

ICSE BOARD
Class X Mathematics
Board Paper – 2017(Solution)

SECTION A

1.

(a) Cost price of an article = Rs. 3,450

(i) Marked price of the article = Cost price + 16% of Cost price

$$\begin{aligned} &= 3450 + \frac{16}{100} \times 3450 \\ &= 3450 + 552 \\ &= \text{Rs. } 4002 \end{aligned}$$

(ii) Price paid by the customer = Marked price + Sales Tax

$$\begin{aligned} &= 4002 + \frac{10}{100} \times 4002 \\ &= 4002 + 400.2 \\ &= \text{Rs. } 4402.20 \end{aligned}$$

(b) $13x - 5 < 15x + 4 < 7x + 12$, $x \in \mathbb{R}$

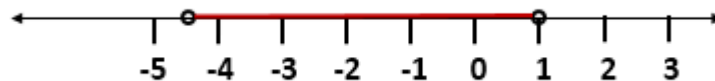
| | | |
|------|---------------------|---------------------|
| Take | $13x - 5 < 15x + 4$ | $15x + 4 < 7x + 12$ |
| | $13x < 15x + 9$ | $15x < 7x + 8$ |
| | $0 < 2x + 9$ | $8x < 8$ |
| | $-9 < 2x$ | $x < 1$ |
| | $-\frac{9}{2} < x$ | $x < 1$ |

$$\therefore -\frac{9}{2} < x < 1$$

$$\text{i.e. } -4.5 < x < 1$$

$$\therefore \text{Solution set} = \{x : -4.5 < x < 1, x \in \mathbb{R}\}$$

The solution on the number line is as follows:



$$\begin{aligned}
(c) \quad & \frac{\sin 65^\circ}{\cos 25^\circ} + \frac{\cos 32^\circ}{\sin 58^\circ} - \sin 28^\circ \cdot \sec 62^\circ + \operatorname{cosec}^2 30^\circ \\
&= \frac{\sin(90^\circ - 25^\circ)}{\cos 25^\circ} + \frac{\cos(90^\circ - 58^\circ)}{\sin 58^\circ} - \sin 28^\circ \times \frac{1}{\cos(90^\circ - 28^\circ)} + \frac{1}{\sin^2 30^\circ} \\
&= \frac{\cos 25^\circ}{\cos 25^\circ} + \frac{\sin 58^\circ}{\sin 58^\circ} - \sin 28^\circ \times \frac{1}{\sin 28^\circ} + \left(\frac{1}{\left(\frac{1}{2}\right)^2} \right) \\
&= 1 + 1 - 1 + 4 \\
&= 5
\end{aligned}$$

2.

$$(a) \text{ Given: } A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix} \text{ and } A^2 = B$$

$$\text{Now, } A^2 = A \times A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 3x+x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix}$$

We have $A^2 = B$

Two matrices are equal if each and every corresponding element is equal.

$$\text{Thus, } \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$$

$$\Rightarrow 4x = 16 \text{ and } 1 = -y$$

$$\Rightarrow x = 4 \text{ and } y = -1$$

$$(b) \text{ Population after } n \text{ years} = \text{Present population} \times \left(1 + \frac{r}{100}\right)^n$$

Present population = 2,00,000

$$\text{After first year, population} = 2,00,000 \times \left(1 + \frac{10}{100}\right)^1$$

$$= 2,00,000 \times \frac{11}{10}$$

$$= 2,20,000$$

$$\text{Population after two years} = 2,20,000 \times \left(1 + \frac{15}{100}\right)^1$$

$$= 2,53,000$$

Thus, the population after two years is 2,53,000.

(c) Three vertices of a parallelogram taken in order are A(3, 6), B(5, 10) and C(3, 2)

(i) We need to find the co-ordinates of D.

We know that the diagonals of a parallelogram bisect each other.

Let (x, y) be the co-ordinates of D.

$$\therefore \text{Mid-point of diagonal AC} = \left(\frac{3+3}{2}, \frac{6+2}{2} \right) \equiv (3, 4)$$

$$\text{And, mid-point of diagonal BD} = \left(\frac{5+x}{2}, \frac{10+y}{2} \right)$$

Thus, we have

$$\frac{5+x}{2} = 3 \quad \text{and} \quad \frac{10+y}{2} = 4$$

$$\Rightarrow 5+x=6 \quad \text{and} \quad 10+y=8$$

$$\Rightarrow x=1 \quad \text{and} \quad y=-2$$

$$\therefore D=(1, -2)$$

$$\begin{aligned} \text{(ii) Length of diagonal BD} &= \sqrt{(1-5)^2 + (-2-10)^2} \\ &= \sqrt{(-4)^2 + (-12)^2} \\ &= \sqrt{16+144} \\ &= \sqrt{160} \\ &= 4\sqrt{10} \end{aligned}$$

$$\text{(iii) } A(3, 6) = (x_1, y_1) \text{ and } B(5, 10) = (x_2, y_2)$$

$$\text{Slope of line AB} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-6}{5-3} = \frac{4}{2} = 2$$

\therefore Equation of line AB is given by

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 6 = 2(x - 3)$$

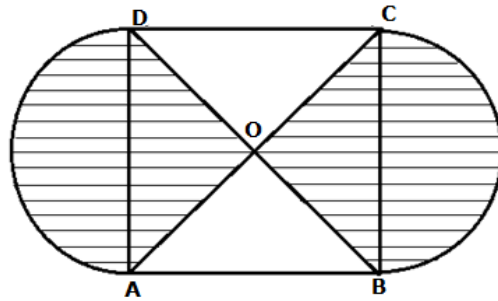
$$\Rightarrow y - 6 = 2x - 6$$

$$\Rightarrow 2x - y = 0$$

$$\Rightarrow 2x = y$$

3.

(a)



$$\text{Area of one semi-circle} = \frac{1}{2} \times \pi \times \left(\frac{21}{2}\right)^2$$

$$\Rightarrow \text{Area of both semi-circles} = 2 \times \frac{1}{2} \times \pi \times \left(\frac{21}{2}\right)^2$$

$$\text{Area of one triangle} = \frac{1}{2} \times 21 \times \frac{21}{2}$$

$$\Rightarrow \text{Area of both triangles} = 2 \times \frac{1}{2} \times 21 \times \frac{21}{2}$$

Area of shaded portion

$$= 2 \times \frac{1}{2} \times \pi \times \left(\frac{21}{2}\right)^2 + 2 \times \frac{1}{2} \times 21 \times \frac{21}{2}$$

$$= \frac{22}{7} \times \frac{441}{4} + \frac{441}{2}$$

$$= \frac{693}{2} + \frac{441}{2}$$

$$= \frac{1134}{2}$$

$$= 567 \text{ cm}^2$$

(b)

| Marks (x) | 0 | 1 | 2 | 3 | 4 | 5 | Total |
|---------------------|---|---|----|----|----|----|------------------|
| No. of Students (f) | 1 | 3 | 6 | 10 | 5 | 5 | n = 30 |
| fx | 0 | 3 | 12 | 30 | 20 | 25 | $\Sigma fx = 90$ |
| c.f. | 1 | 4 | 10 | 20 | 25 | 30 | |

$$\text{Mean} = \frac{\Sigma fx}{n} = \frac{90}{30} = 3$$

Number of observations = 30 (even)

$$\begin{aligned}\therefore \text{Median} &= \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ observation}}{2} \\ &= \frac{\left(\frac{30}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{30}{2} + 1\right)^{\text{th}} \text{ observation}}{2} \\ &= \frac{15^{\text{th}} \text{ observation} + 16^{\text{th}} \text{ observation}}{2} \\ &= \frac{3 + 3}{2} \\ &= 3\end{aligned}$$

Mode = The number (marks) with highest frequency = 3

(c) In the given figure, $TS \perp SP$,

$$m\angle TSR = m\angle OSP = 90^\circ$$

$$\text{In } \triangle TSR, m\angle TSR + m\angle TRS + m\angle RTS = 180^\circ$$

$$\Rightarrow 90^\circ + 65^\circ + x = 180^\circ$$

$$\Rightarrow x = 180^\circ - 90^\circ - 65^\circ$$

$$\Rightarrow x = 25^\circ$$

Now, $y = 2x$ [Angle subtended at the centre is double that of the angle subtended by the arc at the same centre]

$$\Rightarrow y = 2 \times 25^\circ$$

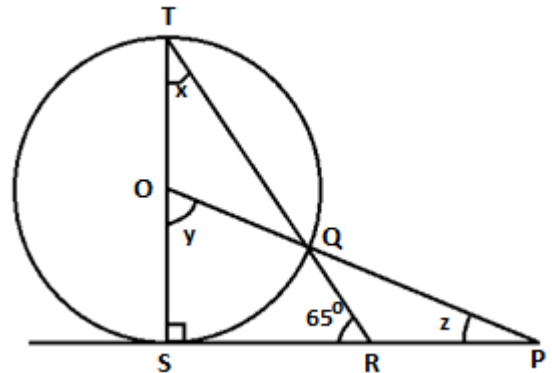
$$\therefore y = 50^\circ$$

$$\text{In } \triangle OSP, m\angle OSP + m\angle SPO + m\angle POS = 180^\circ$$

$$\Rightarrow 90^\circ + z + 50^\circ = 180^\circ$$

$$\Rightarrow z = 180^\circ - 140^\circ$$

$$\therefore z = 40^\circ$$



4.

(a) Given,

$$P = \text{Rs. } 1000$$

$$n = 2 \text{ years} = 24 \text{ months}$$

$$r = 6\%$$

$$\begin{aligned} \text{(i) Interest} &= P \times \frac{n(n+1)}{2} \times \frac{r}{12 \times 100} \\ &= 1000 \times \frac{24 \times 25}{2} \times \frac{6}{12 \times 100} \\ &= 1500 \end{aligned}$$

Thus, the interest earned in 2 years is Rs. 1500.

$$\text{(ii) Sum deposited in two years} = 24 \times 1000 = 24,000$$

$$\begin{aligned} \text{Maturity value} &= \text{Total sum deposited in two years} + \text{Interest} \\ &= 24,000 + 1,500 \\ &= 25,500 \end{aligned}$$

Thus, the maturity value is Rs. 25,500.

$$\text{(b) } (K + 2)x^2 - Kx + 6 = 0 \quad \dots(1)$$

Substituting $x = 3$ in equation (1), we get

$$(K + 2)(3)^2 - K(3) + 6 = 0$$

$$\therefore 9(K + 2) - 3K + 6 = 0$$

$$\therefore 9k + 18 - 3k + 6 = 0$$

$$\therefore 6k + 24 = 0$$

$$\therefore K = -4$$

Now, substituting $K = -4$ in equation (1), we get

$$(-4 + 2)x^2 - (-4)x + 6 = 0$$

$$\therefore -2x^2 + 4x + 6 = 0$$

$$\therefore x^2 - 2x - 3 = 0$$

$$\therefore x^2 - 3x + x - 3 = 0$$

$$\therefore x(x - 3) + 1(x - 3) = 0$$

$$\therefore (x + 1)(x - 3) = 0$$

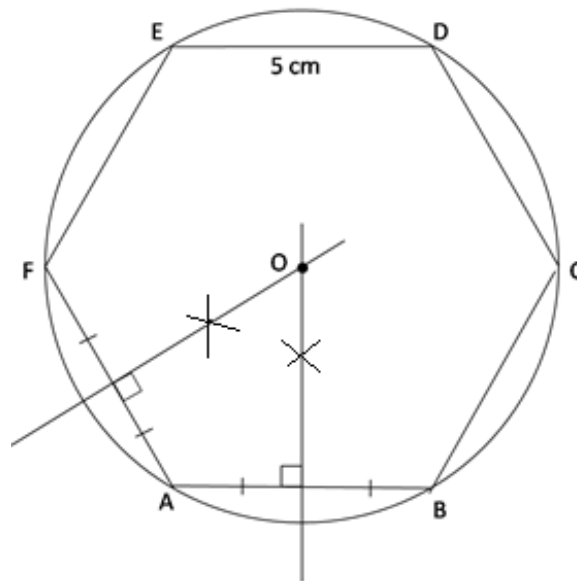
So, the roots are $x = -1$ and $x = 3$.

Thus, the other root of the equation is $x = -1$.

(c) Each interior angle of the regular hexagon = $\frac{(2n-4)}{n} \times 90^\circ = \frac{(2 \times 6 - 4)}{6} \times 90^\circ = 120^\circ$

Steps of construction:

- Construct the regular hexagon ABCDEF with each side equal to 5 cm.
- Draw the perpendicular bisectors of sides AB and AF and make them intersect each other at point O.
- With O as the centre and OA as the radius draw a circle which will pass through all the vertices of the regular hexagon ABCDEF.

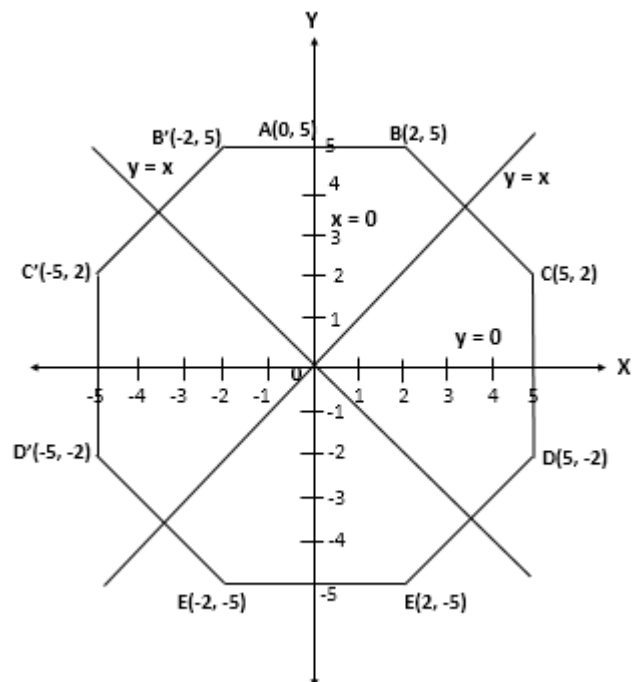


SECTION B

5.

(a)

- See the figure.
- Reflection of points on the y-axis will result in the change of the x-coordinate
- Points will be B'(-2, 5), C'(-5, 2), D'(-5, -2), E'(-2, -5).
- The figure BCDEE'D'C'B' is a hexagon.
- The lines of symmetry is x-axis or y-axis.



- (b) Principal for the month of April = Rs. 0
 Principal for the month of May = Rs. 4650
 Principal for the month of June = Rs. 4750
 Principal for the month of July = Rs. 8950
 Total Principal for one month = Rs. 18,350

$$\text{Time} = \frac{1}{12} \text{ year, Rate} = 4\%$$

$$\text{Interest earned} = \frac{18350 \times 1 \times 4}{100 \times 12} = 61.17$$

$$\begin{aligned} \text{Money received on closing the account on 1st August, 2010} \\ &= \text{Last Balance} + \text{Interest earned} \\ &= \text{Rs. } (8950 + 61.17) \\ &= \text{Rs. } 9011.17 \end{aligned}$$

6.

- (a) Given that a, b and c are in continued proportion.

$$\Rightarrow \frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$$

$$\begin{aligned} \text{L.H.S.} &= (a + b + c)(a - b + c) \\ &= a(a - b + c) + b(a - b + c) + c(a - b + c) \\ &= a^2 - ab + ac + ab - b^2 + bc + ac - bc + c^2 \\ &= a^2 + ac - b^2 + ac + c^2 \\ &= a^2 + b^2 - b^2 + b^2 + c^2 \quad \left[\because b^2 = ac \right] \\ &= a^2 + b^2 + c^2 \\ &= \text{R.H.S.} \end{aligned}$$

(b)

- i. The line intersects the x-axis where, $y = 0$. Thus, the co-ordinates of A are (4, 0).

ii. Length of AB = $\sqrt{(4 - (-2))^2 + (0 - 3)^2} = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5}$ units

Length of AC = $\sqrt{(4 - (-2))^2 + (0 + 4)^2} = \sqrt{36 + 16} = \sqrt{52} = 2\sqrt{13}$ units

- iii. Let Q divides AC in the ratio $m_1 : m_2$. Thus, the co-ordinates of Q are (0, y)

$$\text{Since } x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$\Rightarrow 0 = \frac{m_1(-2) + m_2(4)}{m_1 + m_2} \Rightarrow 2m_1 = 4m_2 \Rightarrow m_1 = 2m_2$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{2}{1}$$

\therefore Required ratio is 2 : 1.

iv. $A(4, 0) = A(x_1, y_1)$ and $B(-2, -4) = b(x_2, y_2)$

$$\text{Slope of AC} = \frac{-4-0}{-2-4} = \frac{-4}{-6} = \frac{2}{3}$$

\therefore Equation of line AC is given by $y - y_1 = m(x - x_1)$

$$\Rightarrow y - 0 = \frac{2}{3}(x - 4)$$

$$\Rightarrow 3y = 2x - 8$$

$$\Rightarrow 2x - 3y = 8$$

(c) Consider the following distribution:

| Class Interval | Frequency (f) | Class mark (x) | fx |
|----------------|--------------------|----------------|------------------|
| 0 - 10 | 8 | 5 | 40 |
| 10 - 20 | 5 | 15 | 75 |
| 20 - 30 | 12 | 25 | 300 |
| 30 - 40 | 35 | 35 | 1225 |
| 40 - 50 | 24 | 45 | 1080 |
| 50 - 60 | 16 | 55 | 880 |
| Total | $n = \sum f = 100$ | | $\sum fx = 3600$ |

$$\text{Mean} = \frac{\sum fx}{n} = \frac{3600}{100} = 36$$

7.

(a) Radius of small sphere = $r = 2$ cm

Radius of big sphere = $R = 4$ cm

$$\text{Volume of small sphere} = \frac{4}{3}\pi r^3 = \frac{4\pi}{3} \times (2)^3 = \frac{32\pi}{3} \text{ cm}^3$$

$$\text{Volume of big sphere} = \frac{4}{3}\pi R^3 = \frac{4\pi}{3} \times (4)^3 = \frac{256\pi}{3} \text{ cm}^3$$

$$\text{Volume of both the spheres} = \frac{32\pi}{3} + \frac{256\pi}{3} = \frac{288\pi}{3} \text{ cm}^3$$

We need to find R_1 . $h = 8$ cm (Given)

$$\text{Volume of the cone} = \frac{1}{3}\pi R_1^2 \times (8)$$

Volume of the cone = Volume of both the sphere

$$\Rightarrow \frac{1}{3}\pi R_1^2 \times (8) = \frac{288\pi}{3}$$

$$\Rightarrow R_1^2 \times (8) = 288$$

$$\Rightarrow R_1^2 = \frac{288}{8} \Rightarrow R_1^2 = 36$$

$$\Rightarrow R_1 = 6 \text{ cm}$$

(b) The given polynomials are $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$.

Let $p(x) = ax^3 + 3x^2 - 9$ and $q(x) = 2x^3 + 4x + a$

Given that $p(x)$ and $q(x)$ leave the same remainder when divided by $(x + 3)$,

Thus by Remainder Theorem, we have

$$p(-3) = q(-3)$$

$$\Rightarrow a(-3)^3 + 3(-3)^2 - 9 = 2(-3)^3 + 4(-3) + a$$

$$\Rightarrow -27a + 27 - 9 = -54 - 12 + a$$

$$\Rightarrow -27a + 18 = -66 + a$$

$$\Rightarrow -27a - a = -66 - 18$$

$$\Rightarrow -28a = -84$$

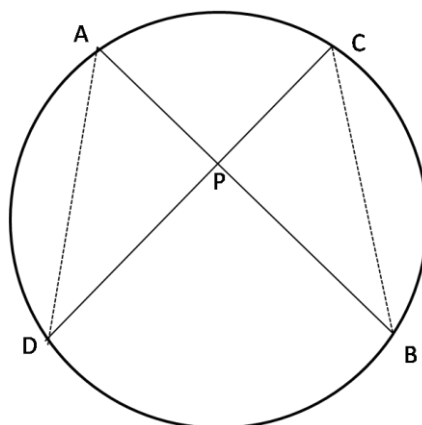
$$\Rightarrow a = \frac{84}{28}$$

$$\therefore a = 3$$

$$\begin{aligned} \text{(c) L.H.S.} &= \frac{\sin \theta}{1 - \cot \theta} + \frac{\cos \theta}{1 - \tan \theta} \\ &= \frac{\sin \theta}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\cos \theta}{1 - \frac{\sin \theta}{\cos \theta}} \\ &= \frac{\sin^2 \theta}{\sin \theta - \cos \theta} + \frac{\cos^2 \theta}{\cos \theta - \sin \theta} \\ &= \frac{\sin^2 \theta}{\sin \theta - \cos \theta} - \frac{\cos^2 \theta}{\sin \theta - \cos \theta} \\ &= \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta - \cos \theta} \\ &= \frac{(\sin \theta - \cos \theta)(\sin \theta + \cos \theta)}{\sin \theta - \cos \theta} \\ &= \sin \theta + \cos \theta \\ &= \text{R.H.S.} \end{aligned}$$

8.

(a) Construction: Join AD and CB.



In $\triangle APD$ and $\triangle CPB$

$\angle A = \angle C$ (Angles in the same segment)

$\angle D = \angle B$ (Angles in the same segment)

$\Rightarrow \triangle APD \sim \triangle CPB$ (By AA Postulate)

$\Rightarrow \frac{AP}{CP} = \frac{PD}{PB}$ (Corresponding sides of similar triangles)

$\Rightarrow AP \times PB = CP \times PD$

(b) Total number of balls = 5 + 6 + 9 = 20

(i) Number of green balls = 9 = Number of favourable cases

$$\therefore P(\text{Green ball}) = \frac{\text{Number of favourable cases}}{\text{Total number of balls}} = \frac{9}{20}$$

(ii) Number of white balls = 5, Number of red balls = 6

Number of favourable cases = 5 + 6 = 11

$$\therefore P(\text{White ball or Red ball}) = \frac{\text{Number of favourable cases}}{\text{Total number of balls}} = \frac{11}{20}$$

(iii) $P(\text{Neither green ball nor white ball}) = P(\text{Red ball})$

$$\begin{aligned} &= \frac{\text{Number of Red balls}}{\text{Total number of balls}} \\ &= \frac{6}{20} \end{aligned}$$

(c)

(i) 100 shares at Rs. 20 premium means:

Nominal value of the share is Rs. 100

Market value of each share = $100 + 20 = \text{Rs. } 120$

Investment = Rs. 9600

$$\therefore \text{Number of shares} = \frac{\text{Investment}}{\text{Market Value of each Share}} = \frac{9600}{120} = 80$$

(ii) Sale price of each share = Rs. 160

$$\therefore \text{The sale proceeds} = 80 \times 160 = \text{Rs. } 12,800$$

(iii) New investment = Rs. 12,800

Market Value of each share = Rs. 40

$$\therefore \text{Number of shares} = \frac{\text{Investment}}{\text{Market Value of each share}} = \frac{12800}{40} = 320$$

(iv) Dividend in the 1st investment:

$$= \text{Number of shares} \times \text{Rate of dividend} \times \text{N.V. of each share}$$

$$= 80 \times 8\% \times 100$$

$$= 80 \times \frac{8}{100} \times 100$$

$$= \text{Rs. } 640$$

Dividend in the 2nd investment

$$= \text{Number of shares} \times \text{Rate of dividend} \times \text{N.V. of each share}$$

$$= 320 \times 10\% \times 50$$

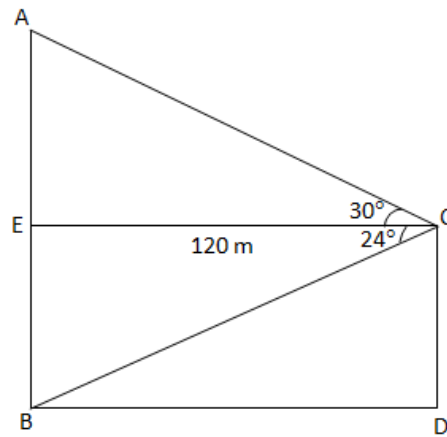
$$= 320 \times \frac{10}{100} \times 50$$

$$= \text{Rs. } 1600$$

$$\text{Thus, change in two dividends} = 1600 - 640 = \text{Rs. } 960$$

9.

(a) Consider the following figure:



In $\triangle AEC$,

$$\tan 30^\circ = \frac{AE}{EC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AE}{120}$$

$$\Rightarrow \frac{120}{\sqrt{3}} = AE$$

$$\therefore AE = 69.28 \text{ m}$$

In $\triangle BEC$,

$$\tan 24^\circ = \frac{EB}{EC}$$

$$\Rightarrow 0.445 = \frac{EB}{120}$$

$$\therefore EB = 53.427 \text{ m}$$

Thus, height of first tower, $AB = AE + EB$

$$= 69.282 + 53.427$$

$$= 122.709 \text{ m}$$

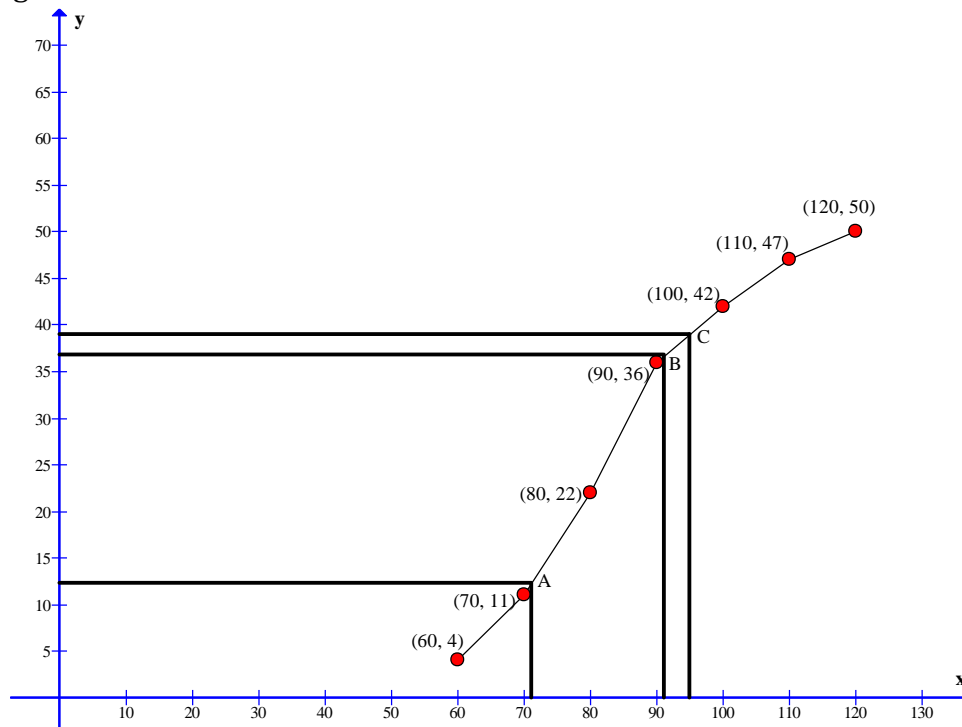
$$= 122 \text{ m (correct to 3 significant figures)}$$

And, height of second tower, $CD = EB = 53.427 \text{ m} = 53.4$ (correct to 3 significant figures)

(b) The cumulative frequency table of the given distribution table is as follows:

| Weight in Kg | Number of workers | Cumulative frequency |
|--------------|-------------------|----------------------|
| 50-60 | 4 | 4 |
| 60-70 | 7 | 11 |
| 70-80 | 11 | 22 |
| 80-90 | 14 | 36 |
| 90-100 | 6 | 42 |
| 100-110 | 5 | 47 |
| 110-120 | 3 | 50 |

The ogive is as follows:



Number of workers = 50

$$(i) \text{ Upper quartile } (Q_3) = \left(\frac{3 \times 50}{4} \right)^{\text{th}} \text{ term} = (37.5)^{\text{th}} \text{ term} = 92$$

$$\text{Lower quartile } (Q_1) = \left(\frac{50}{4} \right)^{\text{th}} \text{ term} = (12.5)^{\text{th}} \text{ term} = 71.1$$

(ii) Through mark of 95 on the x – axis, draw a vertical line which meets the graph at point C.

Then through point C, draw a horizontal line which meets the y-axis at the mark of 39.

Thus, number of workers weighing 95 kg and above = $50 - 39 = 11$

10.

(a) (i) Selling price of the manufacturer = Rs. 25000

Marked price of the wholesaler

$$= 25000 + \frac{20}{100} \times 25000$$

$$= 25000 + 5000$$

$$= \text{Rs. } 30,000$$

(ii) For retailer,

C.P. = Marked price – Discount

$$= \text{Rs. } 30000 - 10\% \text{ of Rs. } 30000$$

$$= \text{Rs. } 30000 - \frac{10}{100} \times \text{Rs. } 30000$$

$$= \text{Rs. } 30000 - \text{Rs. } 3000$$

$$= \text{Rs. } 27,000$$

Now, C.P. inclusive of tax = Rs. 27000 + 8% of Rs. 27000

$$= \text{Rs. } 27000 + \frac{8}{100} \times \text{Rs. } 27000$$

$$= \text{Rs. } 27000 + \text{Rs. } 2160$$

$$= \text{Rs. } 29,160$$

(iii) For wholesaler,

C.P. = Rs. 25000

S.P. = Rs. 27000

Profit = S.P. – C.P. = Rs. (27000 – 25000) = Rs. 2000

VAT paid by wholesaler = $\frac{8}{100} \times \text{Rs. } 2000 = \text{Rs. } 160$

$$(b) AB = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \times 0 + 7 \times 5 & 3 \times 2 + 7 \times 3 \\ 2 \times 0 + 4 \times 5 & 2 \times 2 + 4 \times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + 35 & 6 + 21 \\ 0 + 20 & 4 + 12 \end{bmatrix}$$

$$= \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix}$$

$$5C = 5 \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$$

$$AB - 5C = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix} = \begin{bmatrix} 30 & 52 \\ 40 & -14 \end{bmatrix}$$

(c)

(i) Consider $\triangle ADE$ and $\triangle ACB$.

$$\angle A = \angle A \quad [\text{Common}]$$

$$m\angle B = m\angle E = 90^\circ$$

Thus by Angle-Angle similarity, triangles, $\triangle ACB \sim \triangle ADE$.

(ii) Since $\triangle ADE \sim \triangle ACB$, their sides are proportional.

$$\Rightarrow \frac{AE}{AB} = \frac{AD}{AC} = \frac{DE}{BC} \dots (1)$$

In $\triangle ABC$, by Pythagoras Theorem, we have

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow AB^2 + 5^2 = 13^2$$

$$\Rightarrow AB = 12 \text{ cm}$$

From equation (1), we have,

$$\frac{4}{12} = \frac{AD}{13} = \frac{DE}{5}$$

$$\Rightarrow \frac{1}{3} = \frac{AD}{13}$$

$$\Rightarrow AD = \frac{13}{3} \text{ cm}$$

$$\text{Also } \frac{4}{12} = \frac{DE}{5}$$

$$\Rightarrow DE = \frac{20}{12} = \frac{5}{3} \text{ cm}$$

(iii) We need to find the area of $\triangle ADE$ and quadrilateral BCED.

$$\text{Area of } \triangle ADE = \frac{1}{2} \times AE \times DE = \frac{1}{2} \times 4 \times \frac{5}{3} = \frac{10}{3} \text{ cm}^2$$

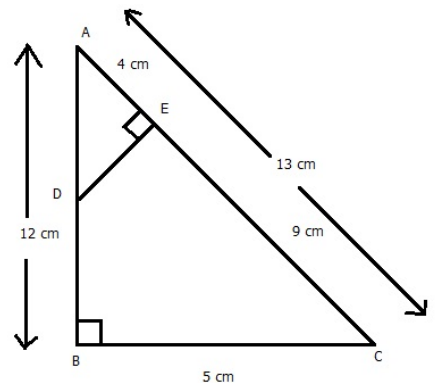
$$\text{Area of quad. BCED} = \text{Area of } \triangle ABC - \text{Area of } \triangle ADE$$

$$= \frac{1}{2} \times BC \times AB - \frac{10}{3}$$

$$= \frac{1}{2} \times 5 \times 12 - \frac{10}{3}$$

$$= 30 - \frac{10}{3}$$

$$= \frac{80}{3} \text{ cm}^2$$



$$\text{Thus ratio of areas of } \triangle ADE \text{ to quadrilateral BCED} = \frac{\frac{10}{3}}{\frac{80}{3}} = \frac{1}{8}$$

11.

(a) Let the two natural numbers be x and $(8 - x)$. Then, we have

$$\frac{1}{x} - \frac{1}{8-x} = \frac{2}{15}$$

$$\Rightarrow \frac{8-x-x}{x(8-x)} = \frac{2}{15}$$

$$\Rightarrow \frac{8-2x}{x(8-x)} = \frac{2}{15}$$

$$\Rightarrow \frac{4-x}{x(8-x)} = \frac{1}{15}$$

$$\Rightarrow 15(4-x) = x(8-x)$$

$$\Rightarrow 60 - 15x = 8x - x^2$$

$$\Rightarrow x^2 - 15x - 8x + 60 = 0$$

$$\Rightarrow x^2 - 23x + 60 = 0$$

$$\Rightarrow x^2 - 20x - 3x + 60 = 0$$

$$\Rightarrow (x-3)(x-20) = 0$$

$$\Rightarrow (x-3) = 0 \text{ or } (x-20) = 0$$

$$\Rightarrow x = 3 \text{ or } x = 20$$

Since sum of two natural numbers is 8, x cannot be equal to 20

$$\Rightarrow x = 3 \text{ and } 8 - x = 8 - 3 = 5$$

Hence, required natural numbers are 3 and 5.

$$(b) \frac{x^3 + 12x}{6x^2 + 8} = \frac{y^3 + 27y}{9y^2 + 27}$$

$$\Rightarrow \frac{x^3 + 12x + 6x^2 + 8}{x^3 + 12x - 6x^2 - 8} = \frac{y^3 + 27y + 9y^2 + 27}{y^3 + 27y - 9y^2 - 27} \quad (\text{Using componendo-dividendo})$$

$$\Rightarrow \frac{(x+2)^3}{(x-2)^3} = \frac{(y+3)^3}{(y-3)^3}$$

$$\Rightarrow \left(\frac{x+2}{x-2} \right)^3 = \left(\frac{y+3}{y-3} \right)^3$$

$$\Rightarrow \frac{x+2}{x-2} = \frac{y+3}{y-3}$$

$$\Rightarrow \frac{2x}{4} = \frac{2y}{6} \quad (\text{Using componendo-dividendo})$$

$$\Rightarrow \frac{x}{2} = \frac{y}{3}$$

$$\Rightarrow \frac{x}{y} = \frac{2}{3} \Rightarrow x : y = 2 : 3$$

(c)

1. Draw a line segment AB of length 5.5 cm.
2. Make an angle $m\angle BAX = 105^\circ$ using a protractor.
3. Draw an arc AC with radius AC = 6 cm on AX with centre at A.
4. Join BC.

Thus $\triangle ABC$ is the required triangle.

- (i) Draw BR, the bisector of $\angle ABC$, which is the locus of points equidistant from BA and BC.
 - (ii) Draw MN, the perpendicular bisector of BC, which is the locus of points equidistant from B and C.
 - (iii) The angle bisector of $\angle ABC$ and perpendicular bisector of BC meet at point P.
- Thus, P satisfies the above two loci.

Length of PC = 4.8 cm

