ICSE Question Paper (2013)

MATHEMATICS

SECTION A [40 Marks]

(Answer all questions from this Section.)

Question 1.

(a) Given \( A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} \), \( B = \begin{bmatrix} -3 & 2 \\ 2 & 0 \end{bmatrix} \), \( C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} \).

Find the matrix \( X \) such that \( A + 2X = 2B + C \).

(b) At what rate \% p.a. will a sum of ₹4000 yield ₹1324 as compound interest in 3 years?

(c) The median of the following observations 11, 12, 14, (\( x - 2 \)), (\( x + 4 \)), (\( x + 9 \)), 32, 38, 47 arranged in ascending order is 24. Find the value of \( x \) and hence find the mean.

Solution:

(a) Given:
\[
A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 2 \\ 2 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}
\]

\( A + 2X = 2B + C \)

Putting the given values, we get
\[
\begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = 2 \begin{bmatrix} -3 & 2 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}
\]

\[
2X - \begin{bmatrix} -6 + 4 & 4 + 0 \\ 8 + 0 & 0 + 2 \end{bmatrix} = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}
\]

\[
X = \frac{1}{2} \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}
\]

\[
X = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix}
\]

(b) Given:
Principal = ₹4,000, C.I. = ₹1,324,
Amount = \( P + C.I. \)
\[
= ₹(4,000 + 1,324) = ₹5,324
\]

Time = 3 years

We know that,
\[
A = P \left( 1 + \frac{r}{100} \right)^t
\]

\[
5,324 = 4,000 \left( 1 + \frac{r}{100} \right)^3
\]

\[
5,324 = \left( 1 + \frac{r}{100} \right)^3
\]
\[
\frac{1,331}{1,000} = \left(1 + \frac{r}{100}\right)^3
\]
\[
\left(\frac{11}{10}\right)^3 = \left(1 + \frac{r}{100}\right)^3
\]

Therefore,
\[
1 + \frac{r}{100} = \frac{11}{10}
\]
\[
\frac{r}{100} = \frac{11}{10} - 1
\]
\[
\frac{r}{100} = \frac{1}{10}
\]
\[
\frac{r}{10} = \frac{100}{10} = 10
\]
\[
r = 100
\]
\[
r = 10\%
\]

(c) Given observation are 11, 12, 14, (\(x - 2\)), (\(x + 4\)), (\(x + 9\)), 32, 38, 47 and median = 24.

\[
n = 9 \text{ (odd)}
\]

Median = \(\frac{n + 1}{2}\) th term
\[
= \frac{9 + 1}{2} \text{ th term}
\]
\[
24 = 5\text{th term}
\]
\[
x + 4 = 24
\]
\[
x = 24 - 4
\]
\[
x = 20
\]

Therefore, 11, 12, 14, (20 - 2), (20 + 4), (20 + 9), 32, 38, 47
\[
= 11, 12, 14, 18, 24, 29, 32, 38, 47
\]

Now
\[
\text{Mean} = \frac{\sum x}{n}
\]
\[
= \frac{11 + 12 + 14 + 18 + 24 + 29 + 32 + 38 + 47}{9}
\]
\[
= \frac{225}{9} = 25
\]

Question 2.

(a) \textbf{What number must be added to each of the number 6, 15, 20 and 43 to make them proportional?} \hspace{1cm} [3]

(b) \textbf{If } (x - 2) \text{ is a factor of the expression } 2x^3 + ax^2 + bx - 14 \text{ and when the expression is divided by } (x - 3), \text{ it leaves a remainder 52, find the values of } a \text{ and } b. \hspace{1cm} [3]

(c) \textbf{Draw a histogram from the following frequency distribution and find the mode from the graph:} \hspace{1cm} [4]

<table>
<thead>
<tr>
<th>Class</th>
<th>0-5</th>
<th>5-10</th>
<th>10-15</th>
<th>15-20</th>
<th>20-25</th>
<th>25-30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>2</td>
<td>5</td>
<td>18</td>
<td>14</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>
Solution:

(a) Let the number must be added be $x$, then

the new number = $6 + x$, $15 + x$, $20 + x$, $43 + x$

\[ \therefore \] These are proportionals.

\[ \frac{6 + x}{15 + x} :: \frac{20 + x}{43 + x} \]

or

\[ (6 + x)(43 + x) = (15 + x)(20 + x) \]

or

\[ 258 + 6x + 43x + x^2 = 300 + 20x + 15x + x^2 \]

or

\[ 49x - 35x = 300 - 258 \]

or

\[ 14x = 42 \]

or

\[ x = 3. \]

Ans.

(b) Let $(x - 2)$ is a factor of the given expression.

\[ x - 2 = 0 \]

\[ x = 2 \]

Given expression,

\[ 2x^3 + ax^2 + bx - 14 = 0 \]

\[ 2(2)^3 + a(2)^2 + b(2) - 14 = 0 \]

\[ 16 + 4a + 2b - 14 = 0 \]

\[ 4a + 2b + 2 = 0 \]

\[ 4a + 2b = -2 \]

\[ 2a + b = -1 \] \( \text{...(i)} \)

and when given expression is divided by $(x - 3)$

\[ x - 3 = 0 \]

\[ x = 3 \]

\[ 2x^3 + ax^2 + bx - 14 = 52 \]

\[ 2(3)^3 + a(3)^2 + b(3) - 66 = 0 \]

\[ 54 + 9a + 3b - 66 = 0 \]

\[ 9a + 3b = 12 \]

\[ 3a + b = 4 \] \( \text{...(ii)} \)

Solving equation (i) and (ii),

\[ 2a + b = -1 \]

\[ 3a + b = 4 \]

\[ (-) \quad (-) \quad (+) \]

\[ -a = -5 \]

\[ a = 5 \]

from (ii),

\[ 3 \times 5 + b = 4 \]

\[ b = 4 - 15 \]

\[ b = -11 \]

\[ a = 5 \text{ and } b = -11 \]

Ans.
Question 3.

(a) Without using tables evaluate $3 \cos 80^\circ \cdot \csc 10^\circ + 2 \sin 59^\circ \cdot \sec 31^\circ$.

(b) In the given figure,

\[ \angle BAD = 65^\circ, \]
\[ \angle ABD = 70^\circ, \]
\[ \angle BDC = 45^\circ \]

(i) Prove that $AC$ is a diameter of the circle.

(ii) Find $\angle ACB$.

(c) $AB$ is a diameter of a circle with centre $C = (-2, 5)$. If $A = (3, -7)$. Find:

(i) The length of radius $AC$

(ii) The coordinates of $B$.

Solution:

(a) Given:

\[
3 \cos 80^\circ \cdot \csc 10^\circ + 2 \sin 59^\circ \cdot \sec 31^\circ
= 3 \cos 80^\circ \cdot \csc (90^\circ - 80^\circ) + 2 \sin 59^\circ \cdot \sec (90^\circ - 59^\circ)
= 3 \cos 80^\circ \cdot \csc 80^\circ + 2 \sin 59^\circ \cdot \sec 59^\circ
= 3 \cos 80^\circ \cdot \frac{1}{\cos 80^\circ} + 2 \sin 59^\circ \cdot \frac{1}{\sin 59^\circ}
= 3 + 2 = 5. \quad \text{Ans.}
\]

(b) Given: $\angle BAD = 65^\circ$, $\angle ABD = 70^\circ$, $\angle BDC = 45^\circ$.

(i) \[ \therefore \text{ABCD is a cyclic quadrilateral.} \]

In $\triangle ABD$,

\[ \angle BDA + \angle DAB + \angle ABD = 180^\circ \]

By using sum property of $\triangle$

\[ \therefore \angle BDA = 180^\circ - (65^\circ + 70^\circ) \]
\[ = 180^\circ - 135^\circ \]
\[ = 45^\circ \]
Now from $\triangle ACD$,
\[ \angle ADC = \angle ADB + \angle BDC \]
\[ = 45^\circ + 45^\circ \]
\[ = 90^\circ \]
\[ (\because \angle BDA = \angle ADB = 45^\circ) \]

Hence, $\angle D$ makes right angle belongs in semi-circle therefore $AC$ is a diameter of the circle.

(ii) $\angle ACB = \angle ADB$ (Angles in the same segment of a circle)
\[ \angle ACB = 45^\circ \]
\[ (\because \angle ADB = 45^\circ) \]

Ans.

(c) (i) The length of radius $AC = \sqrt{(-2 -3)^2 + (5 + 7)^2}$
\[ = \sqrt{(-5)^2 + (12)^2} \]
\[ = \sqrt{25 + 144} \]
\[ = \sqrt{169} \]
\[ = 13. \]

Ans.

(ii) Let the point of $B$ be $(x, y)$.
Given $C$ is the mid-point of $AB$. Therefore
\[ -2 = \frac{3 + x}{2} \]
\[ \Rightarrow \]
\[ 3 + x = -4 \]
\[ \Rightarrow \]
\[ x = -4 - 3 = -7 \]

and
\[ 5 = \frac{-7 + y}{2} \]
\[ \Rightarrow \]
\[ 10 = -7 + y \]
\[ y = 17 \]

Hence, the co-ordinate of $B$ $(-7, 17)$.

Ans.

Question 4.

(a) Solve the following equation and calculate the answer correct to two decimal places:
\[ x^2 - 5x - 10 = 0. \] [3]

(b) In the given figure, $AB$ and $DE$ are perpendicular to $BC$.

(i) Prove that $\triangle ABC \sim \triangle DEC$

(ii) If $AB = 6$ cm, $DE = 4$ cm and $AC = 15$ cm. Calculate $CD$.

(iii) Find the ratio of the area of $\triangle ABC$ : area of $\triangle DEC$. [3]

(c) Using graph paper, plot the points $A(6, 4)$ and $B(0, 4)$.

(i) Reflect $A$ and $B$ in the origin to get the images $A'$ and $B'$.

(ii) Write the co-ordinates of $A'$ and $B'$.

(iii) State the geometrical name for the figure $ABA'B'$.

(iv) Find its perimeter. [4]
Solution:
(a) Given: \( x^2 - 5x - 10 = 0 \)

Here, \( a = 1 \), \( b = -5 \) and \( c = -10 \)

\[
D = b^2 - 4ac = (-5)^2 - 4 \times 1 \times -10 = 25 + 40 = 65
\]

\[
x = \frac{-b \pm \sqrt{D}}{2a} = \frac{5 \pm \sqrt{65}}{2} = \frac{5 \pm 8.06}{2}
\]

\[
x = \frac{13.06}{2} = 6.53, \quad \frac{3.06}{2} = -1.53
\]

(b) (i) From \( \triangle ABC \) and \( \triangle DEC \),

\[
\angle ABC = \angle DEC = 90^\circ \quad ( \text{Given} )
\]

and

\[
\angle ACB = \angle DCE = \text{Common}
\]

\[
\therefore \quad \triangle ABC \sim \triangle DEC \quad ( \text{By AA similarity} )
\]

(ii) In \( \triangle ABC \) and \( \triangle DEC \),

\[
\frac{AB}{DE} = \frac{AC}{CD}
\]

\[
\text{Given} : AB = 6 \text{ cm}, DE = 4 \text{ cm}, AC = 15 \text{ cm},
\]

\[
\therefore \quad \frac{6}{4} = \frac{15}{CD}
\]

\[
\therefore \quad 6 \times CD = 15 \times 4
\]

\[
\therefore \quad CD = \frac{60}{6} = 10 \text{ cm.}
\]

(iii) \[
\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEC} = \frac{AB^2}{DE^2} \quad (\because \triangle ABC \sim \triangle DEC)
\]

\[
= \frac{(6)^2}{(4)^2} = \frac{36}{16} = \frac{9}{4}
\]

\[
\therefore \quad \text{Area of } \triangle ABC : \text{Area of } \triangle DEC = 9 : 4.
\]

Ans.
(c) (i) Please See Graph.

(ii) Reflection of $A'$ and $B'$ in the origin $= A' (-6, -4)$ and $B' (0, -4)$

(iii) The geometrical name for the figure $AB A'B'$ is a parallelogram.

(iv) From the graph, $AB = 6\, \text{cm}$, $BB' = 8\, \text{cm}$.

In $\triangle ABB'$

$$AB' = 10 = A'B'$$

(AB $A'$ $B'$ is a parallelogram)

Perimeter of $AB A'B' = A'B' + AB' + AB + A'B$

$= 6 + 10 + 6 + 10$

$= 32$ units.

SECTION B [40 Marks]
Answer any four Questions in this Section.

Question 5.

(a) Solve the following inequation, write the solution set and represent it on the number line:

$$-\frac{x}{3} \leq \frac{x}{2} - 1 \frac{1}{3} < \frac{1}{6}, x \in R$$

(b) Mr. Britto deposits a certain sum of money each month in a Recurring Deposit Account of a bank. If the rate of interest is of 8% per annum and Mr. Britto gets ₹ 8088 from the bank after 3 years, find the value of his monthly instalment.

(c) Salman buys 50 shares of face value ₹ 100 available at ₹ 132.

(i) What is his investment?
(ii) If the dividend is 7.5%, what will be his annual income?
(iii) If he wants to increase his annual income by ₹ 150, how many extra shares should he buy?
Solution:

(a) Given:
\[-\frac{x}{3} \leq \frac{x}{2} - \frac{1}{3} < \frac{1}{6}\]

Taking L.C.M. of 3, 2 and 6 is 6.
\[-\frac{2x}{3} \leq \frac{x}{2} \times 6 - \frac{4}{3} \times 6 < \frac{1}{6} \times 6\]
\[-2x \leq 3x - 8 < 1\]
\[\Rightarrow -2x \leq 3x - 8 \quad \text{and} \quad 3x - 8 < 1\]
\[\Rightarrow 8 \leq 3x + 2x \quad \Rightarrow \quad 3x < 1 + 8\]
\[\Rightarrow 8 \leq 5x \quad \Rightarrow \quad 3x < 9\]
\[\Rightarrow -\frac{5}{3} \leq x \quad \Rightarrow \quad x < 3\]

\[\therefore \text{The solution set is } \{x : 1.6 \leq x \leq 3, x \in \mathbb{R}\}\]

(b) Let the monthly instalment be ₹x

Given: Maturity amount = ₹ 8,088, Time (n) = 3 years = 3 \times 12 \text{ months} = 36 \text{ months}, Rate (R) = 8\% \text{ p.a.}

\[\text{Principle for one month} = P \times \frac{n(n+1)}{2}\]
\[= x \times 36 \times 37\]
\[= \frac{18 \times 37x}{2}\]

Interest = \[\frac{18 \times 37x \times 1}{100 \times 12} = \frac{444x}{100}\]

Actual sum deposited = 36x

Maturity amount = Interest + Actual sum deposited

\[8,088 = \frac{444x}{100} + 36x\]
\[8,088 = \frac{4044x}{100}\]
\[x = \frac{8,088 \times 100}{4,044} = 200\]

Hence, the monthly instalment be ₹ 200. \hspace{1cm} \text{Ans.}

(c) Number of shares = 50
Face value of each share = ₹ 100
Market value of each share = ₹ 132

Total face value = ₹ 100 \times 50
= ₹ 5,000

(i) Total investment = ₹ 132 \times 50
= ₹ 6,600 \hspace{1cm} \text{Ans.}
(ii) Rate of dividend = 7.5%
\[
\text{Annual income} = \frac{\text{₹} 5,000 \times 7.5}{100} = \text{₹} 375
\]

(iii) Let extra share should be buy be \(x\).
then total number of shares = 50 + \(x\)
Total face value = \(\text{₹} 100 \times (50 + x)\)

\[
\text{Annual income} = \frac{\text{₹} 100 \times (50 + x) \times 7.5}{100} = (50 + x) \times 7.5
\]

\[
(50 + x) \times 7.5 = 375 + 150
\]

\[
50 + x = \frac{525}{7.5} = 70
\]

\[
x = 70 - 50
\]

\[
x = 20
\]

Hence, the extra shares should be buy = 20.

Question 6.

(a) Show that \[\frac{1 - \cos A}{1 + \cos A} = \frac{\sin A}{1 + \cos A}\]

(b) In the given circle with centre \(O\), \(\angle ABC = 100^\circ, \angle ACD = 40^\circ\) and \(CT\) is a tangent to the circle at \(C\). Find \(\angle ADC\) and \(\angle DCT\).

(c) Given below are the entries in a Savings Bank A/c pass book:

<table>
<thead>
<tr>
<th>Date</th>
<th>Particulars</th>
<th>Withdrawals</th>
<th>Deposit</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feb. 8</td>
<td>B/F</td>
<td>—</td>
<td>—</td>
<td>₹8,500</td>
</tr>
<tr>
<td>Feb. 18</td>
<td>To self</td>
<td>₹4,000</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>April 12</td>
<td>By cash</td>
<td>—</td>
<td>₹2,230</td>
<td>—</td>
</tr>
<tr>
<td>June 15</td>
<td>To self</td>
<td>₹5,000</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>July 8</td>
<td>By cash</td>
<td>₹6,000</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Calculate the interest for six months from February to July at 6% p.a.

Solution:

(a) L.H.S. = \[\sqrt{\frac{1 - \cos A}{1 + \cos A}}\]

Multiplying by \(\sqrt{1 + \cos A}\) in numerator and denominator

\[
= \sqrt{\frac{1 - \cos A}{1 + \cos A} \times \frac{1 + \cos A}{1 + \cos A}}
\]
\[= \sqrt{(1 - \cos A)(1 + \cos A)} \]
\[= \sqrt{\frac{1 - \cos^2 A}{(1 + \cos A)^2}} \]
\[= \sqrt{\frac{\sin^2 A}{(1 + \cos A)^2}} \]
\[= \frac{\sin A}{1 + \cos A} = \text{R.H.S.} \]

**Proved**

(b) \(\angle ABC = 100^\circ\)

We know that,
\[\angle ABC + \angle ADC = 180^\circ \quad \text{(The sum of opposite angles in a cyclic quadrilateral = 180°)}\]
\[100^\circ + \angle ADC = 180^\circ \]
\[\angle ADC = 180^\circ - 100^\circ = 80^\circ \]

Join OA and OC, we have a isosceles \(\triangle OAC,\)
\[\angle AOC = 2 \times \angle ADC \quad \text{(by theorem)}\]

or
\[\angle AOC = 2 \times 80^\circ = 160^\circ \]

In \(\triangle AOC,\)
\[\angle AOC + \angle OAC + \angle OCA = 180^\circ \]
\[160^\circ + \angle OCA + \angle OCA = 180^\circ \quad [\therefore \angle OAC = \angle OCA] \]
\[2 \angle OCA = 20^\circ \]
\[\angle OCA = 10^\circ \]
\[\angle OCA + \angle OCD = 40^\circ \]
\[10^\circ + \angle OCD = 40^\circ \]
\[\angle OCD = 30^\circ \]

Hence,
\[\angle OCD + \angle DCT = \angle OCT \]
\[\angle OCT = 90^\circ \]

(The tangent at a point to circle is \(\perp\) to the radius through the point to contant)
\[30^\circ + \angle DCT = 90^\circ \]
\[\angle DCT = 60^\circ \]

<table>
<thead>
<tr>
<th>Date</th>
<th>Particulars</th>
<th>Withdrawals</th>
<th>Deposit</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feb. 8</td>
<td>B/F</td>
<td>—</td>
<td>—</td>
<td>8,500</td>
</tr>
<tr>
<td>Feb. 18</td>
<td>To self</td>
<td>4,000</td>
<td>—</td>
<td>4,500</td>
</tr>
<tr>
<td>April 12</td>
<td>By cash</td>
<td>—</td>
<td>2,230</td>
<td>6,730</td>
</tr>
<tr>
<td>June 15</td>
<td>To self</td>
<td>5,000</td>
<td>—</td>
<td>1,730</td>
</tr>
<tr>
<td>July 8</td>
<td>By cash</td>
<td>—</td>
<td>6,000</td>
<td>7,730</td>
</tr>
</tbody>
</table>

Principal for the month of Feb. = ₹ 4,500
Principal for the month of March = ₹ 4,500
Principal for the month of April = ₹ 4,500
Principal for the month of May = ₹ 6,730
Principal for the month of June = ₹ 1,730
Principal for the month of July = ₹ 7,730
Total principal from the month of Feb. to July = ₹ 29,690

Time = $\frac{1}{12}$ years
Rate of interest = 6%
Interest = $\frac{P \times R \times T}{100}$

\[= \frac{29690 \times 6 \times 1}{100 \times 12} = ₹ 148.45 \text{ Ans.}\]

Question 7.
(a) In $\triangle ABC$, A(3, 5), B(7, 8) and C(1, -10). Find the equation of the median through A. \[\text{[3]}\]
(b) A shopkeeper sells an article at the listed price of ₹ 1,500 and the rate of VAT is 12% at each stage of sale. If the shopkeeper pays a VAT of ₹ 36 to the Government, what was the price, inclusive of Tax, at which the shopkeeper purchased the article from the wholesaler? \[\text{[3]}\]
(c) In the figure given, from the top of a building $AB = 60$ m high, the angles of depression of the top and bottom of a vertical lamp post $CD$ are observed to be $30^\circ$ and $60^\circ$ respectively. Find:
(i) The horizontal distance between $AB$ and $CD$.
(ii) The height of the lamp post. \[\text{[4]}\]

Solution:
(a) Here D is mid point of BC.

\[\text{The co-ordinate of } D = \left( \frac{7 + 1}{2}, \frac{8 - 10}{2} \right) = (4, -1)\]

Now equation of median AD,

\[y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)\]

Here, $x_1 = 3, y_1 = 5, x_2 = 4, y_2 = -1$

\[y - 5 = \frac{-1 - 5}{4 - 3} (x - 3)\]
\[y - 5 = \frac{-6}{1} (x - 3)\]
\[y - 5 = -6x + 18\]
\[ y = -6x + 18 + 5 \]
\[ y = -6x + 23 \]
\[ 6x + y - 23 = 0 \]

(b) Listed price of an article = \( \text₹1,500 \)
Rate of VAT = 12%
VAT on the article = \( \frac{12}{100} \times 1500 \)
\[ = \text₹180 \]

Let C.P. of this article be \( x \), then
\[ \text{VAT} = \frac{12}{100} \times x \]
\[ = \frac{12x}{100} \]

If the shopkeeper pays a VAT = \( \text₹36 \)
Then
\[ 180 - \frac{12x}{100} = 36 \]
\[ \frac{18000 - 12x}{100} = 36 \]
\[ 18000 - 12x = 3600 \]
\[ 12x = 18000 - 3600 = 14,400 \]
\[ x = \text₹1,200 \]

\[ \therefore \text{The price at which the shopkeeper purchased the article inclusive of sales tax} \]
\[ = 1,200 + \frac{12}{100} \times 1,200 \]
\[ = 1,200 + 144 \]
\[ = \text₹1,344 \]

(c) \textbf{Given}: \( AB = 60 \text{ m} \)
\[ \therefore \angle PAC = 60^0 \]
\[ \therefore \angle PAC = \angle BCA \]

(i) \textbf{Now in } \triangle ABC, \]
\[ \tan 60^0 = \frac{AB}{BC} \]
\[ \sqrt{3} = \frac{60}{BC} \]
\[ \Rightarrow \sqrt{3} BC = 60 \]
\[ \Rightarrow BC = \frac{60}{\sqrt{3}} \times \frac{1}{\sqrt{3}} \]
\[ = \frac{60\sqrt{3}}{3} = 20\sqrt{3} \]

\[ \text{Hence, the horizontal distance between AB and CD = } 20\sqrt{3} \text{ m. } \]

(ii) \textbf{Let } AE = x \text{ and proved above } BC = 20\sqrt{3} \text{ m} \]
\[ \therefore \quad BC = ED = 20\sqrt{3} \]
Now in $\triangle AED$, 
\[
\tan 30^\circ = \frac{AE}{ED} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AE}{20\sqrt{3}}
\]
\[
\Rightarrow \quad \sqrt{3} \ AE = 20 \ \sqrt{3} \quad \Rightarrow \quad AE = 20 \ \text{m}
\]

now \quad EB = AB - AE
\[
\Rightarrow \quad EB = 60 - 20 \quad \Rightarrow \quad 40 \ \text{m}
\]

Hence, the height of the lamp post = 40 m. 

**Answer.**

**Question 8.**
(a) Find $x$ and $y$ if 
\[
\begin{bmatrix}
x & 3x \\
y & 4y
\end{bmatrix} \begin{bmatrix}
2 \\
1
\end{bmatrix} = \begin{bmatrix}
5 \\
12
\end{bmatrix}
\]

(b) A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast.

(c) Without solving the following quadratic equation, find the value of ‘$p$’ for which the given equation has real and equal roots:
\[
x^2 + (p - 3)x + p = 0
\]

**Solution:**
(a) Given:
\[
\begin{bmatrix}
x & 3x \\
y & 4y
\end{bmatrix} \begin{bmatrix}
2 \\
1
\end{bmatrix} = \begin{bmatrix}
5 \\
12
\end{bmatrix}
\]
\[
\begin{bmatrix}
2x + 3x \\
2y + 4y
\end{bmatrix} = \begin{bmatrix}
5 \\
12
\end{bmatrix}
\]
\[
\begin{bmatrix}
5x \\
6y
\end{bmatrix} = \begin{bmatrix}
5 \\
12
\end{bmatrix}
\]
\[
\Rightarrow \quad 5x = 5 \quad \Rightarrow \quad x = 1
\]
and \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 6y = 12 \quad \Rightarrow \quad y = 2
\]

Hence, $x = 1$ and $y = 2$. 

(b) Radius of a solid sphere, $r = 15$ cm

Volume of a solid sphere = \[
\frac{4}{3} \pi r^3
\]
\[
= \frac{4}{3} \times \pi (15)^3 \text{ cm}^3.
\]

Now, \quad radius of right circular cone = 2.5 cm
and \quad height, $h = 8$ cm.

Volume of right circular cone = \[
\frac{1}{3} \pi r^2h
\]
\[
= \frac{1}{3} \pi (2.5)^2 \times 8
\]
The number of cones = \( \frac{\text{Volume of a sphere}}{\text{Volume of a cone}} \)

\[ = \frac{\frac{4}{3} \pi \times (15)^3}{\frac{1}{3} \pi (2.5)^2 \times 8} \]

\[ = \frac{15 \times 15 \times 15}{2.5 \times 2.5 \times 2} \]

\[ = 270 \]

(c) Given equation \( x^2 + (p - 3)x + p = 0 \)

\[ \therefore \] Roots are real and equal, then

\[ b^2 - 4ac = 0 \]

Here we compare the coefficients of \( a, b \) and \( c \) with the equation \( ax^2 + bx + c = 0 \).

\[ a = 1, b = p - 3 \text{ and } c = p \]

Now putting the values of \( a, b \) and \( c \) in equation

\[ (p - 3)^2 - 4 \times 1 \times p = 0 \]

\[ p^2 + 9 - 6p - 4p = 0 \]

\[ p^2 + 9 - 10p = 0 \]

\[ p^2 - 10p + 9 = 0 \]

\[ p^2 - 9p - p + 9 = 0 \]

\[ p (p - 9) - 1(p - 9) = 0 \]

\[ \Rightarrow \]

\[ (p - 9)(p - 1) = 0 \]

Hence,

\[ p = 9 \text{ or } 1 \]

Question 9.

(a) In the figure alongside, \( OAB \) is a quadrant of a circle. The radius \( OA = 3.5 \text{ cm} \) and \( OD = 2 \text{ cm} \). Calculate the area of the shaded portion. (Take \( \pi = \frac{22}{7} \) ) [3]

(b) A box contains some black balls and 30 white balls. If the probability of drawing a black ball is two-fifths of a white ball, find the number of black balls in the box. [3]

(c) Find the mean of the following distribution by step deviation method:

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>20–30</th>
<th>30–40</th>
<th>40–50</th>
<th>50–60</th>
<th>60–70</th>
<th>70–80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>10</td>
<td>6</td>
<td>8</td>
<td>12</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>
Solution:
(a) Radius of quadrant OACB, \( r = 3.5 \) cm

{\[
\text{Area of quadrant OACB} = \frac{1}{4} \pi r^2
\]}

{\[
= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5
\]}

{\[
= 9.625 \text{ cm}^2.
\]}

Here, \( \angle AOD = 90^\circ \)

Then, area of \( \triangle AOD = \frac{1}{2} \times \text{base} \times \text{height} \)

Base = 3.5 cm and height = 2 cm

{\[
= \frac{1}{2} \times 3.5 \times 2 = 3.5 \text{ cm}^2.
\]}

Area of shaded portion = Area of quadrant - Area of triangle

{\[
= 9.625 - 3.5
\]}

{\[
= 6.125 \text{ cm}^2.
\]}

(b) Let the number of black balls be \( x \), then

Total number of balls = 30 + \( x \)

Thus, the probability of black balls = \( \frac{x}{30 + x} \)

and the probability of white balls = \( \frac{30}{30 + x} \)

Given: Probability of black ball = \( \frac{2}{5} \times \) probability of white ball

{\[
\frac{x}{30 + x} = \frac{2}{5} \times \frac{30}{x + 30}
\]}

{\[
5x = 60
\]}

{\[
x = 12
\]}

Hence, the number of black balls = 12.

(c) C.I. | Frequency \((f_i)\) | Mid-value \((x)\) | \(d_i = \frac{x - a}{h}\) | \(f_i \cdot d_i\) | \(\Sigma f_i = 50\) | \(\Sigma f_i d_i = 23\)
---|---|---|---|---|---|---
20–30 | 10 | 25 | -2 | -20 | 
30–40 | 6 | 35 | -1 | -6 | 
40–50 | 8 | 45 | 0 | 0 | 
50–60 | 12 | 55 | 1 | 12 | 
60–70 | 5 | 65 | 2 | 10 | 
70–80 | 9 | 75 | 3 | 27 | 

Here, \( a = 45 \) and \( h = 10 \)

{\[
\text{Mean} = a + \frac{\Sigma f_i d_i \times h}{\Sigma f_i}
\]}

{\[
= 45 + \frac{23 \times 10}{50}
\]}

{\[
= 45 + 4.6 = 49.6.
\]}

Ans.
Question 10.

(a) Using a ruler and compasses only:
   (i) Construct a triangle ABC with the following data:
   \[ AB = 3.5 \text{ cm}, \ BC = 6 \text{ cm and } \angle ABC = 120^\circ \]
   (ii) In the same diagram, draw a circle with BC as diameter. Find a point P on the circumference of the circle which is equidistant from AB and BC.
   (iii) Measure \( \angle BCP \).

(b) The mark obtained by 120 students in a test are given below:

<table>
<thead>
<tr>
<th>Marks</th>
<th>No. of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–10</td>
<td>5</td>
</tr>
<tr>
<td>10–20</td>
<td>9</td>
</tr>
<tr>
<td>20–30</td>
<td>16</td>
</tr>
<tr>
<td>30–40</td>
<td>22</td>
</tr>
<tr>
<td>40–50</td>
<td>26</td>
</tr>
<tr>
<td>50–60</td>
<td>18</td>
</tr>
<tr>
<td>60–70</td>
<td>11</td>
</tr>
<tr>
<td>70–80</td>
<td>6</td>
</tr>
<tr>
<td>80–90</td>
<td>4</td>
</tr>
<tr>
<td>90–100</td>
<td>3</td>
</tr>
</tbody>
</table>

Draw an ogive for the given distribution on a graph sheet.
Using suitable scale for ogive to estimate the following:
(i) The median.
(ii) The number of students who obtained more than 75% marks in the test.
(iii) The number of students who did not pass the test if minimum marks required to pass is 40.

Solution:
(a) Steps of Construction:
   (i) Draw a line BC = 6 cm.
   (ii) With the help of the point B, draw \( \angle ABC = 120^\circ \)
   (iii) Taking radius 3.5 cm cut BA = 3.5 cm.
   (iv) Join A to C.
   (v) Draw \( \perp \) bisector MN of BC.
   (vi) Draw a circle O as centre and OC as radius.
   (vii) Draw angle bisector of \( \angle ABC \) which intersects circle at P.
   (viii) Join BP and CP.
   (ix) Now, \( \angle BCP = 30^\circ \).
(b) | Marks   | No. of Students \((f)\) | Cumulative Frequency |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0–10</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>10–20</td>
<td>9</td>
<td>14</td>
</tr>
<tr>
<td>20–30</td>
<td>16</td>
<td>30</td>
</tr>
<tr>
<td>30–40</td>
<td>22</td>
<td>52</td>
</tr>
<tr>
<td>40–50</td>
<td>26</td>
<td>78</td>
</tr>
<tr>
<td>50–60</td>
<td>18</td>
<td>96</td>
</tr>
<tr>
<td>60–70</td>
<td>11</td>
<td>107</td>
</tr>
<tr>
<td>70–80</td>
<td>6</td>
<td>113</td>
</tr>
<tr>
<td>80–90</td>
<td>4</td>
<td>117</td>
</tr>
<tr>
<td>90–100</td>
<td>3</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>(n = 120)</td>
<td></td>
</tr>
</tbody>
</table>

On the graph paper, we plot the following points:

\((10, 5), (20, 14), (30, 30), (40, 52), (50, 78), (60, 96), (70, 107), (80, 113), (90, 117), (100, 120)\).

(i) \(\text{Median} = \left(\frac{n}{2}\right)\text{th term} \)  
\[= \frac{120}{2} = 60\text{th term} \] 
From the graph 60th term = 42  
Ans.

(ii) The number of students who obtained more than 75\% marks in test  
\[= 120 - 110 \]  
= 10.  
Ans.

(iii) The number of students who did not pass the test if the minimum pass marks 40 = 52.  
Ans.
Question 11.

(a) In the figure given below, the line segment AB meets X-axis at A and Y-axis at B. The point P(-3, 4) on AB divides it in the ratio 2 : 3. Find the coordinates of A and B.

(b) Using the properties of proportion, solve for x, given

\[ \frac{x^4 + 1}{2x^2} = \frac{17}{8} \]  

(c) A shopkeeper purchases a certain number of books for ₹960. If the cost per book was ₹8 less, the number of books that could be purchased for ₹960 would be 4 more. Write an equation, taking the original cost of each book to be ₹x, and solve it to find the original cost of the books.

Solution:

(a) Let the co-ordinates of A and B be (x, 0) and (0, y).

:. The co-ordinates of a point P (-3, 4) on AB divides it in the ratio 2 : 3.

i.e., \[ \frac{AP}{PB} = \frac{2}{3} \]

By using section formula, we get

\[ -3 = \frac{2 \times 0 + 3 \times x}{2 + 3} \]

\[ -3 = \frac{3x}{5} \Rightarrow 3x = -15 \]

\[ x = -5 \]

and

\[ 4 = \frac{2 \times y + 3 \times 0}{2 + 3} \]

\[ 4 = \frac{2y}{5} \Rightarrow 2y = 20 \]

\[ y = 10 \]

Hence, the co-ordinates of A and B are (-5, 0) and (0, 10).

(b) Given:

\[ \frac{x^4 + 1}{2x^2} = \frac{17}{8} \]

By using componendo and dividendo, we get

\[ \frac{x^4 + 1 + 2x^2}{x^4 + 1 - 2x^2} = \frac{17 + 8}{17 - 8} \]
\[
\left( \frac{x^2 + 1}{x^2 - 1} \right)^2 = \frac{25}{9}
\]
\[
\left( \frac{x^2 + 1}{x^2 - 1} \right)^2 = \left( \frac{5}{3} \right)^2
\]

Taking square root on both sides, we get

\[
x^2 + 1 = 5 \\
x^2 - 1 = 3
\]

\[
\Rightarrow 5x^2 - 5 = 3x^2 + 3
\]

\[
\Rightarrow 5x^2 - 3x^2 = 3 + 5
\]

\[
\Rightarrow 2x^2 = 8 \quad \Rightarrow \quad x^2 = 4
\]

\[
\Rightarrow x = \pm 2
\]

\[\text{Ans.}\]

(c) Given the original cost of each book be ₹ \(x\).

Total cost = ₹ 960  \hspace{1cm} \text{(Given)}

Number of books for 960 =

If the cost per book was ₹ 8 less, \(i.e., x - 8\) then

Number of books = \[\frac{960}{x - 8}\]

According to question,

\[
\frac{960}{x - 8} = \frac{960}{x} + 4
\]

\[
\frac{960}{x - 8} - \frac{960}{x} = 4
\]

\[
960 \left( \frac{x - x + 8}{x(x-8)} \right) = 4
\]

\[
\frac{8}{x^2 - 8x} = \frac{1}{240}
\]

\[
\Rightarrow x^2 - 8x = 1,920
\]

\[
x^2 - 8x - 1,920 = 0
\]

\[
x^2 - 48x + 40x - 1,920 = 0
\]

\[
x(x - 48) + 40(x - 48) = 0
\]

\[
(x - 48)(x + 40) = 0
\]

\[
x - 48 = 0 \quad \text{or} \quad x + 40 = 0
\]

\[
x = 48 \quad \text{or} \quad x = -40
\]

\[\therefore -40 \text{ is not possible.}\]

Hence, the original cost of each book = ₹ 48.  \[\text{Ans.}\]