ICSE QUESTION PAPER

Class X

MATHS

(2015)

(Two and a half hours)

Answers to this Paper must be written on the paper provided separately.
You will not be allowed to write during the first 15 minutes.
This time is to be spent in reading the Question Paper.
The time given at the head of this Paper is the time allowed for writing the answers.

*Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*
All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.
Omission of essential working will result in loss of marks.
The intended marks for questions or parts of questions are given in brackets [ ].
Mathematical tables are provided.

SECTION A (40 Marks)

*Attempt **all** questions from this Section.*

Question 1

(a) A shopkeeper bought an article for Rs. 3,450. He marks the price of the article 16% above the cost price. The rate of sales tax charged on the article is 10%. Find the:

(i) marked price of the article.

(ii) price paid by a customer who buys the article. [3]

(b) Solve the following inequation and write the solution set:

$13x - 5 < 15x + 4 < 7x + 12, x \varepsilon R$

Represent the solution on a real number line. [3]

(c) Without using trigonometric tables evaluate: [4]

$$\frac{\sin 65°}{\cos 25°} + \frac{\cos 32°}{\sin 58°} - \sin 28°.\sec 62° + \operatorname{cosec}^2 30°$$

Question 2

(a) If $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$, find x and y where $A^2 = B$. [3]

(b) The present population of a town is 2,00,000. The population is increased by 10% in the first year and 15% in the second year. Find the population of the town at the end of two years. [3]

(c) Three vertices of a parallelogram ABCD taken in order are A(3, 6), B(5, 10) and C(3, 2)

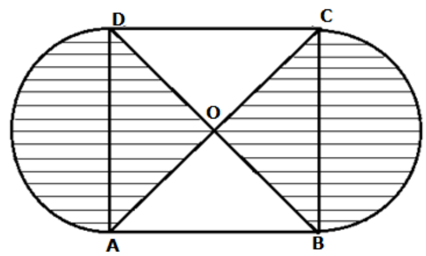
(i) the coordinate of the fourth vertex D

(ii) length of diagonal BD

(iii) equation of the side AD of the parallelogram ABCD [4]

Question 3

(a) In the given figure, ABCD is a square of side 21 cm. AC and BD are two diagonals of the square. Two semicircles are drawn with AD and BC as diameters. Find the area of the shaded region. $\left(\text{Take } \pi = \frac{22}{7}\right)$ [3]

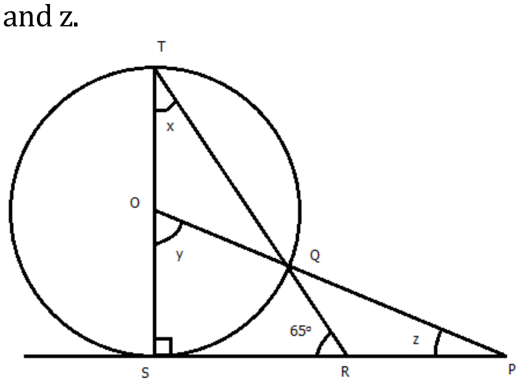


(b) The marks obtained by 30 students in a class assignment of 5 marks are given below. [3]

Marks	0	1	2	3	4	5
No. of Students	1	3	6	10	5	5

Calculate the mean, median and mode of the above distribution.

(c) In the figure given below, O is the centre of the circle and SP is a tangent. If ∠SRT = 65°, find the value of x, y and z. [4]



Question 4

(a) Katrina opened a recurring deposit account with a Nationalised Bank for a period of 2 years. If the bank pays interest at the rate 6% per annum and the monthly instalment is Rs. 1,000, find the:

(i) Interest earned in 2 years.

(ii) Matured value [3]

(b) Find the value of 'K' for which $x = 3$ is a solution of the quadratic equation, $(K + 2)x^2 - Kx + 6 = 0$.
Thus find the other root of the equation. [3]

(c) Construct a regular hexagon of side 5 cm. Construct a circle circumscribing the hexagon. All traces of construction must be clearly shown. [4]

SECTION B (40 Marks)

*Attempt any **four** questions from this Section*

Question 5

(a) Use a graph paper for this question taking 1 cm = 1 unit along both the x and y axis :

(i) Plot the points A(0, 5), B(2, 5), C(5, 2), D(5, -2), E(2, -5) and F(0, -5).

(ii) Reflect the points B, C, D and E on the y-axis and name them respectively as B', C', D' and E'.

(iii) Write the coordinates of B', C', D' and E'.

(iv) Name the figure formed by B C D E E' D' C' B'.

(v) Name a line of symmetry for the figure formed. [5]

(b) Virat opened a Savings Bank account in a bank on 16th April 2010. His pass book shows the following entries :

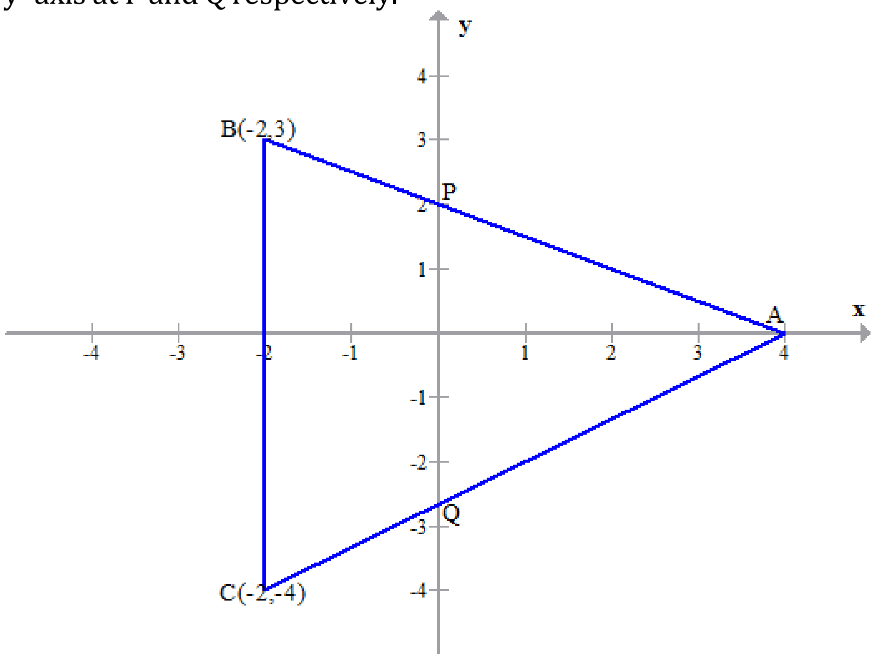
Date	Particulars	Withdrawal (Rs.)	Deposit (Rs.)	Balance (Rs.)
April 16, 2010	By cash	-	2500	2500
April 28th	By cheque	-	3000	5500
May 9th	To cheque	850	-	4650
May 15th	By cash	-	1600	6250
May 24th	To cash	1000	-	5250
June 4th	To cash	500	-	4750
June 30th	To cheque	-	2400	7150
July 3rd	By cash	-	1800	8950

Calculate the interest Virat earned at the end of 31st July, 2010 at 4% per annum interest. What sum of money will he receive if he closed the account on 1st August, 2010? [5]

Question 6

(a) If a, b, c are in continued proportion, prove that $(a + b + c)(a - b + c) = a^2 + b^2 + c^2$. [3]

(b) In the given figure ABC is a triangle and BC is parallel to the y – axis. AB and AC intersect the y–axis at P and Q respectively.



(i) Write the coordinates of A.

(ii) Find the length of AB and AC.

(iii) Find the ratio in which Q divides AC.

(iv) Find the equation of the line AC [4]

(c) Calculate the mean of the following distribution : [3]

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	8	5	12	35	24	16

Question 7

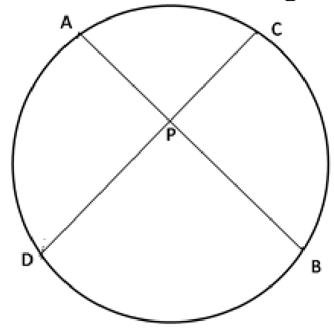
(a) Two solid spheres of radii 2 cm and 4 cm are melted and recast into a cone of height 8 cm. Find the radius of the cone so formed. [3]

(b) Find 'a' of the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leaves the same remainder when divided by $x + 3$. [3]

(c) Prove that $\dfrac{\sin\theta}{1-\cot\theta} + \dfrac{\cos\theta}{1-\tan\theta} = \cos\theta + \sin\theta$ [4]

Question 8

(a) AB and CD are two chords of a circle intersecting at P. Prove that AP × PB = CP × PD



[3]

(b) A bag contains 5 white balls, 6 red balls and 9 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is:

(i) a green ball

(ii) a white or a red ball

(iii) is neither a green ball nor a white ball. [3]

(c) Rohit invested Rs. 9,600 on Rs. 100 shares at Rs. 20 premium paying 8% dividend. Rohit sold the shares when the price rose to Rs. 160. He invested the proceeds (excluding dividend) in 10% Rs. 50 shares at Rs. 40. Find the:

(i) original number of shares

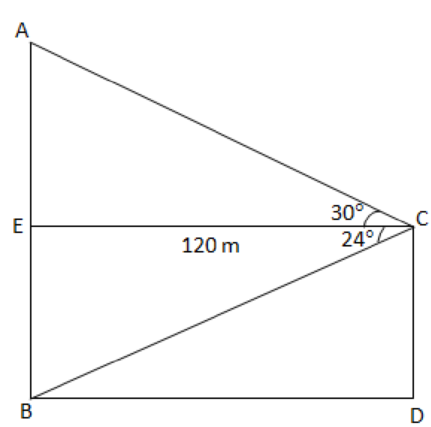
(ii) sale proceeds

(iii) new number of shares.

(iv) change in the two dividends. [4]

Question 9

(a) The horizontal distance between two towers is 120 m. The angle of elevation of the top and angle of depression of the bottom of the first tower as observed from the second tower is 30° and 24° respectively. [4]



Find the height of the two towers. Give your answer correct to 3 significant figures.

(b) The weight of 50 workers is given below : [6]

Weight in Kg	50-60	60-70	70-80	80-90	90-100	100-110	110-120
No. of Workers	4	7	11	14	6	5	3

Draw an ogive of the given distribution using a graph sheet. Take 2 cm = 10 kg on one axis and 2 cm = 5 workers along the other axis. Use a graph to estimate the following:

(i) The upper and lower quartiles.

(ii) If weighing 95 kg and above is considered overweight, find the number of workers who are overweight.

Question 10

(a) A wholesaler buys a TV from the manufacturer for Rs. 25,000. He marks the price of TV 20% above his cost price and sells it to a retailer at a 10% discount on the market price. If the rate of VAT is 8%, find the :

(i) Market price

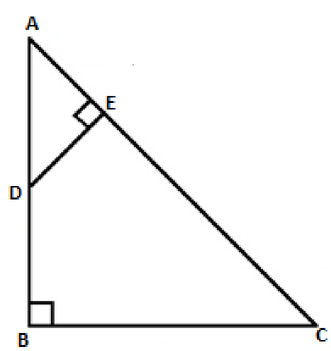
(ii) Retailer's cost price inclusive of tax.

(iii) VAT paid by the wholesaler. [3]

(b) If $A=\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}, B=\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and $C=\begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

Find $AB-5C$. [3]

(c) ABC is a right angled triangle with $\angle ABC = 90°$. D is any point on AB and DE is perpendicular to AC. Prove that:



(i) $\Delta ADE \sim \Delta ACB$

(ii) If AC = 13 cm, BC = 5 cm and AE = 4 cm. Find DE and AD.

(iii) Find. Area of ΔADE : Area of quadrilateral BCED. [4]

Question 11

(a) Sum of two natural numbers is 8 and the difference of their reciprocal is $\frac{2}{15}$.

Find the numbers. [3]

(b) Given $\frac{x^3+12x}{6x^2+8}=\frac{y^3+27y}{9y^2+27}$. Using componendo and dividendo find $x:y$. [3]

(c) Construct a triangle ABC with AB = 5.5 cm, AC = 6 cm and $\angle BAC = 105°$.

Hence:

(i) Construct the locus of points equidistant from BA and BC.

(ii) Construct the locus of points equidistant from B and C.

(iii) Mark the point which satisfies the above two loci as P. Measure and write the length of PC. [4]

Class X Mathematics

Board Paper 2015 (Solution)

SECTION A

1.

(a)

Given, b is the mean proportion between a and c.

$\Rightarrow \frac{a}{b}=\frac{b}{c}=k(\text{say})$

$\Rightarrow a=bk,\ b=ck$

$\Rightarrow a=(ck)k=ck^2, b=ck$

$$\text{L.H.S.}=\frac{a^4+a^2b^2+b^4}{b^4+b^2c^2+c^4}$$

$$=\frac{\left(ck^2\right)^4+\left(ck^2\right)^2\left(ck\right)^2+\left(ck\right)^4}{\left(ck\right)^4+\left(ck\right)^2c^2+c^4}$$

$$=\frac{c^4k^8+\left(c^2k^4\right)\left(c^2k^2\right)+c^4k^4}{c^4k^4+\left(c^2k^2\right)c^2+c^4}$$

$$=\frac{c^4k^8+c^4k^6+c^4k^4}{c^4k^4+c^4k^2+c^4}$$

$$=\frac{c^4k^4\left(k^4+k^2+1\right)}{c^4\left(k^4+k^2+1\right)}$$

$$=k^4$$

$$\text{R.H.S.}=\frac{a^2}{c^2}$$

$$=\frac{\left(ck^2\right)^2}{c^2}$$

$$=\frac{c^2k^4}{c^2}$$

$$=k^4$$

Hence, L.H.S. = R.H.S.

(b)

Given equation is $4x^2 - 5x - 3 = 0.$

Comparing with $ax^2 + bx + c = 0,$ we get

$a = 4,\ b = -5$ and $c = -3$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(-3)}}{2 \times 4}$$

$$= \frac{5 \pm \sqrt{25 + 48}}{8}$$

$$= \frac{5 \pm \sqrt{73}}{8}$$

$$= \frac{5 \pm 8.54}{8}$$

$$= \frac{13.54}{8} \text{ or } \frac{-3.54}{8}$$

$$= 1.6925 \text{ or } -0.4425$$

$$= 1.69 \text{ or } -0.44$$

(c)

Join OA and OC.

Since the perpendicular from the centre of the circle to a chord bisects the chord.

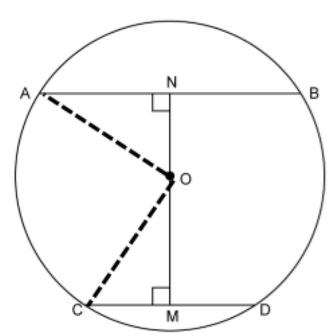
Therefore, N and M are the mid-points of AB and CD respectively.

Consequently,

$AN = NB = \frac{1}{2} AB = \frac{1}{2} \times 24 = 12$ cm and

$CM = MD = \frac{1}{2} CD = \frac{1}{2} \times 10 = 5$ cm

In right-angled triangles ANO and CMO, we have



$OA^2 = ON^2 + AN^2$ and $OC^2 = OM^2 + CM^2$

$\Rightarrow 13^2 = ON^2 + 12^2$ and $13^2 = OM^2 + 5^2$

$\Rightarrow ON^2 = 13^2 - 12^2$ and $OM^2 = 13^2 - 5^2$

$\Rightarrow ON^2 = 169 - 144$ and $OM^2 = 169 - 25$

$\Rightarrow ON^2 = 25$ and $OM^2 = 144$

$\Rightarrow ON = 5$ and $OM = 12$

Now, $NM = ON + OM = 5 + 12 = 17$ cm

Hence, the distance between the two chords is 17 cm.

2.

(a)

$$\sin^2 28° + \sin^2 62° + \tan^2 38° - \cot^2 52° + \frac{1}{4}\sec^2 30°$$

$$= \sin^2 28° + \sin^2(90° - 28°) + \tan^2 38° - \cot^2(90° - 38°) + \frac{1}{4}\sec^2 30°$$

$$= \left(\sin^2 28° + \cos^2 28°\right) + \tan^2 38° - \tan^2 38° + \frac{1}{4} \times \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= 1 + 0 + \frac{1}{4} \times \frac{4}{3}$$

$$= 1 + \frac{1}{3}$$

$$= \frac{4}{3}$$

(b)

$$\text{Given}: A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}, \ B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} \text{ and } A^2 - 5B^2 = 5C$$

$$\text{Now, } A^2 = A \times A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1 + 3\times3 & 1\times3 + 3\times4 \\ 3\times1 + 4\times3 & 3\times3 + 4\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+9 & 3+12 \\ 3+12 & 9+16 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix}$$

$$\text{And, } B^2 = B \times B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} \times \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2\times(-2) + 1\times(-3) & -2\times1 + 1\times2 \\ -3\times(-2) + 2\times(-3) & -3\times1 + 2\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & -2+2 \\ 6-6 & -3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{Now, } A^2 - 5B^2 = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix} = 5\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} = 5C$$

$$\text{Hence, } C = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$

(c)

For 1^{st} year:

$P = \text{Rs. } 50{,}000;\ R = 12\%$ and $T = 1$ year

$\therefore \text{Interest} = \text{Rs. } \dfrac{50{,}000 \times 12 \times 1}{100} = \text{Rs. } 6{,}000$

And, $\text{Amount} = \text{Rs. } 50{,}000 + \text{Rs. } 6{,}000 = \text{Rs. } 56{,}000$

Since $\text{Money repaid} = \text{Rs. } 33{,}000$

$\therefore \text{Balance} = \text{Rs. } 56{,}000 - \text{Rs. } 33{,}000 = \text{Rs. } 23{,}000$

For 2^{nd} year:

$P = \text{Rs. } 23{,}000;\ R = 15\%$ and $T = 1$ year

$\therefore \text{Interest} = \text{Rs. } \dfrac{23{,}000 \times 15 \times 1}{100} = \text{Rs. } 3{,}450$

And, $\text{Amount} = \text{Rs. } 23{,}000 + \text{Rs. } 3{,}450 = \text{Rs. } 26{,}450$

Thus, Jaya must pay Rs. 26,450 at the end of 2nd year to clear her debt.

3.

(a)

$\text{List price} = \text{Rs.} 42{,}000$

$\text{Discount} = 10\% \text{ of Rs. } 42{,}000$

$= \dfrac{10}{100} \times \text{Rs. } 42{,}000$

$= \text{Rs. } 4{,}200$

$\Rightarrow \text{Discounted price} = \text{Rs. } 42{,}000 - \text{Rs. } 4{,}200 = \text{Rs. } 37{,}800$

$\text{Off-season discount} = 5\% \text{ of Rs. } 37{,}800$

$= \dfrac{5}{100} \times \text{Rs. } 37{,}800$

$= \text{Rs. } 1{,}890$

$\therefore \text{Sale-price} = \text{Rs. } 37{,}800 - \text{Rs. } 1{,}890 = \text{Rs. } 35{,}910$

(i) The amount of sales tax a customer has to pay $= 8\% \text{ of Rs. } 35{,}910$

$= \dfrac{8}{100} \times \text{Rs. } 35{,}910$

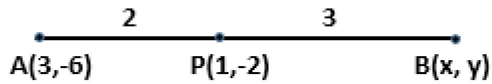
$= \text{Rs. } 2872.80$

(ii) The total price, a customer has to pay for the computer $= \text{Sale-price} + \text{Sales Tax}$

$= \text{Rs. } 35{,}910 + \text{Rs. } 2872.80$

$= \text{Rs. } 38782.80$

(b)



Given, P(1,−2), A(3,−6) and B(x,y)

$AP : PB = 2 : 3$

Hence, coordinates of $P = \left(\frac{2 \times x + 3 \times 3}{2+3}, \frac{2 \times y + 3 \times (-6)}{2+3} \right) = \left(\frac{2x+9}{5}, \frac{2y-18}{5} \right)$

But, the coordinates of P are $(1, -2)$.

$\therefore \frac{2x+9}{5} = 1 \quad \text{and} \quad \frac{2y-18}{5} = -2$

$\Rightarrow 2x + 9 = 5 \quad \text{and} \quad 2y - 18 = -10$

$\Rightarrow 2x = -4 \quad \text{and} \quad 2y = 8$

$\Rightarrow x = -2 \quad \text{and} \quad y = 4$

Hence, the coordinates of B are $(-2, 4)$.

(c)

Data in ascending order:

13, 35, 43, 46, x, x + 4, 55, 61, 71, 80

Median = 48

Number of observations = n = 10 (even)

$$\therefore \text{Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{ term} + \left(\frac{n}{2}+1\right)^{th} \text{ term}}{2}$$

$$\Rightarrow 48 = \frac{\left(\frac{10}{2}\right)^{th} \text{ term} + \left(\frac{10}{2}+1\right)^{th} \text{ term}}{2}$$

$$\Rightarrow 48 = \frac{5^{th} \text{ term} + 6^{th} \text{ term}}{2}$$

$$\Rightarrow 48 = \frac{x + x + 4}{2}$$

$$\Rightarrow 48 = \frac{2x+4}{2}$$

$\Rightarrow 48 = x + 2$

$\Rightarrow x = 46$

$\Rightarrow x + 4 = 46 + 4 = 50$

Thus, the observations are 13, 35, 43, 46, 46, 50, 55, 61, 71, 80

Observation 46 is appearing twice.

Hence, the mode of the data is 46.

4.

(a)

Let the number to be subtracted from the given polynomial be k.

Let $f(y) = 16x^3 - 8x^2 + 4x + 7 - k$

It is given that $(2x+1)$ is a factor of $f(y)$.

$\therefore\ f\left(-\frac{1}{2}\right) = 0$

$\Rightarrow 16\left(-\frac{1}{2}\right)^3 - 8\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 7 - k = 0$

$\Rightarrow 16 \times \left(-\frac{1}{8}\right) - 8 \times \frac{1}{4} - 2 + 7 - k = 0$

$\Rightarrow -2 - 2 - 2 + 7 - k = 0$

$\Rightarrow 1 - k = 0$

$\Rightarrow k = 1$

Thus, 1 should be subtracted from the given polynomial.

(b)

$$\begin{aligned}\text{Length of a rectangle} &= \text{Radius of two semi-circles} + \text{Diameter of a circle}\\ &= 5 + 5 + 10\\ &= 20 \text{ cm}\end{aligned}$$

Breadth of a rectangle $=$ Diameter of a circle $= 2 \times 5 = 10$ cm

$$\begin{aligned}\therefore \text{Area of a rectangle} &= \text{Length} \times \text{Breadth}\\ &= 20 \times 10\\ &= 200 \text{ sq. cm}\end{aligned}$$

Area of a circle $= \frac{22}{7} \times 5 \times 5 = 78.571$ sq. cm

And, area of two semi-circles each of radius 5 cm $= 2\left(\frac{1}{2} \times 78.571\right) = 78.571$ sq. cm

Now,

$$\begin{aligned}\text{Area of shaded region} &= \text{Area of a rectangle} - \text{Area of a circle} - \text{Area of two semi-circles}\\ &= 200 - 78.571 - 78.571\\ &= 200 - 157.142\\ &= 42.858 \text{ sq. cm}\end{aligned}$$

(c)

$$-8\frac{1}{2} < -\frac{1}{2} - 4x \le 7\frac{1}{2},\ x \in I$$

$$\Rightarrow -\frac{17}{2} < -\frac{1}{2} - 4x \le \frac{15}{2},\ x \in I$$

Take

$$-\frac{17}{2} < -\frac{1}{2} - 4x \qquad\qquad -\frac{1}{2} - 4x \le \frac{15}{2}$$

$$-\frac{17}{2} + \frac{1}{2} < -4x \qquad\qquad -4x \le \frac{15}{2} + \frac{1}{2}$$

$$-\frac{16}{2} < -4x \qquad\qquad -4x \le \frac{16}{2}$$

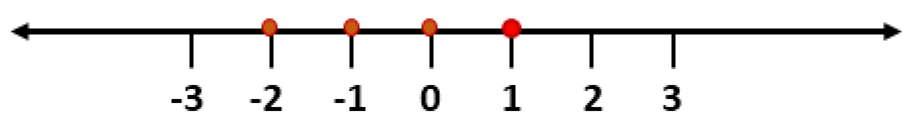
$$-8 < -4x \qquad\qquad -4x \le 8$$

$$2 > x \qquad\qquad x \ge -2$$

Thus, on simplifying, the given inequation reduces to $-2 \le x < 2$.

Since $x \in I$, the solution set is $\{-2, -1, 0, 1\}$.

The required graph on number line is as follows:



SECTION B (40 Marks)

Attempt any four questions from this section

5.

(a)

$$\text{Given: } B=\begin{bmatrix}1 & 1\\ 8 & 3\end{bmatrix} \text{ and } X=B^2-4B$$

$$\text{Now, } B^2=B\times B$$

$$=\begin{bmatrix}1 & 1\\ 8 & 3\end{bmatrix}\times\begin{bmatrix}1 & 1\\ 8 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+1\times8 & 1\times1+1\times3\\ 8\times1+3\times8 & 8\times1+3\times3\end{bmatrix}$$

$$=\begin{bmatrix}1+8 & 1+3\\ 8+24 & 8+9\end{bmatrix}$$

$$=\begin{bmatrix}9 & 4\\ 32 & 17\end{bmatrix}$$

$$X=B^2-4B=\begin{bmatrix}9 & 4\\ 32 & 17\end{bmatrix}-4\begin{bmatrix}1 & 1\\ 8 & 3\end{bmatrix}=\begin{bmatrix}9 & 4\\ 32 & 17\end{bmatrix}-\begin{bmatrix}4 & 4\\ 32 & 12\end{bmatrix}=\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}$$

$$\text{Now, } X\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}5\\ 50\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}5\\ 50\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}5a+0b\\ 0a+5b\end{bmatrix}=\begin{bmatrix}5\\ 50\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}5a\\ 5b\end{bmatrix}=\begin{bmatrix}5\\ 50\end{bmatrix}$$

$$\Rightarrow 5a=5 \text{ and } 5b=50$$

$$\Rightarrow a=1 \text{ and } b=10$$

(b)

$$\text{Since Dividend on 1 share}=10\% \text{ of Rs. } 50=\frac{10}{100}\times\text{Rs. }50=\text{Rs. }5$$

$$\therefore \text{Number of shares bought}=\frac{\text{Total dividend}}{\text{Dividend on 1 share}}=\frac{\text{Rs. }450}{\text{Rs. }5}=90$$

$$\text{Since market value of each share}=\text{Rs. }60$$

$$\therefore \text{Sum invested by the man}=90\times\text{Rs. }60=\text{Rs. }5,400$$

$$\text{Percentage return}=\frac{\text{Total return}}{\text{Sum invested}}\times100\%=\frac{\text{Rs. }450}{\text{Rs. }5400}\times100\%=8.33\%=8\%$$

(c)

Outcomes: a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p

Total number of all possible outcomes $= 16$

(i) When the selected card has a vowel,

the possible outcomes are a, e, i, o.

Number of favourable outcomes $= 4$

$\therefore$ Required probability $= \frac{4}{16} = \frac{1}{4}$

(ii) When the selected card has a consonant,

Number of favourable outcomes $= 16 - 4 = 12$

$\therefore$ Required probability $= \frac{12}{16} = \frac{3}{4}$

(iii) When the selected card has none of the letters from the word median,

the possible outcomes are b, c, f, g, h, j, k, l, o, p.

Number of favourable outcomes $= 10$

$\therefore$ Required probability $= \frac{10}{16} = \frac{5}{8}$

6.

(a) **Steps of construction:**

(i) Draw line AC = 5 cm and $\angle CAB = 60^o$. Cut off AB = 7 cm. Join BC, ΔABC is the required triangle.

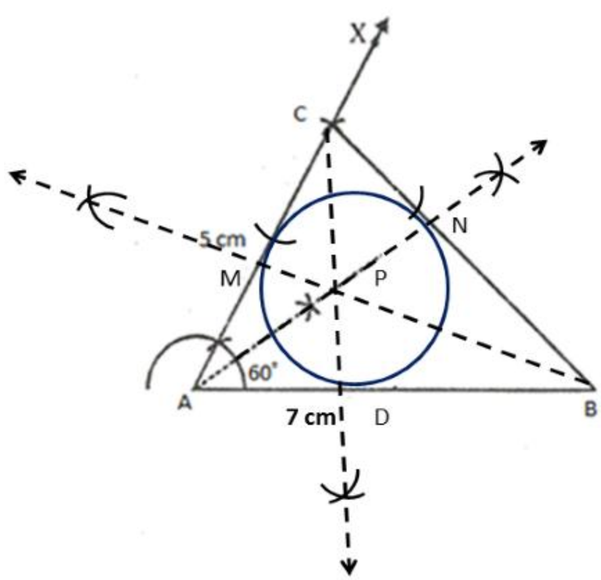
(ii) Draw angle bisectors of $\angle A$ and $\angle B$.

(iii) Bisector of $\angle B$ meets AC at M and bisector of $\angle A$ meets BC at N.

(iv) Similarly, draw the angle bisector of $\angle C$ which meets AB at D.

(v) P is the point which is equidistant from AB, BC and AC.

(vi) With DP as the radius, draw a circle touching the three sides of the triangle (incircle.)



(b)

Let h be the height and r be the radius of the base of the conical tent.

According to the given information,

$$77 \times 16 = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow 77 \times 16 = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times h$$

$$\Rightarrow 77 \times 16 = \frac{1}{3} \times 22 \times 7 \times h$$

$$\Rightarrow h = \frac{77 \times 16 \times 3}{22 \times 7} \Rightarrow h = 24 \text{ m}$$

Now, $l^2 = r^2 + h^2$

$$\Rightarrow l^2 = 7^2 + 24^2 = 625$$

$$\Rightarrow l = 25 \text{ m}$$

$$\therefore \text{Curved surface area} = \pi r l = \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

Hence, the height of the tent is 24 m and the curved surface area of the tent is 550 m^2.

(c)

(i) $$\frac{7m + 2n}{7m - 2n} = \frac{5}{3}$$

By Componendo – Divinendo, we get

$$\frac{7m + 2n + (7m - 2n)}{7m + 2n - (7m - 2n)} = \frac{5 + 3}{5 - 3}$$

$$\Rightarrow \frac{14m}{4n} = \frac{8}{2}$$

$$\Rightarrow \frac{7m}{2n} = \frac{4}{1}$$

$$\Rightarrow \frac{m}{n} = \frac{8}{7}$$

$$\Rightarrow m : n = 8 : 7$$

(ii) $$\frac{m}{n} = \frac{8}{7} \Rightarrow \frac{m^2}{n^2} = \frac{8^2}{7^2}$$

Applying Componendo – Divinendo, we get

$$\Rightarrow \frac{m^2 + n^2}{m^2 - n^2} = \frac{8^2 + 7^2}{8^2 - 7^2}$$

$$\Rightarrow \frac{m^2 + n^2}{m^2 - n^2} = \frac{64 + 49}{64 - 49}$$

$$\Rightarrow \frac{m^2 + n^2}{m^2 - n^2} = \frac{113}{15}$$

7.

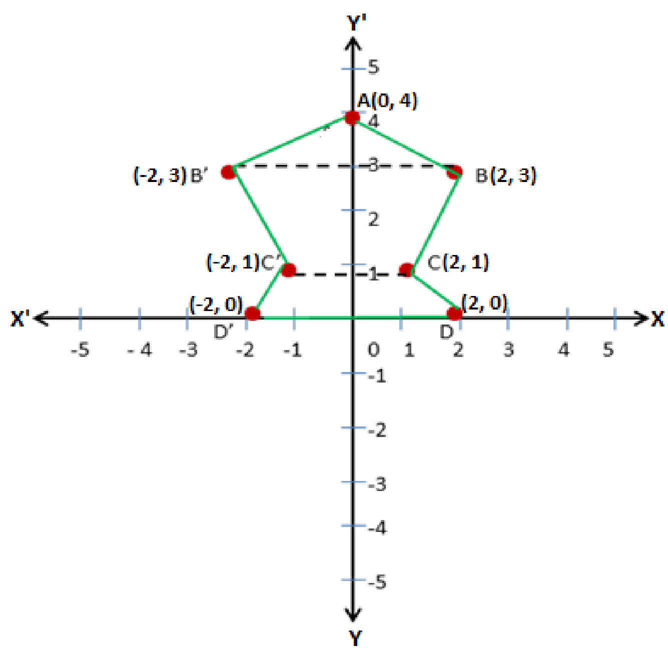
(a) Principal for the month of Jan = Rs. 5600
Principal for the month of Feb = Rs. 4100
Principal for the month of Mar = Rs. 4100
Principal for the month of Apr = Rs. 2000
Principal for the month of May = Rs. 8500
Principal for the month of June = Rs. 10000
Total Principal for one month = Rs. 34300

Rate of interest = 6% pa

(i) Simple interest $= \frac{PRT}{100} = \frac{34300 \times 6 \times 1}{100 \times 12} = Rs. 171.50$

(ii) Total amount $= Rs. 10000 + Rs. 171.50 = Rs. 10171.50$

(b)



The image of point (x, y) on Y-axis has the coordinates (−x, y).
Thus, we have
Coordinates of B' = (−2, 3)
Coordinates of C' = (−1, 1)
Coordinates of D' = (−2, 0)

Since, Y-axis is the line of symmetry of the figure formed, the equation of the line of symmetry is x = 0.

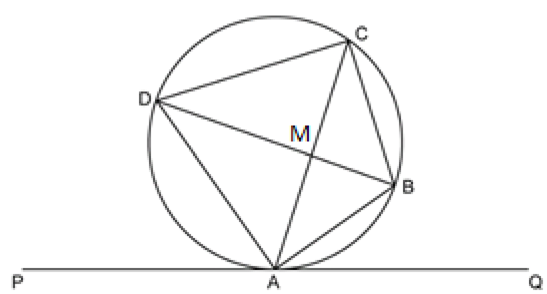
8.

(a) Let the assumed mean A = 25

Marks	Mid-value x	f	$d = x - A$	$t = \frac{x-A}{i} = \frac{x-25}{10}$	ft
$0-10$	5	10	-20	-2	-20
$10-20$	15	9	-10	-1	-9
$20-30$	25	25	0	0	0
$30-40$	35	30	10	1	30
$40-50$	45	16	20	2	32
$50-60$	55	10	30	3	30
		$\sum f = 100$			$\sum ft = 63$

$$\therefore \text{Mean} = A + \frac{\sum ft}{\sum f} \times i = 25 + \frac{63}{100} \times 10 = 25 + \frac{63}{10} = 25 + 6.3 = 31.3$$

(b)



(i) $\angle BAQ = 30^\circ$

Since AB is the bisector of $\angle CAQ$

$\Rightarrow \angle CAB = \angle BAQ = 30^\circ$

AD is the bisector of $\angle CAP$ and $P - A - Q$,

$\angle DAP + \angle CAD + \angle CAQ = 180^\circ$

$\Rightarrow \angle CAD + \angle CAD + 60^\circ = 180^\circ$

$\Rightarrow \angle CAD = 60^\circ$

So, $\angle CAD + \angle CAB = 60^\circ + 30^\circ = 90^\circ$

Since angle in a semi-circle $= 90^\circ$

$\Rightarrow$ Angle made by diameter to any point on the circle is 90°

So, BD is the diameter of the circle.

(ii) Since BD is the diameter of the circle, so it will pass through the centre.

By Alternate segment theorem,

$\angle ABD = \angle DAP = 60^\circ$

So, in ΔBMA,

$\angle AMB = 90^\circ$ ....(Use Angle Sum Property)

We know that perpendicular drawn from the centre to a chord of a circle bisects the chord.

$\Rightarrow \angle BMA = \angle BMC = 90^\circ$

In ΔBMA and ΔBMC,

$\angle BMA = \angle BMC = 90^\circ$

$BM = BM$ (common side)

$AM = CM$ (perpendicular drawn from the centre to a chord of a circle bisects the chord.)

$\Rightarrow \Delta BMA \cong \Delta BMC$

$\Rightarrow AB = BC$ (SAS congruence criterion)

$\Rightarrow \Delta ABC$ is an isosceles triangle.

(c)

(i) Printed price of an air conditioner = Rs. 45000

Discount = 10%

$$\therefore \text{C.P. of the air conditioner} = \text{Rs.}\ \frac{45000 \times (100 - 10)}{100}$$

$$= \text{Rs.}\ \frac{45000 \times 90}{100}$$

$$= \text{Rs.}\ 40500$$

$$\text{VAT (12\%)} = 40500 \times \frac{12}{100} = \text{Rs.}\ 4860$$

So, the shopkeeper paid VAT of Rs. 4860 to the government.

(ii) Discount = 5% of the marked price

$$\therefore \text{C.P. of the air conditioner} = \text{Rs.}\ \frac{45000 \times (100 - 5)}{100}$$

$$= \text{Rs.}\ \frac{45000 \times 95}{100}$$

$$= \text{Rs.}\ 42750$$

$$\text{VAT (12\%)} = 42750 \times \frac{12}{100} = \text{Rs.}\ 5130$$

So, the total amount paid by the customer inclusive of tax

$= \text{Rs.}\ 42750 + \text{Rs.}\ 5130$

$= \text{Rs.}\ 47880$

9.

(a)

(i) $\angle DAE = 70^\circ$ ....(given)

$\angle BAD + \angle DAE = 180^\circ$ ....(linear pair)

$\Rightarrow \angle BAD + 70^\circ = 180^\circ$

$\Rightarrow \angle BAD = 110^\circ$

Since ABCD is a cyclic quadrilateral, sum of the measures of the opposite angles are supplementary.

So, $\angle BCD + \angle BAD = 180^\circ$

$\Rightarrow \angle BCD + 110^\circ = 180^\circ$

$\Rightarrow \angle BCD = 70^\circ$

(ii) $\angle BOD = 2\angle BCD$ (Inscribed angle theorem)

$\Rightarrow \angle BOD = 2(70^\circ) = 140^\circ$

(iii) In ΔOBD,

$OB = OD$ ....(radii of same circle)

$\Rightarrow \angle OBD = \angle ODB$

By Angle Sum property,

$\angle OBD + \angle ODB + \angle BOD = 180^\circ$

$\Rightarrow 2\angle OBD + \angle BOD = 180^\circ$

$\Rightarrow 2\angle OBD + 140^\circ = 180^\circ$

$\Rightarrow 2\angle OBD = 40^\circ$

$\Rightarrow \angle OBD = 20^\circ$

(b)

Given vertices: $A(-1, 3)$, $B(4, 2)$ and $C(3, -2)$

(i) Coordinates of the centroid G of ΔABC are given by

$$G = \left(\frac{-1+4+3}{3}, \frac{3+2-2}{3}\right) = \left(\frac{6}{3}, \frac{3}{3}\right) = (2, 1)$$

(ii) Since the line through G is parallel to AC, the slope of the lines are the same.

$$\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-3}{3-(-1)} = \frac{-5}{4}$$

So, equation of the line passing through $G(2, 1)$ and with slo pe $\frac{-5}{4}$ is given by,

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 1 = \frac{-5}{4}(x-2)$

$\Rightarrow 4y - 4 = -5x + 10$

$\Rightarrow 5x + 4y = 14$ is the required equation.

(c)

$$\text{L.H.S.} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta\,[2(1-\sin^2\theta)-1]}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2-2\sin^2\theta-1)}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(1-2\sin^2\theta)}$$

$$= \tan\theta$$

$$= \text{R.H.S.} \quad \text{(proved)}$$

10.

(a)

Let Vivek's age be x years and Amit's age be (47-x) years.

According to the given information,

$x(47-x) = 550$

$\Rightarrow 47x - x^2 = 550$

$\Rightarrow x^2 - 47x + 550 = 0$

$\Rightarrow (x-25)(x-22) = 0$

$\Rightarrow x = 25 \text{ or } x = 22$

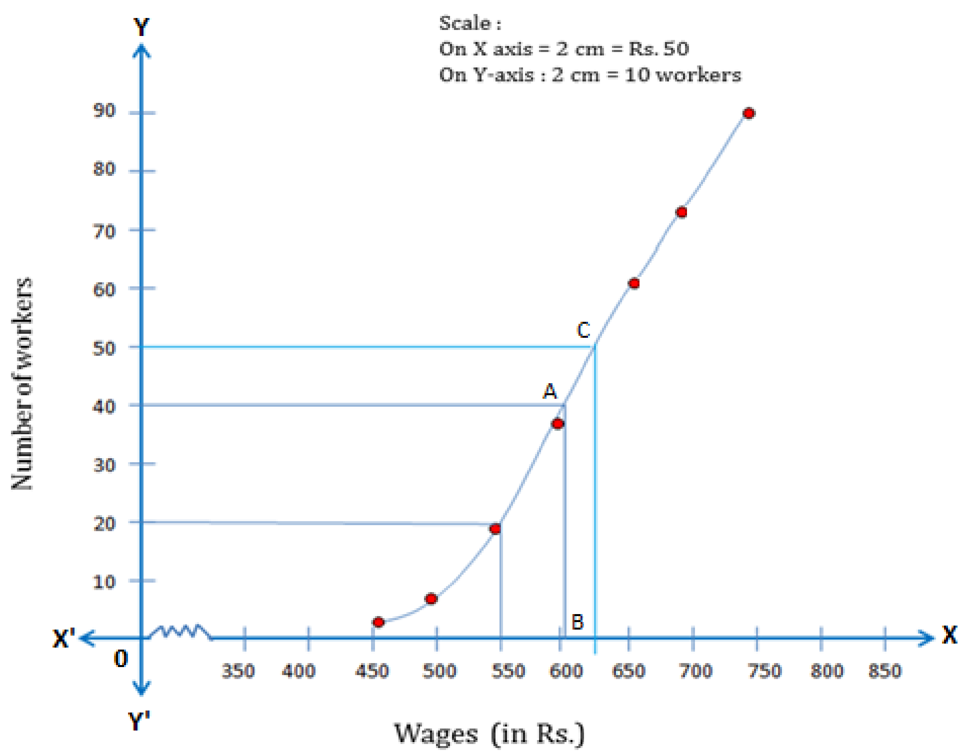
So, Vivek's age is 25 years and Amit's age is 22 years.

(b)

The cumulative frequency table of the given distribution is as follows:

Wages in Rs.	Upper Limit	No. of workers	Cumulative frequency
400-450	450	2	2
450-500	500	6	8
500-550	550	12	20
550-600	600	18	38
600-650	650	24	62
650-700	700	13	75
700-750	750	5	80

The ogive is as follows:



Number of workers = n = 80

(i) Median $= \left(\frac{n}{2}\right)^{th}$ term $= 40^{th}$ term

Through mark 40 on the Y-axis, draw a horizontal line which meets the curve at point A.

Through point A, on the curve draw a vertical line which meets the X-axis at point B.

The value of point B on the X-axis is the median, which is 605.

(ii) Lower quartile $(Q_1) = \left(\frac{80}{4}\right)^{th}$ term $= 20^{th}$ term $= 550$

(ii) Through mark of 625 on X-axis, draw a verticle line which meets the graph at point C.

Then through point C, draw a horizontal line which meets the Y-axis at the mark of 50.

Thus, number of workers that earn more than Rs. 625 daily $= 80 - 50 = 30$

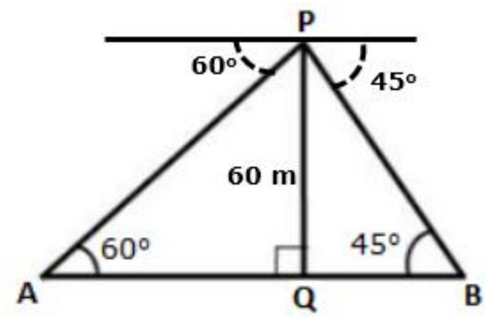
11.

(a)

Let PQ be the lighthouse.

$\Rightarrow PQ = 60$

In ΔPQA,



$\tan 60° = \frac{PQ}{AQ}$

$\Rightarrow \sqrt{3} = \frac{60}{AQ}$

$\Rightarrow AQ = \frac{60}{\sqrt{3}}$

$\Rightarrow AQ = \frac{20\times 3}{\sqrt{3}}$

$\Rightarrow AQ = \frac{20\times\sqrt{3}\times\sqrt{3}}{\sqrt{3}}$

$\Rightarrow AQ = 20\sqrt{3}$ m

In ΔPQB,

$\tan 45° = \frac{PQ}{QB}$

$\Rightarrow 1 = \frac{60}{QB}$

$\Rightarrow QB = 60$ m

Now,

$AB = AQ + QB$

$= 20\sqrt{3} + 60$

$= 20\times 1.732 + 60$

$= 94.64$

$= 95$ m

(b)

(i) In ΔPQR and ΔSPR, we have

$\angle QPR = \angle PSR$ ....(given)

$\angle PRQ = \angle PRS$ ....(common)

So, by AA-axiom similarity, we have

$\Delta PQR \sim \Delta SPR$ ....(proved)

(ii) Since $\Delta PQR \sim \Delta SPR$ ....(proved)

$$\Rightarrow \frac{PQ}{SP} = \frac{QR}{PR} = \frac{PR}{SR}$$

Consider $\frac{QR}{PR} = \frac{PR}{SR}$ ....[From (1)]

$$\Rightarrow \frac{QR}{6} = \frac{6}{3}$$

$$\Rightarrow QR = \frac{6 \times 6}{3} = 12 \text{ cm}$$

Also, $\frac{PQ}{SP} = \frac{PR}{SR}$

$$\Rightarrow \frac{8}{SP} = \frac{6}{3}$$

$$\Rightarrow \frac{8}{SP} = 2$$

$$\Rightarrow SP = \frac{8}{2} = 4 \text{ cm}$$

(iii) $\frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta SPR} = \frac{PQ^2}{SP^2} = \frac{8^2}{4^2} = \frac{64}{16} = 4$

(c)

(i) Let the deposit per month $=$ Rs. P

Number of months $(n) = 36$

Rate of interest $(r) = 7.5\%$ p.a.

$$\therefore \text{S.I.} = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow 8325 = P \times \frac{36 \times 37}{2 \times 12} \times \frac{7.5}{100}$$

$$\Rightarrow 8325 = P \times \frac{3 \times 37}{2} \times \frac{7.5}{100}$$

$$\Rightarrow P = \frac{8325 \times 2 \times 100}{3 \times 37 \times 7.5} = \text{Rs. } 2000$$

(ii) Maturity value $= P \times n + \text{S.I.} = \text{Rs.}(2000 \times 36 + 8325) = \text{Rs. } 80{,}325$