# ICSE Question Paper (2008) MATHEMATICS 

SECTION A [40 MARKS]<br>(Answer all questions from this Section.)

## Question 1.

(a) The simple interest on a sum of money for 2 years at $4 \%$ per annum is ₹ 340. Find:
(i) the sum of money and
(ii) the compound interest on this sum for one year payable half yearly at the same rate.
(b) If $\begin{aligned} & 8 a-5 b \\ & 8 c-5 d\end{aligned}=8 a+5 b$, prove that $\frac{a}{b}=\frac{c}{d}$
(c) If $(x-2)$ is a factor of $2 x^{3}-x^{2}-p x-2$
$\therefore$ (i) find the value of $p$.
(ii) with the value of $p$, factorize the above expression completely.

## Solution.

(a) (i) Given: S.I. $=$ ₹ 340, $\mathrm{T}=2$ Years, $\mathrm{R}=4 \%, \mathrm{P}=$ ?

$$
\begin{aligned}
\text { S.I. } & =\frac{P \times R \times T}{100} \\
P & =\frac{\text { S.I. } \times 100}{R \times T}=\frac{340 \times 100}{4 \times 2}
\end{aligned}
$$

$$
=₹ 4250
$$

Ans.
(ii) Given : $\mathrm{T}=1$ Years $=1 \times 2=2$ times, $\mathrm{R}=4 \%, \Rightarrow \mathrm{R}=2 \%$ (for half yearly.)

$$
\begin{aligned}
\text { C.I. } & =P\left[\left(1+\frac{R}{100}\right)^{T}-1\right] \\
& =4250\left[\left(1+\frac{2}{100}\right)^{2}-1\right] \\
& =4250\left[\frac{101}{2500}\right]=₹ 171 \cdot 70
\end{aligned}
$$

Ans.
(b) Given:

$$
\begin{array}{rl}
\frac{8 a+5 b}{8 c+5 d}= & 8 a-5 b \\
\frac{8 a+5 d}{8 a-5 b}= & 8 c+5 d \\
8 c-5 d \\
\frac{8 a+5 b+8 a-5 b}{8 a+5 b-8 a+5 b}= & 8 c+5 d+8 c-5 d \\
8 c+5 d-8 c+5 d \\
& \quad \text { (Apply componendo and dividendo rule) } \\
169 & 16 c \\
10 b & 10 d \\
\frac{a}{b}= & \frac{c}{d} \quad \text { (Apply alternendo) } \\
\text { Proved }
\end{array}
$$

(c) $(x-2)$ is a factor of $2 x^{3}-x^{2}-p x-2$
(i) $\Rightarrow x=2$ will satisfy this equation.

$$
\begin{array}{rlrl}
\Rightarrow & 2 \cdot 2^{3}-2^{2}-2 p-2 & =0 \\
\Rightarrow & 16-4-2 p-2 & =0 \\
& \Rightarrow & 10-2 p & =0 \\
\Rightarrow & & 2 p & =10 \\
\Rightarrow & & p & =5
\end{array}
$$

Ans.
(ii) On dividing $2 x^{9}-x^{2}-5 x-2$ by $x-2$,

$$
\text { x-2) } \begin{array}{r}
\frac{\frac{2 x^{2}+3 x+1}{2 x^{3}-x^{2}-5 x-2}}{\frac{2 x^{9}-4 x^{2}}{+}} \\
\frac{3 x^{2}-5 x-2}{\frac{3 x^{2}-6 x}{+}} \\
\frac{x-2}{x-2}
\end{array}
$$

we get

$$
\begin{array}{ll}
\Rightarrow & (x-2)\left(2 x^{2}+3 x+1\right) \\
\Rightarrow & (x-2)\left(2 x^{2}+2 x+x+1\right) \\
\Rightarrow & (x-2)(2 x(x+1)+1(x+1)) \\
\Rightarrow & (x-2)(x+1)(2 x+1)
\end{array}
$$

Ans.
Question 2.
(a) Solve the given inequation and graph the solution on the number line.

$$
2 y-3<y+1 \leq 4 y+7 ; y \in R .
$$

(b) In the given figure, find the area of the unshaded portion within the rectangle.
(Take $\pi=3 \cdot 14$ ) [3]

(c) A shopkeeper buys a camera at a discount of $20 \%$ from the wholesaler, the printed price of the camera being ₹ 1600 and the rate of sales tax is $6 \%$. The shopkeeper sells it to the buyer at the printed price and charges tax at the same rate. Find:
(i) The price at which the camera can be bought.
(ii) The VAT (Value Added Tax) paid by the shopkeeper.

Solution.
(a) Given:

Given: $\quad$|  | $2 y-3<y+1 \leq 4 y+7, y \in R$ |
| ---: | :--- |
|  | $2 y-3$ |
|  | $2 y-y<3+1$ |
| $y+1$ | $\leq 4 y+7$ |
| $4 y-y$ | $\geq 1-7$ |
| $3 y$ | $\geq-6$ |
| $y$ | $y<4$ |

solution set :
\{y. $\mid y \in R,-2 \leq y<4\}$

(b)

$$
\begin{aligned}
\text { Length of rectangle } & =15 \mathrm{~cm} \\
\text { Breadth of rectangle } & =6 \mathrm{~cm} . \\
\text { Area of rectangle } & =15 \times 6 \\
& =90 \mathrm{~cm}^{2}
\end{aligned}
$$

$$
\begin{aligned}
\text { Area of circle } & =\pi r^{2} \\
& =3 \cdot 14 \times 9 \\
& =28.26 \mathrm{~cm}^{2} . \\
\text { Area of shaded portion } & =\text { Area of } 2 \frac{1}{2} \text { circle } \\
& =28.26+28 \cdot 26+14 \cdot 13 \\
& =70.65 \mathrm{~cm}^{2} .
\end{aligned}
$$

Area of unshaded portion in the rectangle $=$ Area of the rectangle

- Area of shaded portion

$$
\begin{aligned}
& =90-70 \cdot 65 \\
& =19 \cdot 35 \mathrm{~cm}^{2}
\end{aligned}
$$

(c). (i) Cost of camera for buyer $=$ Printed Price + Sales tax on it

$$
\begin{aligned}
& =1600+\frac{6}{100} \times 1600 \\
& =₹ 1696
\end{aligned}
$$

Ans.
(ii) Discount on printed price $=\frac{20}{100} \times 1600=₹ 320$.

$$
\text { Cost Price of the camera }=1600-320=₹ 1280 .
$$

$$
\text { Sales tax }=\frac{6}{100} \times 1280=₹ 76.80
$$

Tax paid by shopkeeper $=\frac{6}{100} \times 1600=₹ 96$.
VAT paid by shopkeeper $=96-76.80$

$$
=₹ 19 \cdot 20
$$

Ans.

## Question 3.

(a) David opened a Recurring Deposit Account in a bank and deposited ₹ 300 per month for two years. If he received $\geqslant 7725$ at the time of maturity, find the rate of interest_per annum.
(b) If $\left[\begin{array}{rr}1 & 4 \\ -2 & 3\end{array}\right]+2 M=3\left[\begin{array}{rr}3 & 2 \\ 0 & -3\end{array}\right]$, find the Matrix $M$.

$$
\ldots,-1
$$

(c) Use a graph paper for this question. (Take $1 \mathrm{~cm}=1$ unit on both the axes). Plot the points $A(-2,0), B(4,0), C(1,4)$ and $D(-2,4)$.
(i) Draw the line of symmetry of $\triangle A B C$. Name it $L_{1}$.
(ii) Point $D$ is reflected about the Line $L_{1}$ to get the image $E$. Write the coordinates of $E$.
(iii) Name the figure $A B E D$.
(iv) Draw all the lines of symmetry of the figure $A B E D$.

Solution.
(a) Given: Deposited per month $(\mathrm{P})=₹ 300, n=2$ Year $=24$ months, Amount $=$ ₹ $7725, \mathrm{R}=$ ?

- Principal equivalent to 1 month $=P \times \frac{n(n+1)}{2}=\frac{300 \times 24 \times 25}{2}$

$$
=300 \times 12 \times 25=₹ 90,000
$$

$\mathrm{A}=\mathrm{P}+\mathrm{SI}$
$\Rightarrow \quad 7725=24 \times 300+\frac{90000 \times \mathrm{R} \times 1}{12 \times 100}$

$$
\begin{array}{rlrl}
\Rightarrow & 525 & =\frac{900 \times \mathrm{R}}{12} \\
\Rightarrow & \mathrm{R} & =\frac{525 \times 12}{900}=7 \% . \\
\text { (b) Given: } & {\left[\begin{array}{rr}
1 & 4 \\
-2 & 3
\end{array}\right]+2 \mathrm{M}} & \left.=3\left[\begin{array}{rr}
3 & 2 \\
0 & -3
\end{array}\right] \right\rvert\, \\
\Rightarrow & 2 \mathrm{M} & =\left[\begin{array}{rr}
9 & 6 \\
0 & -9
\end{array}\right]-\left[\begin{array}{rr}
1 & 4 \\
-2 & 3
\end{array}\right] \\
\Rightarrow & & =\left[\begin{array}{rr}
8 & 2 \\
2 & -12
\end{array}\right] \\
& & \mathrm{M} & =\left[\begin{array}{rr}
4 & 1 \\
1 & -6
\end{array}\right]
\end{array}
$$

Ans.

Ans.
(c) (i)

(ii) Co-ordinate of $E(4,4)$
(iii) ABED is rectangle.
(iv) See figure ( $L_{1}$ and $L_{2}$ are the line of symmetry of ractangle ABED)

## Question 4.

(a) Without using tables, evaluate $: \frac{\sin 25^{\circ}}{\sec 65^{\circ}}+\frac{\cos 25^{\circ}}{\operatorname{cosec} 65^{\circ}}$
(b) In the alongside figure, $A B$ is parallel to $D C, \angle B C E=80^{\circ}$ and $\angle B A C=25^{\circ}$. Find:
(i) $\angle \mathrm{CAD}$,
(ii) $\angle \mathrm{CBD}$,
(ii) $\angle \mathrm{ADC}$.
(c) Mr. Dhoni has an account in the Union Bank of India. The following entries are from his pass book :


| Date | Particulars | Withdrawals <br> (in ₹) | Deposits <br> (in ₹) | Balance <br> (in ₹) |
| :--- | :--- | :---: | :---: | :---: |
| Jan 3, 07 | B/F | - | - | - |
| Jan 16 | To Self | 640.00 | - | 2642.00 |
| March 5 | By Cash | - | 850.00 | 2002.00 |
| April 10 | To Self | 1130.00 | - | 1722.00 |
| April 25 | By Cheque | - | 650.00 | 2372.00 |
| June 15 | By Cash | 577.00 | - | 1795.00 |

Calculate the interest from January 2007 to June 2007 at the rate of $4 \%$ per annum.

## Solution.

(a) Given: $\quad \frac{\sin 25^{\circ}}{\sec 65^{\circ}}+\frac{\cos 25^{\circ}}{\operatorname{cosec} 65^{\circ}}$
$=\sin 25^{\circ} \cos 65^{\circ}+\cos 25^{\circ} \sin 65^{\circ}$
$=\sin 25^{\circ} \cos \left(90^{\circ}-25^{\circ}\right)+\cos 25^{\circ} \sin \left(90^{\circ}-25^{\circ}\right)$
$=\sin 25^{\circ} \sin 25^{\circ}+\cos 25^{\circ} \cos 25^{\circ}$
$=\sin ^{2} 25^{\circ}+\cos ^{2} 25^{\circ}=1 \quad\left(\because \sin ^{2} \theta+\cos ^{2} \theta=1\right)$ Ans.
(b) (i) $\quad \angle \mathrm{BCE}=\angle \mathrm{BAD}=80^{\circ}$
(.. ext. of cyclic quad. is equal to opp. int. angle.)

$$
\angle \mathrm{CAD}=\angle \mathrm{BAD}-\angle \mathrm{BAC}=80^{\circ}-25^{\circ}=55^{\circ}
$$

Ans.
(ii)

$$
\begin{aligned}
\angle \mathrm{CBD} & =\angle \mathrm{CAD} \\
& =55^{\circ}
\end{aligned}
$$

( $\because$ Angle of the same segment are equal.)
Ans.
(iii)
$A B \| D C$ (given)

$$
\begin{aligned}
\angle \mathrm{BAD}+\angle \mathrm{ADC} & =180^{\circ} \\
\Rightarrow \quad(25+55)+\angle \mathrm{ADC} & =180^{\circ} \\
& =180-80=100^{\circ}
\end{aligned}
$$

( $\because \mathrm{ABCD}$ is cyclic quad.)
Ans.
(c) Minimum Balance for January = ₹ 2,002

Minimum Balance for February $=\boldsymbol{₹} \quad 2,002$
Minimum Balance for March $=$ ₹ 2,852
Minimum Balance for April $=\boldsymbol{₹} \quad 1,722$
Minimum Balance for May $=$ ₹ 2,372
Minimum Balance for June $=\boldsymbol{₹} \quad 1,795$
Total $=$ ₹ 12,745

$$
\begin{aligned}
\text { S.I. } & =\frac{P \times R \times T}{100} \\
& =\frac{12745 \times 4 \times 1}{100 \times 12}
\end{aligned}
$$

$$
=₹ 42 \cdot 48
$$

Ans.

## SECTION B [40 Marks]

## Answer any Four Questions in this Section.

## Question 5.

(a) A function in $x$ is defined as:*

$$
f(x)=\frac{x+2}{2 x-1} ; x \in R \text { and } x \neq \frac{1}{2},
$$

Find: (i) $\quad f(-3)$,
(ii) $f(x-1)$,
(iii) $x$ if $f(x)=1$.
(b) Prove the identity: $\frac{\sin A}{1+\cos A}=\operatorname{cosec} A-\cot A$.
(c) If $A=(-4,3)$ and $B=(8,-6)$
(i) Find the length of $A B$.
(ii) In what ratio is the line joining $A B$, divided by the $x$-axis?

Solution.
(b)

$$
\text { L.H.S. }=\frac{\sin \mathrm{A}}{1+\cos \mathrm{A}}
$$

$$
=\frac{\sin A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}
$$

$$
=\frac{\sin A(1-\cos A)}{1-\cos ^{2} A}
$$

$$
=\frac{\sin A(1-\cos A)}{\sin ^{2} A}
$$

$$
\Rightarrow \quad=\frac{1}{\sin A}-\frac{\cos A}{\sin A}
$$

$$
=\operatorname{cosec} A-\cot A=\text { R.H.S. }
$$

Proved
(c) (i) Given: $\mathrm{A}=(-4,3), \mathrm{B}=(8,-6)$

$$
\begin{aligned}
\mathrm{AB} & =\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \\
& =\sqrt{(8+4)^{2}+(-6-3)^{2}} \\
& =\sqrt{144+81} \\
& =\sqrt{225}=15 .
\end{aligned}
$$

(ii) Let any point on $x$-axis will be $\mathrm{P}(x, 0)$.

$$
\begin{aligned}
x & =\frac{m x_{1}+n x_{2}}{m+n}, \\
y & =\frac{m y_{1}+n y_{2}}{m+n} \\
0 & =\frac{m \cdot 3+n \cdot(-6)}{m+n} \\
\Rightarrow \quad 3 m & =6 n \\
\Rightarrow \quad \frac{m}{n} & =\frac{6}{3}=\frac{2}{1}
\end{aligned}
$$

The ratio will be $2: 1$.

## Question 6.

(a) Solve the following quadratic equation for $x$ and give your answer correct to two decimal places:

$$
5 x(x+2)=3
$$

[3]
(b) In the figure given alongside $P Q=Q R, \angle$ $R Q P=68^{\circ}, P C$ and $C Q$ are tangents to the circle with centre O. Calculate the values of: (i) $\angle Q O P$,
(ii) $\angle Q C P$.
[3]

(c) A company with 4000 shares of nominal value of $₹ 110$ each declares an annual dividend of $15 \%$. Calculate :
(i) The total amount of dividend paid by the company.
(ii) The annual income of Shah Rukh who holds 88 shares in the company.
(iii) If he received only $10 \%$ on his investment, find the price Shah Rukh paid for each share.

## Solution.

(a) Given:

$$
\begin{array}{r}
5 x(x+2)=3 \\
5 x^{2}+10 x-3=0
\end{array}
$$

Camparing with $a x^{2}+b x+c=0$, we get $a=5, b=10$ and $c=-3$.
We know that

$$
\begin{aligned}
x & =\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \\
& =\frac{-10 \pm \sqrt{(10)^{2}-4 \times 5 \times(-3)}}{2 \times 5} \\
& =\frac{-10 \pm \sqrt{100+60}}{10} \\
& =\frac{-10 \pm 4 \sqrt{10}}{10}=\frac{-5 \pm 2 \sqrt{10}}{5} \\
& =\frac{-5 \pm 2 \times 3.16-5 \pm 6.32}{5}
\end{aligned}
$$

Hence

$$
x=0.26 \text { or }-2.26
$$

Ans.
(b) Given: $\mathrm{PQ}=\mathrm{QR}, \angle \mathrm{RQP}=68^{\circ}$
(i) $\ln \triangle \mathrm{PQR}$,
$P Q=R Q$
$\therefore \quad \angle \mathrm{PRQ}=\angle \mathrm{QPR}$
$\Rightarrow \quad \angle \mathrm{PRQ}+\angle \mathrm{QPR}+68^{\circ}=180^{\circ} \quad$ [sum of the angle of a $\triangle$ is $180^{\circ}$ ]
$\Rightarrow \quad 2 \angle \mathrm{PRQ}=180-68=112^{\circ}$
$\Rightarrow \quad \angle \mathrm{PRQ}=56^{\circ}$
$\therefore \quad \angle \mathrm{QOP}=2 \times \angle \mathrm{PRQ}$
[angle at centre of the circle is twice the angle of at the remaining circumference]

$$
=2 \times 56=112^{\circ}
$$

Ans.
(ii)

$$
\begin{aligned}
& \angle \mathrm{PQC}=\angle \mathrm{PRQ} \\
& \angle \mathrm{QPC}=\angle \mathrm{PRQ} \\
& \Rightarrow \quad \angle \mathrm{PQC}=\angle \mathrm{QPC}=56^{\circ} \\
& \Rightarrow \quad \angle \mathrm{PQC}+\angle \mathrm{QPC}+\angle \mathrm{PCQ}=180^{\circ} \\
& \Rightarrow \quad 56^{\circ}+56^{\circ}+\angle \mathrm{PCQ}=180^{\circ} \\
& \Rightarrow \quad \angle \mathrm{QCP}=180^{\circ}-56^{\circ}-56^{\circ} \\
& =68^{\circ}
\end{aligned}
$$

Ans.
(c) Given: No. of shares $=4000, \mathrm{NV}=₹ 110$, Dividend $=15 \%$
(i)

$$
\text { (i) } \begin{aligned}
& \text { Dividend on } 1 \text { share }=\frac{15}{100} \times 110 \\
&=₹ 16 \cdot 50 \\
& \text { Total dividend }=4000 \times 16 \cdot 50 \\
&=₹ 66000 \\
& \text { (ii) } \quad=88 \times 16.50 \\
&=₹ 1,452 \\
& \text { (iii) } \quad \begin{aligned}
\text { Income on } 88 \text { shares } & \\
& =\{1,452 . \\
\text { Annual income of Shah Rukh } & =₹ \\
\text { Let his investment } & =x \\
\frac{10}{100} \times x & =₹ 1452 \\
x & =₹ 14520 \\
\therefore \quad & \text { Price for each share }
\end{aligned}=\frac{14520}{88}=₹ 165 .
\end{aligned}
$$

Ans.
(ii)

Ans.

Ans.

## Question 7.

(a) The income of Mr. Bachhan was as follows : **

- Basic Salary
: ₹ 20,000 per month
- Dearness Allowance : ₹ 12,000 per month
- Interest from Bank
: ₹ 16,000 for the whole year.

Savings:

- Contribution towards Provident Fund: 15\% of Basic salary
- National Savings Certificate : ₹40,000
- Contribution towards LIC premium : श30,000 per year Donations
- To National Defence Fund : ₹ 12,000 (eligible for $100 \%$ exemption)

If a sum of $₹ 3,000$ was deducted every month towards Income tax from his salary for the first 11 months of the year, calculate the tax Mr. Bachhan has to pay in the last month of the financial year:
Tax slab:
Upto ₹ $1,00,000$ : No tax.
From ₹ $1,00,001$ to $₹ 1,50,000$ : $10 \%$ of the income exceeding $₹ 1,00,000$
From ₹ $1,50,001$ to $₹ 2,50,000$ : $₹ 5,000+20 \%$ of the income exceeding ₹ $1,50,000$
Above ₹ $2,50,000 \quad: ~ ₹ ~ 25,000+30 \%$ of the income exceeding ₹ $2,50,000$
Deductions against savings : Upto a maximum amount of $₹ 1,00,000$
Education Cess . : $2 \%$ of the tax payable.
**. Solution has not given due to out of present syllabus.
(b) A vertical pole and a vertical tower are on the same level ground. From the top of the pole the angle of elevation of the top of the tower is $60^{\circ}$ and the angle of depression of the foot of the tower is $30^{\circ}$. Find the height of the tower if the height of the pole is 20 m .
Solution.
(b) Let $h$ be the height of tower and $x$ be the distance between tower and pole.

In rt. $\triangle \mathrm{BCE}, \quad \tan 30^{\circ}=\frac{20}{x}$

$$
\begin{array}{lrl}
\Rightarrow & \frac{1}{\sqrt{3}} & =\frac{20}{x} \\
\Rightarrow & x & =20 \sqrt{3} \mathrm{~m} \\
\text { In rt. } \triangle \text { ECA, } & \tan 60^{\circ} & =\frac{h-20}{x} \\
\Rightarrow & \frac{\sqrt{3}}{1} & =\frac{h-20}{x} \\
\Rightarrow & h-20 & =x \sqrt{3} \\
\Rightarrow & h-20 & =20 \sqrt{3} \times \sqrt{3}=60 \mathrm{~m} \\
\Rightarrow & h & =80 \mathrm{~m} . \quad \text { Ans. }
\end{array}
$$



## Question 8.

(a) Find the H.C.F. of the given polynomials : **

$$
\begin{equation*}
x^{2}-\frac{1}{a^{2}} \text { and } x^{2}+\frac{2 x}{a}+\frac{1}{a^{2}} \tag{3}
\end{equation*}
$$

(b) Using a ruler and a pair of compasses only, construct:
(i) A triangle $A B C$, given $A B=4 \mathrm{~cm}, B C=6 \mathrm{~cm}$ and $\angle A B C=90^{\circ}$.
(ii) A circle which passes through the points $A, B$ and $C$ and mark its centre as 0 .
(c) Points $A$ and $B$ have coordinates $(7,-3)$ and $(1,9)$ respectively. Find 子'
(i) The slope of $A B$.
(ii) The equation of the perpendicular bisector of the line segment $A B$.
(iii) The value of ' $p$ ' if $(-2, p)$ lies on it.

## Solution.

(b) Steps of Construction :
(1) Draw side BAC, 6 cm .
(2) Draw a $\angle B=90^{\circ}$, and cut $A B=4 \mathrm{~cm}$.
(3) Meet AC.
(4) Draw Bisector of $B C$ and $A B$. Which meet at point ' $O$ '.
(5) Now draw a circle as centre ' $O$ '.
(6) This circle passes through the point A, B, and ' $C$ '.

** Solution has not given due to out of present syllabus.
(c) (i) Slope of $\mathrm{AB}\left(m_{1}\right)=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$

$$
=\frac{9+3}{1-7}=\frac{12}{-6}=-2
$$

(ii) The mid point of line $\mathrm{AB}=\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)$


$$
=\left(\frac{7+1}{2}, \frac{-3+9}{2}\right)=(4,3)
$$

Slope of perpendicular bisector of AB

$$
m_{2}=\frac{-1}{m_{1}}=\frac{-1}{-2}=\frac{1}{2}
$$

Equation of perpendicular bisector

$$
\begin{array}{rlrl} 
& & y-y_{2} & =m\left(x-x_{1}\right) \\
\Rightarrow & y-3 & =\frac{1}{2}(x-4) \\
\Rightarrow & 2 y-6 & =x-4 \\
\Rightarrow & x-2 y+2 & =0
\end{array}
$$

(iii) Given point $(-2, p)$ lies on the equation $x-2 y+2=0$

$$
\begin{array}{rlrl} 
& & -2-2 p+2 & =0 \\
\Rightarrow & -2 p & =0 \\
\Rightarrow & p & =0
\end{array}
$$

Ans.

## Question 9.

(a) Given $A=\left[\begin{array}{ll}p & 0 \\ 0 & 2\end{array}\right], B=\left[\begin{array}{rr}0 & -q \\ 1 & 0\end{array}\right], C=\left[\begin{array}{rr}2 & -2 \\ 2 & 2\end{array}\right]$ and $B A=C^{2}$.

Find the values of $p$ and $q$.
(b) In $\triangle A B C, A P: P B=2: 3$. $P O$ is parallel to $B C$ and is extended to $Q$ so that $C Q$ is parallel to BA. Find :
(i) Area $\triangle A P O$ : area $\triangle A B C$.
(ii) Area $\triangle A P O$ : area $\triangle C Q O$.
(c) The volume of a conical tent is $1232 \mathrm{~m}^{3}$ and the area of the bare floor is $154 \mathrm{~m}^{2}$ : Calculate the:
(i) Radius of the floor.
(ii) Height of the tent.
(iii) Length of the canvas required to cover this conical tent if its width is 2 m .

Solution.
(a) Given: $\mathrm{A}=\left[\begin{array}{ll}p & 0 \\ 0 & 2\end{array}\right], \mathrm{B}=\left[\begin{array}{rr}0 & -q \\ 1 & 0\end{array}\right], \mathrm{C}=\left[\begin{array}{rr}2 & -2 \\ 2 & 2\end{array}\right]$

$$
\mathrm{BA}=\mathrm{C}^{2}
$$

$$
\left.\begin{array}{l}
\Rightarrow \quad\left[\begin{array}{rr}
0 & -q \\
1 & 0
\end{array}\right]\left[\begin{array}{ll}
p & 0 \\
0 & 2
\end{array}\right]
\end{array}\right]=\left[\begin{array}{rr}
2 & -2 \\
2 & 2
\end{array}\right]\left[\begin{array}{rr}
2 & -2 \\
2 & 2
\end{array}\right] .
$$

(b) (i)

$$
\frac{\mathrm{AP}}{\mathrm{~PB}}=\frac{2}{3}=\frac{\mathrm{AO}}{\mathrm{OC}}
$$

$$
\frac{\mathrm{AP}}{\mathrm{AP}+\mathrm{PB}}=\frac{2}{2+3}
$$

$$
\Rightarrow \quad \frac{\mathrm{AP}}{\mathrm{AB}}=\frac{2}{5}
$$

PO is parallel to BC and CQ is parallel to BA .
So, PBCQ is a parallelogram.

$$
\begin{array}{ll}
\Rightarrow & \mathrm{PB}=\mathrm{CQ} \\
\Rightarrow & \frac{\mathrm{AP}}{\mathrm{~PB}}=\frac{2}{3}=\frac{\mathrm{AP}}{\mathrm{CQ}}
\end{array}
$$

In $\triangle A P O$ and $\triangle A B C$,

$$
\begin{array}{rlrl}
\angle \mathrm{APO} & =\angle \mathrm{ABC} \\
\angle \mathrm{~A} & =\angle \mathrm{A} \\
\Rightarrow & & \triangle \mathrm{APO} & \sim \triangle \mathrm{ABC} \\
& & \frac{\mathrm{AP}}{\mathrm{AB}}=\frac{\mathrm{AO}}{\mathrm{AC}} & =\frac{2}{5} \\
\Rightarrow \quad & \frac{\text { area of } \triangle \mathrm{APO}}{\text { area of } \triangle \mathrm{ABC}} & =\frac{\mathrm{AP}^{2}}{\mathrm{AB}^{2}}
\end{array}
$$

[Ratio between the areas of two similar $\Delta$ 's is equal to the ratio between the squares of their corresponding sides]

$$
=\left(\frac{2}{5}\right)^{2}=\frac{4}{25}
$$

Ans.
(ii) In $\triangle A P O$ and $\triangle C Q O, \quad \angle A O P=\angle C O Q$
(vertically opp.)
(By A.A axiom)

$$
\begin{array}{ll}
\Rightarrow & \triangle \mathrm{AOP} \sim \triangle \mathrm{COQ} \\
\text { So } & \frac{\text { area of } \triangle \mathrm{APO}}{\text { area of } \triangle \mathrm{CQO}}=\frac{\mathrm{AP}^{2}}{\mathrm{CQ}^{2}}=\left(\frac{2}{3}\right)^{2}=\frac{4}{9}
\end{array}
$$

Ans.
(c) Given: Volume $(\mathrm{V})=1232 \mathrm{~m}$, Area of base $=154 \mathrm{~m}^{2}$

Let $r$ be the radius and $h$ be the height.
(i)

$$
\begin{aligned}
\text { Area }=\pi r^{2} & =154 \\
\frac{22}{7} r^{2} & =154 \\
r^{2} & =49 \quad \Rightarrow \quad r=7 \mathrm{~m}
\end{aligned}
$$

Ans.
(ii) Volume $(\mathrm{V})=\frac{1}{3} \pi r^{2} h=1232$

$$
\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times(7)^{2} \times h=1232
$$

$$
h=\frac{1232 \times 3}{22 \times 7}=24 \mathrm{~m} .
$$

Ans.
(iii) Let $l$ be the slant height of the conical tent, then

$$
l=\sqrt{ } h^{2}+r^{2}=\sqrt{24^{2}+7^{2}}=25 \mathrm{~m}
$$

Area of canvas required $=$ Curved surface area of the tent $\Rightarrow$ Length $\times$ width of canvas $=\pi l$ (for tent)
$\Rightarrow \quad$ Length of canvas $\times 2=\frac{22}{7} \times 7 \times 25$

$$
\text { Length of canvas }=\frac{550}{2}=275 \mathrm{~m} .
$$

Ans.

## Question 10.

(a) In the given figure, $A E$ and $B C$ intersect each other at point D. If $\angle C D E=90^{\circ}$, $A B=5 \mathrm{~cm}, B D=4 \mathrm{~cm}$ and $C D=9 \mathrm{~cm}$, find $D E$.


0

(b) A straight line $A B$ is 8 cm long. Locate by construction the locus of a point. which is :
(i) Equidistant from $A$ and $B$.
(ii) Always 4 cm from the line $A B$.
(iii) Mark two points $X$ and $Y$, which are 4 cm from $A B$ and equidistant from $A$ and $B$, Name the figure $A X B Y$.
(c) Some students planned a picnic. The budget for the food was $₹ 480$. As eight of them failed to join the party, the cost of the food for each member increased by $₹$ 10. Find how many students went for the picnic.

## Solution.

(a) Given: $\angle \mathrm{CDE}=90^{\circ}, \mathrm{AB}=5 \mathrm{~cm}, \mathrm{BD}=4 \mathrm{~cm}, \mathrm{CD}=9 \mathrm{~cm}, \mathrm{DE}=$ ?

$$
\mathrm{AD}=\sqrt{ } \mathrm{AB}^{2}-\mathrm{DB}^{2}=\sqrt{25}-16=3
$$

$\mathrm{DA} \times \mathrm{DE}=\mathrm{DB} \times \mathrm{DC}$ (Product of the length of their segment is equal.)

$$
\Rightarrow \quad 3 \times \mathrm{DE}=4 \times 9
$$

$$
\Rightarrow \quad D E=12 \mathrm{~cm}
$$

(b) Steps of Construction :
(i) Draw AB is 8 cm , Draw PQ the perpen-dicular bisector of $A B$.
(ii) Draw CD and EF both parallel to AB and each at a distance of 4 cm from AB .
(iii) AXBX is a square.

(c) Let the no. of students planned a picnic $=x$.

$$
\begin{aligned}
\text { Budget for food } & =₹ 480 \\
\text { Budget for each student } & =\frac{480}{x}
\end{aligned}
$$

If eight student failed to join the party, the cost for each student $=\frac{480}{x-8}$

$$
\begin{array}{rlrl}
\frac{480}{x-8}-\frac{480}{x} & =10 \\
& & \frac{480 x-480(x-8)}{x(x-8)} & =10 \\
\Rightarrow & & \frac{480 x-480 x+3840}{x(x-8)} & =10 \\
\Rightarrow & x^{2}-8 x-384 & =0 \\
\Rightarrow & x(x-24)+16(x-24) & =0 \\
\Rightarrow & x-24)(x+16) & =0 \\
\Rightarrow & x & =24,-16
\end{array}
$$

But the number of students can not be negative.
$\because \quad x=24$
$\therefore$ The number of student who went for picnic $=x-8=24-8=16$
Ans.

## Question 11.

(a) The weight of 50 apples were recorded as given below. Calculate the mean weight, to the nearest gram, by the Step Deviation Method.
[5]

| Weight in grams | $80-85$ | $85-90$ | $90-95$ | $95-100$ | $100-105$ | $105-110$ | $110-115$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of apples | 5 | 8 | 10 | 12 | 8 | 4 | $3 /$ |

(b) Using a graph paper, draw an ogive for the following distribution which shows the marks obtained in the General Knowledge paper by 100 students.

| Marks | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of students | 5 | 10 | 20 | 25 | 15 | 12 | 9 | 4 |

Use the ogive to estimate :
(i) The median.
(ii) The number of students who score marks above 65.

Solution.
(a)

| Weight | No. of apples <br> $\left(f_{i}\right)$ | Mid Value <br> $(x)$ | $u_{i}=\frac{x-A}{C}$ | $f_{i} u_{i}$ |
| :---: | :---: | :---: | :---: | :---: |
| $80-85$ | 5 | 82.5 | -3 | -15 |
| $85-90$ | 8 | 87.5 | -2 | -16 |
| $90-95$ | 10 | 92.5 | -1 | -10 |
| $95-100$ | 12 | 97.5 | 0 | 0 |
| $100-105$ | 8 | 102.5 | 1 | 8 |
| $105-110$ | 4 | 107.5 | 2 | 8 |
| $110-115$ | 3 | 112.5 | 3 | 9 |
|  | $\Sigma f_{i}=50$ |  |  | $\Sigma f_{i} u_{i}=-16$ |

Taking assumed mean $(\mathrm{A})=97 \cdot 5, \mathrm{C}=5$

$=97 \cdot 5-1 \cdot 6=95 \cdot 9=96$
Ans.
(b)

| Marks | No. of stiudents | c.f. |
| :---: | :---: | :---: |
| $0-10$ | 5 | 5 |
| $10-20$ | 10 | 15 |
| $20-30$ | 20 | 35 |
| $30-40$ | 25 | 60 |
| $40-50$ | 15 | 75 |
| $50-60$ | 12 | 87 |
| $60-70$ | 9 | 96 |
| $70-80$ | 4 | 100 |
|  | 100 |  |



Here $n=100$ which is even.
(i)

$$
\text { Median }=\left(\frac{n}{2}\right)^{\text {th }} \text { term }=50^{\text {th }} \text { term }
$$

From the graph, we get

$$
\text { The required median }=36
$$

(ii) The number of student who score above $65=100-92=8$.

