# **ICSE Question Paper (2011)**

# **MATHEMATICS**

#### SECTION A [40 Marks]

(Answer all questions from this Section.)

#### Question 1.

(a) Find the value of 'k' if (x - 2) is a factor of :

$$x^3 + 2x^2 - kx + 10$$

Hence determine whether (x + 5) is also a factor.

(b) If  $A = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ , is the product AB possible ? Give a reason. If yes, find AB. [3]

[3]

(c) Mr. Kumar borrowed ₹ 25,000 for two years. The rate of interest for the two successive years are 8% and 10% respectively. If he repays ₹ 6,200 at the end of the first year, find the outstanding amount at the end of the second year. [4]

#### Solution :

(a) Let  $f(x) = x^3 + 2x^2 - kx + 10$ 

 $\therefore$  (x – 2) is a factor,

$$f(2) = 0$$
  

$$f(2) = 8 + 8 - 2k + 10 = 0$$
  

$$k = 13$$

To check for (x + 5) is a factor,

$$f(-5) = (-5)^3 + 2(-5)^2 - 13(-5) + 10$$
$$= -125 + 50 + 65 + 10 = 0$$

 $\therefore$  (x + 5) is a factor.

**(b)**  $A_{2 \times 2} \cdot B_{2 \times 1}$ 

From the order of both the matrix A and B, it is clear that AB is possible because the number of columns of A are equal to the number of rows of B.

$$AB = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$
$$= \begin{bmatrix} 6+20 \\ 8-8 \end{bmatrix}$$
$$= \begin{bmatrix} 26 \\ 0 \end{bmatrix}$$
Ans.

(c) Given : Principal =  $\overline{15,000}$ 

We know that 
$$A = P\left(1 + \frac{r}{100}\right)^{t}$$
  
Amount after 1<sup>st</sup> year = 15,000  $\left(1 + \frac{8}{100}\right)$   
= ₹ 16,200  
Principal after repayment = 16,200 - 6,200 = ₹ 10,000  
Amount outstanding at the end of second year  
= 10,000  $\left(1 + \frac{10}{100}\right)$   
= ₹ 11,000 Ans.

#### Question 2.

(a) From a pack of 52 playing cards all cards whose numbers are multiples of 3 are removed. A card is now drawn at random.
 What is the probability that the card drawn is

What is the probability that the card drawn is

- (i) a face card (King, Jack or Queen)
- (ii) an even numbered red card ? [3]
- (b) Solve the following equation : x - <sup>18</sup>/<sub>x</sub> = 6. Give your answer correct to two significant figures. [3]
   (c) In the given figure O is the centre of the circle.
- Tangents at A and B meet at C. If  $\angle AOC \approx 30^{\circ}$ , find
  - (i) ∠*BCO*
  - (ii) ∠AOB
  - (iii) ∠APB

#### Solution :

(a) Number of cards which are multiples of 3 = 12

Cards left in the pack = 40

- (i) Number of face cards = 12 P (face card) =  $\frac{12}{40} = \frac{3}{10}$  Ans. (ii) Even numbered red cards = 10
- P (even number red card) =  $\frac{10}{40} = \frac{1}{4}$  Ans.

(b) Let 
$$x^2 - 6x - 18 = 0$$
  
Compare with equation  $ax^2 + bx + c = 0$ , we get

a = 1, b = -6, c = -18

Now,

$$x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{6 \pm \sqrt{36} + 72}{2}$$

30°

0

B

 $= \frac{6\pm 6\sqrt{3}}{2} \Rightarrow x = 3\pm 3\sqrt{3}$  $x = 3 \pm 5.196$ Taking +ve and -ve sign respectively, we get x = 8.196 or x = -2.196Ans.  $\triangle ACO \cong \triangle BCO$ (R.H.S.) (c)  $\angle BCO = \angle ACO$ (C.P.C.T.) (i)  $\angle BCO = 30^{\circ}$  $\angle OAC = 90^{\circ}$  $In \triangle ACO$ , (Radius is perpendicular to tangent)  $\angle AOC = 60^{\circ}$  $\angle BOC = 60^{\circ}$ (C.P.C.T.) Also  $\angle AOB = 120^{\circ}$ (ii)  $\angle APB = 60^{\circ}$ (Angle at circumference is half the (iii) angle at the centre)

#### **Question 3.**

- (a) Ahmed has a recurring deposit account in a bank. He deposits ₹ 2,500 per month for 2 years. If he gets ₹ 66,250 at the time of maturity, find
  - (i) The interest paid by the bank.
  - (ii) The rate of interest.
- (b) Calculate the area of the shaded region, if the diameter of the semi circle is equal to 14 cm.

[3]

Take 
$$\pi = \frac{22}{7}$$
 [3]

(c) ABC is a triangle and G(4, 3) is the centroid of the triangle. If A = (1, 3), B = (4, b) and C = (a, 1), find 'a' and 'b'. Find the length of side BC. [4]

(a) (i) Interest =  $66,250 - 2,500 \times 24$ = 66,250 - 60,000=  $\overline{<} 6,250$  'Ans. (ii) Principal =  $\frac{n(n+1)}{2} \times \text{sum of deposited per month}$ =  $\frac{24(24+1)}{2} \times 2,500$ =  $\frac{24 \times 25}{2} \times 2,500$ I = Principal  $\times \frac{R}{100} \times \frac{1}{12}$ 

$$R = \frac{6,250 \times 2 \times 100 \times 12}{2,500 \times 24 \times 25}$$
  
= 10% p.a. Ans.

(b) Area of shaded portion = Complete area - area of the two quadrants
 = (Area of ACDE + Area of semi circle EFD)

– (Area of Quadrant ABE +

Area of Quadrant BCD)

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$$= \left\{ 14 \times 7 + \frac{\pi}{2} (7)^2 \right\} - \left\{ \frac{\pi}{4} (7)^2 + \frac{\pi}{4} (7)^2 \right\}$$
$$= \left\{ 14 \times 7 + \frac{\pi}{2} (7)^2 \right\} - \left\{ \frac{\pi}{2} (7)^2 \right\}$$
$$= 98 \text{ cm}^2$$



#### **Question 4.**

- (a) Solve the following inequation and represent the solution set on the number line  $2x-5 \le 5x+4 < 11$ , where  $x \in I$ . [3]
- (b) Evaluate without using trigonometric tables :

$$2\left(\frac{\tan 35^{\circ}}{\cot 55^{\circ}}\right)^{2} + \left(\frac{\cot 55^{\circ}}{\tan 35^{\circ}}\right)^{2} - 3\left(\frac{\sec 40^{\circ}}{\csc 50^{\circ}}\right)$$
[3]

(c) A Mathematics aptitude test of 50 students was recorded as follows :

Marks	50-60	60-70	70-80	80-90	90-100
No. of students	4	8	14	19	5

Draw a histogram for the above data using a graph paper and locate the mode. [4]

#### Solution :

(a) Given:  $2x-5 \le 5x+4$  and 5x+4 < 11  $-3x \le 9$   $x \ge -3$   $-3 \le x$ Solution set,  $x \in \{-3, -2, -1, 0, 1\}$ 

(b) Given : 
$$2\left(\frac{\tan 35^{\circ}}{\cot 55^{\circ}}\right)^{2} + \left(\frac{\cot 55^{\circ}}{\tan 35^{\circ}}\right) - 3\left(\frac{\sec 40^{\circ}}{\csc 50^{\circ}}\right)$$
  
 $= 2\left(\frac{\tan (90^{\circ} - 55^{\circ})}{\cot 55^{\circ}}\right)^{2} + \left(\frac{\cot (90^{\circ} - 35^{\circ})}{\tan 35^{\circ}}\right) - 3\left(\frac{\sec (90^{\circ} - 50^{\circ})}{\csc 50^{\circ}}\right)$   
 $= 2\left(\frac{\cot 55^{\circ}}{\cot 55^{\circ}}\right)^{2} + \left(\frac{\tan 35^{\circ}}{\tan 35^{\circ}}\right) - 3\left(\frac{\csc 50^{\circ}}{\csc 50^{\circ}}\right)$   
 $= 2 + 1 - 3 = 0$  Ans.  
(c)  $16^{5}$ 

Mode from graph = 82.5.

Marks -

### SECTION B [40 Marks]

Answer any four Questions in this Section.

# **Question 5.**

- (a) A manufacturer sells a washing machine to a wholesaler for ₹ 15,000. The wholesaler sells it to a trader at a profit of ₹ 1,200 and the trader in turn sells it to a consumer at a profit of ₹ 1,800. If the rate of VAT is 8% find :
  - (i) The amount of VAT received by the State Government on the sale of this machine from the manufacturer and the wholesaler.
  - (ii) The amount that the consumer pays for the machine. [3]
- (b) A solid cone of radius 5 cm and height 8 cm is melted and made into small spheres of radius 0.5 cm. Find the number of spheres formed. [3]
- (c) ABCD is a parallelogram where A(x, y), B(5, 8), C(4, 7) and D(2, -4). Find
  - (i) Coordinates of A
  - (ii) Equation of diagonal BD.

Solution :

(a) (i) VAT received by Govt. from manufacturer =  $15,000 \times \frac{8}{100} = ₹ 1,200$ 

VAT from wholesaler = 
$$1200 \times \frac{6}{100} = ₹96$$

Total VAT from manufacturer and wholesaler

[4]

(ii) Amount that customer pays = 
$$(15000 + 1200 + 1800) + VAT$$
  
=  $18,000 + \frac{18,000 \times 8}{100}$   
=  $18,000 + 1440 = ₹ 19,440$  Ans.  
(b) Number of spheres =  $\frac{Volume of cone}{Volume of each sphere}$   
=  $\frac{1}{3}\pi (5)^2 (8) - \frac{50 \times 10^3}{5 \times 5 \times 5}$   
=  $400$  Ans.

(c) In a parallelogram, mid point of diagonal BD co-incides with the mid point of diagonal AC.

 $\frac{x+4}{2} = \frac{7}{2}$ Equating,

Mid point of BD = 
$$5 + 2, 8 - 4 = 7$$
  
Mid point of AC =  $x + 4, y + 7 = 7$   
 $\frac{x + 4}{2} = \frac{7}{2} \Rightarrow x = 3$   
 $\frac{y + 7}{2} = 2 \Rightarrow y = -3$ 

Co-ordinates of A (3, -3) (i)

Mid point of AC =

m of BD = 
$$\frac{8 - (-4)}{5 - 2} = \frac{12}{3} = 4$$

Equation of BD,

$$y-y_1 = m(x-x_1)$$
  
 $y+4 = 4(x-2)$   
 $y = 4x-4$  Ans.

D

C

Ans.

[5]

(4, 7)

#### Question 6.

- (a) Use a graph paper to answer the following questions. (Take 1 cm = 1 unit on both axes).
  - (i) Plot A(4, 4), B(4, -6) and C(8, 0), the vertices of a triangle ABC.
  - Reflect ABC on the y-axis and name it as ABC'. (ii)
  - (iii) Write the coordinates of the image A', B' and C'.
  - Give a geometrical name for the figure AA'C'B'BC. (iv)
  - (v) Identify the line of symmetry of AA' C' B' BC.
- (b) Mr. Choudhury opened a Saving's Bank Account at State Bank of India on 1st April 2007. The entries of one year as shown in his pass book are given below :

Date	Particulars	Withdrawals	Deposits	Balance
		(in 🕄	(in <b>र</b> )	(in 🕄
1st April 2007	By Cash	-	8550.00	8550.00
12th April 2007	To Self	1200.00		7350.00
24th April 2007	By Cash	_	4550.00	11900.00

8th July 2007	By Cheque		1500.00	13400.00
10th Sept. 2007	By Cheque		3500.00	16900.00
17th Sept. 2007	To Cheque	2500.00		14400.00
11th Oct. 2007	By Cash		800.00	15200.00
6th Jan. 2008	To Self	2000.00		13200.00
9th March 2008	By Cheque		950.00	14150.00

If the bank pays interest at the rate of 5% per annum, find the interest paid on 1st April, 2008. Give your answer correct to the nearest rupee. [5]

## Solution :

**(b)** 

- (a) (i) In the given diagram.
  - (ii) In the given diagram.
  - (iii) A' (-4, 4)

B' (-4, -6)

- C' (-8, 0)
- (iv) Hexagon.
- (v) Y-Axis is the line of symmetry.

	Minimum Balanc	es
April	7,350.00	
Мау	11,900.00	
June	11,900.00	
July	13,400.00	
August	13,400.00	
September	14,400.00	
October	14,400.00	
November	15,200.00	
December	15,200.00	
January	13,200.00	
February	13,200.00	
March	14,150.00	
	1,57,700.00	⇒ P = ₹ 1,57,700
	$I = \frac{1,57,700 \times 5}{100} \times \frac{5}{100}$	1 12
	= $\frac{7885}{12}$ = ₹ 657.0	)8

# **Question 7.**

(a) Using componendo and dividendo, find the value of x

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}} = 9$$
 [3]

Ans.

- **(b)** If  $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$  and I is the identity matrix of the same order and  $A^{1}$  is the transpose of matrix A, find  $A^{1}.B + BI$ . [3]
- (c) In the adjoining figure ABC is a right angled triangle with  $\angle BAC = 90^{\circ}$ .
  - (i) Prove  $\triangle ADB \sim \triangle CDA$ .
  - (ii) If BD = 18 cm, CD = 8 cm, find AD.
  - (iii) Find the ratio of the area of  $\triangle$  ADB is to area of  $\triangle$  CDA. [4]

Solution :

(a) Given: 
$$\sqrt[4]{\frac{3x+4}{3x+4}} \sqrt[4]{\frac{3x-5}{3x-5}} 9$$

Applying componendo and Dividendo,

$$\frac{\sqrt{3x} + 4 + \sqrt{3x} - 5 + \sqrt{3x} + 4 - \sqrt{3x} - 5}{\sqrt{3x} + 4 + \sqrt{3x} - 5} = \frac{9 + 1}{9 - 1}$$

$$\frac{2\sqrt{3x} + 4 + \sqrt{3x} - 5 - \sqrt{3x} + 4 + \sqrt{3x} - 5}{2\sqrt{3x} - 5} = \frac{10}{8}$$

$$\frac{\sqrt{3x} + 4}{\sqrt{3x} - 5} = \frac{10}{4}$$
Squaring both sides,
$$\frac{3x + 4}{3x - 5} = \frac{25}{16}$$

Applying Componendo and Dividendo,

(b) Transpose of matrix A,

$$\frac{3x+4+3x-5}{3x+4-3x+5} = \frac{25+16}{25-16}$$
$$\frac{6x-1}{9} = \frac{41}{9}$$
$$6x = 42$$
$$x = 7$$

Ans.

$$A^{t} = \begin{bmatrix} 2 & 1 \\ 3 \end{bmatrix}$$

$$A^{t} \cdot B = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 - 1 & -4 + 4 \\ 20 - 3 & -10 + 9 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix}$$

$$B.I = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$





#### **Question 8.**

- (a) (i) Using step-deviation method, calculate the mean marks of the following distribution.
  - (ii) State the modal class :

Class Interval	Frequency
5055	5
55-60	20
6065	10
65–70	10
70–75	9
75-80	6
8085	12
85-90	8

[5]

	tion are given below :	in a	0 students	y 200	lained by	Marks ol	<b>(b)</b>
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Marks	No. of Students
0–10	5
10–20	11
20–30	10
3040	20
40–50	28
5060	37
6070	40
7080	29
80–90	14
90-100	6

Draw an ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis. Using the graph, determine

- (i) The median marks.
- The number of students who failed if minimum marks required to pass is (ii) 40.
- (iii) If scoring 85 and more marks is considered as grade one, find the number of students who secured grade on in the examination. [5]

Sol	ution :						1
(a)	(i)	` C.I.	f	x	d = x - 67.5	u	f.u
		50-55	5	52.5	-15	-3	-15
		55-60	20	57·5	-10	-2	-40
		6065	10	62·5	-5	-1	-10
		65–70	10	67-5	0	0	0
		70–75	9	<b>72</b> ∙5	5	1	9
		75-80	6	77-5	10	2	12
		8085	12	82.5	15	3	36
		85–90	8	<b>87</b> ∙5	20	4	32
			$\Sigma f = 80$				$\Sigma f u = 24$

A.M. = 67.5

$$\overline{x} = A.M. + \frac{\Sigma f u}{\Sigma f} \times i$$

$$= 67.5 + \frac{24}{80} \times 5$$

$$= 67.5 + 1.5 = 69$$
Ans.
odal class is 55-60 (class with heighest freq.)
Ans.

(ii) Modal class is 55-60 (class with heighest freq.)

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C.I.	f	C.F.	Scale : 2 em = 10 maña, on t-scala
0-10	5	5	200
10-20	11	16	160
20-30	10	26	140
30-40	20	46	120
40-50	28	- 74	2100
50-60	37	111	80 BO
60-70	40	151	
70-80	29	180	857.02 85
80-90	14	194	2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
90-100	6	200	Marks

Median = 
$$\left(\frac{n}{2}\right)^{\text{th}}$$
 observation  
=  $\left(\frac{200}{2}\right)^{\text{th}}$  observation

=  $100^{\text{th}}$  observation = 57.02 Ans.

- (ii) Number of students who failed = 46 Ans.
- (iii) Number of students who secured grade one = 200 1888 = 12. Ans.

#### **Question 9.**

- (a) Mr. Parekh invested ₹ 52,000 on 100 shares at a discount of ₹ 20 paying 8% dividend. At the end of one year he sells the shares at a premium of ₹ 20. Find
  - (i) The annual dividend.
  - (ii) The profit earned including his dividend. [3]
- (b) Draw a circle of radius 3.5 cm. Mark a point P outside the circle at a distance of 6 cm from the centre. Construct two tangents from P to the given circle. Measure and write down the length of one tangent. [3]
- (c) Prove that (cosec  $A \sin A$ ) (sec  $A \cos A$ ) sec<sup>2</sup>  $A = \tan A$ . [4]

#### Solution :

(a) Investment = ₹ 52,000, N.V = ₹ 100, M.V. of one share = ₹ (100 - 20) = ₹ 80, Dividend = 8%

No. of shares = 
$$\frac{\text{Investment}}{\text{MV}}$$
  
=  $\frac{52,000}{80}$   
=  $\mathbf{\overline{\xi}} 650$   
(i) Annual Dividend =  $\frac{8}{100} \times 650 \times 100$   
=  $\mathbf{\overline{\xi}} 5,200$  Ans.

(b) Length of the tangent = 4.8 cm.



**(c)** 

L.H.S. =  $(\operatorname{cosec} A - \sin A) (\operatorname{sec} A - \cos A) \cdot \operatorname{sec}^2 A$  $= \left(\frac{1}{\sin A} - \sin A\right) \left(\frac{1}{\cos A} - \cos A\right) \cdot \operatorname{sec}^2 A$   $= \left(\frac{1 - \sin^2 A}{\sin A}\right) \left(\frac{1 - \cos^2 A}{\cos A}\right) \cdot \operatorname{sec}^2 A$   $= \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} \times \frac{1}{\cos^2 A}$ 

 $= \frac{\sin A}{\cos A}$ 

= tan A = R.H.S.

#### Hence Proved

Ans.

#### Question 10.

- (a) 6 is the mean proportion between two numbers x and y and 48 is the third proportional of x and y. Find the numbers.
   [3]
- (b) In what period of time will ₹ 12,000 yield ₹ 3,972 as compound interest at 10%.
   per annum, if compounded on an yearly basis ?
   [3]
- (c) A man observes the angle of elevation of the top of a building to be 30°. He walks towards it in a horizontal line through its base. On covering 60 m the angle of elevation changes to 60°. Find the height of the building correct to the nearest metre.
  [4]

#### Solution:

(a)  

$$xy = 6^{2}$$

$$\Rightarrow \qquad xy = 36 \qquad \dots(1)$$

$$x: y :: y: 48$$

$$\frac{x}{y} = \frac{y}{48}$$

$$\Rightarrow \qquad y^{2} = 48x \qquad \dots(2)$$
Subtituting the value of x from (1),  

$$y^{2} = 48 \times \frac{36}{y}$$

(ii)



- Question 11.
- (a) ABC is a triangle with AB = 10 cm, BC = 8 cm and AC = 6 cm (not drawn to scale). Three circles are drawn touching each other with the vertices as their centres. Find the radii of the three circles. [3]



- (b) ₹ 480 is divided equally among 'x' children. If the number of children were 20 more then each would have got ₹ 12 less. Find 'x'.
   [3]
- (c) Given equation of line  $L_1$  is y = 4.



- (i) Write the slope of line  $L_2$  if  $L_2$  is the bisector of angle O.
- (ii) Write the co-ordinates of point P.
- (iii) Find the equation of  $L_2$ .

[4]

#### Solution :

(a) Let the three radii be x, y, z respectively.

x + y = 10 ...(1)

y + z = 8 ...(2)

 $x+z = 6 \qquad \dots (3)$ 

Adding equation's (1), (2) and (3), 
$$2x + 2y + 2z = 24$$
  
 $x + y + z = 12$  ...(4)

Subtracting each equation (1), (2) and (3) from equation (4), we get

$$z = 2 \text{ cm}, x = 4 \text{ cm}, y = 6 \text{ cm}.$$
 Ans.

(b) Initial share of each child = 
$$\frac{480}{\pi}$$

New share of each child =  $\frac{480}{x+20}$ 

Difference in share is ₹ 12

$$\frac{480}{x} - \frac{480}{x+20} = 12$$

$$\frac{1}{x} - \frac{1}{x+20} = \frac{12}{480} = \frac{1}{40}$$

$$\frac{x+20-x}{x(x+20)} = \frac{1}{40}$$

$$x^2 + 20x = 800$$

$$x^2 + 20x - 800 = 0$$

$$x^2 + 40x - 20x - 800 = 0$$

$$x(x+40) - 20(x+40) = 0 \text{ (not possible)}$$

$$x = 20 \text{ or } x = -40$$

$$x = 20$$

Ans.

(c)	(i)	Slope of $L_2$ is	m = tan 45°	
			m = 1 (L <sub>2</sub> makes an angle of 4	5° with X axis)
	(ii)	Equation of line $L_2$		
			y-0 = 1 (x-0) It passes the	rough (0, 0)
		$\therefore$ Equation of L <sub>2</sub> is	y = x	
		P can be obtained by	solving $L_1$ and $L_2$ simultaneously,	
		$L_1$	y = 3	
		$L_2$	y = x	
		On solving, we get	x = 3, y = 3	
		Co-ordinate of P (3, 3	3)	Ans.
	(jii)	Equation of $L_2$ is $y =$	x [as solved above part (ii)].	Ans ••