# ICSE Question Paper (2011) MATHEMATICS 

SECTION A [40 Marks]<br>(Answer all questions from this Section.)

## Question 1.

(a) Find the value of ' $k$ ' if $(x-2)$ is a factor of:

$$
x^{3}+2 x^{2}-k x+10
$$

Hence determine whether $(x+5)$ is also a factor.
(b) If $A=\left[\begin{array}{rr}3 & 5 \\ 4 & -2\end{array}\right]$ and $B=\left[\begin{array}{l}2 \\ 4\end{array}\right]$, is the product $A B$ possible? Give a reason. If yes, find $A B$.
(c) Mr. Kumar borrowed ₹ 25,000 for two years. The rate of interest for the two successive years are $8 \%$ and $10 \%$ respectively. If he repays $₹ 6,200$ at the end of the first year, find the outstanding amount at the end of the second year. [4]

## Solution :

(a) Let

$$
f(x)=x^{3}+2 x^{2}-k x+10
$$

$\because(x-2)$ is a factor,

$$
\begin{aligned}
f(2) & =0 \\
f(2) & =8+8-2 k+10=0 \\
k & =13
\end{aligned}
$$

To check for $(x+5)$ is a factor,

$$
\begin{aligned}
f(-5) & =(-5)^{3}+2(-5)^{2}-13(-5)+10 \\
& =-125+50+65+10=0
\end{aligned}
$$

$\therefore(x+5)$ is a factor.
(b) $\mathrm{A}_{2 \times 2} \cdot \mathrm{~B}_{2 \times 1}$

From the order of both the matrix $A$ and $B$, it is clear that $A B$ is possible because the number of columns of $A$ are equal to the number of rows of $B$.

$$
\begin{aligned}
\mathrm{AB} & =\left[\begin{array}{rr}
3 & 5 \\
4 & -2
\end{array}\right] \cdot\left[\begin{array}{l}
2 \\
4
\end{array}\right] \\
& =\left[\begin{array}{l}
6+20 \\
8-8
\end{array}\right] \\
& =\left[\begin{array}{r}
26 \\
0
\end{array}\right]
\end{aligned}
$$

Ans.
(c) Given: Principal $=\mathbf{₹} \mathbf{1 5 , 0 0 0}$

We know that

$$
\mathrm{A}=\mathrm{P}\left(1+\frac{r}{100}\right)
$$

$$
\begin{aligned}
\text { Amount after } 1^{\text {st }} \text { year } & =15,000\left(1+\frac{8}{100}\right) \\
& =श 16,200
\end{aligned}
$$

Principal after repayment $=16,200-6,200=₹ 10,000$
Amount outstanding at the end of second year

$$
\begin{aligned}
& =10,000\left(1+\frac{10}{100}\right) \\
& =₹ 11,000
\end{aligned}
$$

Ans.

## Question 2.

(a) From a pack of 52 playing cards all cards whose numbers are multiples of 3 are removed. A card is now drawn at random.
What is the probability that the card drawn is
(i) a face card (King, Jack or Queen)
(ii) an even numbered red card?
[3]
(b) Solve the following equation:
$x-\frac{18}{x}=6$. Give your answer correct to two significant figures.
(c) In the given figure $O$ is the centre of the circle. Tangents at $A$ and $B$ meet at C. If $\angle A O C=30^{\circ}$, find
(i) $\angle B C O$
(ii) $\angle A O B$
(iii) $\angle A P B$

## Solution :

(a) Number of cards which are multiples of $3=12$
(i)

$$
\text { Cards left in the pack }=40
$$



$$
\text { Number of face cards }=12
$$

$$
P(\text { face card })=\frac{12}{40}=\begin{gathered}
3 \\
10
\end{gathered}
$$

Ans.
(ii)

$$
\text { Even numbered red cards }=10
$$

$$
P(\text { even number red card })=\frac{10}{40}=\frac{1}{4}
$$

Ans.
(b) Let

$$
x^{2}-6 x-18=0
$$

Compare with equation $a x^{2}+b x+c=0$, we get $a=1, b=-6, c=-18$

Now,

$$
\begin{aligned}
& x=\frac{b \pm \sqrt{b^{2}-4 a c}}{2 a} \\
&=6 \pm \sqrt{36+72} \\
& 2
\end{aligned}
$$

$$
\begin{aligned}
& =\begin{array}{c}
6 \pm 6 \sqrt{3} \\
2
\end{array} \Rightarrow x=3 \pm 3 \sqrt{3} \\
x & =3 \pm 5 \cdot 196
\end{aligned}
$$

Taking +ve and $-v e$ sign respectively, we get
(c)

$$
x=8.196 \text { or } x=-2.196
$$

Ans.

$$
\triangle \mathrm{ACO} \cong \triangle \mathrm{BCO}
$$

(R.H.S.)

$$
\begin{equation*}
\angle \mathrm{BCO}=\angle \mathrm{ACO} \tag{С.Р.С.Т.}
\end{equation*}
$$

(i)

$$
\angle B C O=30^{\circ}
$$

In $\triangle \mathrm{ACO}, \quad \quad \angle \mathrm{OAC}=90^{\circ} \quad$ (Radius is perpendicular to tangent) $\angle \mathrm{AOC}=60^{\circ}$
Also $\quad \angle B O C=60^{\circ}$
(C.P.C.T.)
(ii)

$$
\angle \mathrm{AOB}=120^{\circ}
$$

(iii)
$\angle \mathrm{APB}=60^{\circ} \quad$ (Angle at circumference is half the angle at the centre)

## Question 3.

(a) Ahmed has a recurring deposit account in a bank. He deposits ₹ 2,500 per month for 2 years. If he gets ₹ 66,250 at the time of maturity, find
(i) The interest paid by the bank.
(ii) The rate of interest.
[3]
(b) Calculate the area of the shaded region, if the diameter of the semi circle is equal to 14 cm .
Take $\pi=\frac{22}{7}$
[3]

(c) $A B C$ is a triangle and $G(4,3)$ is the centroid of the triangle. If $A=(1,3), B=$ $(4, b)$ and $C=(a, 1)$, find ' $a$ ' and ' $b$ '. Find the length of side $B C$.

## Solution :

(a) (i)

$$
\begin{aligned}
\text { Interest } & =66,250-2,500 \times 24 \\
& =66,250-60,000 \\
& =₹ 6,250
\end{aligned}
$$

'Ans.
(ii)

$$
\begin{aligned}
\text { Principal } & =\begin{array}{c}
n(n+1) \\
2
\end{array}{ }^{24} \text { sum of del } \\
& \left.=\frac{(24}{2}+1\right) \times 2,500 \\
& =\frac{24 \times 25}{2} \times 2,500 \\
I & =\text { Principal } \times \frac{\mathrm{R}}{100} \times \frac{1}{12}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{R} & =\frac{6,250 \times 2 \times 100 \times 12}{2,500 \times 24 \times 25} \\
& =10 \% \text { p.a. }
\end{aligned}
$$

Ans.
(b) Area of shaded portion = Complete area - area of the two quadrants
$=$ (Area of ACDE + Area of semi circle EFD)

- (Area of Quadrant ABE +

Area of Quadrant BCD)
$=\left\{14 \times 7+\frac{\pi}{2}(7)^{2}\right\}-\left\{\frac{\pi}{4}(7)^{2}+\frac{\pi}{4}(7)^{2}\right\}$
$=\left\{14 \times 7+\frac{\pi}{2}(7)^{2}\right\}-\left\{\frac{\pi}{2}(7)^{2}\right\}$
$=98 \mathrm{~cm}^{2}$.
Ans.
(c) Coordinate of centroid G $\left(x_{1}+\frac{x_{2}}{3}+x_{3}, \underline{y}_{1}+\underline{y}_{2}+y_{3}\right)$

$$
\begin{array}{cccc}
\Rightarrow & \begin{array}{c}
1+4+a \\
3
\end{array} & \Rightarrow & a=7 \\
3+b+1=3 & \Rightarrow & b=5
\end{array}
$$

Now,

$$
\begin{aligned}
\mathrm{BC} & =\sqrt{ }(4-7)^{2}+(5-1)^{2} \\
& =\sqrt{ } 9+16=5 \text { units. }
\end{aligned}
$$



## Question 4.

(a) Solve the following inequation and represent the solution set on the number line $2 x-5 \leq 5 x+4<11$, where $x \in I$.
(b) Evaluate without using trigonometric tables:

$$
\begin{equation*}
2\left(\frac{\tan 35^{\circ}}{\cot 55^{\circ}}\right)^{2}+\left(\frac{\cot }{\tan } \frac{55^{\circ}}{35^{\circ}}\right)^{2}-3\left(\frac{\sec 40^{\circ}}{\operatorname{cosec} 50^{\circ}}\right) \tag{3}
\end{equation*}
$$

(c) A Mathematics aptitude test of 50 students was recorded as follows:

| Marks | $50-60$ | $60-70$ | $70-80$ | $80-90$ | $90-100$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| No. of students | 4 | 8 | 14 | 19 | 5 |

Draw a histogram for the above data using a graph paper and locate the mode.

## Solution:

(a) Given: $2 x-5 \leq 5 x+4$ and $5 x+4<11$

$$
\begin{aligned}
& -3 x \leq 9 \\
& x \geq-3
\end{aligned}
$$

Solution set,

$$
\begin{aligned}
& x \in\{-3,-2,-1,0,1\}
\end{aligned}
$$

(b) Given: $\quad 2\left(\frac{\tan }{\cot } \frac{35^{\circ}}{55^{\circ}}\right)^{2}+\left(\frac{\cot }{\tan } \frac{55^{\circ}}{35^{\circ}}\right)-3\left(\frac{\sec 40^{\circ}}{\operatorname{cosec}} 50^{\circ}\right)$

$$
=2\left(\frac{\tan \left(90^{\circ}-55^{\circ}\right)}{\cot 55^{\circ}}\right)^{2}+\left(\frac{\cot \left(90^{\circ}-35^{\circ}\right)}{\tan 35^{\circ}}\right)=3\left(\frac{\sec \left(90^{\circ}-50^{\circ}\right)}{\operatorname{cosec} 50^{\circ}}\right)
$$

$$
=2\left(\frac{\cot }{\cot } \frac{55^{\circ}}{55^{\circ}}\right)^{2}+\left(\frac{\tan }{\tan } \frac{35^{\circ}}{35^{\circ}}\right)-3\left(\frac{\operatorname{cosec}}{\operatorname{cosec}} \frac{50^{\circ}}{50^{\circ}}\right)
$$

$$
=2+1-3=0
$$

Ans.
(c)


Mode from graph $=82 \cdot 5$.
SECTION B [40 Marks]
Answer any four Questions in this Section.
Question 5.
(a) A manufacturer sells a washing machine to a wholesaler for $₹ 15,000$. The wholesaler sells it to a trader at a profit of $₹ 1,200$ and the trader in turn sells it to a consumer at a profit of $₹ 1,800$. If the rate of VAT is $8 \%$ find :
(i) The amount of VAT received by the State Government on the sale of this machine from the manufacturer and the wholesaler.
(ii) The amount that the consumer pays for the machine.
(b) A solid cone of radius 5 cm and height 8 cm is melted and made into small spheres of radius 0.5 cm . Find the number of spheres formed.
(c) $A B C D$ is $\alpha$ parallelogram where $A(x, y), B(5,8), C(4,7)$ and $D(2,-4)$. Find
(i) Coordinates of $A$
(ii) Equation of diagonal BD.

## Solution:

(a) (i) VAT received by Govt. from manufacturer $=15,000 \times{ }_{100}^{8}=₹ 1,200$

$$
\text { VAT from wholesaler }=1200 \times{ }_{100}^{8}=₹ 96
$$

Total VAT from manufacturer and wholesaler

$$
=1200+96=₹ 1296 \text { Ans. }
$$

(ii) Amount that customer pays $=(15000+1200+1800)+$ VAT

$$
\begin{aligned}
& =18,000+\frac{18,000 \times 8}{100} \\
& =18,000+1440=₹ 19,440
\end{aligned}
$$

Ans.
(b)

$$
\begin{aligned}
\text { Number of spheres } & =\begin{array}{c}
\text { Volume of cone } \\
\text { Volume of each sphere }
\end{array} \\
& \frac{1}{3} \pi(5)^{2}(8) \\
& -\frac{4}{3} \pi(0.5)^{8}-5 \times 10^{9} \\
& =400
\end{aligned}
$$

Ans.
(c) In a parallelogram, mid point of diagonal BD co-incides with the mid point of diagonal AC.

Mid point of $\left.\mathrm{BD}=5 \frac{ \pm}{2}, \frac{8-4}{2}\right)=\left(\frac{7}{2}, 2\right)$
Mid point of $\left.\mathrm{AC}=\frac{x+4}{2} ; \frac{y+7}{2}\right)$
Equating, $\quad \frac{x+4}{2}=\frac{7}{2} \Rightarrow x=3$

$$
\frac{y+7}{2}=2 \Rightarrow y=-3
$$


(i) Co-ordinates of A $(3,-3)$.

Ans.

$$
m \text { of } \mathrm{BD}=\begin{gather*}
8-(-4)  \tag{ii}\\
5-2
\end{gather*}=\frac{12}{3}=4
$$

Equation of BD,

$$
\begin{aligned}
y-y_{1} & =m\left(x-x_{1}\right) \\
y+4 & =4(x-2) \\
y & =4 x-4
\end{aligned}
$$

Ans.

## Question 6.

(a) Use a graph paper to answer the following questions. (Take $1 \mathrm{~cm}=1$ unit on both axes).
(i) Plot $A(4,4), B(4,-6)$ and $C(8,0)$, the vertices of a triangle $A B C$.
(ii) Reflect $A B C$ on the $y$-axis and name it as $A B^{\prime} C^{\prime}$.
(iii) Write the coordinates of the image $A^{\prime}, B^{\prime}$ and $C^{\prime}$.
(iv) Give a geometrical name for the figure $A A^{\prime} C^{\prime} B^{\prime} B C$.
(v) Identify the line of symmetry of $A A^{\prime} C^{\prime} B^{\prime} B C$.
(b) Mr. Choudhury opened a Saving's Bank Account at State Bank of India on 1st

April 2007. The entries of one year as shown in his pass book are given below :

| Date | Particulars | Withdrawals <br> (in ₹) | Deposits <br> (in ₹) | Balance <br> (in ₹) |
| :--- | :--- | :---: | :---: | :---: |
| 1st April 2007 | By Cash | - | 8550.00 | 8550.00 |
| 12th April 2007 | To Self | 1200.00 | - | 7350.00 |
| 24th April 2007 | By Cash | - | 4550.00 | 11900.00 |


| 8th July 2007 | By Cheque |  | 1500.00 | 13400.00 |
| :--- | :--- | :--- | :--- | :--- |
| 10th Sept. 2007 | By Cheque |  | 3500.00 | 16900.00 |
| 17th Sept. 2007 | To Cheque | 2500.00 |  | 14400.00 |
| 11th Oct. 2007 | By Cash |  | 800.00 | 15200.00 |
| 6th Jan. 2008 | To Self | 2000.00 |  | 13200.00 |
| 9th March 2008 | By Cheque |  | 950.00 | 14150.00 |

If the bank pays interest at the rate of $5 \%$ per annum, find the interest paid on 1st April, 2008. Give your answer correct to the nearest rupee.

## Solution :

(a) (i) In the given diagram.
(ii) In the given diagram.
(iii) $\mathrm{A}^{\prime}(-4,4)$
$B^{\prime}(-4,-6)$
$C^{\prime}(-8,0)$
(iv) Hexagon.
(v) Y-Axis is the line of symmetry.
(b)

## Minimum Balances

| April | $7,350.00$ |  |
| :--- | ---: | :--- |
| May | $11,900.00$ |  |
| June | $11,900.00$ |  |
| July | $13,400.00$ |  |
| August | $13,400.00$ |  |
| September | $14,400.00$ |  |
| October | $14,400.00$ |  |
| November | $15,200.00$ |  |
| December | $15,200.00$ |  |
| January | $13,200.00$ |  |
| February | $13,200.00$ |  |
| March | $14,150.00$ |  |
|  | $1,57,700.00$ |  |
|  | 1 | $=\frac{1,57,700 \times 5}{100} \times 1$ |
|  | $=\frac{7885}{12}=₹ 657.08$ |  |
|  |  |  |

Ans.

## Question 7.

(a) Using componendo and dividendo, find the value of $x$

$$
\frac{\sqrt{ } 3 x+4+\sqrt{3 x-5}}{\sqrt{3 x+4-\sqrt{3 x-5}}=9}
$$

(b) If $A=\left[\begin{array}{ll}2 & 5 \\ 1 & 3\end{array}\right], B=\left[\begin{array}{rr}4 & -2 \\ -1 & 3\end{array}\right]$ and $I$ is the identity matrix of the same order and $A^{1}$ is the transpose of matrix $A$, find $A^{1} \cdot B+B I$.
(c) In the adjoining figure $A B C$ is a right angled triangle with $\angle B A C=90^{\circ}$.
(i) Prove $\triangle A D B \sim \triangle C D A$.
(ii) If $B D=18 \mathrm{~cm}, C D=8 \mathrm{~cm}$, find $A D$.
(iii) Find the ratio of the area of $\triangle A D B$ is to area of $\triangle C D A$.


## Solution :

(a) Given:

$$
\begin{aligned}
& \sqrt{3 x+4}+\sqrt{3 x-5} \\
& \sqrt{3 x+4}-\sqrt{3 x-5}
\end{aligned}
$$

Applying componendo and Dividendo,

$$
\begin{aligned}
\left.\sqrt{3 x+4+\sqrt{3 x-5+\sqrt{3 x+4}}-\sqrt{3 x-5}} \begin{array}{rl}
9+1 \\
\left.\sqrt{3 x+4+\sqrt{3 x-5-\sqrt{3 x+4}+\sqrt{3 x-5}}} \begin{array}{rl}
9-1 \\
2 \sqrt{3 x+4} & 10 \\
2 \sqrt{3 x-5} & 8 \\
\sqrt{3 x+4} & 5 \\
\sqrt{3 x-5} & 4 \\
& 3 x+4
\end{array}\right) 25 \\
\text { Squaring both sides, } & 3 x-5
\end{array}\right) 16
\end{aligned}
$$

Applying Componendo and Dividendo,

$$
\begin{aligned}
\frac{3 x+4+3 x-5}{3 x+4-3 x+5} & =\frac{25+16}{25-16} \\
\frac{6 x-1}{9} & =\frac{41}{9} \\
6 x & =42 \\
x & =7
\end{aligned}
$$

Ans.
(b) Transpose of matrix $A, \quad A^{t}=\left[\begin{array}{ll}2 & 1 \\ & 3\end{array}\right]$

$$
\begin{aligned}
A^{t} \cdot B & =\left[\begin{array}{ll}
2 & 1 \\
5 & 3
\end{array}\right] \cdot\left[\begin{array}{rr}
4 & -2 \\
-1 & 3
\end{array}\right] \\
& =\left[\begin{array}{rr}
8-1 & -4+4 \\
20-3 & -10+9
\end{array}\right]=\left[\begin{array}{rr}
7 & -1 \\
17 & -1
\end{array}\right] \\
\text { B.I } & =\left[\begin{array}{rr}
4 & -2 \\
-1 & 3
\end{array}\right] \cdot\left[\begin{array}{lr}
1 & 0 \\
0 & 1
\end{array}\right]=\left[\begin{array}{rr}
4 & -2 \\
-1 & 3
\end{array}\right]
\end{aligned}
$$

$$
\begin{aligned}
A^{\ell} \cdot B+B . I & =\left[\begin{array}{rr}
7 & -1 \\
17 & -1
\end{array}\right]+\left[\begin{array}{rr}
4 & -2 \\
-1 & 3
\end{array}\right] \\
& =\left[\begin{array}{rr}
11 & -3 \\
6 & 2
\end{array}\right]
\end{aligned}
$$

Ans.
(c) Let

$$
\begin{aligned}
& \angle \mathrm{DAB}=\theta \\
& \angle \mathrm{DAC}=90-\theta \\
& \angle \mathrm{DBA}=90-\theta \\
& \angle \mathrm{DCA}=\theta
\end{aligned}
$$



B
$\because$ All three angles of $\triangle \mathrm{ADB}$ are equal to all angles of $\triangle$ CDA.
(i)

$$
\triangle \mathrm{ADB} \sim \triangle \mathrm{CDA}
$$

Proved
(ii)

$$
\begin{aligned}
& C D \\
& A D
\end{aligned}=\begin{aligned}
& A D \\
& B D
\end{aligned}
$$

$$
\mathrm{AD}^{2}=\mathrm{CD} \times \mathrm{BD}
$$

$$
=8 \times 18 \Rightarrow A D=12
$$

Ans.
(iii)

$$
\begin{aligned}
& \triangle \mathrm{ADB}=\frac{1}{2} \mathrm{AD} \times \mathrm{BD} \\
& \triangle \mathrm{CDA}=\frac{1}{2} \mathrm{AD} \times \mathrm{CD} \\
&=\frac{\mathrm{BD}}{\mathrm{CD}}=\frac{18}{8} \\
& 9
\end{aligned}
$$

Ans.

## Question 8.

(a) (i) Using step-deviation method, calculate the mean marks of the following distribution.
(ii) State the modal class:

Class Interval
50-55
55-60
60-65
65-70
70-75
75-80
80-85
85-90

## Frequency

5
20
10
10
9
6
12
8
(b) Marks obtained by 200 students in an examination are given below :

| Marks | No. of Students |
| :---: | :---: |
| $0-10$ | 5 |
| $10-20$ | 11 |
| $20-30$ | 10 |
| $30-40$ | 20 |
| $40-50$ | 28 |
| $50-60$ | 37 |
| $60-70$ | 40 |
| $70-80$ | 29 |
| $80-90$ | 14 |
| $90-100$ | 6 |

Draw an ogive for the given distribution taking $2 \mathrm{~cm}=10$ marks on one axis and $2 \mathrm{~cm}=20$ students on the other axis. Using the graph, determine
(i) The median marks.
(ii) The number of students who failed if minimum marks required to pass is 40.
(iii) If scoring 85 and more marks is considered as grade one, find the number of students who secured grade on in the examination.
[5]

## Solution :

| (a) (i) | C.I. | $f$ | $x$ | $d=x-67.5$ | $u$ | f.u |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 50-55 | 5 | 52.5 | -15 | -3 | -15 |
|  | 55-60 | 20 | 57.5 | -10 | -2 | -40 |
|  | 60-65 | 10 | 62.5 | -5 | -1 | -10 |
|  | 65-70 | 10 | 67.5 | 0 | 0 | 0 |
|  | 70-75 | 9 | $72 \cdot 5$ | 5 | 1 | 9 |
|  | 75-80 | 6 | $77 \cdot 5$ | 10 | 2 | 12 |
|  | 80-85 | 12 | 82.5 | 15 | 3 | 36 |
|  | 85-90 | 8 | $87 \cdot 5$ | 20 | 4 | 32 |
|  |  | $=80$ |  |  |  | $=24$ |

(a) (i)
A.M. $=67.5$

$$
\begin{aligned}
\bar{x} & =\text { A.M. }+\frac{\Sigma f u}{\Sigma f} \times i \\
& =67 \cdot 5+\frac{24}{80} \times 5 \\
& =67 \cdot 5+1.5=69
\end{aligned}
$$

(ii) Modal class is 55-60 (class with heighest freq.)

Ans.
Ans.
(b)

| C.I | $f$ | C.F. |
| ---: | ---: | ---: |
| $0-10$ | 5 | 5 |
| $10-20$ | 11 | 16 |
| $20-30$ | 10 | 26 |
| $30-40$ | 20 | 46 |
| $40-50$ | 28 | 74 |
| $50-60$ | 37 | 111 |
| $60-70$ | 40 | 151 |
| $70-80$ | 29 | 180 |
| $80-90$ | 14 | 194 |
| $90-100$ | 6 | 200 |


(i) $n=200$

$$
\begin{aligned}
\text { Median } & =\left(\frac{n}{2}\right)^{\mathrm{th}} \text { observation } \\
& =\left(\frac{200}{2}\right)^{\mathrm{h}^{\mathrm{h}}} \text { observation } \\
& =100^{\text {th }} \text { observation }=57.02
\end{aligned}
$$

Ans.
(ii) Number of students who failed $=46$

Ans.
(iii) Number of students who secured grade one $=200-1888=12$. Ans.

## Question 9.

(a) Mr. Parekh invested ₹ 52,000 on 100 shares at a discount of $₹ 20$ paying $8 \%$ dividend. At the end of one year he sells the shares at a premium of $\geqslant 20$. Find
(i) The annual dividend.
(ii) The profit earned including his dividend.
[3]
(b) Draw a circle of radius 3.5 cm . Mark a point $P$ outside the circle at a distance of 6 cm from the centre. Construct two tangents from $P$ to the given circle. Measure and write down the length of one tangent.
(c) Prove that $(\operatorname{cosec} A-\sin A)(\sec A-\cos A) \sec ^{2} A=\tan A$

Solution :
 Dividend $=8 \%$

$$
\begin{aligned}
\text { No. of shares } & =\frac{\text { Investment }}{\mathrm{MV}} \\
& =\begin{array}{c}
52,000 \\
80 \\
\\
\end{array}=₹ 650
\end{aligned}
$$

(i)

$$
\begin{aligned}
\text { Annual Dividend } & =\frac{8}{100} \times 650 \times 100 \\
& =₹ 5,200
\end{aligned}
$$

(ii)

$$
\begin{aligned}
\text { Profit } & =\text { Total S.P. }+ \text { Dividend }- \text { Investment } \\
& =650 \times 120+5,200-52,000 \\
& =78,000+5,200-52,000 \\
& =₹ 31,200
\end{aligned}
$$

Ans.
(b) Length of the tangent $=4.8 \mathrm{~cm}$.

(c)

$$
\begin{aligned}
\text { L.H.S. } & =(\operatorname{cosec} A-\sin A)(\sec A-\cos A) \cdot \sec ^{2} A \\
& =\left(\frac{1}{\sin A}-\sin A\right)\left(\begin{array}{c}
1 \\
\cos A
\end{array}-\cos A\right) \cdot \sec ^{2} A \\
& =\binom{1-\sin ^{2} A}{\sin A}\binom{1-\cos ^{2} A}{\cos A} \cdot \sec ^{2} A \\
& =\frac{\cos ^{2} A}{\sin A} \cdot \frac{\sin ^{2} A}{\cos A} \times \cos ^{2} A \\
& =\frac{\sin A}{\cos A} \\
& =\tan A=\text { R.H.S. }
\end{aligned}
$$

## Hence Proved

## Question 10.

(a) 6 is the mean proportion between two numbers $x$ and $y$ and 48 is the third proportional of $x$ and $y$. Find the numbers.
(b) In what period of time will $₹ 12,000$ yield $₹ 3,972$ as compound interest at $10 \%$. per annum, if compounded on an yearly basis?
(c) A man observes the angle of elevation of the top of a building to be $30^{\circ}$. He walks towards it in a horizontal line through its base. On covering 60 m the angle of elevation changes to $60^{\circ}$. Find the height of the building correct to the nearest metre.

## Solution :

(a)

$$
\begin{align*}
& x y=6^{2} \\
& \Rightarrow \quad x y=36  \tag{I}\\
& x: y:: y: 48 \\
& \frac{x}{y}=\begin{array}{c}
y \\
48
\end{array} \\
& \Rightarrow \quad y^{2}=48 x \tag{2}
\end{align*}
$$

Subtituting the value of $x$ from (1),

$$
y^{2}=48 \times \frac{36}{y}
$$

$$
\Rightarrow \quad \begin{aligned}
y^{3} & =48 \times 36 \\
y^{3} & =6 \times 8 \times 6 \times 6 \\
y & =12 \\
x & =3
\end{aligned}
$$

Ans.
(b) Given : $\mathrm{P}=\mathrm{P} 12,000$, C.I. $=\mathbf{₹} \mathbf{3 , 9 7 2 , R \%}=\mathbf{1 0 \%}$ p.a.

Ans.
(c) Let $\mathrm{BC}=x$ and $\mathrm{AB}=h$

In right angled $\triangle \mathrm{ADB}$

$$
\begin{array}{ll} 
& \tan 30^{\circ}=\frac{h}{60+x} \\
\Rightarrow \quad & 60+x=h \sqrt{3}
\end{array}
$$

Now right angled $\triangle \mathrm{ACB}$

$$
\begin{aligned}
& \tan 60^{\circ}=\frac{h}{x} \\
& x=h \\
&
\end{aligned}
$$

$$
\Rightarrow \quad \begin{aligned}
3 h-60 \sqrt{3} & =h \\
2 h & =60 \sqrt{3} \\
h & =30 \sqrt{3} \\
& =51 \cdot 96 \mathrm{~m} .
\end{aligned}
$$



$$
\text { Equating ' } x \text { ', } \quad h \sqrt{3}-60=\begin{gathered}
h \\
\sqrt{3}
\end{gathered}
$$

## Question 11.

(a) $A B C$ is a triangle with $A B=10 \mathrm{~cm}, B C=8 \mathrm{~cm}$ and $A C=6 \mathrm{~cm}$ (not drawn to scale). Three circles are drawn touching each other with the vertices as their centres. Find the radii of the three circles.


$$
\begin{aligned}
& \text { Let } \\
& A=P\left(1+\frac{R}{100}\right)^{2} \\
& \Rightarrow \quad 15,972=12,000\left(1+\frac{10}{100}\right)^{p} \\
& \begin{array}{l}
1331 \\
1000
\end{array}=\binom{11}{10}^{p} \\
& \left(\frac{11}{10}\right)^{9}=\left(\frac{11}{10}\right)^{2} \\
& \Rightarrow \quad n=3 \text { years. }
\end{aligned}
$$

(b) $₹ 480$ is divided equally among ' $x$ ' children. If the number of children were 20 more then each would have got $₹ 12$ less. Find ' $x$ '.
(c) Given equation of line $L_{1}$ is $y=4$.

$y^{\prime}$
(i) Write the slope of line $L_{2}$ if $L_{2}$ is the bisector of angle $O$.
(ii) Write the co-ordinates of point $P$.
(iii) Find the equation of $L_{2}$.

## Solution :

(a) Let the three radii be $x, y, z$ respectively.

$$
\begin{align*}
& x+y=10  \tag{1}\\
& y+z=8  \tag{2}\\
& x+z=6 \tag{3}
\end{align*}
$$

Adding equation's (1), (2) and (3), $\quad 2 x+2 y+2 z=24$

$$
\begin{equation*}
x+y+z=12 \tag{4}
\end{equation*}
$$

Subtracting each equation (1), (2) and (3) from equation (4), we get

$$
z=2 \mathrm{~cm}, x=4 \mathrm{~cm}, y=6 \mathrm{~cm} .
$$

(b) Initial share of each child $=\begin{gathered}480 \\ x\end{gathered}$

$$
\text { New share of each child }=\begin{gathered}
480 \\
x+20
\end{gathered}
$$

Difference in share is ₹ 12

$$
\begin{aligned}
\frac{480}{x}-\frac{480}{x+20} & =12 \\
\frac{1}{x}-\frac{1}{x+20} & =12=180=40 \\
\frac{x+20-x}{x(x+20)} & =1 \\
x^{2}+20 x & =80 \\
x^{2}+20 x-800 & =0 \\
x^{2}+40 x-20 x-800 & =0 \\
x(x+40)-20(x+40) & =0 \text { (not possible) } \\
x=20 & \text { or } x=-40 \\
x & =20
\end{aligned}
$$

Ans.
(c) (i). Slope of $\mathrm{L}_{2}$ is
$m=\tan 45^{\circ}$
$m=\tan 45^{\circ}$
$m=1$ ( $\mathrm{L}_{2}$ makes an angle of $45^{\circ}$ with X axis)
(ii) Equation of line $L_{2}$
$y-0=1(x-0)$ It passes through ( 0,0 )
$\therefore$ Equation of $\mathrm{L}_{2}$ is ..... $y=x$
P can be obtained by solving $\mathrm{L}_{1}$ and $\mathrm{L}_{2}$ simultaneously,
$\mathrm{L}_{1}$
$y=3$
$\mathrm{L}_{2}$
$y=x$
On solving, we get ..... $x=3, y=3$
Co-ordinate of $\mathrm{P}(3,3)$
Ans.
(iii) Equation of $\mathrm{L}_{2}$ is $y=x$ [as solved above part (ii)]. ..... Ans

