ICSE Paper Question (2009)

MATHEMATICS

SECTION A   [40 Marks]

(Answer all questions from this Section.)

Question 1.

(a) *Mr. Dubey borrows ₹ 1,00,000 from State Bank of India at 11% per annum compound interest. He repays ₹ 41,000 at the end of the first year and ₹ 47,700 at the end of the second year. Find the amount outstanding at the beginning of the third year.* **[3]**

(b) *A dice is thrown once. What is the probability that the*
   (i)   *number is even*
   (ii) *number is greater than 2 ?* **[3]**

(c) *Find the HCF and LCM of the following polynomials :* **
$$3x^3 - 27x^2 + 60x \quad and \quad x^2 - 16$$ **[4]**

Solution :

(a) *Given* : P = ₹ 1,00,000, R = 11%

$$\text{Interest for first year} = \frac{PRT}{100} = \frac{1,00,000 \times 11 \times 1}{100} = ₹\ 11,000$$

$$\text{Amount after first year} = 1,00,000 + 11,000 = ₹\ 1,11,000$$

$$\text{Principal for second year} = ₹\ 1,11,000 - ₹\ 41,000$$
$$= ₹\ 70,000$$

$$\text{Interest for second year} = \frac{70,000 \times 11 \times 1}{100} = ₹\ 7,700$$

$$\text{Amount after second year} = ₹\ 70,000 + 7,700 = 77,700$$

Amount outstanding for beginning of third year
$$= ₹\ 77,700 - ₹\ 47,700$$
$$= ₹\ 30,000. \qquad \textbf{Ans.}$$

(b) Dice is thrown once.

$$\text{Sample space} = \{1, 2, 3, 4, 5, 6\}$$
$$\therefore \qquad n\,(S) = 6$$

(i) $$\text{Number is even} = \{2, 4, 6\}$$
$$\therefore \qquad n\,(E) = 3$$
$$P\,(\text{Even number}) = \frac{n\,(E)}{n\,(S)} = \frac{3}{6} = 1/2 \qquad \textbf{Ans.}$$

(ii) $$\text{Number is greater than 2} = \{3, 4, 5, 6\}$$
$$n\,(E) = 4$$
$$P\,(> 2) = \frac{n\,(E)}{n\,(S)} = \frac{4}{6} = \frac{2}{3} \qquad \textbf{Ans.}$$

** *Solution has not given due to out of present syllabus.*

Question 2.

(a) *Find x and y, if* $\begin{bmatrix} 2x & x \\ y & 3y \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$ [3]

(b) *What least number must be added to each of the numbers 5, 11, 19 and 37 so that they are in proportion ?* [3]

(c) *Given that x + 2 and x + 3 are factors of $2x^3 + ax^2 + 7x - b$. Determine the values of a and b.* [4]

Solution :

(a) *Given :* $\begin{bmatrix} 2x & x \\ y & 3y \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$

$$\begin{bmatrix} 6x & + & 2x \\ 3y & + & 6y \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 8x \\ 9y \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

$$\left.\begin{array}{l} 8x = 16 \Rightarrow x = 2 \\ 9y = 9 \Rightarrow y = 1 \end{array}\right]$$ **Ans.**

(b) Let the number x be added to each number.

$$5 + x : 11 + x = 19 + x : 37 + x$$

$$\frac{5 + x}{11 + x} = \frac{19 + x}{37 + x}$$

By componendo and dividendo,

$$\Rightarrow \qquad \frac{5 + x + 11 + x}{5 + x - 11 - x} = \frac{19 + x + 37 + x}{19 + x - 37 - x}$$

$$\frac{16 + 2x}{-6} = \frac{56 + 2x}{-18}$$

$$\Rightarrow \qquad 3(16 + 2x) = 56 + 2x$$

$$48 + 6x = 56 + 2x$$

$$\Rightarrow \qquad 4x = 8$$

$$\Rightarrow \qquad x = 2 \qquad \textbf{Ans.}$$

(c) Given : $(x + 2)$ and $(x + 3)$ are the factors of $2x^3 + 9x^2 + 7x - b$.

$\therefore f(-2)$ and $f(-3)$ will be zero.

$$f(x) = 2x^3 + ax^2 + 7x - b$$

$$f(-2) = 2(-2)^3 + a(-2)^2 + 7(-2) - b = 0$$

$$-16 + 4a - 14 - b = 0$$

$$4a - b = 30 \qquad \ldots(1)$$

$$f(-3) = 2(-3)^3 + a(-3)^2 + 7(-3) - b = 0$$

$$-54 + 9a - 21 - b = 0$$

$$9a - b = 75 \qquad \ldots(2)$$

Solving (1) and (2), we get

$$a = 9, \; b = 6 \qquad \textbf{Ans.}$$

Question 3.
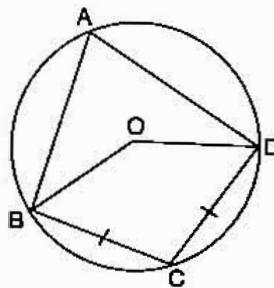
(a) *Solve the inequation and represent the solution set on the number line.*

$$-3 + x \leq \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x, \text{ where } x \in I \qquad [3]$$

(b) *Find the value of p for which the lines*

$2x + 3y - 7 = 0$ and $4y - px - 12 = 0$ are perpendicular to each other. [3]

(c) *In the given figure O is the centre of the circle, $\angle BAD = 75°$ and chord BC = chord CD. Find : (i) $\angle BOC$ (ii) $\angle OBD$ (iii) $\angle BCD$.* [4]
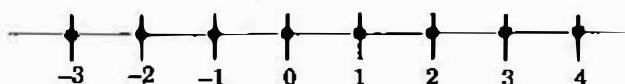

Solution :

(a) $$-3 + x \leq \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x, x \in I$$

$-3 + x \leq \dfrac{8x}{3} + 2$	$\dfrac{8x}{3} + 2 \leq \dfrac{14}{3} + 2x$
$\dfrac{8x}{3} - \dfrac{x}{1} \geq -3 - 2$	$\dfrac{8x}{3} - 2x \leq \dfrac{14}{3} - \dfrac{2}{1}$
$\dfrac{8x - 3x}{3} \geq -5$	$\dfrac{8x - 6x}{3} \leq \dfrac{14 - 6}{3}$
$5x \geq -15$	$\dfrac{2x}{3} \leq \dfrac{8}{3}$
$x \geq -3$	$x \leq 4$

Solution set : $\{-3 \leq x \leq 4\}$

(b) Given equation is $2x + 3y - 7 = 0$

$\Rightarrow$ $3y = -2x + 7$

$\Rightarrow$ $y = -\dfrac{2}{3}x + \dfrac{7}{3}$

Slope of the line $(m_1) = -\dfrac{2}{3}$

Another equation is $4y - px - 12 = 0$

$\Rightarrow$ $4y = px + 12$

$\Rightarrow$ $y = \dfrac{p}{4}x + 3$

Slope of the line $(m_2) = \dfrac{p}{4}$

As per the question, lines are perpendicular.

$$m_1 \times m_2 = -1$$

$$-\frac{2}{3} \times \frac{p}{4} = -1$$

$$\Rightarrow \qquad -2p = -12$$

$$\Rightarrow \qquad p = 6 \qquad \textbf{Ans.}$$

(c) *Given* : $\angle$ BAD = 75°, chord BC = chord CD

$$\angle \text{BOD} = 2 \times \angle \text{BAD} = 2 \times 75° = 150°$$

(i)
$$\angle \text{BOC} = \frac{1}{2} \angle \text{BOD}$$

$$= \frac{1}{2} \times 150° = 75° \qquad \textbf{Ans.}$$
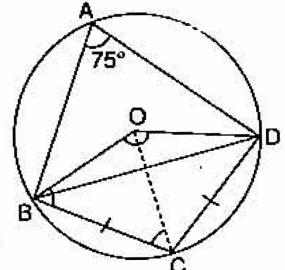
(ii)
$$\angle \text{OBD} = \frac{1}{2}(180° - 150°)$$

$$= 15° \qquad \textbf{Ans.}$$

(iii)
$$\angle \text{BCD} = 180° - 75° = 105° \qquad \textbf{Ans.}$$

Question 4.

(a) *Find the mean, median and mode of the following distribution :*

$$8, 10, 7, 6, 10, 11, 6, 13, 10 \qquad \text{[3]}$$

(b) *Without using trigonometric tables evaluate the following :*
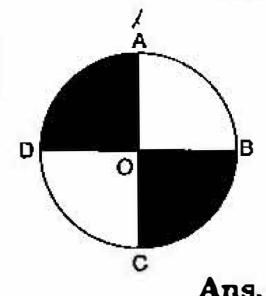
$$\frac{\sec 17°}{\csc 73°} + \frac{\tan 68°}{\cot 22°} + \cos^2 44° + \cos^2 46° \qquad \text{[3]}$$

(c) *AC and BD are two perpendicular diameters of a circle with centre O. If AC = 16 cm, calculate the area and perimeter of the shaded part. (Take π = 3.14)* [4]

Solution :

(a)
$$\text{Mean} = \frac{\Sigma x}{n} = \frac{8 + 10 + 7 + 6 + 10 + 11 + 6 + 13 + 10}{9}$$

$$= \frac{81}{9} = 9 \qquad \textbf{Ans.}$$

For the median, we arrange the data in ascending order

$$6, 6, 7, 8, 10, 10, 10, 11, 13$$

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{term} = \left(\frac{9+1}{2}\right)^{th} \text{term}$$

$$= 5^{th} \text{term} = 10$$

In the given data, 10 occurs maximum number of times, therefore

$$\text{Mode} = 10 \qquad \textbf{Ans.}$$
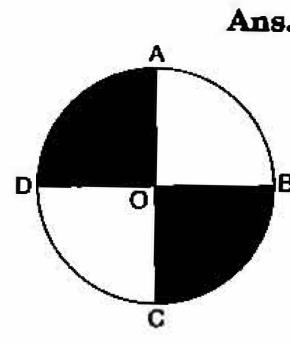
(b) *Given* :
$$\frac{\sec 17°}{\csc 73°} + \frac{\tan 68°}{\cot 22°} + \cos^2 44° + \cos^2 46°$$

$$= \frac{\sec(90° - 73°)}{\text{cosec }73°} + \frac{\tan(90° - 22°)}{\cot 22°} + \cos^2(90° - 46°) + \cos^2 46°$$

$$= \frac{\text{cosec }73°}{\text{cosec }73°} + \frac{\cot 22°}{\cot 22°} + \sin^2 46° + \cos^2 46°$$

$$= 1 + 1 + 1 = 3 \qquad \textbf{Ans.}$$

(c) *Given* : AC = 16 cm = diameter of the circle, $\pi = 3.14$.

Area of shaded portion = 2 quadrant

$$= \frac{1}{2} \times \pi r^2$$

$$= \frac{1}{2} \times 3.14 \times (8)^2$$

$$= 100.48 \text{ cm}^2 \quad \textbf{Ans.}$$

Perimeter of the shaded portion = πr + DB + AC

$$= 3.14 \times 8 + 16 + 16$$

$$= 57.12 \text{ cm.} \qquad \textbf{Ans.}$$

SECTION B [40 Marks]

*Answer any **four** Questions in this Section.*

Question 5.

(a) *A shopkeeper bought a TV at a discount of 30% of the listed price of ₹ 24,000. The shopkeeper offers a discount of 10% of the listed price to his customer. If the VAT (Value Added Tax) is 10%.*

Find : (i) *the amount paid by the customer.*

(ii) *the VAT to be paid by the shopkeeper.* [3]

(b) *Solve the following quadratic equation and give the answer correct to two significant figures.*

$$4x^2 - 7x + 2 = 0 \qquad [3]$$

(c) *Use graph paper to answer this question.*

(i) *Plot the points A (4, 6) and B (1, 2)*

(ii) *A′ is the image of A when reflected in X-axis.*

(iii) *B′ is the image of B when B is reflected in the line AA′.*

(iv) *Give the geometrical name for the figure AB A′B′.* [4]

Solution :

(a) (i)

$$\text{Discount} = 30\% \text{ on } 24,000$$

$$= \frac{30}{100} \times 24,000 = ₹\ 7,200$$

$$\text{Cost price of shopkeeper} = 24,000 - 7,200$$

$$= ₹\ 16,800$$

$$\text{Tax @ }10\% = \frac{10 \times 16800}{100} = ₹\ 1680$$

Amount paid by shopkeeper = 16,800 + 1,680 = ₹ 18,480

$$\text{Discount on customer } = \text{10\% on 24,000} = \frac{10}{100} \times 24,000$$
$$= ₹\,2,400$$
$$\text{Selling price of shopkeeper } = 24,000 - 2,400 = ₹\,21,600$$
$$\text{Tax @ 10\% } = \frac{10 \times 21,600}{100} = ₹\,2,160$$
$$\text{The amount paid by customer } = 21,600 + 2,160 = ₹\,23,760 \qquad \textbf{Ans.}$$

(ii)   VAT to be paid by shopkeeper $= ₹\,2,160 - ₹\,1,680$
$$= ₹\,480 \qquad\qquad \textbf{Ans.}$$

(b) Given equation $4x^2 - 7x + 2 = 0$ comparing with $ax^2 + bx + c = 0$, we have $a = 4$, $b = -7, c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{+7 \pm \sqrt{(7)^2 - 4 \times 4 \times 2}}{2 \times 4} = \frac{+7 \pm \sqrt{49 - 32}}{8}$$
$$= \frac{7 \pm 4.123}{8}$$

Taking $+ve$ sign
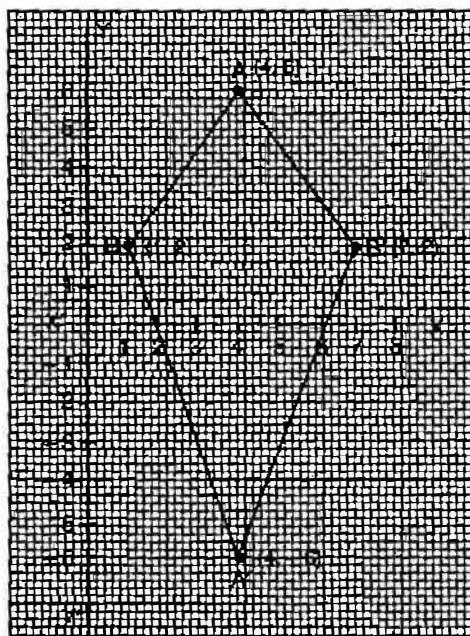
$$x = \frac{7 + 4.123}{8} = \frac{11.123}{8} = 13.9$$

Taking $-ve$ sign

$$x = \frac{7 - 4.123}{8} = \frac{2.877}{8} = 0.36$$
$$= 1.390 \text{ and } 0.3596$$
$$\Rightarrow \qquad\qquad = 1.4 \text{ and } 0.36 \qquad\qquad \textbf{Ans.}$$
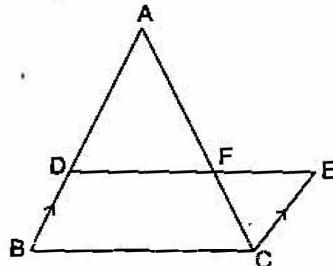
(c) (i)   See figure.
      (ii)   See figure.
      (iii)   See figure.

(iv)   Geometrical name is Kite.

Question 6.

(a) *In the given figure, ABC and CEF are two triangles where BA is parallel to CE and AF : AC = 5 : 8.*

  (i) . *Prove that $\triangle ADF \sim \triangle CEF$.*

  (ii) *Find AD if CE = 6 cm.*

  (iii) *If DF is parallel to BC find area of $\triangle ADF$: area of $\triangle ABC$.* **[3]**

(b) *Prove the following identity :*

$$\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A} = 2\cosec A$$ **[3]**

(c) *The following table gives the wages of workers in a factory :*
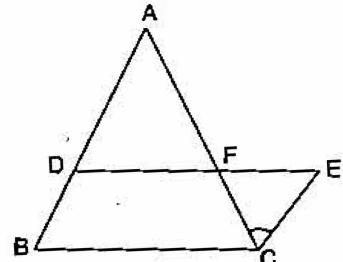
Wages in ₹	45–50	50–55	55–60	60–65	65–70	70–75	75–80
No. of workers	5	8	30	25	14	12	6

Calculate the mean by the short cut method. **[4]**

Solution :

(a) In $\triangle$ ABC and $\triangle$ CEF,

$$\text{BA} \parallel \text{CE (given)}$$

and

$$\frac{\text{AF}}{\text{AC}} = \frac{5}{8}\ \text{(given)}$$

$$\Rightarrow \quad \frac{\text{AF}}{\text{AF} + \text{FC}} = \frac{5}{5 + 3}$$

$$\Rightarrow \quad \frac{\text{AF}}{\text{FC}} = \frac{5}{3}$$

(i)
$$\angle \text{DAF} = \angle \text{FCE} \qquad \text{(Int. } \angle)$$
$$\angle \text{AFD} = \angle \text{CFE} \qquad \text{(Vert.)}$$
$$\therefore \triangle \text{ADF} \sim \triangle \text{CEF} \qquad \text{(AA similarity)}$$
Proved

(ii)
$$\frac{\text{AD}}{\text{CE}} = \frac{\text{AF}}{\text{FC}} \quad \Rightarrow \quad \frac{\text{AD}}{6} = \frac{5}{3}$$

$$\Rightarrow \quad \text{AD} = \frac{5}{3} \times 6 = 10 \text{ cm} \qquad \textbf{Ans.}$$

(iii) *Given :* DF ∥ BC

$$\triangle \text{ADF} \sim \triangle \text{ABC}$$
$$\frac{\text{AF}}{\text{AC}} = \frac{\text{AD}}{\text{AB}} = \frac{5}{8}$$
$$\frac{\text{Area of } \triangle \text{ADF}}{\text{Area of } \triangle \text{ABC}} = \frac{\text{AD}^2}{\text{AB}^2} = \frac{(5)^2}{(8)^2} = \frac{25}{64}$$

Ans.

(b)

$$\text{L.H.S.} = \frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A}$$

$$= \frac{\sin^2 A + 1 + \cos^2 A + 2\cos A}{\sin A\,(1 + \cos A)}$$

$$= \frac{2 + 2\cos A}{\sin A\,(1 + \cos A)} = \frac{2\,(1 + \cos A)}{\sin A\,(1 + \cos A)}$$

$$= 2\,\text{cosec}\,A = \text{R.H.S.} \qquad\qquad \textbf{Proved}$$

(c)

Class interval	Class Marks x_i	$d_i = x_i - A$	Frequency f_i	$f_i.d_i$
45–50	47·5	– 15	5	– 75
50–55	52·5	–10	8	– 80
55–60	57·5	–5	30	– 150
60–65	62·5 (A)	0	25	0
65–70	67·5	5	14	70
70–75	72·5	10	12	120
75–80	77·5	15	6	90
			$\Sigma f_i = 100$	$\Sigma f_i d_i = -25$

Here, A = 62·5

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i} = 62\cdot5 + \left(\frac{-25}{100}\right)$$

$$= 62\cdot5 - 0\cdot25 = 62\cdot25 \qquad\qquad \textbf{Ans.}$$

Question 7.

(a) *Amit Kumar invests ₹ 36,000 in buying ₹ 100 shares at ₹ 20 premium. The dividend is 15% per annum. Find :*

(i)   *The number of shares he buys*

(ii) *His yearly dividend*

(iii) *The percentage return on his investment.*

Give your answer correct to the nearest whole number.   **[3]**

(b) *What sum of money will amount to ₹ 9,261 in 3 years at 5% per annum compound interest ?*   **[3]**

(c) *Mr. Mishra has a Savings Bank Account in Allahabad Bank. His pass book entries are as follows :*

Date	Particulars	Withdrawals (in ₹)	Deposits (in ₹)	Balance (in ₹)
Jan. 4, 2007	By Cash	—	1000.00	1000.00
Jan. 11, 2007	By Cheque	—	3000.00	4000.00
Feb. 3, 2007	By Cash	—	2500.00	6500.00
Feb. 7, 2007	To Cheque	2000.00	—	4500.00
March 3, 2007	By Cash	—	5000.00	9500.00
March 25, 2007	By Cash	—	2000.00	11,500.00
June 7, 2007	By Cash	—	3500.00	15000.00
Aug. 29, 2007	To Cheque	1000.00	—	14000.00

Rate of interest paid by the bank is 4.5% per annum. Mr. Mishra closes his account on 30th October, 2007. Find the interest he receives.   **[4]**

Solution :

(a)   MV of 1 share = ₹ (100 + 20) = ₹ 120

Given : Dividend = 15%, Investment = 36,000

(i)   Number of shares buys $= \dfrac{\text{Investment}}{\text{MV}}$

$= \dfrac{36,000}{120} = 300$ **Ans.**

(ii)   Dividend on 1 share $= ₹ \dfrac{15}{100} \times 100 = ₹ 15$

Dividend on 300 shares $= 15 \times 300$

$= ₹ 4,500$ **Ans.**

(iii)   Rate of interest $= \dfrac{4,500}{36,000} \times 100$

$= 12{\cdot}5\%$ **Ans.**

(b) *Given* A = ₹ 9,261, T = 3 year, R = 5%

$$A = P\left(1 + \frac{R}{100}\right)^{T}$$

$\Rightarrow$ $9,261 = P\left(1 + \dfrac{5}{100}\right)^{3}$

$\Rightarrow$ $9,261 = P\left(\dfrac{21}{20}\right)^{3}$

$\Rightarrow$ $P = \dfrac{9,261 \times 20 \times 20 \times 20}{21 \times 21 \times 21}$

$= ₹ 8,000$ **Ans.**

(c) Qualifying amounts for interest for various months

| January | ₹ | 1,000 | P = ₹ 92,000 |
February	₹	4,500	T = 1 month
March	₹	9,500	R = 4·5%
April	₹	9,500	Interest $= \dfrac{P \times R \times T}{100}$
May	₹	9,500	
June	₹	15,000	$\dfrac{92,000 \times 4{\cdot}5 \times 1}{100 \times 12}$
July	₹	15,000	
August	₹	14,000	= ₹ 345 **Ans.**
September	₹	14,000	
	₹	92,000	
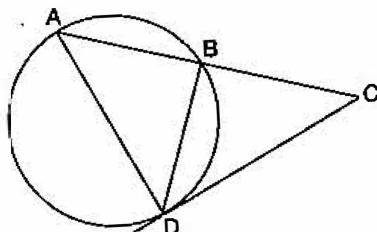
Question 8.

(a) *Given that* $\dfrac{a^3 + 3ab^2}{b^3 + 3a^2b} = \dfrac{63}{62}$.

Using Componendo and Dividendo find $a : b$. [3]

(b)

In the above figure AB = 7 cm and BC = 9 cm.
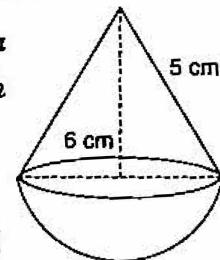
(i) *Prove* $\triangle ACD \sim \triangle DCB$.   (ii) *Find the length of CD.* [3]

(c) *The given figure represents a hemisphere surmounted by a conical block of wood. The diameter of their bases is 6 cm each and the slant height of the cone is 5 cm. Calculate :*

(i) *the height of the cone.*

(ii) *the volume of the solid.* [4]

Solution :

(a) *Given :*
$$\frac{a^3 + 3ab^2}{b^3 + 3a^2b} = \frac{63}{62}$$

By componendo and dividendo,
$$\frac{a^3 + 3ab^2 + b^3 + 3a^2b}{a^3 + 3ab^2 - b^3 - 3a^2b} = \frac{63 + 62}{63 - 62}$$

$$\Rightarrow \qquad \frac{(a+b)^3}{(a-b)^3} = \frac{125}{1} = \left(\frac{5}{1}\right)^3$$

$$\Rightarrow \qquad \frac{a+b}{a-b} = \frac{5}{1}$$

Again componendo and dividendo,

$$\Rightarrow \qquad \frac{a+b+a-b}{a+b-a+b} = \frac{5+1}{5-1} = \frac{6}{4}$$

$$\Rightarrow \qquad \frac{2a}{2b} = \frac{3}{2}$$

$$\Rightarrow \qquad a : b = 3 : 2 \qquad\qquad \textbf{Ans.}$$

(b) *Given :* AB = 7 cm, BC = 9 cm.
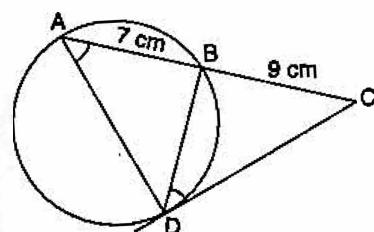
(i)   In $\triangle$ ACD and $\triangle$ DCB,

$$\angle C = \angle C \qquad\qquad \text{(common)}$$

$$\angle CDB = \angle BAD \ (\angle\text{'s alternate segment})$$

$$\triangle ACD \sim \triangle DCB \qquad \text{(By A·A similarity)}$$

Proved

(ii) $\qquad AC \times BC = CD^2 \qquad\qquad (\because \triangle ACD \sim \triangle DCB)$

$$CD^2 = 16 \times 9 = 144$$
$$CD = 12 \text{ cm.} \qquad\qquad \textbf{Ans.}$$

(c) *Given* : Diameter (AB) = 6 cm, *r* (OB) = 3 cm, *l* (DB) = 5 cm.

(i) $\qquad\qquad OD^2 = DB^2 - OB^2$
$$= 25 - 9 = 16$$
$$OD = 4 \text{ cm} \qquad\qquad \textbf{Ans.}$$
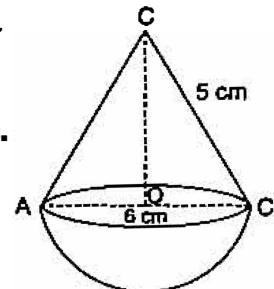
(ii) $\qquad$ Volume of hemisphere $= \dfrac{2}{3}\pi r^3$

$$= \dfrac{2}{3} \times \dfrac{22}{7} \times (3)^3$$
$$= 56.57 \text{ cm}^3.$$

$$\text{Volume of cone} = \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3} \times \dfrac{22}{7} \times 9 \times 4 = 37.71 \text{ cm}^3$$

$$\text{Volume of the solid} = 37.71 + 56.57$$
$$= 94.28 \text{ cm}^3. \qquad\qquad \textbf{Ans.}$$

Question 9.

(a) *Attempt this question on graph paper.*

Marks obtained by 200 students in examination are given below

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of Students	5	10	14	21	25	34	36	27	16	12

Draw an Ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis.

From the graph find

(i) *the Median*

(ii) *the Upper Quartile*
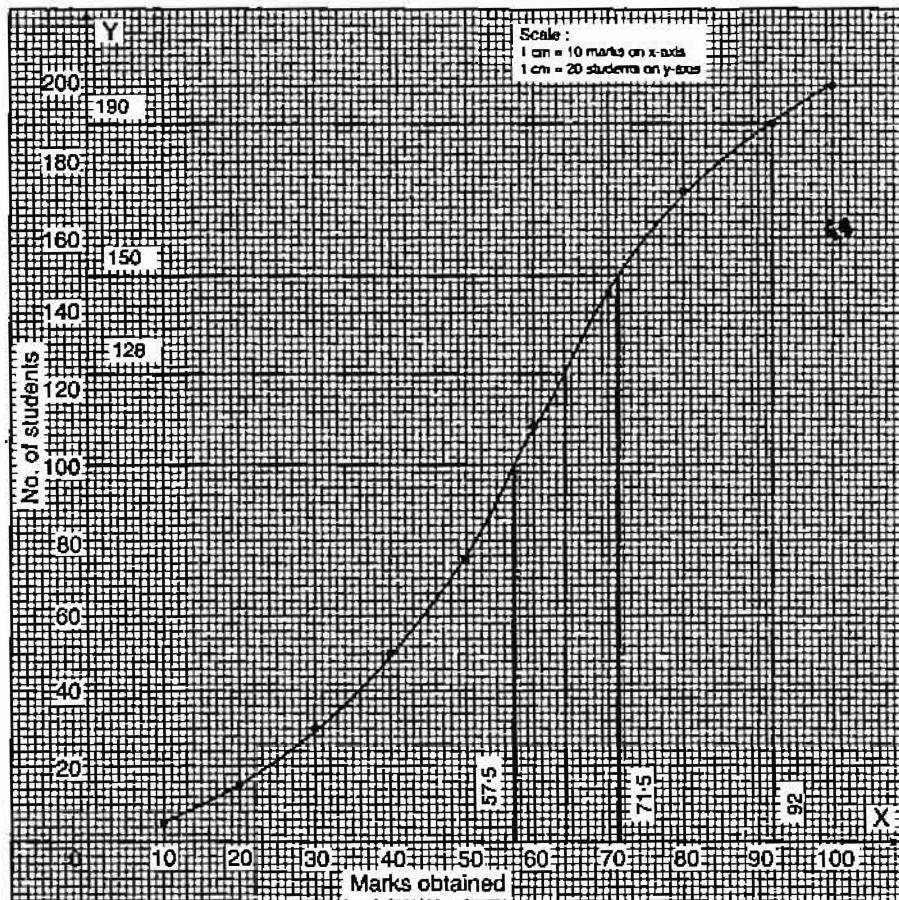
(iii) *Number of students scoring above 65 marks.*

(iv) *If 10 students qualify for merit scholarship, find the minimum marks required to qualify.* **[6]**

(b) *From two points A and B on the same side of a building, the angles of elevation of the top of the building are 30° and 60° respectively. If the height of the building is 10m, find the distance between A and B correct to two decimal places.* **[4]**

Solution :

(a)

Marks	No. of students	cf
0–10	5	5
10–20	10	15
20–30	14	29
30–40	21	50
40–50	25	75
50–60	34	109
60–70	36	145
70–80	27	172
80–90	16	188
90–100	12	200

Scale:
1 cm = 10 marks on x-axis
1 cm = 20 students on y-axis

(i) Here n (no. of students) = 200 (even)

$\therefore$ Median = $\left(\dfrac{n}{2}\right)^{th}$ term = 100th term.

From the graph 100th term is 57·5.

Median = 57·5 **Ans.**

(ii) Upper quartile (Q_3) = $\dfrac{3n}{4} = \dfrac{3 \times 200}{4}^{th}$ term

= $3 \times 50 = 150$ term

From Graph 150th term = 71·5

The upper quartile = 71·5 **Ans.**

(iii) Number of students scoring above 65 marks = 200 − 128 = 72 **Ans.**
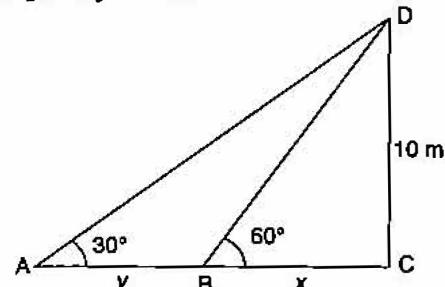
(iv) Minimum marks to qualify = 92 **Ans.**

(b) In $\triangle$ DBC, $\tan 60° = \dfrac{10}{x}$

$\Rightarrow$ $\sqrt{3} = \dfrac{10}{x}$

$\Rightarrow$ $x = \dfrac{10}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

= $\dfrac{10\sqrt{3}}{3}$ m

In $\triangle$ DAC, $\qquad \tan 30° = \dfrac{10}{x+y} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \qquad\qquad\qquad x + y = 10\sqrt{3}$

$\Rightarrow \qquad\qquad\qquad\quad y = 10\sqrt{3} - \dfrac{10}{\sqrt{3}}$

$$= \dfrac{30 - 10}{\sqrt{3}} = \dfrac{20}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$$

$$= \dfrac{20}{3}\sqrt{3}$$

$$= 11.55 \text{ m} \qquad\qquad \textbf{Ans.}$$

Question 10.

(a) *Mrs. Goswami deposits ₹ 1000 every month in a recurring deposit account for 3 years at 8% interest per annum. Find the matured value.* **[3]**

(b) *Find the equation of a line with x intercept = 5 and passing through the point (4, –7).* **[3]**

(c) *In a school the weekly pocket money of 50 students is as follows :*

Weekly pocket money in ₹	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	2	8	12	14	8	6

Draw a histogram and a frequency polygon on the same graph. Find the mode from the graph. **[4]**

Solution :

(a) Total Principal (P) for 1 month

$$P = x \times \dfrac{n\,(n+1)}{2} = 1000 \times \dfrac{36 \times 37}{2}$$

$$= ₹\,6,66,000$$

$$\text{Interest for 1 month} = \dfrac{\text{PRT}}{100} = \dfrac{6,66,000 \times 8 \times 1}{100 \times 12}$$

$$= ₹\,4,440$$
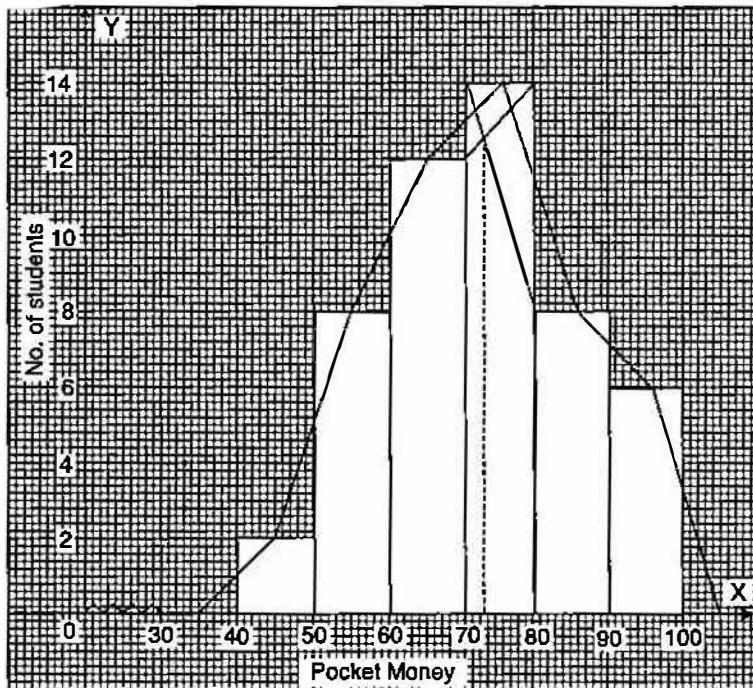
$$\text{Total amount deposited by Mr. Goswami} = 36 \times 1000 = ₹\,36,000$$

$$\text{Maturity value} = ₹\,36,000 + 4,440$$

$$= ₹\,40,440 \qquad\qquad \textbf{Ans.}$$

(b) Equation of the line passing through (5, 0) and (4, –7) :

$$y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \dfrac{-7 - 0}{4 - 5}(x - 5)$$

$$y = \dfrac{-7}{-1}(x - 5)$$

$\Rightarrow \qquad\qquad\qquad y = 7\,(x - 5)$

$\Rightarrow \qquad\qquad\qquad y = 7x - 35$

$\Rightarrow \qquad\qquad 7x - y - 35 = 0 \qquad\qquad\qquad \textbf{Ans.}$

(c)

So, from the graph,   Mode = 73   **Ans.**

Question 11.

(a) *The model of a building is constructed with scale factor 1 : 30.*

   (i) *If the height of the model is 80 cm, find the actual height of the building in metres.*

   (ii) *If the actual volume of a tank at the top of the building is 27m³, find the volume of the tank on the top of the model.*   **[2]**

(b) *The speed of an express train is x km/h and the speed of an ordinary train is 12 km/h less than that of the express train. If the ordinary train takes one hour longer than the express train to cover a distance of 240 km, find the speed of the express train.*   **[4]**

(c) *Using ruler and compasses construct*

   (i) *a triangle ABC in which AB = 5.5 cm, BC = 3.4 cm and CA = 4.9 cm.*

   (ii) *the locus of points equidistant from A and C.*

   (iii) *a circle touching AB at A and passing through C.*   **[4]**

Solution :

(a)   Scale factor $k = \dfrac{1}{30}$

   (i)   Height of the model = k (times the height of the building)

      ⇒   Height of building = 80 × 30

                          = 2400 cm

                          = 24 m   **Ans.**

(ii) Volume of model $= k^3$ times volume of the building

$$= \frac{1}{30} \times \frac{1}{30} \times \frac{1}{30} \times 27 \text{ m}^3$$

$$= \frac{1}{1000} \text{ m}^3 = 0.001 \text{ m}^3 \qquad \textbf{Ans.}$$

(b) Time taken by express train $= \dfrac{240}{x} \text{ h}$

Speed of ordinary train $= (x - 12) \text{ km/h}$

Time taken by ordinary train $= \dfrac{240}{x - 12}$

According to the question, $\dfrac{240}{x} = \dfrac{240}{x - 12} - 1$

$\Rightarrow \qquad 240\,(x - 12) = 240\,x - x\,(x - 12)$

$\Rightarrow \qquad 240x - 2880 = 240\,x - x^2 + 12x$

$\Rightarrow \qquad x^2 - 12x - 2880 = 0$

$\Rightarrow \qquad x^2 - 60x + 48x - 2880 = 0$

$\Rightarrow \qquad x\,(x - 60) + 48\,(x - 60) = 0$

$\Rightarrow \qquad (x - 60)\,(x + 48) = 0$

either $\qquad x - 60 = 0 \quad \text{or} \quad x + 48 = 0$
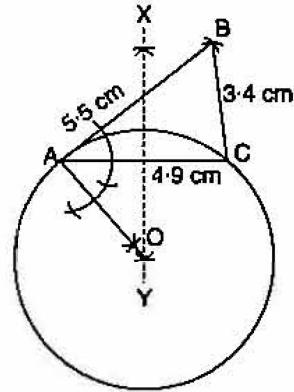
$\Rightarrow \qquad x = 60 \quad \text{or} \quad x = -48 \qquad \textbf{Ans.}$

Hence, the speed of express train is 60 km/h.

(c) Steps of construction :

1. Draw $\triangle$ ABC with given values.

2. Draw XY perpendicular bisector of AC.

3. Draw perpendicular of AB at A which cuts perpendicular XY at O.

4. Draw a circle at centre O which touching AB at A and passing through C i.e., required circle.

●●