ICSE Paper Question (2009)

MATHEMATICS

SECTION A [40 Marks]
(Answer all questions from this Section.)

Question 1.
(a) Mr. Dubey borrows ₹ 1,00,000 from State Bank of India at 11% per annum compound interest. He repays ₹ 41,000 at the end of the first year and ₹ 47,700 at the end of the second year. Find the amount outstanding at the beginning of the third year. [3]

(b) A dice is thrown once. What is the probability that the
(i) number is even
(ii) number is greater than 2? [3]

(c) Find the HCF and LCM of the following polynomials: **
   \[3x^3 - 27x^2 + 60x\] and \[x^2 - 16\] [4]

Solution:
(a) Given : \(P = ₹ 1,00,000, R = 11\%

Interest for first year = \(\frac{PRT}{100} = \frac{1,00,000 \times 11 \times 1}{100} = ₹ 11,000\)

Amount after first year = \(₹ 1,00,000 + ₹ 11,000 = ₹ 1,11,000\)

Principal for second year = \(₹ 1,11,000 - ₹ 41,000\)
   = \(₹ 70,000\)

Interest for second year = \(\frac{70,000 \times 11 \times 1}{100} = ₹ 7,700\)

Amount after second year = \(₹ 70,000 + ₹ 7,700 = ₹ 77,700\)

Amount outstanding for beginning of third year
   = \(₹ 77,700 - ₹ 47,700\)
   = \(₹ 30,000\).
   Ans.

(b) Dice is thrown once.

Sample space = \(\{1, 2, 3, 4, 5, 6\}\)

\(n(S) = 6\)

(i) Number is even = \(\{2, 4, 6\}\)

\(n(E) = 3\)

\(P(\text{Even number}) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}\)

Ans.

(ii) Number is greater than 2 = \(\{3, 4, 5, 6\}\)

\(n(E) = 4\)

\(P(> 2) = \frac{n(E)}{n(S)} = \frac{4}{6} = \frac{2}{3}\)

Ans.

** Solution has not given due to out of present syllabus.
Question 2.

(a) Find x and y, if \[
\begin{bmatrix}
2x & x \\
y & 3y
\end{bmatrix}
\begin{bmatrix}
3 \\
2
\end{bmatrix}
= \begin{bmatrix}
16 \\
9
\end{bmatrix}
\]

(b) What least number must be added to each of the numbers 5, 11, 19 and 37 so that they are in proportion?

(c) Given that \(x + 2\) and \(x + 3\) are factors of \(2x^3 + ax^2 + 7x - b\). Determine the values of \(a\) and \(b\).

Solution:

(a) Given:
\[
\begin{bmatrix}
2x & x \\
y & 3y
\end{bmatrix}
\begin{bmatrix}
3 \\
2
\end{bmatrix}
= \begin{bmatrix}
16 \\
9
\end{bmatrix}
\]

\[
8x = 16 \Rightarrow x = 2
\]

\[
9y = 9 \Rightarrow y = 1
\]

Ans.

(b) Let the number \(x\) be added to each number.

\[
5 + x : 11 + x = 19 + x : 37 + x
\]

\[
5 + x = 19 + x
\]

\[
11 + x = 37 + x
\]

By componendo and dividendo,

\[
\begin{align*}
\Rightarrow & \quad \frac{5 + x + 11 + x}{5 + x - 11 - x} = \frac{19 + x + 37 + x}{19 + x - 37 - x} \\
\Rightarrow & \quad \frac{16 + 2x}{-6} = \frac{56 + 2x}{-18} \\
\Rightarrow & \quad 3 (16 + 2x) = 56 + 2x \\
\Rightarrow & \quad 48 + 6x = 56 + 2x \\
\Rightarrow & \quad 4x = 8 \\
\Rightarrow & \quad x = 2
\end{align*}
\]

Ans.

(c) Given: \((x + 2)\) and \((x + 3)\) are the factors of \(2x^3 + 9x^2 + 7x - b\).

\[
\therefore f(-2) \text{ and } f(-3) \text{ will be zero.}
\]

\[
f(x) = 2x^3 + ax^2 + 7x - b
\]

\[
f(-2) = 2(-2)^3 + a(-2)^2 + 7(-2) - b = 0
\]

\[
-16 + 4a - 14 - b = 0
\]

\[
4a - b = 30 \quad \ldots(1)
\]

\[
f(-3) = 2(-3)^3 + a(-3)^2 + 7(-3) - b = 0
\]

\[
-54 + 9a - 21 - b = 0
\]

\[
9a - b = 75 \quad \ldots(2)
\]

Solving (1) and (2), we get

\[
a = 9, \quad b = 6
\]

Ans.
Question 3.

(a) Solve the inequation and represent the solution set on the number line.

\[-3 + x \leq \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x, \quad x \in \mathbb{I}\]

Solution set:

\[-3 \leq x \leq 4\]

(b) Find the value of \(p\) for which the lines

\[2x + 3y - 7 = 0\] and \[4y - px - 12 = 0\] are perpendicular to each other.

Solution:

Given equation is \(2x + 3y - 7 = 0\)

\[3y = -2x + 7\]

\[y = \frac{-2}{3}x + \frac{7}{3}\]

Slope of the line \(m_1 = \frac{-2}{3}\)

Another equation is \[4y - px - 12 = 0\]

\[4y = px + 12\]

\[y = \frac{p}{4}x + 3\]

Slope of the line \(m_2 = \frac{p}{4}\)

(c) In the given figure \(O\) is the centre of the circle, \(\angle BAD = 75^\circ\) and chord \(BC = \) chord \(CD\). Find: (i) \(\angle BOC\) (ii) \(\angle OBD\) (iii) \(\angle BCD\).
As per the question, lines are perpendicular.
\[ m_1 \times m_2 = -1 \]
\[ \frac{2}{3} \times \frac{-p}{4} = -1 \]
\[ \Rightarrow -2p = -12 \]
\[ \Rightarrow p = 6 \]

(c) Given: \( \angle BAD = 75^\circ \), chord \( BC = \text{chord CD} \)
\[ \angle BOD = 2 \times \angle BAD = 2 \times 75^\circ = 150^\circ \]

(i) \[ \angle BOC = \frac{1}{2} \angle BOD \]
\[ = \frac{1}{2} \times 150^\circ = 75^\circ \]

(ii) \[ \angle OBD = \frac{1}{2} (180^\circ - 150^\circ) \]
\[ = 15^\circ \]

(iii) \[ \angle BCD = 180^\circ - 75^\circ = 105^\circ \]

Question 4.
(a) Find the mean, median and mode of the following distribution:
8, 10, 7, 6, 10, 11, 6, 13, 10

(b) Without using trigonometric tables evaluate the following:
\[ \sec 17^\circ \tan 68^\circ \]
\[ \cosec 73^\circ + \cot 22^\circ + \cos^2 44^\circ + \cos^2 46^\circ \]

(c) AC and BD are two perpendicular diameters of a circle
with centre O. If AC = 16 cm, calculate the area and
perimeter of the shaded part. (Take \( \pi = 3.14 \))

Solution:
(a) Mean = \[ \frac{\Sigma x}{n} = \frac{8 + 10 + 7 + 6 + 10 + 11 + 6 + 13 + 10}{9} \]
\[ = \frac{81}{9} = 9 \]

For the median, we arrange the data in ascending order
6, 6, 7, 8, 10, 10, 10, 11, 13

Median = \( \left( \frac{n+1}{2} \right)^{\text{th}} \) term = \( \left( \frac{9+1}{2} \right)^{\text{th}} \) term
\[ = 5^{\text{th}} \text{ term} = 10 \]

In the given data, 10 occurs maximum number of times, therefore
Mode = 10

(b) Given:
\[ \frac{\sec 17^\circ \tan 68^\circ}{\cosec 73^\circ + \cot 22^\circ + \cos^2 44^\circ + \cos^2 46^\circ} \]
\[
\begin{align*}
&= \frac{\sec(90^\circ - 73^\circ)}{\csc 73^\circ} + \frac{\tan(90^\circ - 22^\circ)}{\cot 22^\circ} + \cos^2(90^\circ - 46^\circ) + \cos^2 46^\circ \\
&= \frac{\csc 73^\circ \cdot \cot 22^\circ}{\csc 73^\circ + \cot 22^\circ} + \sin^2 46^\circ + \cos^2 46^\circ \\
&= 1 + 1 + 1 = 3 \\
\text{(c) Given : } \text{AC} = 16 \text{ cm = diameter of the circle, } \pi = 3.14. \\
\text{Area of shaded portion } = 2 \text{ quadrant} \\
&= \frac{1}{2} \times \pi r^2 \\
&= \frac{1}{2} \times 3.14 \times (8)^2 \\
&= 100.48 \text{ cm}^2 \text{ Ans.} \\
\text{Perimeter of the shaded portion } = \pi r + DB + AC \\
&= 3.14 \times 8 + 16 + 16 \\
&= 57.12 \text{ cm.} \text{ Ans.}
\end{align*}
\]

SECTION B [40 Marks]
Answer any four Questions in this Section.

Question 5.

(a) A shopkeeper bought a TV at a discount of 30% of the listed price of ₹24,000. The shopkeeper offers a discount of 10% of the listed price to his customer. If the VAT (Value Added Tax) is 10%.

Find: (i) the amount paid by the customer. (ii) the VAT to be paid by the shopkeeper. [3]

(b) Solve the following quadratic equation and give the answer correct to two significant figures.

\[4x^2 - 7x + 2 = 0\] [3]

(c) Use graph paper to answer this question.

(i) Plot the points A (4, 6) and B (1, 2)
(ii) A' is the image of A when reflected in X-axis.
(iii) B' is the image of B when B is reflected in the line AA'.
(iv) Give the geometrical name for the figure AB A'B'. [4]

Solution:

(a) (i) Discount = 30% on 24,000
\[\frac{30}{100} \times 24,000 = ₹7,200\]
Cost price of shopkeeper = 24,000 - 7,200
\[= ₹16,800\]
Tax @ 10% = \[\frac{10 \times 16800}{100} = ₹1,680\]
Amount paid by shopkeeper = 16,800 + 1,680 = ₹18,480
Discount on customer = 10% on 24,000 = \(\frac{10}{100} \times 24,000\)
= \(\text{₹} 2,400\)
Selling price of shopkeeper = 24,000 – 2,400 = \(\text{₹} 21,600\)

\[
\text{Tax @ 10%} = \frac{10 \times 21,600}{100} = \text{₹} 2,160
\]

The amount paid by customer = 21,600 + 2,160 = \(\text{₹} 23,760\)  \hspace{1cm} \text{Ans.}

(ii) VAT to be paid by shopkeeper = \(\text{₹} 2,160 - \text{₹} 1,680\)
= \(\text{₹} 480\)  \hspace{1cm} \text{Ans.}

(b) Given equation \(4x^2 - 7x + 2 = 0\) comparing with \(ax^2 + bx + c = 0\), we have \(a = 4, b = -7, c = 2\)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
= \frac{+7 \pm \sqrt{(-7)^2 - 4 \times 4 \times 2}}{2 \times 4} = \frac{+7 \pm \sqrt{49 - 32}}{8}
\]
\[
= \frac{7 \pm 4.123}{8}
\]

Taking \(+ve\) sign
\[
x = \frac{7 + 4.123}{8} = \frac{11.123}{8} = 1.390
\]

Taking \(-ve\) sign
\[
x = \frac{7 - 4.123}{8} = \frac{2.877}{8} = 0.36
\]
\[
= 1.390 \text{ and } 0.3596
\]
\[
= 1.4 \text{ and } 0.36
\]  \hspace{1cm} \text{Ans.}

(c) (i) See figure.

(ii) See figure.

(iii) See figure.

(iv) Geometrical name is Kite.
Question 6.

(a) In the given figure, ABC and CEF are two triangles where BA is parallel to CE and \( AF : AC = 5 : 8 \).

(i) Prove that \( \triangle ADF \sim \triangle CEF \).

(ii) Find \( AD \) if \( CE = 6 \text{ cm} \).

(iii) If \( DF \) is parallel to \( BC \) find area of \( \triangle ADF : \) area of \( \triangle ABC \).

(b) Prove the following identity:

\[
\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A} = 2 \csc A
\]

(c) The following table gives the wages of workers in a factory:

<table>
<thead>
<tr>
<th>Wages in ₹</th>
<th>45-50</th>
<th>50-55</th>
<th>55-60</th>
<th>60-65</th>
<th>65-70</th>
<th>70-75</th>
<th>75-80</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of workers</td>
<td>5</td>
<td>8</td>
<td>30</td>
<td>25</td>
<td>14</td>
<td>12</td>
<td>6</td>
</tr>
</tbody>
</table>

Calculate the mean by the short cut method.

Solution:

(a) In \( \triangle ABC \) and \( \triangle CEF \),

\[ BA \parallel CE \text{ (given)} \]

\[ \frac{AF}{AC} = \frac{5}{8} \text{ (given)} \]

\[ \Rightarrow \frac{AF}{AF + FC} = \frac{5}{5 + 3} \]

\[ \Rightarrow \frac{AF}{FC} = \frac{1}{3} \]

(i) \( \angle DAF = \angle FCE \) \hspace{1cm} (Int. \( \angle \))

\( \angle AFD = \angle CFE \) \hspace{1cm} (Vert.)

\( \triangle ADF \sim \triangle CEF \) \hspace{1cm} (AA similarity)

Proved

(ii) \[ \frac{AD}{CE} = \frac{AF}{FC} \Rightarrow \frac{AD}{6} = \frac{5}{3} \]

\[ \Rightarrow AD = \frac{5}{3} \times 6 = 10 \text{ cm} \hspace{1cm} \text{Ans.} \]

(iii) Given: \( DF \parallel BC \)

\[ \triangle ADF \sim \triangle ABC \]

\[ \frac{AF}{AD} = \frac{5}{8} \]

\[ \frac{AC}{AB} = \frac{8}{6} \]

\[ \text{Area of } \triangle ADF = \frac{AD^2}{2} = \frac{(5)^2}{2} = 25 \]

\[ \text{Area of } \triangle ABC = \frac{AB^2}{2} = \frac{(8)^2}{2} = 64 \]

\[ \text{Ans.} \]
(b) \[ \text{L.H.S.} = \frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A} \]
\[= \frac{\sin^2 A + 1 + \cos^2 A + 2 \cos A}{\sin A (1 + \cos A)} \]
\[= \frac{2 + 2 \cos A}{\sin A (1 + \cos A)} = \frac{2 (1 + \cos A)}{\sin A (1 + \cos A)} \]
\[= 2 \cosec A = \text{R.H.S.} \]

Proved

(c) Class Marks \( x_i \) | Frequency \( f_i \) | \( d_i = x_i - A \) | \( f_i d_i \) | \( \sum f_i = 100 \) | \( \sum f_i d_i = -25 \)
--- | --- | --- | --- | --- | ---
45-50 | 47.5 | -15 | 5 | -75
50-55 | 52.5 | -10 | 8 | -80
55-60 | 57.5 | -5 | 30 | -150
60-65 | 62.5 (A) | 0 | 25 | 0
65-70 | 67.5 | 5 | 14 | 70
70-75 | 72.5 | 10 | 12 | 120
75-80 | 77.5 | 15 | 6 | 90

Here, \( A = 62.5 \)

\[ \text{Mean} = A + \frac{\sum f_i d_i}{\sum f_i} = 62.5 + \left( \frac{-25}{100} \right) \]
\[= 62.5 - 0.25 = 62.25 \]

Ans.

Question 7.

(a) Amit Kumar invests \( \text{₹} \) 36,000 in buying \( \text{₹} \) 100 shares at \( \text{₹} \) 20 premium. The dividend is 15\% per annum. Find:

(i) The number of shares he buys
(ii) His yearly dividend
(iii) The percentage return on his investment.

Give your answer correct to the nearest whole number. [3]

(b) What sum of money will amount to \( \text{₹} \) 9,261 in 3 years at 5\% per annum compound interest? [3]

(c) Mr. Mishra has a Savings Bank Account in Allahabad Bank. His pass book entries are as follows:

<table>
<thead>
<tr>
<th>Date</th>
<th>Particulars</th>
<th>Withdrawals  (in ₹)</th>
<th>Deposits  (in ₹)</th>
<th>Balance  (in ₹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan. 4, 2007</td>
<td>By Cash</td>
<td>—</td>
<td>1000.00</td>
<td>1000.00</td>
</tr>
<tr>
<td>Jan. 11, 2007</td>
<td>By Cheque</td>
<td>—</td>
<td>3000.00</td>
<td>4000.00</td>
</tr>
<tr>
<td>Feb. 3, 2007</td>
<td>By Cash</td>
<td>—</td>
<td>2500.00</td>
<td>6500.00</td>
</tr>
<tr>
<td>Feb. 7, 2007</td>
<td>To Cheque</td>
<td>2000.00</td>
<td>—</td>
<td>4500.00</td>
</tr>
<tr>
<td>March 3, 2007</td>
<td>By Cash</td>
<td>—</td>
<td>5000.00</td>
<td>9500.00</td>
</tr>
<tr>
<td>March 25, 2007</td>
<td>By Cash</td>
<td>—</td>
<td>2000.00</td>
<td>11500.00</td>
</tr>
<tr>
<td>June 7, 2007</td>
<td>By Cash</td>
<td>—</td>
<td>3500.00</td>
<td>15000.00</td>
</tr>
<tr>
<td>Aug. 29, 2007</td>
<td>To Cheque</td>
<td>1000.00</td>
<td>—</td>
<td>14000.00</td>
</tr>
</tbody>
</table>

Rate of interest paid by the bank is 4.5\% per annum. Mr. Mishra closes his account on 30th October, 2007. Find the interest he receives. [4]
Solution:

(a) MV of 1 share = ₹ (100 + 20) = ₹ 120

Given: Dividend = 15%, Investment = 36,000

(i) Number of shares buys = \[ \frac{\text{Investment}}{\text{MV}} \]
\[ = \frac{36,000}{120} = 300 \]

(ii) Dividend on 1 share = \[ ₹ \frac{15}{100} \times 100 = ₹ 15 \]
Dividend on 300 shares = 15 \times 300 = ₹ 4,500

(iii) Rate of interest = \[ \frac{4,500}{36,000 \times 100} \]
\[ = 12.5\% \]

(b) Given A = ₹ 9,261, T = 3 year, R = 5%

\[ A = P \left(1 + \frac{R}{100}\right)^T \]
\[ ⇒ 9,261 = P \left(1 + \frac{5}{100}\right)^3 \]
\[ ⇒ 9,261 = P \left(\frac{21}{20}\right)^3 \]
\[ ⇒ P = \frac{9,261 \times 20 \times 20 \times 20}{21 \times 21 \times 21} \]
\[ = ₹ 8,000 \]

(c) Qualifying amounts for interest for various months

<table>
<thead>
<tr>
<th>Month</th>
<th>Amount</th>
<th>P</th>
<th>T</th>
<th>R</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>₹ 1,000</td>
<td>₹ 92,000</td>
<td>1</td>
<td>4.5%</td>
<td>₹ 345</td>
</tr>
<tr>
<td>February</td>
<td>₹ 4,500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>March</td>
<td>₹ 9,500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>April</td>
<td>₹ 9,500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>May</td>
<td>₹ 9,500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>June</td>
<td>₹ 15,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>July</td>
<td>₹ 15,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>August</td>
<td>₹ 14,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>September</td>
<td>₹ 14,000</td>
<td>₹ 92,000</td>
<td>1</td>
<td>4.5%</td>
<td>₹ 345</td>
</tr>
</tbody>
</table>
Question 8.

(a) Given that \( \frac{a^3 + 3ab^2}{b^3 + 3a^2b} = \frac{63}{62} \).

Using Componendo and Dividendo find \( a : b \). [3]

(b) In the above figure \( AB = 7 \text{ cm} \) and \( BC = 9 \text{ cm} \).

(i) Prove \( \triangle ACD \sim \triangle DCB \).

(ii) Find the length of \( CD \). [3]

(c) The given figure represents a hemisphere surmounted by a conical block of wood. The diameter of their bases is 6 cm each and the slant height of the cone is 5 cm. Calculate:

(i) the height of the cone.

(ii) the volume of the solid. [4]

Solution:

(a) Given:

\[ \frac{a^3 + 3ab^2}{b^3 + 3a^2b} = \frac{63}{62} \]

By componendo and dividendo,

\[ \frac{a^3 + 3ab^2 + b^3 + 3a^2b}{a^3 + 3ab^2 - b^3 - 3a^2b} = \frac{63 + 62}{63 - 62} \]

\[ \Rightarrow \frac{(a + b)^3}{(a - b)^3} = \frac{125}{1} = \left(\frac{5}{1}\right)^3 \]

\[ \Rightarrow \frac{a + b}{a - b} = 5 \]

Again componendo and dividendo,

\[ \frac{a + b + a - b}{a + b - a + b} = \frac{5 + 1}{5 - 1} = \frac{6}{4} \]

\[ \Rightarrow \frac{2a}{2b} = \frac{3}{2} \]

\[ \Rightarrow a : b = 3 : 2 \] Ans.

(b) Given: \( AB = 7 \text{ cm} \), \( BC = 9 \text{ cm} \).

(i) In \( \triangle ACD \) and \( \triangle DCB \),

\[ \angle C = \angle C \] (common)

\[ \angle CDB = \angle BAD \] (\( \angle \)'s alternate segment)

\( \triangle ACD \sim \triangle DCB \) (By A:A similarity)

Proved
(ii) \[ AC \times BC = CD^2 \quad \therefore \triangle ACD \sim \triangle DCB \]
\[ CD^2 = 16 \times 9 = 144 \]
\[ CD = 12 \text{ cm.} \quad \text{Ans.} \]

(c) Given: Diameter (AB) = 6 cm, \( r \) (OB) = 3 cm, \( l \) (DB) = 5 cm.

(i) \[ OD^2 = DB^2 - OB^2 \]
\[ = 25 - 9 = 16 \]
\[ OD = 4 \text{ cm} \quad \text{Ans.} \]

(ii) Volume of hemisphere = \[ \frac{2}{3} \pi r^3 \]
\[ = \frac{2}{3} \times \frac{22}{7} \times (3)^3 \]
\[ = 56.57 \text{ cm}^3. \]
Volume of cone = \[ \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 9 \times 4 = 37.71 \text{ cm}^3 \]
Volume of the solid = \[ 37.71 + 56.57 = 94.28 \text{ cm}^3. \quad \text{Ans.} \]

Question 9.
(a) Attempt this question on graph paper.

Marks obtained by 200 students in examination are given below

<table>
<thead>
<tr>
<th>Marks</th>
<th>No. of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10</td>
<td>5</td>
</tr>
<tr>
<td>10-20</td>
<td>10</td>
</tr>
<tr>
<td>20-30</td>
<td>21</td>
</tr>
<tr>
<td>30-40</td>
<td>34</td>
</tr>
<tr>
<td>40-50</td>
<td>40</td>
</tr>
<tr>
<td>50-60</td>
<td>60</td>
</tr>
<tr>
<td>60-70</td>
<td>80</td>
</tr>
<tr>
<td>70-80</td>
<td>100</td>
</tr>
</tbody>
</table>

Draw an Ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis.
From the graph find
(i) the Median
(ii) the Upper Quartile
(iii) Number of students scoring above 65 marks.
(iv) If 10 students qualify for merit scholarship, find the minimum marks required to qualify. [6]

(b) From two points A and B on the same side of a building, the angles of elevation of the top of the building are 30° and 60° respectively. If the height of the building is 10m, find the distance between A and B correct to two decimal places. [4]

Solution:

(a) | Marks | No. of students | cf |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>10-20</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>20-30</td>
<td>21</td>
<td>50</td>
</tr>
<tr>
<td>30-40</td>
<td>25</td>
<td>75</td>
</tr>
<tr>
<td>40-50</td>
<td>34</td>
<td>109</td>
</tr>
<tr>
<td>50-60</td>
<td>36</td>
<td>145</td>
</tr>
<tr>
<td>60-70</td>
<td>27</td>
<td>172</td>
</tr>
<tr>
<td>70-80</td>
<td>16</td>
<td>188</td>
</tr>
<tr>
<td>80-90</td>
<td>12</td>
<td>200</td>
</tr>
</tbody>
</table>
(i) Here \( n \) (no. of students) = 200 (even)

\[
\text{Median} = \left( \frac{n}{2} \right) \text{ term} = 100\text{th term.}
\]

From the graph 100\text{th term} is 57.5.
\[
\text{Median} = 57.5
\]

(ii) Upper quartile \((Q_3)\) = \(\frac{3n}{4} = \frac{3 \times 200}{4}\)th term

\[
= 3 \times 50 = 150 \text{ term}
\]

From Graph 150\text{th term} = 71.5

The upper quartile = 71.5

(iii) Number of students scoring above 65 marks = 200 - 128 = 72

(iv) Minimum marks to qualify = 92

(b) In \( \triangle DBC \),
\[
\tan 60^\circ = \frac{10}{x}
\]
\[
\Rightarrow \sqrt{3} = \frac{10}{x}
\]
\[
\Rightarrow x = \frac{10 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}
\]
\[
= \frac{10 \sqrt{3}}{3} \text{ m}
\]
In Δ DAC, \( \tan 30^\circ = \frac{10}{x+y} = \frac{1}{\sqrt{3}} \)

\[ \Rightarrow x + y = 10\sqrt{3} \]

\[ \Rightarrow y = \frac{10\sqrt{3}}{\sqrt{3}} - 10 \]

\[ = \frac{30 - 10}{\sqrt{3}} = \frac{20\sqrt{3}}{\sqrt{3}} \]

\[ = \frac{20}{3} \]

\[ = 11.55 \text{ m} \quad \text{Ans.} \]

Question 10.
(a) Mrs. Goswami deposits ₹1000 every month in a recurring deposit account for 3 years at 8% interest per annum. Find the matured value. [3]
(b) Find the equation of a line with x intercept = 5 and passing through the point (4, -7). [3]
(c) In a school the weekly pocket money of 50 students is as follows:

<table>
<thead>
<tr>
<th>Weekly pocket money in ₹</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80-90</th>
<th>90-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of students</td>
<td>2</td>
<td>8</td>
<td>12</td>
<td>14</td>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>

Draw a histogram and a frequency polygon on the same graph. Find the mode from the graph. [4]

Solution:
(a) Total Principal (P) for 1 month

\[ P = x \times \frac{n(n + 1)}{2} = 1000 \times \frac{36 \times 37}{2} \]

\[ = ₹ 6,66,000 \]

Interest for 1 month

\[ \text{Interest for 1 month} = \frac{PRT}{100} = \frac{6,66,000 \times 8 \times 1}{100 \times 12} \]

\[ = ₹ 4,440 \]

Total amount deposited by Mr. Goswami = 36 × 1000 = ₹ 36,000

Maturity value = ₹ 36,000 + 4,440

\[ = ₹ 40,440 \quad \text{Ans.} \]

(b) Equation of the line passing through (5, 0) and (4, -7):

\[ y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1) \]

\[ y - 0 = \frac{-7 - 0}{4 - 5} (x - 5) \]

\[ y = \frac{-7}{-1} (x - 5) \]

\[ \Rightarrow y = 7 (x - 5) \]

\[ \Rightarrow y = 7x - 35 \]

\[ \Rightarrow 7x - y - 35 = 0 \quad \text{Ans.} \]
Question 11.

(a) The model of a building is constructed with scale factor $1:30$.
   
   (i) If the height of the model is $80$ cm, find the actual height of the building in metres.
   
   (ii) If the actual volume of a tank at the top of the building is $27m^3$, find the volume of the tank on the top of the model.

(b) The speed of an express train is $x$ km/h and the speed of an ordinary train is $12$ km/h less than that of the express train. If the ordinary train takes one hour longer than the express train to cover a distance of $240$ km, find the speed of the express train.

(c) Using ruler and compasses construct

   (i) a triangle $ABC$ in which $AB = 5.5$ cm, $BC = 3.4$ cm and $CA = 4.9$ cm.
   
   (ii) the locus of points equidistant from $A$ and $C$.
   
   (iii) a circle touching $AB$ at $A$ and passing through $C$.

Solution:

(a) 

Scale factor $k = \frac{1}{30}$

(i) Height of the model = $k$ (times the height of the building)

\[ \Rightarrow \text{Height of building} = 80 \times 30 \]

\[ = 2400 \text{ cm} \]

\[ = 24 \text{ m} \] Ans.

(b) The speed of an ordinary train is $x - 12$ km/h. Let $t$ be the time taken by the express train to cover $240$ km.

Then, $t + 1 = \frac{240}{x}$

Solving for $x$ gives

\[ x = \frac{240}{t + 1} \]

(c) Using ruler and compasses, construct the required geometric figures as specified in the question.
(ii) Volume of model = \( k^3 \) times volume of the building
\[ = \frac{1}{30} \times \frac{1}{30} \times \frac{1}{30} \times 27 \text{ m}^3 \]
\[ = \frac{1}{1000} \text{ m}^3 = 0.001 \text{ m}^3 \]

(b) Time taken by express train = \( \frac{240}{x} \) h

Speed of ordinary train = \( (x - 12) \) km/h

Time taken by ordinary train = \( \frac{240}{x - 12} \)

According to the question,
\[ \frac{240}{x} = \frac{240}{x - 12} - 1 \]
\[ \Rightarrow 240 (x - 12) = 240 x - x (x - 12) \]
\[ \Rightarrow 240x - 2880 = 240 x - x^2 + 12x \]
\[ \Rightarrow x^2 - 12x - 2880 = 0 \]
\[ \Rightarrow x^2 - 60x + 48x - 2880 = 0 \]
\[ \Rightarrow x (x - 60) + 48 (x - 60) = 0 \]
\[ \Rightarrow (x - 60) (x + 48) = 0 \]

either \( x - 60 = 0 \) or \( x + 48 = 0 \)
\[ \Rightarrow x = 60 \text{ or } x = -48 \]

Hence, the speed of express train is 60 km/h.

(c) Steps of construction:

1. Draw \( \triangle ABC \) with given values.
2. Draw XY perpendicular bisector of AC.
3. Draw perpendicular of AB at A which cuts perpendicular XY at O.
4. Draw a circle at centre O which touching AB at A and passing through C i.e., required circle.