ICSE Board

Class X Mathematics

Board Paper 2015 (Solution)

SECTION A

1.

(a)

Given, b is the mean proportion between a and c.

$$\Rightarrow \frac{a}{b} = \frac{b}{c} = k(\text{say})$$

$$\Rightarrow a = bk,\ b = ck$$

$$\Rightarrow a = (ck)k = ck^2, b = ck$$

$$\text{L.H.S.} = \frac{a^4 + a^2b^2 + b^4}{b^4 + b^2c^2 + c^4}$$

$$= \frac{\left(ck^2\right)^4 + \left(ck^2\right)^2\left(ck\right)^2 + \left(ck\right)^4}{\left(ck\right)^4 + \left(ck\right)^2 c^2 + c^4}$$

$$= \frac{c^4k^8 + \left(c^2k^4\right)\left(c^2k^2\right) + c^4k^4}{c^4k^4 + \left(c^2k^2\right)c^2 + c^4}$$

$$= \frac{c^4k^8 + c^4k^6 + c^4k^4}{c^4k^4 + c^4k^2 + c^4}$$

$$= \frac{c^4k^4\left(k^4 + k^2 + 1\right)}{c^4\left(k^4 + k^2 + 1\right)}$$

$$= k^4$$

$$\text{R.H.S.} = \frac{a^2}{c^2}$$

$$= \frac{\left(ck^2\right)^2}{c^2}$$

$$= \frac{c^2k^4}{c^2}$$

$$= k^4$$

Hence, L.H.S. = R.H.S.

(b)

Given equation is $4x^2 - 5x - 3 = 0$.

Comparing with $ax^2 + bx + c = 0$, we get

$a = 4$, $b = -5$ and $c = -3$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(-3)}}{2 \times 4}$$

$$= \frac{5 \pm \sqrt{25 + 48}}{8}$$

$$= \frac{5 \pm \sqrt{73}}{8}$$

$$= \frac{5 \pm 8.54}{8}$$

$$= \frac{13.54}{8} \text{ or } \frac{-3.54}{8}$$

$$= 1.6925 \text{ or } -0.4425$$

$$= 1.69 \text{ or } -0.44$$

(c)

Join OA and OC.

Since the perpendicular from the centre of the circle to a chord bisects the chord.

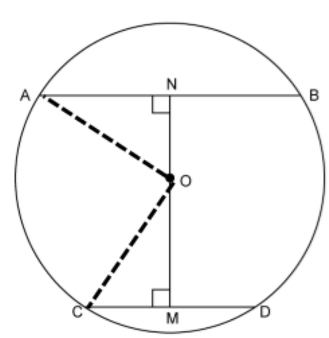
Therefore, N and M are the mid-points of AB and CD respectively.

Consequently,

$$AN = NB = \frac{1}{2}AB = \frac{1}{2} \times 24 = 12 \text{ cm and}$$

$$CM = MD = \frac{1}{2}CD = \frac{1}{2} \times 10 = 5 \text{ cm}$$

In right-angled triangles ANO and CMO, we have



$OA^2 = ON^2 + AN^2$ and $OC^2 = OM^2 + CM^2$

$\Rightarrow 13^2 = ON^2 + 12^2$ and $13^2 = OM^2 + 5^2$

$\Rightarrow ON^2 = 13^2 - 12^2$ and $OM^2 = 13^2 - 5^2$

$\Rightarrow ON^2 = 169 - 144$ and $OM^2 = 169 - 25$

$\Rightarrow ON^2 = 25$ and $OM^2 = 144$

$\Rightarrow ON = 5$ and $OM = 12$

Now, $NM = ON + OM = 5 + 12 = 17 \text{cm}$

Hence, the distance between the two chords is 17 cm.

2.

(a)

$$\sin^2 28° + \sin^2 62° + \tan^2 38° - \cot^2 52° + \frac{1}{4}\sec^2 30°$$

$$= \sin^2 28° + \sin^2(90° - 28°) + \tan^2 38° - \cot^2(90° - 38°) + \frac{1}{4}\sec^2 30°$$

$$= \left(\sin^2 28° + \cos^2 28°\right) + \tan^2 38° - \tan^2 38° + \frac{1}{4} \times \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= 1 + 0 + \frac{1}{4} \times \frac{4}{3}$$

$$= 1 + \frac{1}{3}$$

$$= \frac{4}{3}$$

(b)

$$\text{Given}: A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}, \ B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} \text{ and } A^2 - 5B^2 = 5C$$

$$\text{Now, } A^2 = A \times A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 1 + 3 \times 3 & 1 \times 3 + 3 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 3 + 4 \times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + 9 & 3 + 12 \\ 3 + 12 & 9 + 16 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix}$$

$$\text{And, } B^2 = B \times B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} \times \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \times (-2) + 1 \times (-3) & -2 \times 1 + 1 \times 2 \\ -3 \times (-2) + 2 \times (-3) & -3 \times 1 + 2 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 - 3 & -2 + 2 \\ 6 - 6 & -3 + 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{Now, } A^2 - 5B^2 = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix} = 5\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} = 5C$$

$$\text{Hence, } C = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$

(c)

For 1^{st} year:

$P = \text{Rs. } 50,000;\ R = 12\%$ and $T = 1$ year

$\therefore$ Interest $= \text{Rs. } \dfrac{50,000 \times 12 \times 1}{100} = \text{Rs. } 6,000$

And, Amount $= \text{Rs. } 50,000 + \text{Rs. } 6,000 = \text{Rs. } 56,000$

Since Money repaid $= \text{Rs. } 33,000$

$\therefore$ Balance $= \text{Rs. } 56,000 - \text{Rs. } 33,000 = \text{Rs. } 23,000$

For 2^{nd} year:

$P = \text{Rs. } 23,000;\ R = 15\%$ and $T = 1$ year

$\therefore$ Interest $= \text{Rs. } \dfrac{23,000 \times 15 \times 1}{100} = \text{Rs. } 3,450$

And, Amount $= \text{Rs. } 23,000 + \text{Rs. } 3,450 = \text{Rs. } 26,450$

Thus, Jaya must pay Rs. 26,450 at the end of 2nd year to clear her debt.

3.

(a)

List price $= \text{Rs.}42,000$

Discount $= 10\%$ of Rs. 42,000

$= \dfrac{10}{100} \times \text{Rs. } 42,000$

$= \text{Rs. } 4,200$

$\Rightarrow$ Discounted price $= \text{Rs. } 42,000 - \text{Rs. } 4,200 = \text{Rs. } 37,800$

Off-season discount $= 5\%$ of Rs. 37,800

$= \dfrac{5}{100} \times \text{Rs. } 37,800$

$= \text{Rs. } 1,890$

$\therefore$ Sale-price $= \text{Rs. } 37,800 - \text{Rs. } 1,890 = \text{Rs. } 35,910$

(i) The amount of sales tax a customer has to pay $= 8\%$ of Rs. 35,910

$= \dfrac{8}{100} \times \text{Rs. } 35,910$

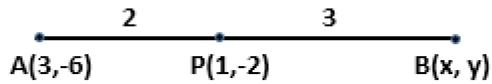
$= \text{Rs. } 2872.80$

(ii) The total price, a customer has to pay for the computer $=$ Sale-price $+$ Sales Tax

$= \text{Rs. } 35,910 + \text{Rs. } 2872.80$

$= \text{Rs. } 38782.80$

(b)



Given, P(1,−2), A(3,−6) and B(x,y)

$AP : PB = 2 : 3$

Hence, coordinates of $P = \left(\frac{2\times x+3\times 3}{2+3}, \frac{2\times y+3\times(-6)}{2+3}\right) = \left(\frac{2x+9}{5}, \frac{2y-18}{5}\right)$

But, the coordinates of P are $(1,-2)$.

$\therefore \frac{2x+9}{5} = 1 \quad \text{and} \quad \frac{2y-18}{5} = -2$

$\Rightarrow 2x+9 = 5 \quad \text{and} \quad 2y-18 = -10$

$\Rightarrow 2x = -4 \quad \text{and} \quad 2y = 8$

$\Rightarrow x = -2 \quad \text{and} \quad y = 4$

Hence, the coordinates of B are $(-2, 4)$.

(c)

Data in ascending order:

13, 35, 43, 46, x, x + 4, 55, 61, 71, 80

Median = 48

Number of observations = n = 10 (even)

$$\therefore \text{Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}}{2}$$

$$\Rightarrow 48 = \frac{\left(\frac{10}{2}\right)^{th} \text{term} + \left(\frac{10}{2}+1\right)^{th} \text{term}}{2}$$

$$\Rightarrow 48 = \frac{5^{th} \text{ term} + 6^{th} \text{term}}{2}$$

$$\Rightarrow 48 = \frac{x+x+4}{2}$$

$$\Rightarrow 48 = \frac{2x+4}{2}$$

$\Rightarrow 48 = x + 2$

$\Rightarrow x = 46$

$\Rightarrow x + 4 = 46 + 4 = 50$

Thus, the observations are 13, 35, 43, 46, 46, 50, 55, 61, 71, 80

Observation 46 is appearing twice.

Hence, the mode of the data is 46.

4.

(a)

Let the number to be subtracted from the given polynomial be k.

Let $f(y) = 16x^3 - 8x^2 + 4x + 7 - k$

It is given that $(2x+1)$ is a factor of $f(y)$.

$\therefore\ f\left(-\frac{1}{2}\right) = 0$

$\Rightarrow 16\left(-\frac{1}{2}\right)^3 - 8\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 7 - k = 0$

$\Rightarrow 16\times\left(-\frac{1}{8}\right) - 8\times\frac{1}{4} - 2 + 7 - k = 0$

$\Rightarrow -2 - 2 - 2 + 7 - k = 0$

$\Rightarrow 1 - k = 0$

$\Rightarrow k = 1$

Thus, 1 should be subtracted from the given polynomial.

(b)

Length of a rectangle = Radius of two semi-circles + Diameter of a circle

$= 5 + 5 + 10$

$= 20$ cm

Breadth of a rectangle = Diameter of a circle $= 2\times5 = 10$ cm

$\therefore$ Area of a rectangle = Length $\times$ Breadth

$= 20\times10$

$= 200$ sq. cm

Area of a circle $= \frac{22}{7}\times5\times5 = 78.571$ sq. cm

And, area of two semi-circles each of radius 5 cm $= 2\left(\frac{1}{2}\times78.571\right) = 78.571$ sq. cm

Now,

Area of shaded region = Area of a rectangle – Area of a circle – Area of two semi-circles

$= 200 - 78.571 - 78.571$

$= 200 - 157.142$

$= 42.858$ sq. cm

(c)

$$-8\frac{1}{2} < -\frac{1}{2} - 4x \le 7\frac{1}{2},\ x \in I$$

$$\Rightarrow -\frac{17}{2} < -\frac{1}{2} - 4x \le \frac{15}{2},\ x \in I$$

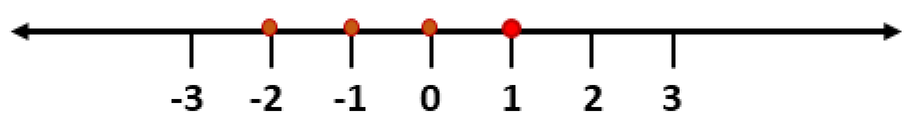
Take

$-\frac{17}{2} < -\frac{1}{2} - 4x$	$-\frac{1}{2} - 4x \le \frac{15}{2}$
$-\frac{17}{2} + \frac{1}{2} < -4x$	$-4x \le \frac{15}{2} + \frac{1}{2}$
$-\frac{16}{2} < -4x$	$-4x \le \frac{16}{2}$
$-8 < -4x$	$-4x \le 8$
$2 > x$	$x \ge -2$

Thus, on simplifying, the given inequation reduces to $-2 \le x < 2$.

Since $x \in I$, the solution set is $\{-2, -1, 0, 1\}$.

The required graph on number line is as follows:



SECTION B (40 Marks)

Attempt any four questions from this section

5.

(a)

$$\text{Given: } B=\begin{bmatrix}1 & 1\\ 8 & 3\end{bmatrix} \text{ and } X=B^2-4B$$

$$\text{Now, } B^2=B\times B$$

$$=\begin{bmatrix}1 & 1\\ 8 & 3\end{bmatrix}\times\begin{bmatrix}1 & 1\\ 8 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+1\times8 & 1\times1+1\times3\\ 8\times1+3\times8 & 8\times1+3\times3\end{bmatrix}$$

$$=\begin{bmatrix}1+8 & 1+3\\ 8+24 & 8+9\end{bmatrix}$$

$$=\begin{bmatrix}9 & 4\\ 32 & 17\end{bmatrix}$$

$$X=B^2-4B=\begin{bmatrix}9 & 4\\ 32 & 17\end{bmatrix}-4\begin{bmatrix}1 & 1\\ 8 & 3\end{bmatrix}=\begin{bmatrix}9 & 4\\ 32 & 17\end{bmatrix}-\begin{bmatrix}4 & 4\\ 32 & 12\end{bmatrix}=\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}$$

$$\text{Now, } X\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}5\\ 50\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}5\\ 50\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}5a+0b\\ 0a+5b\end{bmatrix}=\begin{bmatrix}5\\ 50\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}5a\\ 5b\end{bmatrix}=\begin{bmatrix}5\\ 50\end{bmatrix}$$

$$\Rightarrow 5a=5 \text{ and } 5b=50$$

$$\Rightarrow a=1 \text{ and } b=10$$

(b)

$$\text{Since Dividend on 1 share}=10\% \text{ of Rs. } 50=\frac{10}{100}\times\text{Rs. }50=\text{Rs. }5$$

$$\therefore \text{Number of shares bought}=\frac{\text{Total dividend}}{\text{Dividend on 1 share}}=\frac{\text{Rs. }450}{\text{Rs. }5}=90$$

$$\text{Since market value of each share}=\text{Rs. }60$$

$$\therefore \text{Sum invested by the man}=90\times\text{Rs. }60=\text{Rs. }5{,}400$$

$$\text{Percentage return}=\frac{\text{Total return}}{\text{Sum invested}}\times100\%=\frac{\text{Rs. }450}{\text{Rs. }5400}\times100\%=8.33\%=8\%$$

(c)

Outcomes: a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p

Total number of all possible outcomes = 16

(i) When the selected card has a vowel,

the possible outcomes are a, e, i, o.

Number of favourable outcomes = 4

$\therefore$ Required probability $= \frac{4}{16} = \frac{1}{4}$

(ii) When the selected card has a consonant,

Number of favourable outcomes $= 16 - 4 = 12$

$\therefore$ Required probability $= \frac{12}{16} = \frac{3}{4}$

(iii) When the selected card has none of the letters from the word median,

the possible outcomes are b, c, f, g, h, j, k, l, o, p.

Number of favourable outcomes = 10

$\therefore$ Required probability $= \frac{10}{16} = \frac{5}{8}$

6.

(a) **Steps of construction:**

(i) Draw line AC = 5 cm and $\angle CAB = 60^o$. Cut off AB = 7 cm. Join BC, ΔABC is the required triangle.

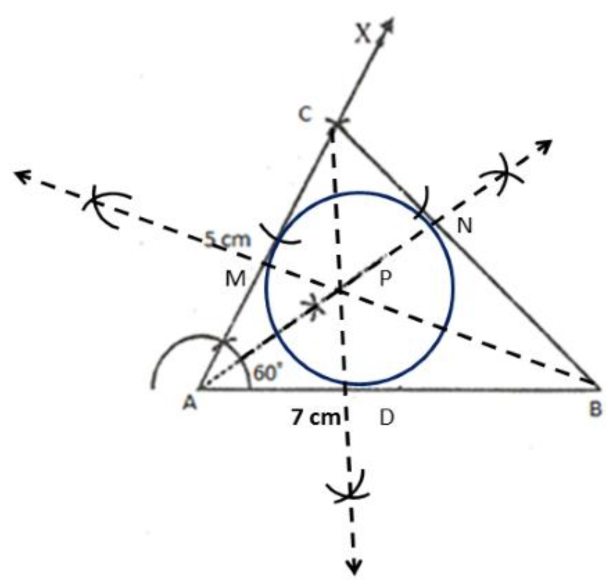
(ii) Draw angle bisectors of $\angle A$ and $\angle B$.

(iii) Bisector of $\angle B$ meets AC at M and bisector of $\angle A$ meets BC at N.

(iv) Similarly, draw the angle bisector of $\angle C$ which meets AB at D.

(v) P is the point which is equidistant from AB, BC and AC.

(vi) With DP as the radius, draw a circle touching the three sides of the triangle (incircle.)



(b)

Let h be the height and r be the radius of the base of the conical tent.

According to the given information,

$$77 \times 16 = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow 77 \times 16 = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times h$$

$$\Rightarrow 77 \times 16 = \frac{1}{3} \times 22 \times 7 \times h$$

$$\Rightarrow h = \frac{77 \times 16 \times 3}{22 \times 7} \Rightarrow h = 24 \text{ m}$$

Now, $l^2 = r^2 + h^2$

$$\Rightarrow l^2 = 7^2 + 24^2 = 625$$

$$\Rightarrow l = 25 \text{ m}$$

$$\therefore \text{Curved surface area} = \pi rl = \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

Hence, the height of the tent is 24 m and the curved surface area of the tent is 550 m^2.

(c)

(i) $$\frac{7m + 2n}{7m - 2n} = \frac{5}{3}$$

By Componendo – Divinendo, we get

$$\frac{7m + 2n + (7m - 2n)}{7m + 2n - (7m - 2n)} = \frac{5+3}{5-3}$$

$$\Rightarrow \frac{14m}{4n} = \frac{8}{2}$$

$$\Rightarrow \frac{7m}{2n} = \frac{4}{1}$$

$$\Rightarrow \frac{m}{n} = \frac{8}{7}$$

$$\Rightarrow m : n = 8 : 7$$

(ii) $$\frac{m}{n} = \frac{8}{7} \Rightarrow \frac{m^2}{n^2} = \frac{8^2}{7^2}$$

Applying Componendo – Divinendo, we get

$$\Rightarrow \frac{m^2 + n^2}{m^2 - n^2} = \frac{8^2 + 7^2}{8^2 - 7^2}$$

$$\Rightarrow \frac{m^2 + n^2}{m^2 - n^2} = \frac{64 + 49}{64 - 49}$$

$$\Rightarrow \frac{m^2 + n^2}{m^2 - n^2} = \frac{113}{15}$$

7.

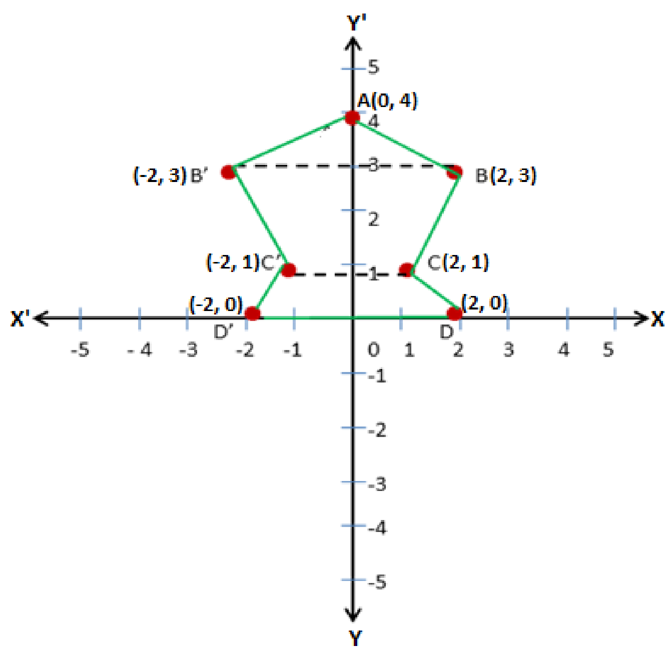
(a) Principal for the month of Jan = Rs. 5600
Principal for the month of Feb = Rs. 4100
Principal for the month of Mar = Rs. 4100
Principal for the month of Apr = Rs. 2000
Principal for the month of May = Rs. 8500
Principal for the month of June = Rs. 10000
Total Principal for one month = Rs. 34300

Rate of interest = 6% pa

(i) $\text{Simple interest} = \frac{PRT}{100} = \frac{34300 \times 6 \times 1}{100 \times 12} = \text{Rs.}171.50$

(ii) $\text{Total amount} = \text{Rs.}10000 + \text{Rs.}171.50 = \text{Rs.}10171.50$

(b)



The image of point (x, y) on Y-axis has the coordinates (−x, y).
Thus, we have
Coordinates of B' = (−2, 3)
Coordinates of C' = (−1, 1)
Coordinates of D' = (−2, 0)

Since, Y-axis is the line of symmetry of the figure formed, the equation of the line of symmetry is x = 0.

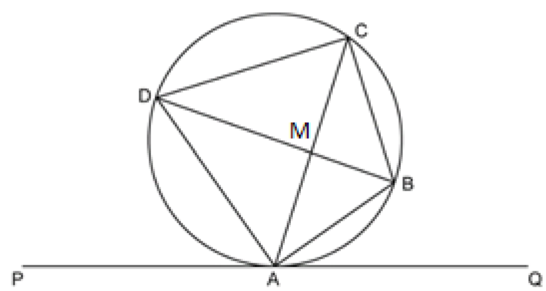
8.

(a) Let the assumed mean A = 25

Marks	Mid-value x	f	$d = x - A$	$t = \frac{x-A}{i} = \frac{x-25}{10}$	ft
$0-10$	5	10	-20	-2	-20
$10-20$	15	9	-10	-1	-9
$20-30$	25	25	0	0	0
$30-40$	35	30	10	1	30
$40-50$	45	16	20	2	32
$50-60$	55	10	30	3	30
		$\sum f = 100$			$\sum ft = 63$

$$\therefore \text{Mean} = A + \frac{\sum ft}{\sum f} \times i = 25 + \frac{63}{100} \times 10 = 25 + \frac{63}{10} = 25 + 6.3 = 31.3$$

(b)



(i) $\angle BAQ = 30°$

Since AB is the bisector of $\angle CAQ$

$\Rightarrow \angle CAB = \angle BAQ = 30°$

AD is the bisector of $\angle CAP$ and $P - A - Q$,

$\angle DAP + \angle CAD + \angle CAQ = 180°$

$\Rightarrow \angle CAD + \angle CAD + 60° = 180°$

$\Rightarrow \angle CAD = 60°$

So, $\angle CAD + \angle CAB = 60° + 30° = 90°$

Since angle in a semi-circle $= 90°$

$\Rightarrow$ Angle made by diameter to any point on the circle is 90°

So, BD is the diameter of the circle.

(ii) Since BD is the diameter of the circle, so it will pass through the centre.

By Alternate segment theorem,

$\angle ABD = \angle DAP = 60°$

So, in ΔBMA,

$\angle AMB = 90°$ ....(Use Angle Sum Property)

We know that perpendicular drawn from the centre to a chord of a circle bisects the chord.

$\Rightarrow \angle BMA = \angle BMC = 90°$

In ΔBMA and ΔBMC,

$\angle BMA = \angle BMC = 90°$

$BM = BM$ (common side)

$AM = CM$ (perpendicular drawn from the centre to a chord of a circle bisects the chord.)

$\Rightarrow \Delta BMA \cong \Delta BMC$

$\Rightarrow AB = BC$ (SAS congruence criterion)

$\Rightarrow \Delta ABC$ is an isosceles triangle.

(c)

(i) Printed price of an air conditioner = Rs. 45000

Discount = 10%

$$\therefore \text{C.P. of the air conditioner} = \text{Rs.}\ \frac{45000 \times (100 - 10)}{100}$$

$$= \text{Rs.}\ \frac{45000 \times 90}{100}$$

$$= \text{Rs.}\ 40500$$

$$\text{VAT } (12\%) = 40500 \times \frac{12}{100} = \text{Rs.} 4860$$

So, the shopkeeper paid VAT of Rs. 4860 to the government.

(ii) Discount = 5% of the marked price

$$\therefore \text{C.P. of the air conditioner} = \text{Rs.}\ \frac{45000 \times (100 - 5)}{100}$$

$$= \text{Rs.}\ \frac{45000 \times 95}{100}$$

$$= \text{Rs.}\ 42750$$

$$\text{VAT } (12\%) = 42750 \times \frac{12}{100} = \text{Rs.} 5130$$

So, the total amount paid by the customer inclusive of tax

$= \text{Rs.}\ 42750 + \text{Rs.} 5130$

$= \text{Rs.} 47880$

9.

(a)

(i) $\angle DAE = 70°$ ....(given)

$\angle BAD + \angle DAE = 180°$ ....(linear pair)

$\Rightarrow \angle BAD + 70° = 180°$

$\Rightarrow \angle BAD = 110°$

Since ABCD is a cyclic quadrilateral, sum of the measures of the opposite angles are supplementary.

So, $\angle BCD + \angle BAD = 180°$

$\Rightarrow \angle BCD + 110° = 180°$

$\Rightarrow \angle BCD = 70°$

(ii) $\angle BOD = 2\angle BCD$ (Inscribed angle theorem)

$\Rightarrow \angle BOD = 2(70°) = 140°$

(iii) In ΔOBD,

$OB = OD$ ....(radii of same circle)

$\Rightarrow \angle OBD = \angle ODB$

By Angle Sum property,

$\angle OBD + \angle ODB + \angle BOD = 180°$

$\Rightarrow 2\angle OBD + \angle BOD = 180°$

$\Rightarrow 2\angle OBD + 140° = 180°$

$\Rightarrow 2\angle OBD = 40°$

$\Rightarrow \angle OBD = 20°$

(b)

Given vertices: $A(-1, 3)$, $B(4, 2)$ and $C(3, -2)$

(i) Coordinates of the centroid G of ΔABC are given by

$$G = \left(\frac{-1+4+3}{3}, \frac{3+2-2}{3}\right) = \left(\frac{6}{3}, \frac{3}{3}\right) = (2, 1)$$

(ii) Since the line through G is parallel to AC, the slope of the lines are the same.

$$\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-3}{3-(-1)} = \frac{-5}{4}$$

So, equation of the line passing through $G(2, 1)$ and with slo pe $\frac{-5}{4}$ is given by,

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 1 = \frac{-5}{4}(x - 2)$

$\Rightarrow 4y - 4 = -5x + 10$

$\Rightarrow 5x + 4y = 14$ is the required equation.

(c)

$$\text{L.H.S.} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta\,[2(1-\sin^2\theta)-1]}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2-2\sin^2\theta-1)}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(1-2\sin^2\theta)}$$

$$= \tan\theta$$

$$= \text{R.H.S.} \quad \text{(proved)}$$

10.

(a)

Let Vivek's age be x years and Amit's age be (47-x) years.

According to the given information,

$x(47-x) = 550$

$\Rightarrow 47x - x^2 = 550$

$\Rightarrow x^2 - 47x + 550 = 0$

$\Rightarrow (x-25)(x-22) = 0$

$\Rightarrow x = 25 \text{ or } x = 22$

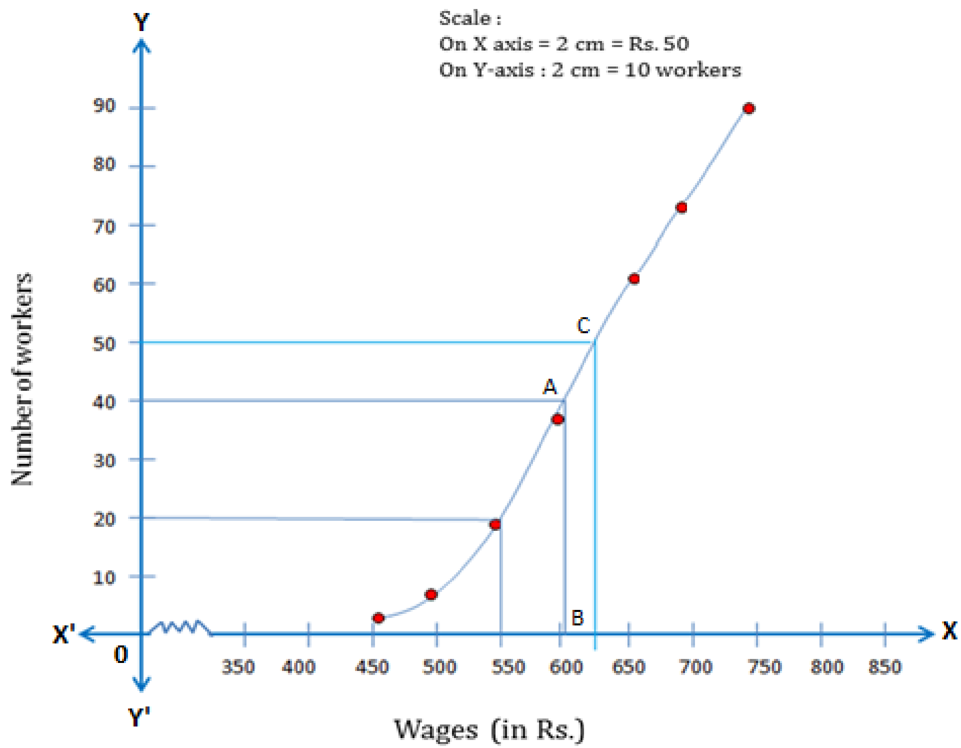
So, Vivek's age is 25 years and Amit's age is 22 years.

(b)

The cumulative frequency table of the given distribution is as follows:

Wages in Rs.	Upper Limit	No. of workers	Cumulative frequency
400-450	450	2	2
450-500	500	6	8
500-550	550	12	20
550-600	600	18	38
600-650	650	24	62
650-700	700	13	75
700-750	750	5	80

The ogive is as follows:



Number of workers = n = 80

(i) Median $= \left(\frac{n}{2}\right)^{th}$ term $= 40^{th}$ term

Through mark 40 on the Y-axis, draw a horizontal line which meets the curve at point A.

Through point A, on the curve draw a vertical line which meets the X-axis at point B.

The value of point B on the X-axis is the median, which is 605.

(ii) Lower quartile $(Q_1) = \left(\frac{80}{4}\right)^{th}$ term $= 20^{th}$ term $= 550$

(ii) Through mark of 625 on X-axis, draw a verticle line which meets the graph at point C.

Then through point C, draw a horizontal line which meets the Y-axis at the mark of 50.

Thus, number of workers that earn more than Rs. 625 daily $= 80 - 50 = 30$

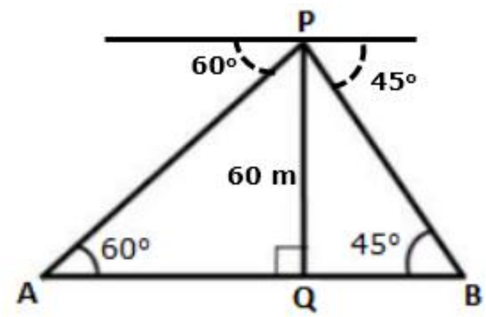
11.

(a)

Let PQ be the lighthouse.

$\Rightarrow PQ = 60$

In ΔPQA,



$\tan 60° = \frac{PQ}{AQ}$

$\Rightarrow \sqrt{3} = \frac{60}{AQ}$

$\Rightarrow AQ = \frac{60}{\sqrt{3}}$

$\Rightarrow AQ = \frac{20 \times 3}{\sqrt{3}}$

$\Rightarrow AQ = \frac{20 \times \sqrt{3} \times \sqrt{3}}{\sqrt{3}}$

$\Rightarrow AQ = 20\sqrt{3}$ m

In ΔPQB,

$\tan 45° = \frac{PQ}{QB}$

$\Rightarrow 1 = \frac{60}{QB}$

$\Rightarrow QB = 60$ m

Now,

$AB = AQ + QB$

$= 20\sqrt{3} + 60$

$= 20 \times 1.732 + 60$

$= 94.64$

$= 95$ m

(b)

(i) In ΔPQR and ΔSPR, we have

$\angle QPR = \angle PSR$ ....(given)

$\angle PRQ = \angle PRS$ ....(common)

So, by AA-axiom similarity, we have

$\Delta PQR \sim \Delta SPR$ ....(proved)

(ii) Since $\Delta PQR \sim \Delta SPR$ ....(proved)

$$\Rightarrow \frac{PQ}{SP} = \frac{QR}{PR} = \frac{PR}{SR}$$

Consider $\frac{QR}{PR} = \frac{PR}{SR}$ ....[From (1)]

$$\Rightarrow \frac{QR}{6} = \frac{6}{3}$$

$$\Rightarrow QR = \frac{6 \times 6}{3} = 12 \text{ cm}$$

Also, $\frac{PQ}{SP} = \frac{PR}{SR}$

$$\Rightarrow \frac{8}{SP} = \frac{6}{3}$$

$$\Rightarrow \frac{8}{SP} = 2$$

$$\Rightarrow SP = \frac{8}{2} = 4 \text{ cm}$$

(iii) $\frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta SPR} = \frac{PQ^2}{SP^2} = \frac{8^2}{4^2} = \frac{64}{16} = 4$

(c)

(i) Let the deposit per month = Rs. P

Number of months (n) = 36

Rate of interest (r) = 7.5% p.a.

$$\therefore S.I. = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow 8325 = P \times \frac{36 \times 37}{2 \times 12} \times \frac{7.5}{100}$$

$$\Rightarrow 8325 = P \times \frac{3 \times 37}{2} \times \frac{7.5}{100}$$

$$\Rightarrow P = \frac{8325 \times 2 \times 100}{3 \times 37 \times 7.5} = \text{Rs. } 2000$$

(ii) Maturity value $= P \times n + S.I. = \text{Rs.}(2000 \times 36 + 8325) = \text{Rs. } 80{,}325$