ICSE Question Paper (2013)

MATHEMATICS

SECTION A [40 Marks]

*(Answer **all** questions from this Section.)*

Question 1.

(a) *Given* $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$.

Find the matrix X such that $A + 2X = 2B + C$. **[3]**

(b) *At what rate % p.a. will a sum of ₹ 4000 yield ₹ 1324 as compound interest in 3 years ?* **[3]**

(c) *The median of the following observations 11, 12, 14, $(x - 2)$, $(x + 4)$, $(x + 9)$, 32, 38, 47 arranged in ascending order is 24. Find the value of x and hence find the mean.* **[4]**

Solution :

(a) *Given :*

$$A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$A + 2X = 2B + C$$

Putting the given values, we get

$$\begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$2X = \begin{bmatrix} -6+4 & 4+0 \\ 8+0 & 0+2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix}$$ **Ans.**

(b) *Given :*

Principal = ₹ 4,000, C.I. = ₹ 1,324,

Amount = P + C.I.

= ₹ (4,000 + 1,324) = ₹ 5,324

Time = 3 years

We know that,

$$A = P\left(1 + \frac{r}{100}\right)^T$$

$$5{,}324 = 4{,}000\left(1 + \frac{r}{100}\right)^3$$

$$\frac{5{,}324}{4{,}000} = \left(1 + \frac{r}{100}\right)^3$$

$$\frac{1,331}{1,000} = \left(1 + \frac{r}{100}\right)^3$$

$$\left(\frac{11}{10}\right)^3 = \left(1 + \frac{r}{100}\right)^3$$

Therefore,
$$1 + \frac{r}{100} = \frac{11}{10}$$

$$\frac{r}{100} = \frac{11}{10} - 1$$

$$\frac{r}{100} = \frac{1}{10}$$

$$r = \frac{100}{10}$$

$$r = 10\%$$ **Ans.**

(c) Given observation are 11, 12, 14, $(x - 2)$, $(x + 4)$, $(x + 9)$, 32, 38, 47 and median = 24.

$$n = 9 \text{ (odd)}$$

$$\text{Median} = \frac{n+1}{2}\text{th term}$$

$$= \frac{9+1}{2}\text{th term}$$

$$24 = 5\text{th term}$$

$$x + 4 = 24$$

$$x = 24 - 4$$

$$x = 20$$

Therefore, 11, 12, 14, (20 – 2), (20 + 4), (20 + 9), 32, 38, 47

= 11, 12, 14, 18, 24, 29, 32, 38, 47

Now
$$\text{Mean} = \frac{\Sigma x}{n}$$

$$= \frac{11 + 12 + 14 + 18 + 24 + 29 + 32 + 38 + 47}{9}$$

$$= \frac{225}{9} = 25$$ **Ans.**

Question 2.

(a) *What number must be added to each of the number 6, 15, 20 and 43 to make them proportional ?* **[3]**

(b) *If $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by $(x - 3)$, it leaves a remainder 52, find the values of a and b.* **[3]**

(c) *Draw a histogram from the following frequency distribution and find the mode from the graph :* **[4]**

Class	*0–5*	*5–10*	*10–15*	*15–20*	*20–25*	*25–30*
Frequency	*2*	*5*	*18*	*14*	*8*	*5*

Solution :

(a) Let the number must be added be x, then

$$\text{the new number} = 6 + x, 15 + x, 20 + x, 43 + x$$

$\because$ These are proportionals.

$$6 + x : 15 + x :: 20 + x : 43 + x$$

or $$(6 + x)(43 + x) = (15 + x)(20 + x)$$

or $$258 + 6x + 43x + x^2 = 300 + 20x + 15x + x^2$$

or $$49x - 35x = 300 - 258$$

or $$14x = 42$$

or $$x = 3.$$ **Ans.**

(b) Let $(x - 2)$ is a factor of the given expression.

$$x - 2 = 0$$
$$x = 2$$

Given expression,

$$2x^3 + ax^2 + bx - 14 = 0$$
$$2(2)^3 + a(2)^2 + b(2) - 14 = 0$$
$$16 + 4a + 2b - 14 = 0$$
$$4a + 2b + 2 = 0$$
$$4a + 2b = -2$$
$$2a + b = -1 \quad \text{...(i)}$$

and when given expression is divided by $(x - 3)$

$$x - 3 = 0$$

$\Rightarrow$ $$x = 3$$

$$2x^3 + ax^2 + bx - 14 = 52$$
$$2(3)^3 + a(3)^2 + b(3) - 66 = 0$$
$$54 + 9a + 3b - 66 = 0$$
$$9a + 3b = 12$$
$$3a + b = 4 \quad \text{...(ii)}$$

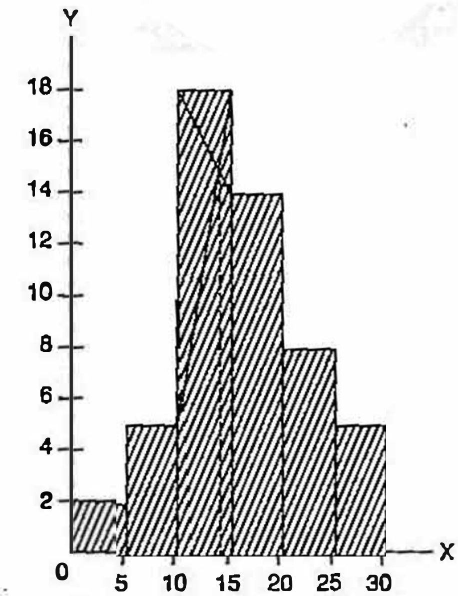
Solving equation (i) and (ii),

$$\begin{aligned} 2a + b &= -1 \\ 3a + b &= 4 \\ (-) \quad (-) & \quad (+) \\ \hline -a &= -5 \\ a &= 5 \end{aligned}$$

from (ii),

$$3 \times 5 + b = 4$$
$$b = 4 - 15$$
$$b = -11$$
$$a = 5 \text{ and } b = -11$$ **Ans.**

(c)



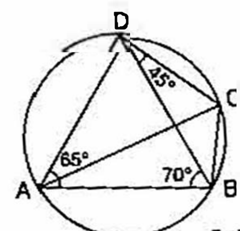
From the Histogram the value of Mode is **13·8.** **Ans.**

Question 3.

(a) *Without using tables evaluate 3 cos 80°.Cosec 10° + 2 sin 59° sec 31°.* **[3]**

(b) *In the given figure,*

$\angle BAD = 65°$,
$\angle ABD = 70°$,
$\angle BDC = 45°$



(i) *Prove that AC is a diameter of the circle.*

(ii) *Find $\angle ACB$.* **[3]**

(c) *AB is a diameter of a circle with centre C = (–2, 5). If A = (3, –7). Find :*

(i) *The length of radius AC*

(ii) *The coordinates of B.* **[4]**

Solution :

(a) *Given* :

$3 \cos 80° \cdot \text{cosec } 10° + 2 \sin 59° \sec 31°$

$= 3 \cos 80° \text{ cosec } (90° - 80°) + 2 \sin 59° \sec (90° - 59°)$

$= 3 \cos 80° \sec 80° + 2 \sin 59° \text{ cosec } 59°$

$= 3 \cos 80° \times \frac{1}{\cancel{\cos 80°}} + 2 \sin 59° \times \frac{1}{\cancel{\sin 59°}}$

$= 3 + 2 = 5.$ **Ans.**

(b) *Given* : $\angle BAD = 65°$, $\angle ABD = 70°$, $\angle BDC = 45°$

(i) $\because$ ABCD is a cyclic quadrilateral.

In Δ ABD,

$\angle BDA + \angle DAB + \angle ABD = 180°$ By using sum property of Δ^s

$\therefore \angle BDA = 180° - (65° + 70°)$

$= 180° - 135°$

$= 45°$

Now from Δ ACD,

$$\angle ADC = \angle ADB + \angle BDC$$
$$= 45° + 45° \qquad (\because \angle BDA = \angle ADB = 45°)$$
$$= 90°$$

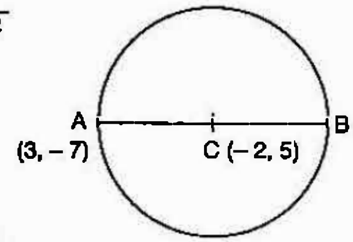
Hence, $\angle D$ makes right angle belongs in semi-circle therefore AC is a diameter of the circle.

(ii) $\angle ACB = \angle ADB$ (Angles in the same segment of a circle)

$\angle ACB = 45°$ $(\because \angle ADB = 45°)$ **Ans.**

(c) (i) The length of radius AC $= \sqrt{(-2-3)^2 + (5+7)^2}$

$$= \sqrt{(-5)^2 + (12)^2}$$
$$= \sqrt{25 + 144}$$
$$= \sqrt{169}$$
$$= 13.$$ **Ans.**



(ii) Let the point of B be (x, y).

Given C is the mid-point of AB. Therefore

$$-2 = \frac{3+x}{2}$$
$$\Rightarrow \quad 3 + x = -4$$
$$\Rightarrow \quad x = -4 - 3 = -7$$

and
$$5 = \frac{-7+y}{2}$$
$$\Rightarrow \quad 10 = -7 + y$$
$$y = 17$$

Hence, the co-ordinate of B $(-7, 17)$. **Ans.**

Question 4.

(a) *Solve the following equation and calculate the answer correct to two decimal places :*

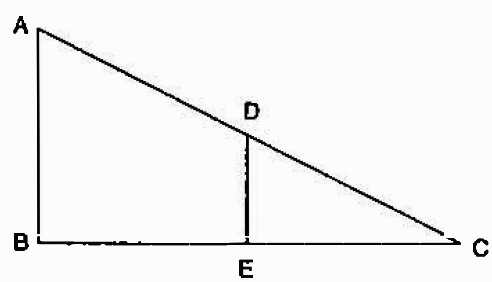
$$x^2 - 5x - 10 = 0.$$ **[3]**

(b) *In the given figure, AB and DE are perpendicular to BC.*

(i) *Prove that $\Delta ABC \sim \Delta DEC$*

(ii) *If AB = 6 cm, DE = 4 cm and AC = 15 cm. Calculate CD.*

(iii) *Find the ratio of the area of Δ ABC : area of Δ DEC.* **[3]**



(c) *Using graph paper, plot the points A(6, 4) and B(0, 4).*

(i) *Reflect A and B in the origin to get the images A′ and B′.*

(ii) *Write the co-ordinates of A′ and B′.*

(iii) *State the geometrical name for the figure ABA′B′.*

(iv) *Find its perimeter.* **[4]**

Solution :

(a) *Given* : $x^2 - 5x - 10 = 0$

Here, $a = 1$, $b = -5$ and $c = -10$

$$\therefore \quad D = b^2 - 4ac$$

$$= (-5)^2 - 4 \times 1 \times -10$$

$$D = 25 + 40 = 65$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{5 \pm \sqrt{65}}{2 \times 1} = \frac{5 \pm 8{\cdot}06}{2}$$

$$= \frac{5 + 8{\cdot}06}{2}, \frac{5 - 8{\cdot}06}{2}$$

$$= \frac{13{\cdot}06}{2}, -\frac{3{\cdot}06}{2}$$

$$x = 6{\cdot}53, -1{\cdot}53$$ **Ans.**

(b) (i) From Δ ABC and Δ DEC,

$\angle ABC = \angle DEC = 90°$ (Given)

and $\angle ACB = \angle DCE =$ Common

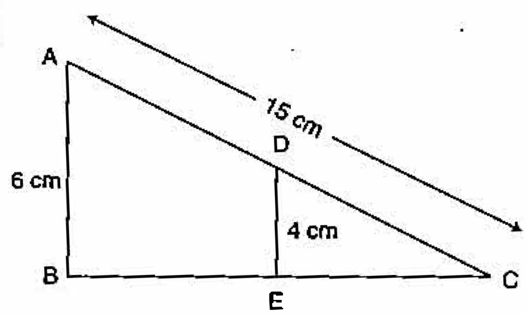
$\therefore \Delta ABC \sim \Delta DEC$ (By AA similarity)

(ii) In Δ ABC and Δ DEC,

$\Delta ABC \sim \Delta DEC$ (proved in (i) part)

$$\therefore \quad \frac{AB}{DE} = \frac{AC}{CD}$$

Given : AB = 6 cm, DE = 4 cm, AC = 15 cm,



$$\therefore \quad \frac{6}{4} = \frac{15}{CD}$$

$$\Rightarrow \quad 6 \times CD = 15 \times 4$$

$$\Rightarrow \quad CD = \frac{60}{6}$$

$$\Rightarrow \quad CD = 10 \text{ cm.}$$ **Ans.**

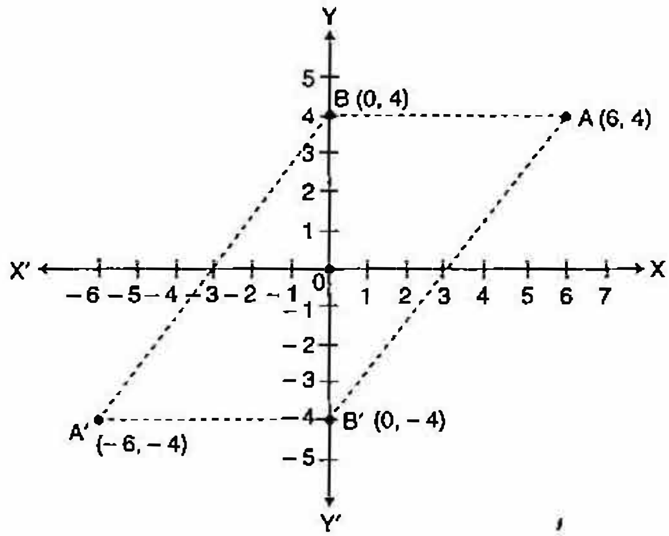
(iii) $$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEC} = \frac{AB^2}{DE^2}$$ $(\because \Delta ABC \sim \Delta DEC)$

$$= \frac{(6)^2}{(4)^2}$$

$$= \frac{3{\cdot}6}{16} = \frac{9}{4}$$

$\therefore$ Area of Δ ABC : Area of Δ DEC = 9 : 4. **Ans.**

(c) (i) Please See Graph.



(ii) Reflection of A′ and B′ in the origin = A′ (–6, –4) and B′ (0, –4)

(iii) The geometrical name for the figure AB A′B′ is a parallelogram.

(iv) From the graph, AB = 6 cm, BB′ = 8 cm.

In Δ A BB′

$$(AB')^2 = AB^2 + (BB')^2$$
$$= (6)^2 + (8)^2 = 36 + 64$$
$$= 100$$
$$AB' = 10 = A'B \qquad \text{(AB A' B' is a parallelogram)}$$
$$\text{Perimeter of AB A'B'} = A'B' + AB' + AB + A'B$$
$$= 6 + 10 + 6 + 10$$
$$= 32 \text{ units.}$$

Ans.

SECTION B [40 Marks]

*Answer any **four** Questions in this Section.*

Question 5.

(a) *Solve the following inequation, write the solution set and represent it on the number line :*

$$-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in R$$ **[3]**

(b) *Mr. Britto deposits a certain sum of money each month in a Recurring Deposit Account of a bank. If the rate of interest is of 8% per annum and Mr. Britto gets ₹ 8088 from the bank after 3 years, find the value of his monthly instalment.*

[3]

(c) *Salman buys 50 shares of face value ₹ 100 available at ₹ 132.*

(i) *What is his investment ?*

(ii) *If the dividend is 7.5%, what will be his annual income ?*

(iii) *If he wants to increase his annual income by ₹ 150, how many extra shares should he buy ?* **[4]**

Solution :

(a) *Given* : $$-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$$

Taking L.C.M. of 3, 2 and 6 is 6.

$$-\frac{x}{3} \times 6 \le \frac{x}{2} \times 6 - \frac{4}{3} \times 6 < \frac{1}{6} \times 6$$

$$-2x \le 3x - 8 < 1$$

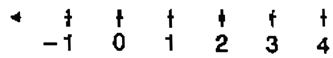
$\Rightarrow \quad -2x \le 3x - 8 \quad$ and $\quad 3x - 8 < 1$

$\Rightarrow \quad 8 \le 3x + 2x \quad \Rightarrow \quad 3x < 1 + 8$

$\Rightarrow \quad 8 \le 5x \quad\quad 3x < 9$

$\Rightarrow \quad \frac{\ }{5} \le x \quad \Rightarrow \quad x < 3$

$\therefore$ The solution set is $\{x : 1{\cdot}6 \le x \le 3, x \in R\}$



Number line

(b) Let the monthly instalment be ₹ x

Given : Maturity amount = ₹ 8,088, Time (n) = 3 years = 3 × 12 months = 36 months, Rate (R) = 8% p.a.

$$\text{Principle for one month} = P \times \frac{n(n+1)}{2}$$

$$= \frac{x \times 36 \times 37}{2}$$

$$= 18 \times 37x$$

$$\text{Interest} = \frac{18 \times 37x \times 8 \times 1}{100 \times 12} \qquad \left[\because I = \frac{PRT}{100}\right]$$

$$= \frac{444x}{100}$$

$$\text{Actual sum deposited} = 36x$$

$$\text{Maturity amount} = \text{Interest} + \text{Actual sum deposited}$$

$$8{,}088 = \frac{444x}{100} + 36x$$

$$8{,}088 = \frac{4{,}044x}{100}$$

$$x = \frac{8{,}088 \times 100}{4{,}044} = 200$$

Hence, the monthly instalment be ₹ 200. **Ans.**

(c)

Number of shares = 50

Face value of each share = ₹ 100

Market value of each share = ₹ 132

Total face value = ₹ 100 × 50

= ₹ 5,000

(i) Total investment = ₹ 132 × 50

= ₹ 6,600 **Ans.**

(ii) Rate of dividend $= 7{\cdot}5\%$

$$\text{Annual income} = ₹\,\frac{5{,}000 \times 7{\cdot}5}{100}$$

$$= ₹\,375$$ **Ans.**

(iii) Let extra share should he buy be x.

then total number of shares $= 50 + x$

Total face value $= ₹\,100 \times (50 + x)$

$$\therefore \quad \text{Annual income} = ₹\,\frac{100 \times (50 + x) \times 7{\cdot}5}{100}$$

$$= (50 + x) \times 7{\cdot}5$$

$$\therefore \quad (50 + x) \times 7{\cdot}5 = 375 + 150$$

$$50 + x = \frac{525}{7{\cdot}5} = 70$$

$$x = 70 - 50$$

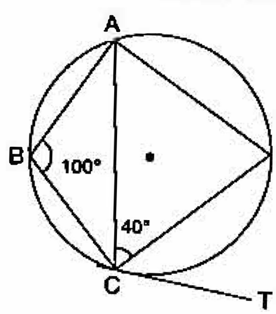
$$x = 20$$

Hence, the extra shares should be buy $= 20$. **Ans.**

Question 6.

(a) *Show that* $\sqrt{\dfrac{1 - \cos A}{1 + \cos A}} = \dfrac{\sin A}{1 + \cos A}$ **[3]**

(b) *In the given circle with centre O, $\angle ABC = 100°$, $\angle ACD = 40°$ and CT is a tangent to the circle at C. Find $\angle ADC$ and $\angle DCT$.* **[3]**



(c) *Given below are the entries in a Savings Bank A/c pass book :*

Date	Particulars	Withdrawals	Deposit	Balance
Feb. 8.	*B/F*	—	—	*₹ 8,500*
Feb. 18	*To self*	*₹ 4,000*	—	—
April 12	*By cash*	—	*₹ 2,230*	—
June 15	*To self*	*₹ 5,000*	—	—
July 8	*By cash*	—	*₹ 6,000*	—

Calculate the interest for six months from February to July at 6% p.a. **[4]**

Solution :

(a) $\text{L.H.S.} = \sqrt{\dfrac{1 - \cos A}{1 + \cos A}}$

Multiplying by $\sqrt{1 + \cos A}$ in numerator and denominator

$$= \sqrt{\frac{1 - \cos A}{1 + \cos A}} \times \sqrt{\frac{1 + \cos A}{1 + \cos A}}$$

$$= \sqrt{\frac{(1-\cos A)(1+\cos A)}{(1+\cos A)(1+\cos A)}}$$

$$= \sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}}$$

$$= \sqrt{\frac{\sin^2 A}{(1+\cos A)^2}}$$

$$= \frac{\sin A}{1+\cos A} = \text{R.H.S.}$$ **Proved**

(b) *Given* : $\angle ABC = 100°$

We know that,

$\angle ABC + \angle ADC = 180°$ (The sum of opposite angles in a cyclic quadrilateral = 180°)

$\therefore$ $100° + \angle ADC = 180°$

$\angle ADC = 180° - 100°$

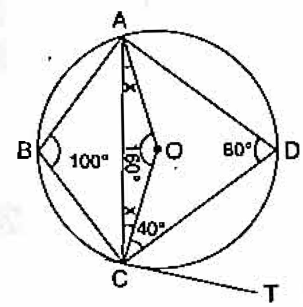
$\angle ADC = 80°$

Join OA and OC, we have a isosceles Δ OAC,

$\because$ $OA = OC$ (Radii of a circle)

$\therefore$ $\angle AOC = 2 \times \angle ADC$ (by theorem)

or $\angle AOC = 2 \times 80° = 160°$



In Δ AOC,

$\angle AOC + \angle OAC + \angle OCA = 180°$

$160° + \angle OCA + \angle OCA = 180°$ [$\because \angle OAC = \angle OCA$]

$2\angle OCA = 20°$

$\angle OCA = 10°$

$\angle OCA + \angle OCD = 40°$

$10° + \angle OCD = 40°$

$\therefore$ $\angle OCD = 30°$

Hence, $\angle OCD + \angle DCT = \angle OCT$

$\because$ $\angle OCT = 90°$

(The tangent at a point to circle is $\perp$ to the radius through the point to contant)

$30° + \angle DCT = 90°$

$\therefore$ $\angle DCT = 60°$ **Ans.**

(c)

Date	Particulars	Withdrawals	Deposit	Balance
Feb. 8	B/F	—	—	₹ 8,500
Feb. 18	To self	₹ 4,000	—	₹ 4,500
April 12	By cash	—	₹ 2,230	₹ 6,730
June 15	To self	₹ 5,000	—	₹ 1,730
July 8	By cash	—	₹ 6,000	₹ 7,730

Principal for the month of Feb. = ₹ 4,500

Principal for the month of March = ₹ 4,500

Principal for the month of April	= ₹	4,500
Principal for the month of May	= ₹	6,730
Principal for the month of June	= ₹	1,730
Principal for the month of July	= ₹	7,730
Total principal from the month of Feb. to July	= ₹	29,690

$$\text{Time} = \frac{1}{12} \text{ years}$$

$$\text{Rate of interest} = 6\%$$

$$\text{Interest} = \frac{P \times R \times T}{100}$$

$$= \frac{29690 \times 6 \times 1}{100 \times 12}$$

$$= ₹\ 148{\cdot}45$$

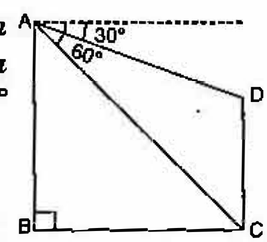
Ans.

Question 7.

(a) *In Δ ABC, A(3, 5), B(7, 8) and C(1, –10). Find the equation of the median through A.* **[3]**

(b) *A shopkeeper sells an article at the listed price of ₹ 1,500 and the rate of VAT is 12% at each stage of sale. If the shopkeeper pays a VAT of ₹ 36 to the Government, what was the price, inclusive of Tax, at which the shopkeeper purchased the article from the wholesaler ?* **[3]**

(c) *In the figure given, from the top of a building AB = 60 m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find :*

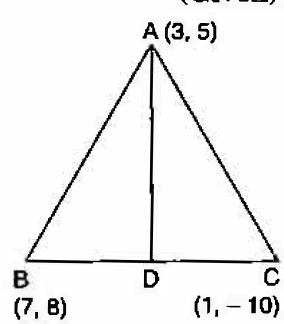


(i) *The horizontal distance between AB and CD.*

(ii) *The height of the lamp post.* **[4]**

Solution :

(a) Here D is mid point of BC. (Given)



$$\therefore \quad \text{The co-ordinate of D} = \left(\frac{7+1}{2}, \frac{8-10}{2}\right)$$

$$= (4, -1)$$

Now equation of median AD,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Here, $x_1 = 3, y_1 = 5, x_2 = 4, y_2 = -1$

$$y - 5 = \frac{-1-5}{4-3}(x-3)$$

$$y - 5 = \frac{-6}{1}(x-3)$$

$$y - 5 = -6x + 18$$

$$y = -6x + 18 + 5$$
$$y = -6x + 23$$
$$6x + y - 23 = 0$$ **Ans.**

(b) Listed price of an article = ₹ 1,500

Rate of VAT = 12%

$$\text{VAT on the article} = \frac{12}{100} \times 1500$$
$$= ₹\ 180$$

Let C.P. of this article be x, then

$$\text{VAT} = \frac{12}{100} \times x$$
$$= ₹\ \frac{12x}{100}$$

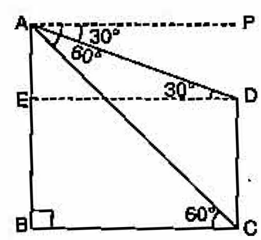
If the shopkeeper pays a VAT = ₹ 36

Then $$180 - \frac{12x}{100} = 36$$
$$\frac{18000 - 12x}{100} = 36$$
$$18000 - 12x = 3600$$
$\therefore$ $$12x = 18000 - 3600 = 14{,}400$$
$$x = ₹\ 1{,}200$$

$\therefore$ The price at which the shopkeeper purchased the article inclusive of sales tax

$$= 1{,}200 + \frac{12}{100} \times 1{,}200$$
$$= 1{,}200 + 144$$
$$= ₹\ 1{,}344$$ **Ans.**

(c) *Given* : AB = 60 m



$\because$ $$\angle PAC = 60°$$
$\therefore$ $$\angle PAC = \angle BCA$$

(i) Now in Δ ABC,

$$\tan 60° = \frac{AB}{BC}$$
$$\sqrt{3} = \frac{60}{BC}$$
$\Rightarrow$ $$\sqrt{3}\,BC = 60$$
$\Rightarrow$ $$BC = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$BC = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$$

Hence, the horizontal distance between AB and CD = $20\sqrt{3}$ m. **Ans.**

(ii) Let AE = x and proved above BC = $20\sqrt{3}$m

$\therefore$ $$BC = ED = 20\sqrt{3}$$

Now in Δ AED,

$$\tan 30^\circ = \frac{AE}{ED}$$

$$\frac{1}{\sqrt{3}} = \frac{AE}{20\sqrt{3}}$$

$\Rightarrow$ $$\sqrt{3}\,AE = 20\sqrt{3}$$

$\Rightarrow$ $$AE = 20 \text{ m}$$

now $$EB = AB - AE$$

$\therefore$ $$EB = 60 - 20 \quad \Rightarrow \quad 40 \text{ m}$$

$\because$ $$EB = CD$$

$\therefore$ $$CD = 40 \text{ m}$$

Hence, the height of the lamp post = 40 m. **Ans.**

Question 8.

(a) *Find x and y if* $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$ **[3]**

(b) *A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast.* **[3]**

(c) *Without solving the following quadratic equation, find the value of 'p' for which the given equation has real and equal roots :*

$$x^2 + (p-3)x + p = 0$$ **[4]**

Solution :

(a) *Given* :

$$\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 2x + 3x \\ 2y + 4y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 5x \\ 6y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$\therefore$ $$5x = 5 \quad \Rightarrow \quad x = 1$$

and $$6y = 12 \quad \Rightarrow \quad y = 2$$

Hence, $x = 1$ and $y = 2$ **Ans.**

(b) Radius of a solid sphere, $r = 15$ cm

Volume of a solid sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \pi (15)^3 \text{ cm}^3.$$

Now, radius of right circular cone $= 2{\cdot}5$ cm

and height, $h = 8$ cm.

Volume of right circular cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi (2{\cdot}5)^2 \times 8$$

$\therefore$ The number of cones $= \dfrac{\text{Volume of a sphere}}{\text{Volume of a cone}}$

$$= \frac{\frac{4}{3}\pi \times (15)^3}{\frac{1}{3}\pi (2\cdot5)^2 \times 8}$$

$$= \frac{15 \times 15 \times 15}{2\cdot5 \times 2\cdot5 \times 2}$$

$$= 270$$ **Ans.**

(c) Given equation $x^2 + (p-3)x + p = 0$

$\because$ Roots are real and equal, then

$$b^2 - 4ac = 0$$

Here we compare the coefficients of a, b and c with the equation $ax^2 + bx + c = 0$.

$$a = 1, b = p - 3 \text{ and } c = p$$

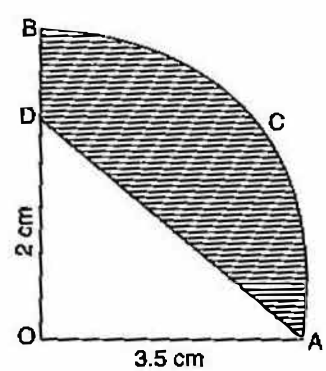
Now putting the values of a, b and c in equation

$$(p-3)^2 - 4 \times 1 \times p = 0$$
$$p^2 + 9 - 6p - 4p = 0$$
$$p^2 + 9 - 10p = 0$$
$$p^2 - 10p + 9 = 0$$
$$p^2 - 9p - p + 9 = 0$$
$$p(p-9) - 1(p-9) = 0$$
$$\Rightarrow (p-9)(p-1) = 0$$

Hence, $p = 9$ or 1 **Ans.**

Question 9.

(a) *In the figure alongside, OAB is a quadrant of a circle. The radius OA = 3.5 cm and OD = 2 cm. Calculate the area of the shaded portion.* $\left(\textit{Take } \pi = \frac{22}{7}\right)$ **[3]**



(b) *A box contains some black balls and 30 white balls. If the probability of drawing a black ball is two-fifths of a white ball, find the number of black balls in the box.* **[3]**

(c) *Find the mean of the following distribution by step deviation method :* **[4]**

Class Interval	*20–30*	*30–40*	*40–50*	*50–60*	*60–70*	*70–80*
Frequency	*10*	*6*	*8*	*12*	*5*	*9*

Solution :

(a) Radius of quadrant OACB, $r = 3{\cdot}5$ cm

$$\text{Area of quadrant OACB} = \frac{1}{4}\pi r^2$$
$$= \frac{1}{4} \times \frac{22}{7} \times 3{\cdot}5 \times 3{\cdot}5$$
$$= 9{\cdot}625 \text{ cm}^2.$$

Here, $\angle \text{AOD} = 90°$

Then area of Δ AOD $= \frac{1}{2} \times \text{base} \times \text{height}$

Base = 3·5 cm and height = 2 cm

$$= \frac{1}{2} \times 3{\cdot}5 \times 2 = 3{\cdot}5 \text{ cm}^2.$$

$$\text{Area of shaded portion} = \text{Area of quadrant} - \text{Area of triangle}$$
$$= 9{\cdot}625 - 3{\cdot}5$$
$$= 6{\cdot}125 \text{ cm}^2.$$ **Ans.**

(b) Let the number of black balls be x, then

Total number of balls $= 30 + x$

Thus, the prabability of black balls $= \frac{x}{30+x}$

and the probability of white balls $= \frac{30}{30+x}$

Given : Probability of black ball $= \frac{2}{5} \times$ probability of white ball

$$\frac{x}{30+x} = \frac{2}{5} \times \frac{30}{x+30}$$
$$5x = 60$$
$$x = 12$$ **Ans.**

Hence, the number of black balls = 12.

(c)

C.I.	Frequency (f_i)	Mid-value (x)	$d_i = \frac{x-a}{h}$	$f_i\, d_i$
20–30	10	25	–2	–20
30–40	6	35	–1	–6
40–50	8	45	0	0
50–60	12	55	1	12
60–70	5	65	2	10
70–80	9	75	3	27
	$\Sigma f_i = 50$			$\Sigma f_i d_i = 23$

Here, $a = 45$ and $h = 10$

$$\text{Mean} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} \times h$$
$$= 45 + \frac{23}{50} \times 10$$
$$= 45 + 4{\cdot}6 = 49{\cdot}6.$$ **Ans.**

Question 10.

(a) *Using a ruler and compasses only :*

(i) *Construct a triangle ABC with the following data :*
AB = 3.5 cm, BC = 6 cm and ∠ABC = 120°

(ii) *In the same diagram, draw a circle with BC as diameter. Find a point P on the circumference of the circle which is equidistant from AB and BC.*

(iii) *Measure ∠BCP.* **[3]**

(b) *The mark obtained by 120 students in a test are given below :*

Marks	No. of Students
0–10	5
10–20	9
20–30	16
30–40	22
40–50	26
50–60	18
60–70	11
70–80	6
80–90	4
90–100	3

Draw an ogive for the given distribution on a graph sheet.

Using suitable scale for ogive to estimate the following :

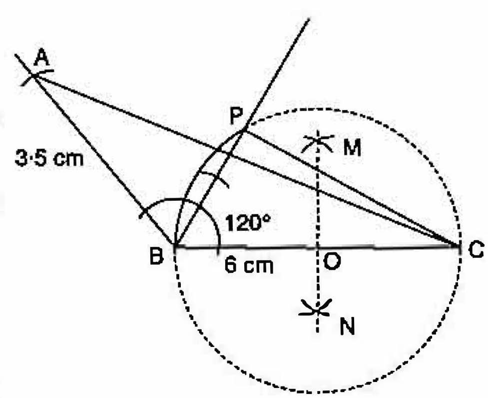
(i) *The median.*

(ii) *The number of students who obtained more than 75% marks in the test.*

(iii) *The number of students who did not pass the test if minimum marks required to pass is 40.* **[6]**

Solution :

(a) Steps of Construction :



(i) Draw a line BC = 6 cm.

(ii) With the help of the point B, draw ∠ ABC = 120°

(iii) Taking radius 3·5 cm cut BA = 3·5 cm.

(iv) Join A to C.

(v) Draw ⊥ bisector MN of BC.

(vi) Draw a circle O as centre and OC as radius.

(vii) Draw angle bisector of ∠ ABC which intersects circle at P.

(viii) Join BP and CP.

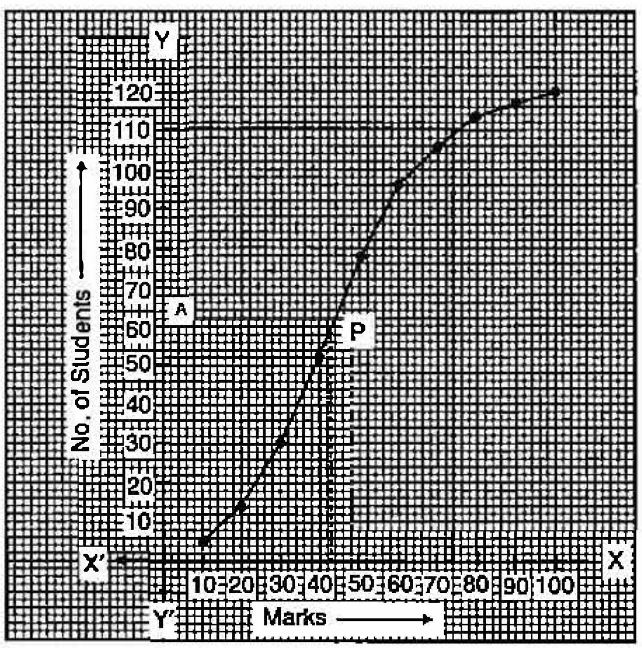
(ix) Now, ∠ BCP = 30°.

(b)

Marks	No. of Students (f)	Cumulative Frequency
0–10	5	5
10–20	9	14
20–30	16	30
30–40	22	52
40–50	26	78
50–60	18	96
60–70	11	107
70–80	6	113
80–90	4	117
90–100	3	120
	$n = 120$	

On the graph paper, we plot the following points :

(10, 5), (20, 14), (30, 30), (40, 52), (50, 78), (60, 96), (70, 107), (80, 113), (90, 117), (100, 120).



(i) $$\text{Median} = \left(\frac{n}{2}\right)^{th} \text{ term} \qquad [\because n = 120, \text{ even}]$$

$$= \frac{120}{2} = 60^{th} \text{ term}$$

From the graph 60th term = 42 **Ans.**

(ii) The number of students who obtained more than 75% marks in test

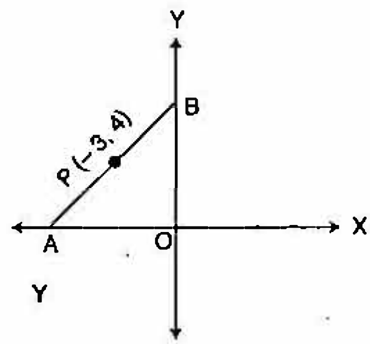
$= 120 - 110$

$= 10.$ **Ans.**

(iii) The number of students who did not pass the test if the minimum pass marks 40 = 52. **Ans.**

Question 11.

(a) *In the figure given below, the line segment AB meets X-axis at A and Y-axis at B. The point P(–3, 4) on AB divides it in the ratio 2 : 3. Find the coordinates of A and B.*



(b) *Using the properties of proportion, solve for x, given*

$$\frac{x^4+1}{2x^2} = \frac{17}{8}$$ **[3]**

(c) *A shopkeeper purchases a certain number of books for ₹ 960. If the cost per book was ₹ 8 less, the number of books that could be purchased for ₹ 960 would be 4 more. Write an equation, taking the original cost of each book to be ₹ x, and solve it to find the original cost of the books.* **[4]**

Solution :

(a) Let the co-ordinates of A and B be $(x, 0)$ and $(0, y)$

$\because$ The co-ordinates of a point P (–3, 4) on AB divides it in the ratio 2 : 3.

i.e., $\quad AP : PB = 2 : 3$

By using section formula, we get

$$-3 = \frac{2\times 0 + 3\times x}{2+3} \qquad \left[\because x = \frac{m_1x_2 + m_2x_1}{m_1+m_2}\right]$$

$$-3 = \frac{3x}{5} \Rightarrow 3x = -15$$

$$\Rightarrow \quad x = -5$$

and

$$4 = \frac{2\times y + 3\times 0}{2+3} \qquad \left[\because y = \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right]$$

$$4 = \frac{2y}{5} \Rightarrow 2y = 20$$

$$\Rightarrow \quad y = 10$$

Hence, the co-ordinates of A and B are (–5, 0) and (0, 10). **Ans.**

(b) *Given* : $\quad \dfrac{x^4+1}{2x^2} = \dfrac{17}{8}$

By using componendo and dividendo, we get

$$\frac{x^4+1+2x^2}{x^4+1-2x^2} = \frac{17+8}{17-8}$$

$$\left(\frac{x^2+1}{x^2-1}\right)^2 = \frac{25}{9}$$

$$\left(\frac{x^2+1}{x^2-1}\right)^2 = \left(\frac{5}{3}\right)^2$$

Taking square root on both sides, we get

$$\frac{x^2+1}{x^2-1} = \frac{5}{3}$$

$$\Rightarrow \quad 5x^2-5 = 3x^2+3$$

$$\Rightarrow \quad 5x^2-3x^2 = 3+5$$

$$\Rightarrow \quad 2x^2 = 8 \quad \Rightarrow \quad x^2 = 4$$

$$\Rightarrow \quad x = \pm 2$$ **Ans.**

(c) Given the original cost of each book be ₹ x.

$$\text{Total cost} = ₹\,960 \qquad \text{(Given)}$$

$$\text{Number of books for } 960 = 960$$

If the cost per book was ₹ 8 less, (*i.e.,* $x - 8$) then

$$\text{Number of books} = \frac{960}{x-8}$$

According to question,

$$\frac{960}{x-8} = \frac{960}{x} + 4$$

$$\frac{960}{x-8} - \frac{960}{x} = 4$$

$$960\left[\frac{x-x+8}{x(x-8)}\right] = 4$$

$$\frac{8}{x^2-8x} = \frac{1}{240}$$

$$\Rightarrow \quad x^2-8x = 1{,}920$$

$$x^2-8x-1{,}920 = 0$$ **Ans.**

$$\Rightarrow \quad x^2-48x+40x-1{,}920 = 0$$

$$\Rightarrow \quad x(x-48)+40(x-48) = 0$$

$$\Rightarrow \quad (x-48)(x+40) = 0$$

$$x-48 = 0 \quad \text{or} \quad x+40 = 0$$

$$x = 48 \quad \text{or} \quad x = -40$$

$\because$ -40 is not possible.

Hence, the original cost of each book = ₹ 48. **Ans**