3}{x^3 - y^3} = \frac{125 + 27}{125 - 27}$$

$$\Rightarrow \quad \frac{x^3 + y^3}{x^3 - y^3} = \frac{152}{98}$$

$$\frac{x^3 + y^3}{x^3 - y^3} = \frac{76}{49}$$ **Ans.**

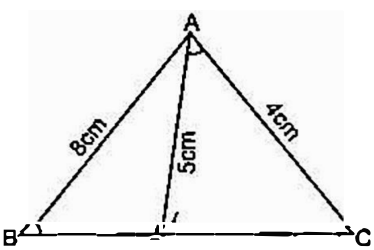
(c) (i) In Δ ACD and Δ BCA

$\angle C = \angle C$ (common)

$\angle ABC = \angle CAD$ (given)

$\Delta ACD \sim \Delta BCA$ (AA postulates)

(ii) $\Delta ACD \sim \Delta BCA$



$$\frac{AC}{BC} = \frac{CD}{CA} = \frac{AD}{BA}$$

$$\frac{4}{BC} = \frac{CD}{4} = \frac{5}{8}$$

$$\frac{4}{BC} = \frac{5}{8} \quad \text{and} \quad \frac{CD}{4} = \frac{5}{8}$$

$\Rightarrow \quad BC = \frac{4 \times 8}{5} = \frac{32}{5} = 6{\cdot}4$ cm. and $CD = \frac{5}{8} \times 4 = \frac{5}{2} = 2{\cdot}5$ cm. **Ans.**

(iii) $\Delta ACD \sim \Delta ABC$

$$\frac{\text{area } (\Delta ACD)}{\text{area } (\Delta ABC)} = \frac{AC^2}{AB^2}$$

$$= \frac{(4)^2}{(8)^2} = \frac{16}{64} = \frac{1}{4}$$

area (Δ ACD) : area (Δ ABC) = 1 : 4. **Ans.**

Question 6.

(a) *Find the value of 'a' for which the following points A (a, 3), B (2, 1) and C (5, a) are collinear. Hence find the equation of the line.* **[3]**

(b) *Salman invests a sum of money in ₹ 50 shares, paying 15% dividend quoted at 20% premium. If his annual dividend is ₹ 600, calculate :*

(i) *the number of shares he bought.*

(ii) *his total investment.*

(iii) *the rate of return on his investment.* **[3]**

(c) *The surface area of a solid metallic sphere is 2464 cm^2. It is melted and recast into solid right circular cones of radius 3.5 cm and height 7 cm. Calculate :*

(i) *the radius of the sphere.*

(ii) *the number of cones recast. (Take $\pi = 22/7$)* **[4]**

Solution :

(a) *Given* : A $(a, 3)$, B$(2, 1)$ and C$(5, a)$ are collinear.

$$\text{Slope of AB} = \text{Slope of BC}$$

$$\frac{1-3}{2-a} = \frac{a-1}{5-2}$$

$$\frac{-2}{2-a} = \frac{a-1}{3}$$

$$-6 = (2-a)(a-1)$$

$$-6 = 2a - 2 - a^2 + a$$

$$a^2 - 3a - 4 = 0$$

$$a^2 - 4a + a - 4 = 0$$

$$(a-4)(a+1) = 0$$

$$a = 4, -1$$

Rejecting, $a = -1$ $\because$ does not satisfy the equation

$$a = 4$$

$$\text{Slope of BC} = \frac{a-1}{5-2} = \frac{4-1}{3} = \frac{3}{3} = 1 = \text{m}$$

Equation of BC; $(y-1) = 1(x-2)$

$$y - 1 = x - 2$$

$$x - y = 1$$ **Ans.**

(b) Nominal value of 1 share = ₹ 50

Dividend on 1 share = $\frac{15}{100} \times 50 =$ ₹ 7·50

Total Dividend of Salman = ₹ 600

(i) No. of shares Salman bought = $\frac{600}{7{\cdot}50} = 80$ **Ans.**

(ii) Premium on 1 share = $\frac{20}{100} \times 50 =$ ₹ 10

Market value of 1 share = $50 + 10 =$ ₹ 60

Total investment for 80 shares = $80 \times 60 =$ ₹ 4,800. **Ans.**

(iii) Rate of return = $\frac{600}{4800} \times 100 = 12{\cdot}5\%$. **Ans.**

(c) (i) Let the radius of sphere = r cm

Surface area of sphere = $4\pi r^2 = 2464 \text{ cm}^2$ (given)

$$r^2 = \frac{2464}{4\pi}$$

$$r^2 = \frac{2464 \times 7}{4 \times 22}$$

$$= 196$$

$$r = 14 \text{ cm.}$$ **Ans.**

(ii) Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (14)^3$

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (3\cdot5)^2 \times 7$

No. of cones recast $= \frac{\text{Volume of sphere}}{\text{Volume of cone}}$

$$= \frac{\frac{4\pi}{3}(14)^3}{\frac{1}{3}\pi (3\cdot5)^2 \times 7} = \frac{4 \times 14 \times 14 \times 14}{3\cdot5 \times 3\cdot5 \times 7} \quad \frac{3200}{25}$$

$= 128.$ **Ans.**

Question 7.

(a) *Calculate the mean of the distribution given below using the short cut method.*

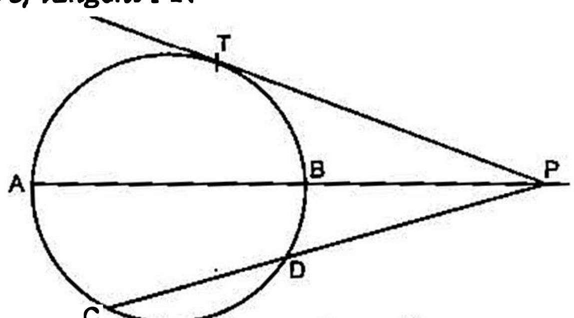
Marks	11-20	21-30	31-40	41-50	51-60	61-70	71-80
No. of students	2	6	10	12	9	7	4

[3]

(b) *In the figure given below, diameter AB and CD of a circle meet at P. PT is a tangent to the circle at T. CD = 7.8 cm, PD = 5 cm, PD = 4 cm. Find :*

(i) *AB.*

(ii) *the length of tangent PT.* **[3]**



(c) *Let* $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$ *and* $C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$.

Find $A^2 + AC - 5B$. **[4]**

Solution :

(a)

Marks (C.I.)	f	**Mean Value** x	**A = 45·5** $d = x - A$	$f \times d$
11–20	2	15·5	– 30	– 60
21–30	6	25·5	– 20	– 120
31–40	10	35·5	– 10	– 100
41–50	12	45·5	0	0
51–60	9	55·5	10	90
61–70	7	65·5	20	140
71–80	4	75·5	30	120
	$\Sigma f = 50$			$\Sigma fd = 70$

$$\text{Mean} = A + \frac{\Sigma fd}{\Sigma f}$$

$$= 45\cdot5 + \frac{70}{50} = 45\cdot5 + 1\cdot4$$

$$= 46\cdot9$$ **Ans.**

(b) (i) Since chord CD and tangent at point T intersect each other at P,

$$PC \times PD = PT^2 \quad \text{...(1)}$$

Since chord AB and tangent at point T intersect each other at P,

$$PA \times PB = PT^2 \quad \text{...(2)}$$

From (1) and (2), $$PC \times PD = PA \times PB \quad \text{...(3)}$$

Given : CD = 7·8 cm; PD = 5 cm, PB = 4 cm.

$\therefore$ PA = PB + AB = 4 + AB, PC = PD + CD = 5 + 7·8 = 12·8 cm.

Putting these values in eq. (3)

$$12\cdot8 \times 5 = (4 + AB) \times 4$$

$$4 + AB = \frac{12\cdot8 \times 5}{4}$$

$\Rightarrow$ $$4 + AB = 16$$

$\Rightarrow$ $$AB = 12 \text{ cm.}$$

Hence, $$AB = 12 \text{ cm.}$$ **Ans.**

(ii) From eq. (1), $$PT^2 = PA \times PB = 12\cdot8 \times 5$$

$\Rightarrow$ $$PT^2 = 64$$

$\Rightarrow$ $$PT = 8 \text{ cm.} = \text{Length of tangent.}$$ **Ans**

(c) *Given* : $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}, C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$

$$A^2 = A\cdot A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 2-2 \\ 0+0 & 0+4 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -6-1 & 4+4 \\ 0+2 & 0-8 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix}$$

and $$5B = 5\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$$

Now, $$A^2 + AC - 5B = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix} - \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 4-7-20 & 0+8-5 \\ 0+2+15 & 4-8+10 \end{bmatrix}$$

$$= \begin{bmatrix} -23 & 3 \\ 17 & 6 \end{bmatrix}$$ **Ans.**

Question 8.

(a) *The compound interest, calculated yearly, on a certain sum of money for the second year is ₹ 1320 and for the third year is ₹ 1452. Calculate the rate of interest and the original sum of money.* [3]

(b) *Construct a Δ ABC with BC = 6.5 cm, AB = 5.5 cm, AC = 5 cm. Construct the incircle of the triangle. Measure and record the radius of the incircle.* [3]

(c) *(Use a graph paper for this question.) The daily pocket expenses of 200 students in a school are given below :*

Pocket expenses (in ₹)	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40
Number of students (frequency)	10	14	28	42	50	30	14	12

Draw a histogram representing the above distribution and estimate the mode from the graph. [4]

Solution :

(a)

$$\text{C.I. for the third year} = ₹\,1{,}452.$$
$$\text{C.I. for the second year} = ₹\,1{,}320$$
$$\text{S.I. on ₹ 1,320 for one year} = ₹\,1{,}452 - ₹\,1{,}320 = ₹\,132.$$
$$\text{Rate of interest} = \frac{132 \times 100}{1{,}320} = 10\%.$$ **Ans.**

Let the original money be ₹ P.

Amount after 2 year – amount after one year = C.I. for second year.

$$P\left(1+\frac{10}{100}\right)^2 - P\left(1+\frac{10}{100}\right) = 1{,}320$$
$$P\left[\left(\frac{110}{100}\right)^2 - \frac{110}{100}\right] = 1{,}320$$
$$P\left[\left(\frac{11}{10}\right)^2 - \frac{11}{10}\right] = 1{,}320 \Rightarrow P\left(\frac{121}{100} - \frac{11}{10}\right) = ₹\,1{,}320$$
$$\Rightarrow \quad P \times \frac{11}{100} = ₹\,1{,}320 \Rightarrow P = \frac{1{,}320 \times 100}{11} = ₹\,12{,}000$$
$$\text{Rate of interest} = 10\%$$
and $$\text{Original sum of money} = ₹\,12{,}000$$ **Ans.**

(b) Steps of construction :

(1) Construct a Δ ABC with the given data.

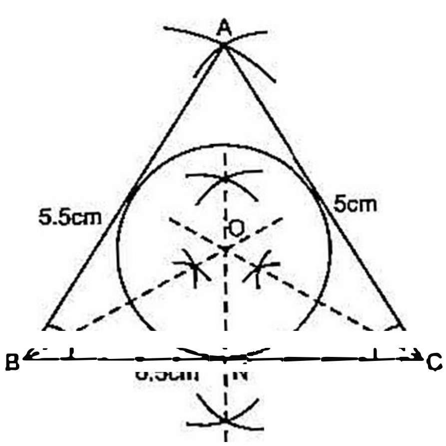
(2) Draw the internal bisectors of $\angle$ B and $\angle$ C. Let these bisectors cut at O.

(3) Taking O as centre. Draw a incircle which touches all the sides of the Δ ABC.

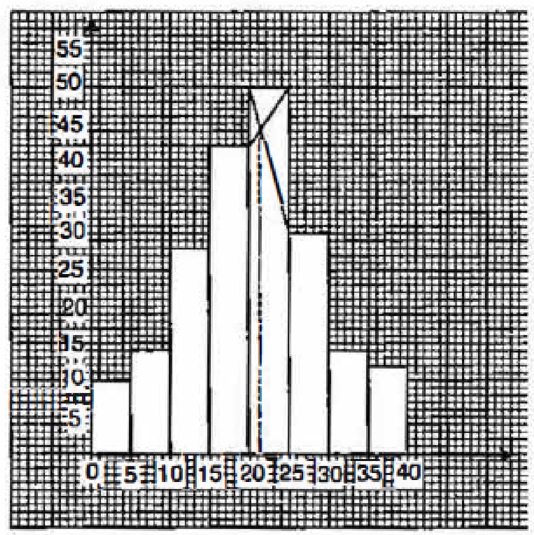
(4) From O draw a perpendicular to side BC which cut at N.

(5) Measure ON which is required radius of the incircle.

ON = 1·5 cm.



(c)



Mode = 21

Question 9.

(a) *If $(x - 9) : (3x + 6)$ is the duplicate ratio of $4 : 9$, find the value of x.* **[3]**

(b) *Solve for x using the quadratic formula. Write your answer correct to two significant figures. $(x - 1)^2 - 3x + 4 = 0$.* **[3]**

(c) *A page from the saving bank account of Priyanka is given below :*

Date	*Particulars*	*Amount withdrawn (₹)*	*Amount deposited (₹)*	*Balance (₹)*
03/04/2006	*B/F*	—	—	*4,000.00*
05/04/2006	*By cash*	—	*2,000.00*	*6,000.00*
18/04/2006	*By cheque*	—	*6,000.00*	*12,000.00*
25/05/2006	*To cheque*	*5,000.00*	—	*7,000.00*
30/05/2006	*By cash*	—	*3,000.00*	*10,000.00*
20/07/2006	*By self*	*4,000.00*	—	*6,000.00*
10/09/2006	*By cash*	—	*2,000.00*	*8,000.00*
19/09/2006	*To cheque*	*1,000.00*	—	*7,000.00*

If the interest earned by Priyanka for the period of ending September, 2006 is ₹ 175, find the rate of interest. **[4]**

Solution :

(a) *Given* : $(x - 9) : (3x + 6)$ is the duplicate ratio of $4 : 9$

$$\Rightarrow \quad \frac{x-9}{3x+6} = \left(\frac{4}{9}\right)^2$$

$$\frac{x-9}{3x+6} = \frac{16}{81}$$

$$81x - 729 = 48x + 96$$

$$81x - 48x = 96 + 729$$

$$\Rightarrow \quad 33x = 825$$

$$x = \frac{825}{33} = 25$$

Ans.

(b) *Given* :

$$(x-1)^2-3x+4 = 0$$
$$x^2+1-2x-3x+4 = 0$$
$$\Rightarrow \quad x^2-5x+5 = 0$$

Comparing $x^2-5x+5=0$ with $ax^2+bx+c=0$, we get $a=1, b=-5, c=5$.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-(-5) \pm \sqrt{(-5)^2-4\,(1)\,(5)}}{2\times 1}$$
$$= \frac{5 \pm \sqrt{25-20}}{2} = \frac{5\pm\sqrt{5}}{2}$$
$$= \frac{5\pm 2{\cdot}236}{2} = \frac{5+2{\cdot}236}{2} \text{ and } \frac{5-2{\cdot}236}{2}$$
$$= \frac{7{\cdot}236}{2} \text{ and } \frac{2{\cdot}764}{2}$$
$$= 3{\cdot}618 \text{ and } 1{\cdot}382$$

Ans.

(c)

Principal for the month of April	= ₹	6,000
Principal for the month of May	= ₹	7,000
Principal for the month of June	= ₹	10,000
Principal for the month of July	= ₹	6,000
Principal for the month of Aug.	= ₹	6,000
Principal for the month of Sep.	= ₹	7,000
Total principal for 1 month	= ₹	42,000

Now, P = ₹ 42,000, I = ₹ 175, T = $\frac{1}{12}$ years, R = ?

$$\text{Interest} = \frac{P\times R\times T}{100}$$
$$175 = \frac{42{,}000\times R\times 1}{100\times 12}$$
$$R = \frac{175\times 100\times 12}{42{,}000\times 1}$$
$$= \frac{2{,}100}{420} = 5\%.$$

Ans.

Question 10.

(a) *A two digit positive number is such that the product of its digits is 6. If 9 is added to the number, the digits interchange their places. Find the number.* **[4]**

(b) *The marks obtained by 100 students in a Mathematics test are given below :*

Marks	No. of Students
0–10	3
10–20	7
20–30	12
30–40	17
40–50	23
50–60	14

60–70	9
70–80	6
80–90	5
90–100	4

Draw an ogive for the given distribution on a graph sheet.
(Use a scale of 2 cm = 10 units on both axis).
Use the ogive to estimate the
(i) *median.*
(ii) *lower quartile.*
(iii) *number of students who obtained more than 85% marks in the test.*
(iv) *number of students who did not pass in the test if the pass percentage was 35.* **[6]**

Solution :

(a) Let the required two digit number be $10x + y$

Given : $xy = 6$ and $10x + y + 9 = 10y + x$

$$10x - x + y - 10y + 9 = 0$$
$$\Rightarrow \quad 9x - 9y + 9 = 0$$
$$\Rightarrow \quad x - y + 1 = 0$$
$$y = x + 1$$
$$xy = 6 \text{ (given)}$$
$$x(x + 1) = 6$$
$$\Rightarrow \quad x^2 + x - 6 = 0$$
$$\Rightarrow \quad x^2 + 3x - 2x - 6 = 0$$
$$\Rightarrow \quad (x + 3)(x - 2) = 0$$
$$\Rightarrow \quad x = -3, 2$$

Rejecting $x = -3$

When $x = 2$, $\quad y = x + 1 = 2 + 1 = 3$

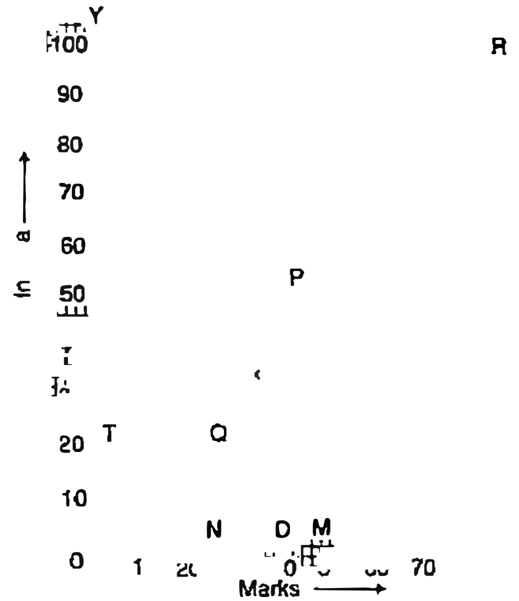
$\therefore$ The required two digit number $= 10x + y$
$= 10 \times 2 + 3$
$= 23.$ **Ans.**

(b)

Marks	**No. of Students**	**Cumulative frequency** (*c.f.*)
0–10	3	3
10–20	7	10
20–30	12	22
30–40	17	39
40–50	23	62
50–60	14	76
60–70	09	85
70–80	06	91
80–90	05	96
90–100	04	100

On the graph paper, we plot the following points

(10, 3), (20, 10), (30, 22), (40, 39), (50, 62), (60, 76), (70, 85), (80, 91), (90, 96), (100, 100)



(i) Median $= \left(\frac{n}{2}\right)^{th}$ term

$= \frac{100}{2} = 50^{th}$ term

From the graph 50^{th} term $=$ 43. **Ans.**

(ii) Lower quartile $= \left(\frac{n}{4}\right)^{th}$ term

$= \frac{100}{4} = 25^{th}$ term.

From the graph 25^{th} term $=$ 31 **Ans.**

(iii) The number of students who obtained more than 85% marks in test

$= 100 - 95$

$=$ 5 students. **Ans.**

(iv) The number of students who did not pass in the test if the pass percentage was 35 = 30. **Ans.**

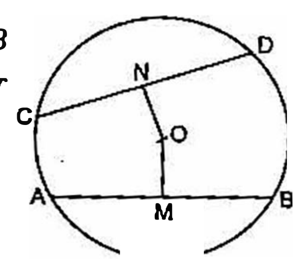
Question 11.

(a) *In the figure given below, O is the centre of the circle. AB and CD are two chords of the circle. OM is perpendicular to AB and ON is perpendicular to CD.*

AB = 24 cm, OM = 5 cm, ON = 12 cm. Find the :

(i) *radius of the circle.*

(ii) *length of chord CD.* **[3]**



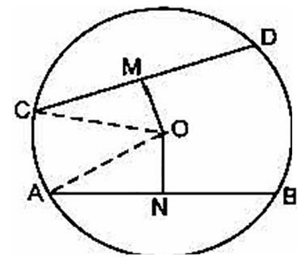
(b) *Prove the identity :*

(sin θ + cos θ) (tan θ + cot θ) = sec θ + cosec θ. **[3]**

(c) *An aeroplane at an altitude of 250 m observes the angle of depression of two boats on the opposite banks of a river to be 45° and 60° respectively. Find the width of the river. Write the answer correct to the nearest whole number.* **[4]**

Solution :

(a) *Given* : AB = 24 cm; OM = 5 cm, ON = 12 cm.



$$OM \perp AB$$

M is mid point of AB.

$$AM = 12 \text{ cm.}$$

(i) Let radius of circle = r

From Δ AMO; $AO^2 = AM^2 + OM^2$ (By Pythagoras theorem)

$$r^2 = (12)^2 + (5)^2$$
$$= 144 + 25$$
$$r^2 = 169$$
$$r = 13 \text{ cm.}$$ **Ans.**

(ii) Now from Δ CNO; $CO^2 = ON^2 + CN^2$

$$r^2 = (12)^2 + CN^2$$ ($\because$ AO = CO = r)

$$(13)^2 - (12)^2 = CN^2$$
$$169 - 144 = CN^2$$
$$CN^2 = 25$$
$$CN = 5$$

As ON ⊥ CD, N is mid point of CD.

$\therefore$ CD = 2 CN = 2 × 5 = 10 cm. **Ans.**

(b)

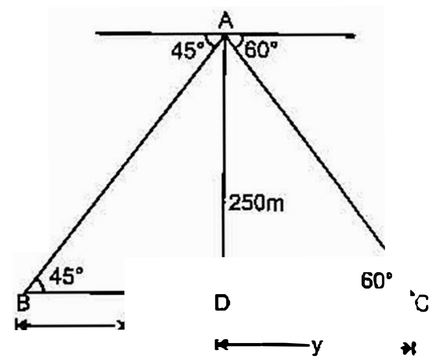
$$\text{L.H.S.} = (\sin\theta + \cos\theta)(\tan\theta + \cot\theta)$$
$$= (\sin\theta + \cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$
$$= (\sin\theta + \cos\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)$$
$$= (\sin\theta + \cos\theta) \times \frac{1}{\cos\theta\sin\theta}$$
$$= \frac{\sin\theta}{\cos\theta\sin\theta} + \frac{\cos\theta}{\cos\theta\sin\theta}$$
$$= \frac{1}{\cos\theta} + \frac{1}{\sin\theta}$$
$$= \sec\theta + \operatorname{cosec}\theta$$
$$= \text{R.H.S.}$$

Hence Proved.

(c) Let $AD = 250$ m height of aeroplane

Two boats are at B and C.

Let $BD = x$ and $DC = y$



From Δ ADB; $\frac{x}{250} = \cot 45°$

$$\frac{x}{250} = 1$$

$$x = 250 \text{ m}$$

From Δ ADC; $\frac{y}{250} = \cot 60°$

$$\frac{y}{250} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $y = 250 \times \frac{1}{\sqrt{3}}$

Width of river $BC = BD + DC = x + y$

$$= 250 + \frac{250}{\sqrt{3}}$$

$$= 250\left(1 + \frac{1}{\sqrt{3}}\right) = 250\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$$ **Ans.**

$$= 250\left(\frac{1{\cdot}732 + 1}{1{\cdot}732}\right) = 250\left(\frac{2{\cdot}732}{1{\cdot}732}\right)$$

$$= 250 \times 1{\cdot}577$$

$$= 394{\cdot}25 \text{ m} = 394 \text{ m.}$$ **Ans.**