MATHEMATICS

SECTION A [40 Marks]
(Answer all questions from this Section.)

Question 1.
(a) Solve the following inequation and represent the solution set on the number line.

\[-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, \quad x \in \mathbb{R}\]  

(b) Tarun bought an article for ₹ 8,000 and spent ₹ 1,000 for transportation. He marked the article at ₹ 11,700 and sold it to a customer. If the customer had to pay 10% sales tax, find
(i) The customer’s price.
(ii) Tarun’s profit percent.

(c) Mr. Gupta opened a recurring deposit account in a bank. He deposited ₹ 2,500 per month for two years. At the time of maturity he got ₹ 67,500. Find:
(i) the total interest earned by Mr. Gupta.
(ii) the rate of interest per annum.

Solution:
(a) Given: \(-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, \quad x \in \mathbb{R}\)

\[-3 < -\frac{1}{2} - \frac{2x}{3} \quad \text{and} \quad -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}\]

\[-3 + \frac{1}{2} < -\frac{2x}{3} \quad \text{and} \quad -\frac{2x}{3} \leq \frac{5}{6} + \frac{1}{2}\]

\[-\frac{5}{2} < -\frac{2x}{3} \quad \text{and} \quad -\frac{2x}{3} \leq \frac{4}{3}\]

\[-\frac{5}{2} > \frac{2x}{3} \quad \text{and} \quad -x \leq 2\]

\[x < -\frac{15}{4} \quad \text{and} \quad x \geq -2\]

Solution set = \(\left\{x : \frac{15}{4} > x \geq -2\right\}\)

(b) Given: C.P. = ₹ 8,000 + ₹ 1,000 = ₹ 9,000, M.P. = ₹ 11,700, S.T. = 10%.
(i) Amount to be paid = M.P. + S.T. % of M.P.

= 11,700 + \(\frac{10}{100} \times 11,700\)

= ₹ 12,870  

Ans.
(ii)

\[
\text{Profit} = \text{M.P.} - \text{C.P.} = 11,700 - 9,000 = \text{₹} 2,700.
\]

Profit percent = \( \frac{\text{Profit}}{\text{C.P.}} \times 100 \)

\[
= \frac{2,700}{9,000} \times 100 = 30\%
\]

(c) Total amount deposited = \( \text{₹} (2,500 \times 24) = \text{₹} 60,000 \)

Equivalent principal for one month = \( \text{₹} 2,500 \times \frac{24 (24 + 1)}{2} = \text{₹} (62,500 \times 12) \)

(i) Total interest = 67,500 - 60,000 = \text{₹} 7,500

(ii) Interest on \( \text{₹} (62,500 \times 12) \) for 1 month

\[
= \text{₹} \left( 62,500 \times 12 \times \frac{R}{100} \times \frac{1}{12} \right)
\]

\[
= 7,500 = 625 \text{ R.} \quad \text{Ans.}
\]

Question 2.

(a) Given \( A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}, B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}, C = \begin{bmatrix} -4 \\ 5 \end{bmatrix} \) and \( D = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \)

Find \( AB + 2C - 4D. \) \([3]\)

(b) Nikita invests \( \text{₹} 6,000 \) for two years at a certain rate of interest compounded annually. At the end of the first year it amounts to \( \text{₹} 6,720 \). Calculate:

(i) the rate of interest.

(ii) the amount at the end of the second year. \([3]\)

(c) A and B are two points on the x-axis and y-axis respectively. P (2, -3) is the mid point of AB. Find the

(i) Coordinates of A and B.

(ii) Slope of line AB.

(iii) Equation of line AB. \([4]\)

Solution:

(a) \( \text{Given : } A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}, B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}, C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}, D = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \)

\[
AB + 2C - 4D = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} + 2 \begin{bmatrix} -4 \\ 5 \end{bmatrix} - 4 \begin{bmatrix} 2 \\ 2 \end{bmatrix}
\]

\[
= \begin{bmatrix} 16 & -8 \\ -2 & 10 \end{bmatrix} - \begin{bmatrix} 8 \\ 8 \end{bmatrix}
\]

\[
\begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0 \quad \text{Ans.} \]
(b) (i) Given: Principal = ₹ 6,000, Time = 2 year, After one year amount = ₹ 6,720.
   For 1st year: 
   \[ P + I = 6,720 \]
   \[ 6,000 + \frac{P \times R \times 1}{100} = 6,720 \]
   \[ \frac{6,000 \times R}{100} = 720 \]
   \[ \Rightarrow R = 12\% \]
   Ans.

(ii)
   \[ A = P \left(1 + \frac{r}{100}\right) \]
   Amount at the end of 2nd year = 6,000 \( \left(1 + \frac{12}{100}\right)^2 \)
   \[ = 6,000 \left(1 + \frac{3}{25}\right)^2 \]
   \[ = 6,000 \left(\frac{28}{25} \times \frac{28}{25}\right) = \frac{37,632}{5} \]
   \[ = ₹ 7,526.40. \]
   Ans.

(c) Given: A \((x_1, 0)\), B \((0, y_1)\)
   (i) Mid point coordinates
   \[ \frac{x_1 + 0}{2} = 2 \quad \Rightarrow \quad x_1 = 4 \]
   \[ \frac{0 + y_1}{2} = -3 \quad \Rightarrow \quad y_1 = -6 \]
   Coordinates of A \((4, 0)\) and B \((0, -6)\)
   Ans.

(ii) Slope of line AB
   \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
   \[ = \frac{-6 - 0}{0 - 4} = \frac{-6}{-4} = \frac{3}{2} \]
   Ans.

(iii) Equation of line
   \[ y - y_1 = m (x - x_1) \]
   \[ y - 0 = \frac{3}{2} (x - 4) \]
   \[ 2y = 3x - 12 \]
   \[ 3x - 2y - 12 = 0 \]
   Ans.

Question 3.
(a) Cards marked with numbers 1, 2, 3, 4 ... 20 are well shuffled and a card is drawn at random. What is the probability that the number of the cards is
   (i) a prime number
   (ii) divisible by 3
   (iii) a perfect square?
   [3]

(b) Without using trigonometric tables evaluate:
   \[ \frac{\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ}{\cosec^2 10^\circ - \tan^2 80^\circ} \]
   [3]
(c) (Use graph paper for this question)

A(0, 3), B(3, -2) and O(0, 0) are the vertices of triangle ABO.

(i) Plot the triangle on a graph sheet taking 2 cm = 1 unit on both the axes.

(ii) Plot D the reflection of B in the Y axis, and write its co-ordinates.

(iii) Give the geometrical name of the figure ABOD.

(iv) Write the equation of the line of symmetry of the figure ABOD.

Solution:

(a) Given : Cards marked with numbers 1, 2, ...... 20.

n(S) = 20

(i) Prime Numbers = 2, 3, 5, 7, 11, 13, 17, 19

n(E) = 8

P (Prime number) = P(A) = \frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5} \hspace{1cm} \text{Ans.}

(ii) No. divided by 3 = 3, 6, 9, 12, 15, 18

n(E) = 6

P (no. divided by 3) = P(A) = \frac{n(E)}{n(S)} = \frac{6}{20} = \frac{3}{10} \hspace{1cm} \text{Ans.}

(iii) No. perfect square = 1, 4, 9, 16

n(E) = 4

P (Perfect square) = P(A) = \frac{n(E)}{n(S)} = \frac{4}{20} = \frac{1}{5} \hspace{1cm} \text{Ans.}

(b) Given : 

\[
\frac{\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ}{\csc^2 10^\circ - \tan^2 80^\circ} = \frac{\sin (90 - 55)^\circ \cos 55^\circ + \cos (90 - 55)^\circ \sin 55^\circ}{\csc^2 10^\circ - \tan^2 (90 - 10)^\circ}
\]

\[
= \frac{\cos 55^\circ \cos 55^\circ + \sin 55^\circ \sin 55^\circ}{(1 + \cot^2 10^\circ) - \cot^2 10^\circ}
\]

\[
= \frac{\cos^2 55^\circ + \sin^2 55^\circ}{1 + \cot^2 10^\circ - \cot^2 10^\circ} = \frac{1}{1} = 1
\hspace{1cm} \text{Ans.}

(c) (i) See graph.
(ii) Coordinate of D = (-3, -2)
(iii) Geometrical name of ABOD is arrow.
(iv) Equation of the line of symmetry is 
    \[ x = 0 \]

Question 4.
(a) When divided by \( x - 3 \) the polynomials \( x^3 - px^2 + x + 6 \) and \( 2x^3 - x^2 - (p + 3) x - 6 \) leave the same remainder. Find the value of \( p \).

(b) In the figure given alongside AB and CD are two parallel chords and O is the centre. If the radius of the circle is 15 cm, find the distance MN between the two chords of length 24 cm and 18 cm respectively.

(c) The distribution given below shows the marks obtained by 25 students in an aptitude test. Find the mean, median and mode of the distribution.

<table>
<thead>
<tr>
<th>Marks obtained</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of students</td>
<td>3</td>
<td>9</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Solution:
(a) Given:
\[ f(x) = x^3 - px^2 + x + 6 \]
\[ g(x) = 2x^3 - x^2 - (p + 3) x - 6 \]
when \( f(x) \) is divided by \( (x - 3) \) remainder \( f(3) \) and \( f(x) \) is divided by \( (x - 3) \) remainder \( g(3) \).

\[ f(3) = g(3) \]
\[ (3)^3 - (3)^2 p + 3 + 6 = 2 (3)^3 - (3)^2 - (p + 3) 3 - 6 \]
\[ 27 - 9p + 3 + 6 = 54 - 9 - (p + 3) 3 - 6 \]
\[ 36 - 9p = 30 - 3p \]
\[ 9p - 3p = 36 - 30 \]
\[ 6p = 6 \]
\[ \Rightarrow p = 1 \]

(b) Given: OA = OC = 15 cm, AB = 24 cm, CD = 18 cm.
Now
\[ AM = 12, \ CN = 9 \]
In \( \triangle OAM, \)
\[ OA^2 = OM^2 + AM^2 \]
\[ OM^2 = OA^2 - AM^2 \]
\[ = 15^2 - 12^2 \]
\[ = 225 - 144 \]
\[ = 81 \]
\[ OM = 9 \]
Similarly, in \( \triangle OCN, \)
\[ OC^2 = ON^2 + CN^2 \]
\[ ON^2 = OC^2 - CN^2 = 15^2 - 9^2 \]
\[ = 225 - 81 = 144 \]
\[ ON = 12 \]
\[ MN = OM + ON = 9 + 12 = 21 \text{ cm}. \]
(c) \[ \begin{array}{cccc}
\text{x}_i & f_i & x_i f_i & \text{cf} \\
5 & 3 & 15 & 3 \\
6 & 9 & 54 & 12 \\
7 & 6 & 42 & 18 \\
8 & 4 & 32 & 22 \\
9 & 2 & 18 & 24 \\
10 & 1 & 10 & 25 \\
\end{array} \]

\[ \Sigma f = 25 \quad \Sigma x_i f_i = 171 \]

Mean \[ = \frac{\Sigma x_i f_i}{N} = \frac{171}{25} = 6.84 \quad \text{Ans.} \]

\[ n = 25 \text{ (odd)} \]

Median \[ = \left( \frac{n + 1}{2} \right)^{\text{th}} \text{term} = 13^{\text{th}} \text{term} = 7 \quad \text{Ans.} \]

Mode = 6 (maximum freq.) \quad \text{Ans.}

SECTION B [40 Marks]

Answer any four Questions in this Section.

Question 5.

(a) Without solving the following quadratic equation, find the value of ‘p’ for which the roots are equal.

\[ px^2 - 4x + 3 = 0 \]  

(b) Rohit borrows ₹ 86,000 from Arun for two years at 5% per annum simple interest. He immediately lends out this money to Akshay at 5% compound interest compounded annually for the same period. Calculate Rohit’s profit in the transaction at the end of the two years.

(c) Mrs. Kapoor opened a Saving Bank Account in State Bank of India on 9th January 2008. Her pass book entries for the year 2008 are given below

<table>
<thead>
<tr>
<th>Date</th>
<th>Particulars</th>
<th>Withdrawals (in ₹)</th>
<th>Deposits (in ₹)</th>
<th>Balance (in ₹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan 9, 2008</td>
<td>By Cash</td>
<td>—</td>
<td>10,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Feb 12, 2008</td>
<td>By Cash</td>
<td>—</td>
<td>15,500</td>
<td>25,500</td>
</tr>
<tr>
<td>April 6, 2008</td>
<td>To Cheque</td>
<td>3,500</td>
<td>—</td>
<td>22,000</td>
</tr>
<tr>
<td>April 30, 2008</td>
<td>To Self</td>
<td>2,000</td>
<td>—</td>
<td>20,000</td>
</tr>
<tr>
<td>July 16, 2008</td>
<td>By Cheque</td>
<td>—</td>
<td>6,500</td>
<td>26,500</td>
</tr>
<tr>
<td>Aug. 4, 2008</td>
<td>To Self</td>
<td>5,500</td>
<td>—</td>
<td>21,000</td>
</tr>
<tr>
<td>Aug. 20, 2008</td>
<td>To Cheque</td>
<td>1,200</td>
<td>—</td>
<td>19,800</td>
</tr>
<tr>
<td>Dec. 12, 2008</td>
<td>By Cash</td>
<td>—</td>
<td>1,700</td>
<td>21,500</td>
</tr>
</tbody>
</table>

Mrs. Kapoor closed the account on 31st December, 2008. If the bank pays interest at 4% per annum, find the interest Mrs. Kapoor receives on closing the account. Give your answer correct to the nearest rupee.

Solution:

(a) Roots are equal \[ \Rightarrow b^2 - 4ac = 0 \]

\[ b^2 = 4ac \]
Given: \( p = 3 \), \( c = \cdot \)

\[
16 = 4p,3
\]

\[
p = \frac{16}{4} = \frac{4}{3}
\]

Ans.

(b) Given: \( P = 86,000, R = 5\%, T = 2 \) years.

\[
\text{S.I.} = \frac{P \times R \times T}{100} = \frac{86,000 \times 5 \times 2}{100} = \text{₹} 8,600
\]

\[
C.I. = P \left(1 + \frac{R}{100}\right)^T - 1
\]

\[
= 86,000 \left[1 + \frac{5}{100}\right]^2 - 1
\]

\[
= 86,000 \left[\frac{21}{20}\right] - 1 = 86,000 \times \frac{41}{400} = \text{₹} 8,815
\]

Profit = C.I. - S.I. = 8,815 - 8,600 = ₹ 215

Ans.

(c) Minimum balance for the month

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>10,000</td>
</tr>
<tr>
<td>February</td>
<td>10,000</td>
</tr>
<tr>
<td>March</td>
<td>25,500</td>
</tr>
<tr>
<td>April</td>
<td>20,000</td>
</tr>
<tr>
<td>May</td>
<td>20,000</td>
</tr>
<tr>
<td>June</td>
<td>20,000</td>
</tr>
<tr>
<td>July</td>
<td>20,000</td>
</tr>
<tr>
<td>August</td>
<td>19,800</td>
</tr>
<tr>
<td>September</td>
<td>19,800</td>
</tr>
<tr>
<td>October</td>
<td>19,800</td>
</tr>
<tr>
<td>November</td>
<td>19,800</td>
</tr>
</tbody>
</table>

\[
\text{Principal} = \text{₹} 2,04,700, R = 4\%
\]

\[
\text{S.I.} = \frac{P \times R \times T}{100} = \frac{2,04,700 \times 4 \times 1}{100} = \text{₹} 682.33 = \text{₹} 682.
\]

Ans.

Question 6.

(a) A manufacturer marks an article for ₹ 5,000. He sells it to a wholesaler at a discount of 25% on the market price and the wholesaler sells it to a retailer at a discount of 15% on the market price. The retailers sells it to a consumer at the market price and at each stage the VAT is 8%. Calculate the amount of VAT received by the Government from:

(i) the wholesaler
(ii) the retailer.

[3]

(b) In the following figure O is the centre of the circle and AB is a tangent to it at point B. \( \angle BDC = 65^\circ \). Find \( \angle BAO \).

[3]
(c) A doorway is decorated as shown in the figure. There are four semi-circles. BC, the diameter of the larger semi-circle is of length 84 cm. Centres of the three equal semi-circles lie on BC. ABC is an isosceles triangle with AB = AC. If BO = OC, find the area of the shaded region. \[ \text{Take } \pi = \frac{22}{7} \]  

Solution:

(a) Given:

Cost of manufacturer = ₹ 5,000
S.P. of manufacturer = C.P. of wholesaler
\[ = 5,000 - \frac{25}{100} \times 5,000 \]
\[ = 5,000 - 1,250 \]
\[ = ₹ 3,750 \]

S.P. of wholesaler = C.P. of retailer
\[ = 5,000 - \frac{15}{100} \times 5,000 \]
\[ = 5,000 - 750 \]
\[ = ₹ 4,250 \]

S.P. of retailer = C.P. of consumer
\[ = ₹ 5,000 \]

(i) VAT by the wholesaler = \[ \frac{8}{100} \times 3,750 \]
\[ = ₹ 300 \] \hspace{1cm} \text{Ans.}

(ii) VAT by retailer = \[ \frac{8}{100} \times (4,250 - 3,750) \]
\[ = \frac{8}{100} \times 500 \]
\[ = ₹ 40 \] \hspace{1cm} \text{Ans.}

(b) AB is tangent \( \Rightarrow \angle ABO = 90^\circ \)

\[ \angle BDC = 65^\circ \text{ (given)} \]
\[ \Rightarrow \quad \angle BCD = 90^\circ - 65^\circ = 25^\circ \]
\[ \angle BOE = 2 \times 25^\circ \quad \text{(angle at centre)} \]
\[ = 50^\circ \]
\[ \angle BAO = 90^\circ - \angle BOE \]
\[ \angle BAO = 90^\circ - 50^\circ \]
\[ = 40^\circ \] \hspace{1cm} \text{Ans.}

(c) Let \( AB = AC = x \text{ cm} \).

As angle in semi circle is \( 90^\circ \)
\[ \text{i.e.,} \quad \angle A = 90^\circ \]

In right angled \( \triangle ABC \), by Pythagoras theorem, we get
\[ AB^2 + AC^2 = BC^2 \]
\[ x^2 + x^2 = 84^2 \]
\[2x^2 = 84 \times 84\]
\[x^2 = 84 \times 42\]

Now
\[
\text{Area of } \triangle ABC = \frac{1}{2} \times AB \times AC
\]
\[
= \frac{1}{2} \times 84 \times 42
\]
\[
= 1764 \text{ cm}^2.
\]

Diameter of semicircle \((2r) = 84 \text{ cm}\)

Radius \((r) = \frac{1}{2} \times 84 = 42 \text{ cm}\)

\[
\text{Area of semicircle} = \frac{1}{2} \pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 42 \times 42
\]
\[
= 2772 \text{ cm}^2.
\]

Diameter of each (three equal) semicircles = \(\frac{1}{3} \times 84 = 28 \text{ cm}\).

Radius of the 3 equal semicircles = \(\frac{1}{2} \times 28 = 14 \text{ cm}\).

\[
\text{Area of three equal semi circles} = 3 \times \frac{1}{2} \pi r^2
\]
\[
= 3 \times \frac{1}{2} \times \frac{22}{7} \times 14 \times 14
\]
\[
= 924 \text{ cm}^2.
\]

Area of shaded region = Area of semicircles + Area of three equal circles
- Area of \(\triangle ABC\)
\[
= 2772 + 924 - 1764
\]
\[
= 3696 - 1764
\]
\[
= 1932 \text{ cm}^2
\]

**Question 7.**

(a) Use ruler and compasses only for this question:

(i) Construct \(\triangle ABC\), where \(AB = 3.5 \text{ cm}, \ BC = 6 \text{ cm}\) and \(\angle ABC = 60^\circ\).

(ii) Construct the locus of points inside the triangle which are equidistant from \(BA\) and \(BC\).

(iii) Construct the locus of points inside the triangle which are equidistant from \(B\) and \(C\).

(iv) Mark the point \(P\) which is equidistant from \(AB, BC\) and also equidistant from \(B\) and \(C\). Measure and record the length of \(PB\). [3]

(b) The equation of a line is \(3x + 4y - 7 = 0\). Find

(i) the slope of the line.

(ii) the equation of a line perpendicular to the given line and passing through the intersection of the lines \(x - y + 2 = 0\) and \(3x + y - 10 = 0\). [3]

(c) The mean of the following distribution is 52 and the frequency of class interval 30-40 is \(f\). Find \(f\).

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>10-20</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
</tr>
</thead>
</table>
| Frequency      | 5     | 3     | \(f\) | 7     | 2     | 6     | 13    | [4]
Solution:

(a) Steps of Construction:

(i) Draw BC = 6 cm and make an angle at B = 60°. Cut BA = 3.5 cm and meet A to C. This is the required ΔABC.

(ii) Draw the bisector of ΔABC and perpendicular bisector of BC, both intersecting at P.

(iii) P is the required point. PB = 3.5 cm.

(b) Given: Equation of the line is

\[3x + 4y - 7 = 0\]

\[4y = -3x + 7\]

\[y = -\frac{3}{4}x + \frac{7}{4}\]

(i) Slope of the line \(m_1 = -\frac{3}{4}\) \hspace{1cm} Ans.

(ii) Slope of the perpendicular line \(m_2 = \frac{-1}{m_1} = -\frac{1}{-\frac{3}{4}} = \frac{4}{3}\)

Intersection of the lines \(x - y + 2 = 0\) \hspace{1cm} ...(i)

and \(3x + y - 10 = 0\) \hspace{1cm} ...(ii)

By Adding equation (i) and (ii) \(4x = 8\)

Put \(x = 2\), in equation (i) we get

\[2 - y + 2 = 0\]

\[y = 4\]

Equation of line \(y - y_1 = m_2 (x - x_1)\)

\[y - 4 = \frac{4}{3}(x - 2)\]

\[4x - 3y + 4 = 0\] \hspace{1cm} Ans.

(c) Interval | Frequency \(f_i\) | \(x_i\) | \(d_i = x_i - \bar{X}\) | \(f_id_i\)
--- | --- | --- | --- | ---
10–20  | 5  | 15  | -30  | -150  |
20–30  | 3  | 25  | -20  | -60   |
30–40  | \(f\) | 35  | -10  | -10f  |
40–50  | 7  | 45  | \(\bar{X}\) | 0     |
50–60  | 2  | 55  | 10   | 20    |
60–70  | 6  | 65  | 20   | 120   |
70–80  | 13 | 75  | 30   | 390   |
\[36 + f\] \hspace{1cm} \(\Sigma f_id_i = 320 - 10f\)
Mean = \( A + \frac{\sum fd_i}{N} \)

\[ 52 = 45 + \frac{320 - 10f}{36 + f} \]

\[ \Rightarrow \]

\[ 7 = \frac{320 - 10f}{36 + f} \]

\[ \Rightarrow \]

\[ 252 + 7f = 320 - 10f \]

\[ \Rightarrow \]

\[ 17f = 68 \]

\[ \Rightarrow \]

\[ f = 4 \]

**Question 8.**

(a) *Use the Remainder Theorem to factorise the following expression:*

\[ 2x^3 + x^2 - 13x + 6 \]

(b) *If \( x, y, z \) are in continued proportion, prove that \( (x+y)^2 = \frac{x}{y+z} \).*

(c) *From the top of a light house 100 m high the angles of depression of two ships on opposite sides of it are 48° and 36° respectively. Find the distance between the two ships to the nearest metre.*

**Solution:**

(a) *Given:*

\[ f(x) = 2x^3 + x^2 - 13x + 6 \]

\[ f(1) = 2 + 1 - 13 + 6 \neq 0 \]

\[ f(-1) = -2 + 1 + 13 + 6 \neq 0 \]

\[ f(2) = 16 + 4 - 26 + 6 = 0 \]

So, \( x - 2 \) is one factor of \( f(x) \) by remainder theorem.

\[ 2x^3 + 5x - 3 \]

\[ x - 2 \)

\[ 2x^3 + x^2 - 13x + 6 \]

\[ 2x^3 + 4x^2 \]

\[ 5x^2 - 13x + 6 \]

\[ 5x^2 - 10x \]

\[ -3x + 6 \]

\[ -3x + 6 \]

\[ \times \]

\[ \therefore \text{ The other factors of } f(x) \text{ are the factors of } 2x^2 + 5x - 3. \]

\[ = 2x^2 + 6x - x - 3 \]

\[ = 2x(x + 3) - 1(x + 3) \]

\[ = (2x - 1)(x + 3) \]

Hence, \( 2x^3 + x^2 - 13x + 6 = (2x - 1)(x + 3)(x - 2) \)
(b) If \(x, y, z\) are in continued proportion

\[
\frac{x}{y} = \frac{y}{z} = k
\]

\[\Rightarrow y = kz\]

and
\[x = xy = k^2z\]

L.H.S. = \[
\frac{(x + y)^2}{(y + z)^2} = \frac{(k^2z + kz)^2}{(kz + z)^2}
\]

\[= \frac{k^2z^2(k + 1)^2}{z^2(k + 1)^2} = k^2\]

R.H.S. = \[
\frac{x}{z} = \frac{k^2z}{z} = k^2
\]

Hence

L.H.S. = R.H.S. \hspace{1cm} \text{Proved}

(c) In \(\triangle ABD\),

\[\tan 48^\circ = \frac{AD}{BD}\]

\[\Rightarrow 1.11 = \frac{100}{BD}\]

\[BD = 100 \times \frac{1}{1.11} = 90.09 \text{ m}\]

In \(\triangle ACD\),

\[\tan 36^\circ = \frac{AD}{DC}\]

\[0.7265 = \frac{100}{DC}\]

\[\Rightarrow DC = \frac{100}{0.7265} = 137.64 \text{ m}\]

\[BC = BD + DC\]

\[= 90.09 + 137.64\]

\[= 227.73 \text{ m}\]

\[\text{Ans.}\]

Question 9.

(a) Evaluate:

\[
\begin{bmatrix}
4 \sin 30^\circ & 2 \cos 60^\circ \\
\sin 90^\circ & 2 \cos 0^\circ
\end{bmatrix}
\begin{bmatrix}
4 & 5 \\
5 & 4
\end{bmatrix}
\]

[3]

(b) In the figure \(\triangle ABC\) is a triangle with \(\angle EDB = \angle ACB\).

Prove that \(\triangle ABC \sim \triangle EBD\).

If \(BE = 6 \text{ cm}, EC = 4 \text{ cm}, BD = 5 \text{ cm}\) and area of \(\triangle BDE\) = 9 cm\(^2\). Calculate the:

(i) length of \(AB\)

(ii) area of \(\triangle ABC\). [4]

(c) Vivek invests \(\text{Rs} 4,500\) in 8%, \(\text{Rs} 10\) shares at \(\text{Rs} 15\). He sells the shares when the price rises to \(\text{Rs} 30\), and invests the proceeds in 12% \(\text{Rs} 100\) shares at \(\text{Rs} 125\).

Calculate:

(i) the sale proceeds.

(ii) the number of \(\text{Rs} 125\) shares he buys.

(iii) the change in his annual income from dividend. [4]
Solution:

(a) Given:

\[
\begin{bmatrix}
4 \sin 30^\circ & 2 \cos 60^\circ \\
\sin 90^\circ & 2 \cos 0^\circ 
\end{bmatrix}
\begin{bmatrix}
4 & 5 \\
5 & 4
\end{bmatrix}
\]

= \[
\begin{bmatrix}
4 \times \frac{1}{2} & 2 \times \frac{1}{2} \\
1 & 2 \times 1
\end{bmatrix}
\begin{bmatrix}
4 & 5 \\
5 & 4
\end{bmatrix}
\]

= \[
\begin{bmatrix}
2 & 1 \\
1 & 2
\end{bmatrix}
\begin{bmatrix}
4 & 5 \\
5 & 4
\end{bmatrix}
\]

= \[
\begin{bmatrix}
13 & 14 \\
14 & 13
\end{bmatrix}
\]

\[
\begin{bmatrix}
13 & 14 \\
14 & 13
\end{bmatrix}
\]

(b) \(\angle EDB = \angle ACB\) (given)
\(\angle DBE = \angle ABC\)
\(\angle DEB = \angle BAC\) (AA axiom)
\(\Delta ABC \sim \Delta EBD\) Proved

(i) Given: \(BE = 6\) cm, \(EC = 4\) cm, \(BD = 5\) cm.

\[
\frac{AB}{BE} = \frac{BC}{BD} = \frac{AC}{ED}
\]

\[
\frac{AB}{BE + EC} = \frac{6 + 4}{5} = \frac{10}{5} = 2
\]

\(AB = 12\) cm Ans.

(ii) \[
\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta EBD} = \frac{AB^2}{EB^2} = \frac{144}{36} = \frac{4^2}{2^2}
\]

\(\Rightarrow\) Area of \(\Delta ABC = \frac{144 \times 9}{36} = 36\) m. Ans.

(c) Number of shares bought = \(\frac{4,500}{15}\) = 300

Total face value = \(\text{Rs} 300 \times 10\)

= \(\text{Rs} 3,000\)

Dividend = \(\frac{8}{100} \times 3,000\)

= \(\text{Rs} 240\).

Amount received on selling 300 shares for \(\text{Rs} 300 \times 30 = \text{Rs} 9,000\)

(i) Sale proceeds = \(\text{Rs} 9,000 - \text{Rs} 4,500 = \text{Rs} 4,500\) Ans.

(ii) Number of shares bought at \(\text{Rs} 125\) = \(\frac{9,000}{125}\) = 72 Ans.
(iii) Total face value of 72 shares = \( \text{₹} 72 \times 100 \)
\[ = \text{₹} 7,200 \]
Dividend = \( \frac{12}{100} \times 7,200 \)
\[ = \text{₹} 864. \]
Change in his annual income = 864 - 240
\[ = \text{₹} 624. \]

**Question 10.**

(a) A positive number is divided into two parts such that the sum of the squares of the two parts is 208. The square of the larger part is 8 times the smaller part. Taking \( x \) as the smaller part of the two parts, find the number.  

(b) The monthly income of a group of 320 employees in a company is given below:

<table>
<thead>
<tr>
<th>Monthly Income</th>
<th>No. of Employees</th>
</tr>
</thead>
<tbody>
<tr>
<td>6000–7000</td>
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</tr>
<tr>
<td>7000–8000</td>
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</tr>
<tr>
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</tr>
<tr>
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</tr>
<tr>
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<td>30</td>
</tr>
<tr>
<td>12000–13000</td>
<td>5</td>
</tr>
</tbody>
</table>

Draw an ogive of the given distribution on a graph sheet taking 2 cm = ₹ 1,000 on one axis and 2 cm = 50 employees on the other axis. From the graph determine:

(i) the median wage.
(ii) the number of employees whose income is below ₹ 8,500
(iii) If the salary of a senior employee is above ₹ 11,500, find the number of senior employees in the company.
(iv) the upper quartile.

**Solution:**

(a) Let \( x \) and \( y \) are the two parts.
\[ x^2 + y^2 = 208 \]  
\[ y^2 = 8x \]
\[ \Rightarrow x^2 + 8x - 208 = 0 \]
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Here, \( a = 1, b = 8, c = -208 \)
\[ x = \frac{-8 \pm \sqrt{(8)^2 - 4 \times 1 \times (-208)}}{2 \times 1} \]
\[ = \frac{-8 \pm \sqrt{64 + 832}}{2} \]
\[ = \frac{-8 \pm \sqrt{912}}{2} \]
\[ = \frac{-8 \pm 30.18}{2} \]
\[ x = 11.09 \, \text{or} \, x = -23.09 \]

(b) The monthly income of a group of 320 employees in a company is given below:

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**Solution:**

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\[ = \frac{-8 \pm \sqrt{912}}{2} \]
\[ = \frac{-8 \pm 30.18}{2} \]
\[ x = 11.09 \, \text{or} \, x = -23.09 \]
-8 ± \sqrt{64 + 832}

\begin{align*}
-8 ± 29·93 &= \frac{-8 ± 29·93}{2} \quad \text{or} \quad \frac{-8 - 29·93}{2} \\
&= -18·96 \text{ or } 10·97
\end{align*}

\begin{align*}
y^2 &= 8x \\
&= 8 \times 10·97 \\
&= 87·76 \\
y &= 9·37
\end{align*}

Number = x + y

\begin{align*}
&= 10·97 + 9·37 \\
&= 20·34
\end{align*}

(ii) Number of employees whose income is below ₨ 8,500 = 92·5 approx.  \textbf{Ans.}
(iii) Number of senior employees in the company = 320 – 302 = 18. Ans.

(iv) Upper quartile = \( \frac{3n}{4} = \frac{3 \times 320}{4} = 240 \)

Upper quartile = 10,200.

**Question 11.**

(a) Construct a regular hexagon of side 4 cm. Construct a circle circumscribing the hexagon. [3]

(b) A hemispherical bowl of diameter 7.2 cm is filled completely with chocolate sauce. This sauce is poured into an inverted cone of radius 4.8 cm. Find the height of the cone. [3]

(c) Given \( x = \frac{\sqrt{a^2 + b^2} + \sqrt{a^2 - b^2}}{\sqrt{a^2 + b^2} - \sqrt{a^2 - b^2}} \)

Use componendo and dividendo to prove that \( b^2 = \frac{2a^2x}{x^2 + 1} \). [4]

**Solution:**

(a) **Steps of Construction:**

(i) Using the given data, construct the regular hexagon ABCDEF with each side equal to 4 cm.

(ii) Draw the perpendicular bisectors of sides AB and AF which intersect each other at point O.

(iii) With O as centre and OA as radius draw a circle which will pass through all the vertices of the regular hexagon ABCDEF.

(b) Given:
- Diameter of hemispherical bowl = 7.2 cm
- Radius of hemispherical bowl = 3.6 cm
- Volume of hemispherical bowl = \( \frac{2}{3} \times \pi r^3 \)
  \[
  = \frac{2}{3} \times \frac{22}{7} \times 3.6 \times 3.6 \times 3.6
  = 97.76 \text{ cm}^3.
  \]
- Volume of cone = \( \frac{1}{3} \times \pi R^2h \)
  \[
  = \frac{1}{3} \times \frac{22}{7} \times 4.8 \times 4.8 \times h
  = 24.14 \times h \text{ cm}^3
  \]
- Volume of cone = Volume of hemispherical bowl
  \[
  24.14 \times h = 97.76
  \]
  \[
  h = \frac{97.76}{24.14}
  = 4.05 \text{ cm.} \quad \text{Ans.}
  \]
(e) \( \text{Given} \quad x = \frac{\sqrt{a^2 + b^2} + \sqrt{a^2 - b^2}}{\sqrt{a^2 + b^2} - \sqrt{a^2 - b^2}} \)

Componendo and dividendo

\[
\frac{x + 1}{x - 1} = \frac{(\sqrt{a^2 + b^2} + \sqrt{a^2 - b^2}) + (\sqrt{a^2 + b^2} - \sqrt{a^2 - b^2})}{(\sqrt{a^2 + b^2} + \sqrt{a^2 - b^2}) - (\sqrt{a^2 + b^2} - \sqrt{a^2 - b^2})}
\]

\[
= \frac{2(\sqrt{a^2 + b^2})}{2\sqrt{a^2 - b^2}}
\]

\[
\frac{(x + 1)^2}{(x - 1)^2} = \frac{a^2 + b^2}{a^2 - b^2}
\]

Again componendo and dividendo

\[
\frac{(x + 1)^2 + (x - 1)^2}{(x + 1)^2 - (x - 1)^2} = \frac{a^2 + b^2 + a^2 - b^2}{a^2 + b^2 - a^2 + b^2}
\]

\[
\frac{2x^2 + 2}{4x} = \frac{2a^2}{2b^2}
\]

\[
\frac{x^2 + 1}{2x} = \frac{a^2}{b^2}
\]

\[
b^2 = \frac{2a^2x}{x^2 + 1} \quad \text{Proved}
\]