

ICSE Board Class X Mathematics Board Paper 2014 Solution (Two and a half hours)

SECTION A

1.

(a)

Given that Ranbir borrows Rs.20000 at 12% compound interest. For the first year, Interest I= $\frac{20000 \ 1 \ 12}{100}$ Rs.2400 Thus, amount after one year = Rs.2000+Rs.2400 Rs.22400 Money repaid = Rs.8400 Balance= Rs.22400 Rs.8400 14000 For the second year, Interest I = $\frac{14000 \ 1 \ 12}{100}$ Rs.1680 Thus the amount =Rs.14000+Rs.1680 Rs.15680 Ranbir paid Rs. 9680 in the second year. The loan outstanding at the beginning of the third year = Rs.15680 Rs. 9680 = Rs.6000



(b)

We need to find the values of x, such that

x satisfies the inequation $2\frac{5}{6}$ $\frac{1}{2}$ $\frac{2x}{3}$ 2,x W Consider the given inequation:

$$2\frac{5}{6} \quad \frac{1}{2} \quad \frac{2x}{3} \quad 2$$

$$\frac{17}{6} \quad \frac{3}{6} \quad \frac{4x}{6} \quad \frac{12}{6}$$

$$\frac{17}{6} \quad \frac{4x}{6} \quad \frac{3}{6} \quad \frac{12}{6}$$

$$17 \quad 4x \quad 3 \quad 12$$

$$12 \quad 4x \quad 3 \quad 17$$

$$12 \quad 3 \quad 4x \quad 3 \quad 3 \quad 17 \quad 3$$

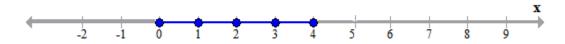
$$9 \quad 4x \quad 20$$

$$\frac{9}{4} \quad \frac{4x}{4} \quad \frac{20}{4}$$

$$\frac{9}{4} \quad x \quad 5$$

Since x W, the Required solution set= 0,1,2,3,4

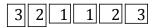
And the required line is





(c)

Given that the die has 6 faces marked by the given numbers as below:



(i)

Let us find the probability of getting a positive integer. When a die is rolled, the total number of possible outcomes =6

For getting a positive integer, the favourable outcomes

are:1,2,3

Number of favourable outcomes = 3

Required probability= $\frac{3}{6} = \frac{1}{2}$

(ii) Let us find the probability of getting an integer greater than 3.

When a die is rolled, the total number of possible

outcomes =6

For getting getting an integer greater than 3, the favourable outcomes are: 2, 1,1,2,3

Number of favourable outcomes = 5

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Required probability=\frac{5}{6}
```

(iii) Let us find the probability of getting a smallest integer

When a die is rolled, the total number of possible

outcomes =6

For getting getting a smallest integer, the favourable outcomes are: 3

Number of favourable outcomes = 1

Required probability= $\frac{1}{6}$



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2.
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(a)

Consider the following equation:

Multiplying and adding the corresponding elements of the matrices, we have

2 1 0 2x 3 2 2y 2 6 2y 4 2y y 2 Similarly, 3 1 1 2x 3 1 2 3 3 2x 3 6 2x 6 x 3 Thus, the values of x and y are: 3, 2

(b)

Shahrukh deposited Rs.800 per month for n=1 $\frac{1}{2}$ years Since 1 $\frac{1}{2}$ years = 18 months, Total money deposited=18 800=Rs.14400 Given that the maturity value = Rs.15084 Interest = Maturity Value Total sum deposited =15084 14400 684 We know that Interest I = P $\frac{n n+1}{2 \ 12} \ \frac{r}{100}$



(c)

Let A 4, 2 and B 3, 6 be two points.

Let P x, 3 be the point which divides the line joining the line segment in the ratio k:1 Thus, we have

$$\frac{3k}{k} \frac{4}{1} x; \qquad \frac{6k}{k} \frac{2}{1} 3$$
For
$$6k 2 3 k 1$$

$$6k 2 3k 3$$

$$3k 3 2$$

$$3k 1$$

$$k \frac{1}{3}$$
Now consider the equation

Now consider the equation,

 $\frac{3k-4}{k-1} \quad x$

Substituting the value of k in the above equation we have,

$$\frac{3 \frac{1}{3} 4}{\frac{1}{3} 1} x$$

$$\frac{3}{\frac{4}{3}} x$$

$$\frac{3}{\frac{4}{3}} x$$

$$\frac{9}{4} x$$

Therefore, coordinate of P is

$$\frac{9}{4}$$
, 3

Now let us find the distance AP:

$$AP = \sqrt{\frac{9}{4}} \frac{4}{4}^{2} \frac{3}{3} \frac{2}{2}^{2}$$

$$AP \sqrt{\frac{49}{16}} \frac{1}{1}$$

$$AP \sqrt{\frac{49}{16}} \frac{16}{16}$$

$$AP \sqrt{\frac{65}{16}} \frac{\sqrt{65}}{4} \text{ units}$$



3.

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(a) Consider the expression \sin^2 34^\circ \ \sin^2 56^\circ \ 2\tan 18^\circ \tan 72^\circ \ \cot^2 30^\circ:

\sin^2 34^\circ \ \sin^2 56^\circ \ 2\tan 18^\circ \tan 72^\circ \ \cot^2 30^\circ

\sin^2 34^\circ \ \sin^2 90^\circ \ 56^\circ \ 2\tan 18^\circ \tan 90^\circ \ 72^\circ \ \cot^2 30^\circ

\sin^2 34^\circ \ \cos^2 34^\circ \ 2\tan 18^\circ \ \cot 18^\circ \ \cot^2 30^\circ

\sin^2 34^\circ \ \cos^2 34^\circ \ 2\tan 18^\circ \ \frac{1}{\tan 18^\circ} \ \cot^2 30^\circ

1 \ 2 \ 1 \ \sqrt{3}^2

1 \ 2 \ 3 \ 3 \ 3
```

(b)

0

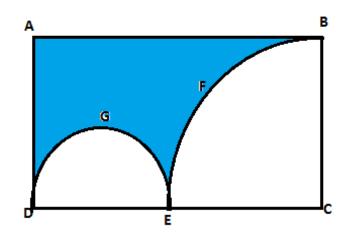
By remainder Theorem,

For x=1, the value of the given expression is the remainder.

Thus, by factor theorem,



(c) Considering the given figure:



Given dimensions of the rectangle: AB=14 cm and BC=7 cm Thus the radius of the quarter circle is 7 cm Area of the quarter circle is $=\frac{1}{4}$ $\frac{22}{7}$ 7² sq. cm Area of the quarter circle $=\frac{77}{2}$ sq. cm Since EC = 7 cm and DC = 14 cm, we have, DE = DC EC 14 7 7 cm Therefore, radius of the semi circle is $=\frac{7}{2}$ cm Thus the area of the semi circle is $=\frac{1}{2}$ $\frac{22}{7}$ $\frac{7}{2}$ sq. cm Area of the semi circle $=\frac{77}{4}$ sq. cm Area of the rectangle is=AB BC=14 7=98 sq.cm Thus, the required area=Area ABCD Area BCEF Area DGE

98
$$\frac{77}{2}$$
 $\frac{77}{4}$ 40.25 sq.cm



4.

(a)

Consider the given set of numbers: 6,8,10,12,13,x There are six numbers and six is even.

Thus the median of the given numbers is $\frac{\frac{N}{2}$ th term $\frac{N}{2}$ 1 th term 2

Median $\frac{\frac{6}{2}$ th term $\frac{6}{2}$ 1 th term $\frac{6}{2}$

Median $\frac{3 \text{rd term } 4 \text{th term}}{2}$ Median $\frac{10 \ 12}{2}$ Median $\frac{22}{2} \ 11$

Given that the mean of 6,8,10,12,13,x is median of 6,8,10,12,13,x Thus, we have

$$\frac{6+8+10+12+13+x}{6} \quad 11$$

$$6+8+10+12+13+x = 66$$

$$49+x = 66$$

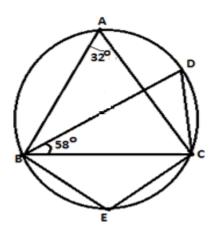
$$x = 66 \quad 49$$

$$x \quad 17$$



(b)

Consider the following figure.



Given that BD is a diameter of the circle.
The angle in a semi circle is a right angle. BCD=90°
Also given that DBC=58°
Consider the triangle BDC:
By angle sum property, we have DBC+ BCD+ BDC=180°

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58° 90°+ BDC=180°
148° BDC=180°
BDC=180° 148°
BDC=32°
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Angles in the same segment are equal.

Thus, BDC=32° BAC=32°

Now, DBACE is a cyclic quadrilateral,

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m BAC + m BEC = 180^{\circ}
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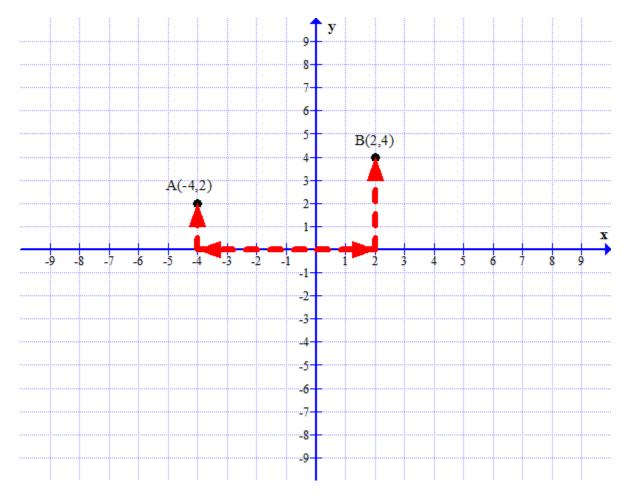
32 + m BEC = 180°

 \square m \square BEC = 180° - 32 = 148°



- (c) Consider 1 unit on the graph to be 2 cm.
 - (i)To plot the point A(-4,2), move 4 units along the negative x-axis. Then move 2 units along the positive y axis.

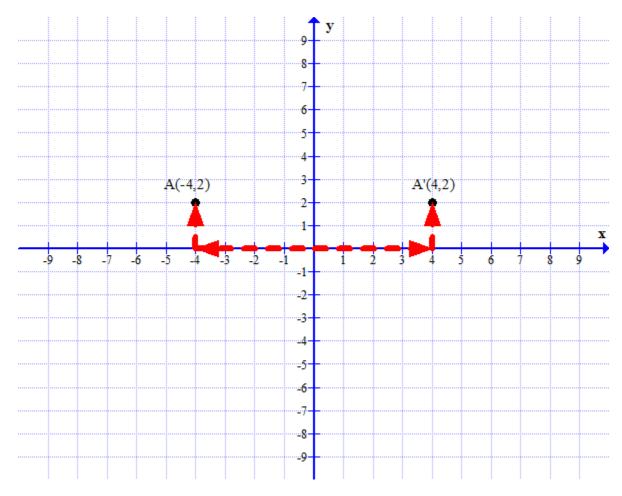
To plot the point B(2,4), move 2 units along the positive x-axis. Then move 4 units along the positive y axis.





(ii) The y-axis acts as a line of symmetry between A and A'. Thus, perpendicular distance of A from y-axis = perpendicular distance of A' from y-axis. Thus, the ycoordinate of A' will be same as A, and the x-coordinate of A' will be the negative of the x-coordinate of A.

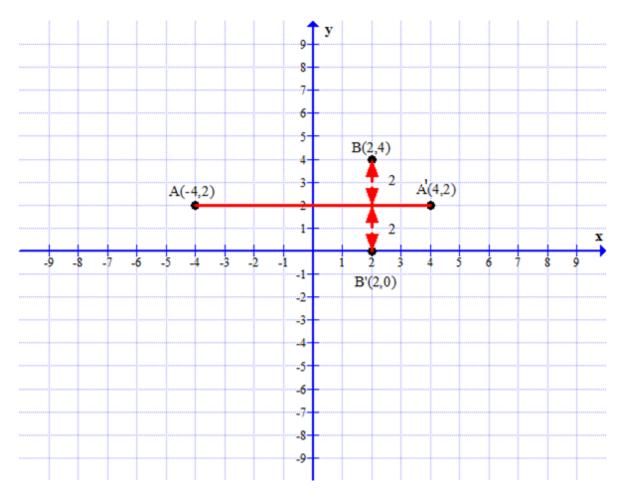
Thus, the coordinates of A' will be (4,2). Plot these in the same way as was done in (i).





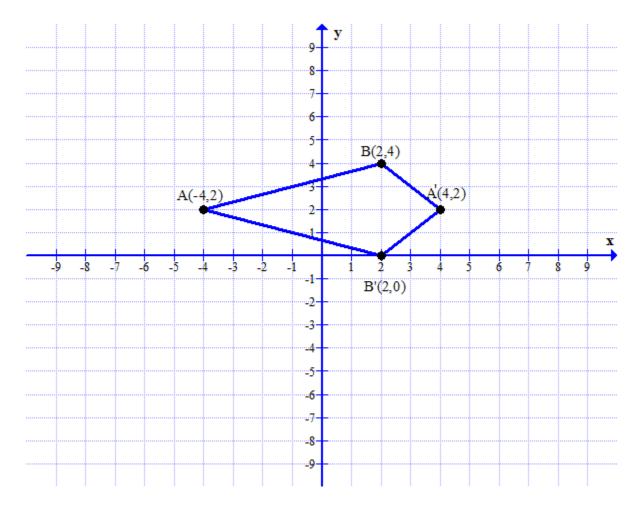
(iii) The line AA' acts as a line of symmetry between B and B'. Thus, perpendicular distance of B from AA' = perpendicular distance of B' from AA'. Thus, the x-coordinate of B' will be same as B, and the y-coordinate of B' will be the same distance away from AA' as B is.

Thus, the coordinates of B' will be (2,0). Plot these in the same way as was done in (i)



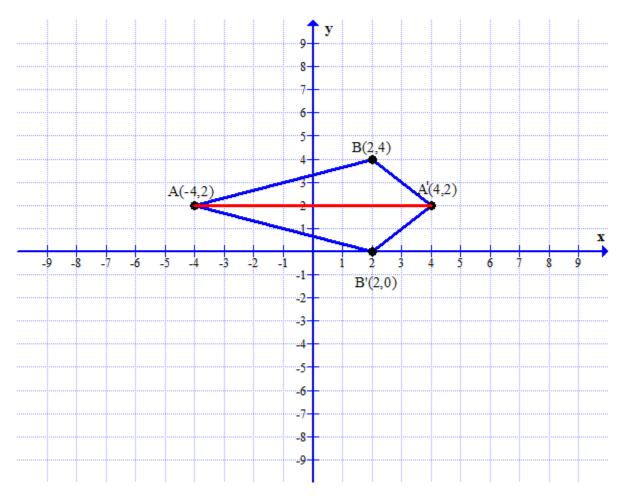


(iv) It can be observed that the figure that is formed by all the 4 points has 4 sides and thus, is a quadrilateral. Since the four sides can be grouped into two pairs of equal-length sides that are adjacent to each other, it is a kite.





(v) A line of symmetry is a line which creates a mirror image on both sides. Thus, in the image, line AA' is the line of symmetry.





SECTION B (40 Marks) Attempt any four questions from this section

5. (a)

Printed price of the washing machine = Rs. 18,000 Discount = 20% of 18,000 = $\frac{20}{100} \times 18000 = 3600$ Thus, sale price for the wholesaler = 18000 - 3600 = Rs. 14,400 Sales tax paid by shopkeeper = 8% of 14,400 = $\frac{8}{100} \times 14400 = 1152$

The shopkeeper sells the washing machine for 10% disount on printed price. Thus, the shopkeeper sells the washing machine to the customer at the price :

 $18000 - \frac{10}{100} \times 18000 = 18000 - 1800 = 16200$

Thus, the tax charged by the shopkeeper = 8% of 16,200 = $\frac{8}{100}$ ×16200=1296

- i. Thus, VAT paid by the shopkeeper = Tax charged Tax paid = 1296 1152 = Rs. 144
- Total amount that the customer pays for the washing machine is: Price at which the shopkeeper sells the washing machine + Tax charged by the shopkeeper = 16,200 + 1296 = Rs. 17,496
- (b) It is given that:

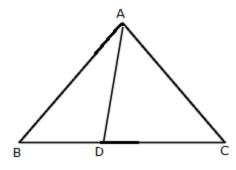
$$\frac{x^2 + y^2}{x^2 - y^2} = \frac{17}{8}$$
Applying componendo and dividendo,

$$\frac{x^2 + y^2}{x^2 + y^2} = \frac{17}{8} + \frac{17}{8} + \frac{8}{17} + \frac{17}{8} + \frac{8}{17} + \frac{17}{2} + \frac{8}{17} + \frac{17}{8} + \frac{8}{17} + \frac{17}{2} + \frac{8}{17} + \frac{17}{8} + \frac{17}{2} + \frac{17}{2} + \frac{17}{2} + \frac{17}{9} + \frac{17}{9}$$

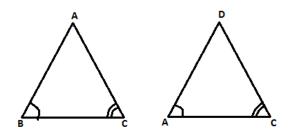


$$\frac{x^{3}}{x^{3}} \frac{y^{3}}{y^{3}} = \frac{125}{125} \frac{27}{27} \text{ or } \frac{125}{125} \frac{27}{27}$$
$$\frac{x^{3}}{x^{3}} \frac{y^{3}}{y^{3}} = \frac{152}{98} = \frac{76}{49} \text{ or } \frac{49}{76}, \text{ depending upon the sign of x and y}$$

(c) Consider the given triangle



Given that ABC= DAC.



(i) Consider the triangles ABC and DAC.

ABC= DAC given

C= C common

By AA-Similarity, $ABC \sim DAC$.

(ii) Hence the corresponding sides are proportional.

$$\frac{AB}{DA} \quad \frac{AC}{DC} \quad \frac{BC}{AC}$$
$$\frac{8}{5} \quad \frac{4}{DC}$$
$$DC \quad \frac{4}{5}$$
$$DC \quad \frac{5}{2} \text{ cm} = 2.5 \text{ cm}$$
$$\therefore \frac{AB}{DA} \quad \frac{BC}{AC}$$



$$\frac{8}{5} \quad \frac{BC}{4}$$

$$BC \quad \frac{8}{5} \quad \frac{4}{5}$$

$$BC \quad \frac{32}{5} \text{ cm} = 6.4 \text{ cm}$$

(iii) We need to find the ratios of the area of the triangles ABC and DAC. Since the triangles ABC and DAC are similar triangles, we have

$$\frac{AB}{DA} \quad \frac{AC}{DC} \quad \frac{BC}{AC}$$

If two similar triangles have sides in the ratio, a:b,

then their areas are in the ratio $a^2 : b^2$

Thus, $\frac{\text{Area}}{\text{Area}} \frac{\text{ACD}}{\text{ABC}} = \frac{\text{AB}^2}{\text{DA}^2} = \frac{\text{AC}^2}{\text{DC}^2} = \frac{\text{BC}^2}{\text{AC}^2}$ So, $\frac{\text{Area}}{\text{Area}} = \frac{\text{ACD}}{\text{ABC}} = \frac{4^2}{8^2}$ $\frac{\text{Area}}{\text{Area}} = \frac{\text{ACD}}{\text{ABC}} = \frac{16}{64} = 1:4$



6.

(a) If 3 points are collinear, the slope between any 2 points is the same. Thus, for A(a,3), B(2,1) and C(5,a) to be collinear, the slope between A and B and between B and C should be the same.

$$\frac{1}{2} \frac{3}{a} \frac{a}{5} \frac{1}{2}$$

$$\frac{2}{2} \frac{a}{a} \frac{1}{3}$$

$$\frac{2}{a} \frac{a}{3} \frac{1}{3}$$

$$\frac{2}{a} \frac{a}{3} \frac{1}{3}$$

$$6 a 2 a 1$$

$$a^{2} 3a 2 6$$

$$a^{2} 3a 4 0$$

$$a 1 \text{ or } 4$$

Rejecting a = -1 as it does not satisfy the equation, we have a = 4

Thus, slope of BC : $\frac{a}{5} \frac{1}{2} \frac{4}{3} \frac{1}{3} \frac{3}{3} 1 \text{ m}$

Thus, the equation of the line can be :

$$\begin{array}{ccc} \underline{y \ 1} \\ x \ 2 \end{array} 1 \\ y \ x \ 1 \\ x \ y \ 1 \end{array}$$



(b) Given :

Nominal Value (NV) of each share = Rs. 50 Since the shares are quoted at 20% premium, the market value of each share is: 20

Market Value (MV) of each share = $50 + \frac{20}{100} \times 50 = \text{Rs.60}$

(i) Dividend = Number of shares × dividend percentage × NV Let n be the number of shares.

Thus, 600 n 15% 50 $600 \text{ n } \frac{15}{100} 50$ $n \frac{600 2}{15}$ n 80

(ii) Total investment = n X MV Thus, total investment is: $80 \times 60 = 4800$ (iii) Rate of interest = $\frac{\text{Dividend}}{\text{Total Investment}}$ 100 $\frac{600}{4800}$ 100 12.5%

(c)

- (i) Total surface area of the sphere = $4 r^2$, where r is the radius of the sphere. Thus,
- (ii) Since the sphere is melted and recast into cones,Volume of a sphere = n × Volume of a cone, where n is the number of cones.

 $\begin{array}{cccccccc} \frac{4}{3} & r^{3} & n & \frac{1}{3} & r_{c}^{2} & h_{c} \text{, where } r_{c} \text{ and } h_{c} \text{ are the radius and height of the cone.} \\ \\ \frac{4}{3} & 14^{3} & n & \frac{1}{3} & 3.5^{2} & 7 \\ \\ 4 & 14^{3} & n & 3.5^{2} & 7 \\ \\ n & 128 \end{array}$



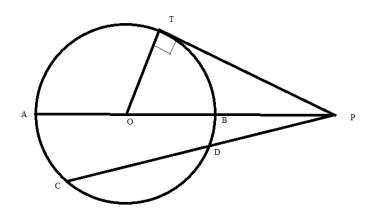
7.

 (a) Let A be the assumed mean and d be the deviation of x from the assumed mean. Let A = 40. d = x - A

Marks (C.I.)	No. of students	Mid-point of	$\mathbf{d} = \mathbf{x} - \mathbf{A}$	f X d
	(Frequency f)	C.I. (x)	A = 45.5	
11-20	2	15.5	-30	-60
21-30	6	25.5	-20	-120
31-40	10	35.5	-10	-100
41-50	12	45.5	0	0
51-60	9	55.5	10	90
61-70	7	65.5	20	140
71-80	4	75.5	30	120
	Total f = 50			Total f _d =70

Mean = A+ $\frac{\text{Total } f_d}{\text{Total } f}$ Mean = 45.5 + $\frac{70}{50}$ Mean = 45.5 + 1.4 Mean = 46.9

(b) Theorem used: Product of the lengths of the segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.



(i) As chord CD and tangent at point T intersect each other at P, $PC \times PD = PT^2$ (i) AB is the diameter and tangent at point T intersect each other at P, $PA \times PB = PT^2$ (ii) From (i) and (ii), PC × PD = PA × PB (iii)



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Given: PD = 5cm, CD = 7.8 cm

PA = PB + AB = 4 + AB, and PC = PD + CD = 12.8 cm

Subs. these values in (iii),

12.8 \times 5 = (4 + AB) \times 4

4 + AB = \frac{12.8 - 5}{4}

4 + AB = 16

AB = 12cm

(ii)

PC PD PT<sup>2</sup>

PT<sup>2</sup> 12.8 5 64

PT 8 cm
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Thus, the length of the tangent is 8 cm.



8.

(a) C. I. for the third year = Rs. 1452. C. I. for the second year = Rs. 1320 S.I. on Rs. 1320 for one year = Rs. 1452 - Rs. 1320 = Rs. 132 Rate of Interest = $\frac{132 \ 100}{1320}$ 10%

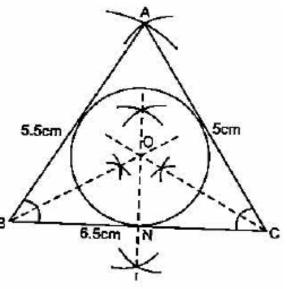
Let P be the original sum of money and r be the rate of interest. Amount after 2 years – Amount after one year = C.I. for second year.

P 1
$$\frac{10}{100}^{2}$$
 P 1 $\frac{10}{100}$ 1320
P $\frac{110}{100}^{2}$ $\frac{110}{100}$ 1320
P $\frac{110}{100}^{2}$ $\frac{110}{100}$ 1320
P $\frac{11}{10}^{2}$ $\frac{11}{10}$ 1320
P $\frac{121}{100}$ $\frac{11}{10}$ 1320
P $\frac{1320}{11}$ Rs.12,000

Thus, rate of interest is 10% and original sum of money is Rs.12,000.

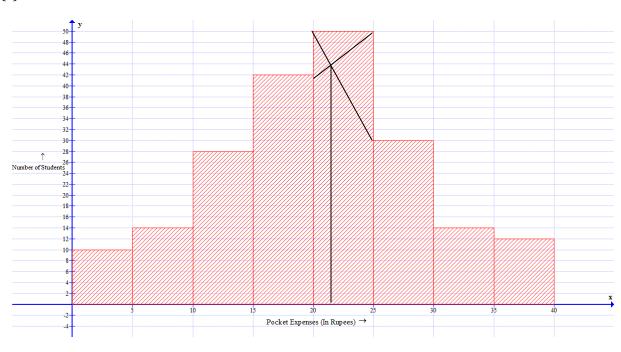
(b) The incircle of the triangle is drawn with the joining point of all angular bisectors as the center.

Construct a \triangle ABC with the given data. Draw the angle bisector of B and C. Let these bisectors cut at O. Taking O as centre. Draw a incircle which touches all the sides of the \triangle ABC From O draw a perpendicular to side BC which cut at N. Measure On which is required radius of the incircle. ON = 1.5cm



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(c)





9.

(a)Duplicate ratio of a:b is $a^2: b^2$

It is given that the duplicate ratio of $(x \ 9):(3x \ 6) \ 4:9$ Thus, $(x \ 9)^2:(3x \ 6)^2 \ 4:9$ $\frac{x \ 9}{3x \ 6} \ \frac{4^2}{9^2}$ $\frac{x \ 9}{3x \ 6} \ \frac{16}{81}$ $81x \ 729 \ 48x \ 96$ $81x \ 48x \ 96+729$ $33x \ 825$ $x \ \frac{825}{33} \ 25$



(b)

The given quadratic equation is :

$$x-1^{2}-3x+4=0$$

 $x^{2}-2x+1-3x+4=0$
 $x^{2}-5x+5=0$

The roots of the quadratic equation $ax^2 + bx + c = 0$ are:

x
$$\frac{b \sqrt{b^2 4ac}}{2a}$$

In the given equation, a = 1, b = -5, c=5

Thus, the roots of the equation are :

$$x = \frac{5 \sqrt{5^2 4 1 5}}{2 1}$$

$$x = \frac{5 \sqrt{25 20}}{2}$$

$$x = \frac{5 \sqrt{5}}{2}$$

$$x = \frac{5 \sqrt{5}}{2} \text{ or } x = \frac{5 \sqrt{5}}{2}$$

$$x = 3.618 \text{ or } x = 1.382$$



(c) Qualifying principal for various months:

Month	Principal		
April	6000		
Мау	7000		
June	10000		
July	6000		
August	6000		
September	7000		
Total	42000		

Here, P= Rs.42,000

Let R% be the rate of interest and $T = \frac{1}{12}$ year

Given that the interest , $I{=}Rs.175$

Thus, we have

$$I = \frac{P R T}{100}$$

$$175 \frac{42000 R 1}{100 12}$$

$$\frac{175 100 12}{42000} R$$

$$R 5\%$$

Thus the rate of interest is 5%



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10.
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(a)

Let the digit at the tens place be 'a' and at units place be 'b'. The two-digit so formed will be 10a+b.

According to given conditions, product of its digits is 6. Thus,

a b 6 a $\frac{6}{b}$ (1)

9 is added to the number = 10a+b+9

It is given that this new value is equal to the value of the reversed number. If the digits are reversed, the new number formed = 10b+a.

```
Thus,

10a b 9 10b a

9a 9b 9

a b 1

Substitute (1) in the above equation

Thus,

a \frac{6}{a} 1

a^2 a 6 0

Thus,

a 3 or 2

Since a digit cannot be negative, a = 2.

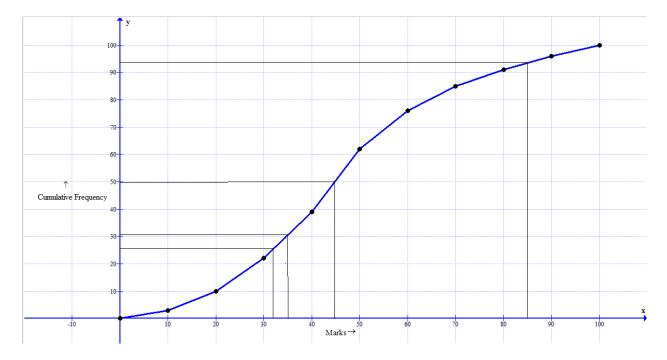
b \frac{6}{a} \frac{6}{2} 3

Thus, the required number is : 10a + b = 23
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Marks	Number of Students (Frequency)	Cumulative Frequency
0-10	3	3
10-20	7	10
20-30	12	22
30-40	17	39
40-50	23	62
50-60	14	76
60-70	9	85
70-80	6	91
80-90	5	96
90-100	4	100

(b) Draw the cumulative frequency table.



Scale : x-axis : 1 unit = 10marks y-axis : 1 unit = 10 students

(i) Median = $\frac{N}{2}^{\text{th}}$ term $\frac{100}{2}^{\text{th}}$ term 50^{th} term

Draw a horizontal line through mark 50 on y-axis. The, draw a vertical line from the point it cuts on the graph. The point this line touches the x-axis is the median. Thus, median = 45

(ii) Lower quartile =
$$\frac{N}{4}^{\text{th}}$$
 term $\frac{100}{4}^{\text{th}}$ term 25^{th} term

Draw a horizontal line through mark 25 on y-axis. The, draw a vertical line from the point it cuts on the graph. The point this line touches the x-axis is the lower quartile.

Thus, lower quartile = 31

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(iii) Draw a vertical line through mark 85 on x-axis. The, draw a horizontal line from the point it cuts on the graph.

The point this line touches the y-axis is the number of students who obtained less than 85% marks =93

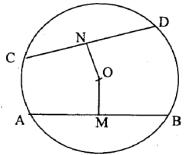
Thus, number of students who obtained more than 85% marks =7

(iv) Draw a vertical line through mark 35 on x-axis. The, draw a horizontal line from the point it cuts on the graph.The point this line touches the y-axis is the number of students who obtained less than 35% marks =21

11.

- (a) A line from center to a chord that is perpendicular to it, bisects it. It is given that AB = 24 cm Thus, MB = 12 cm
 - (i) Applying Pythagoras theorem for ΔOMB , $OM^2 MB^2 OB^2$

 5^2 12^2 OB^2 OB 13 Thus, radius of the circle = 13 cm.



(ii) Similarly, applying Pythagoras theorem for ΔOND ,

 $ON^2 ND^2 OD^2$

OD is the radius of the circle

 $12^2 \quad \text{ND}^2 \quad 13^2$

ND 5

A line from center to a chord that is perpendicular to it, bisects it. ND = 5 cm Thus, CD = 10 cm



(b) Consider LHS:

sin cos tan cot

sin	COS		in os	cos sin
sin	cos	si	$\frac{n^2}{\cos}$	cos ² sin
sin	cos	_		
cos	sin	-		
siı	n	СС)S	
cos	sin	cos	sin	
1	1			
cos	sin	_		
sec	cose	~		

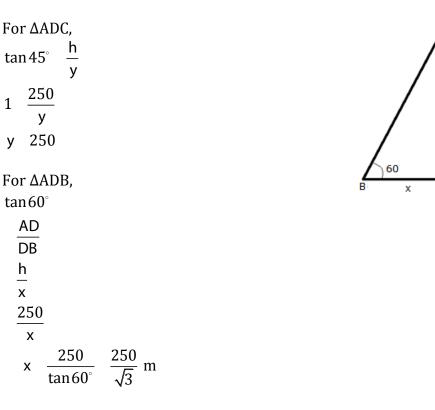
Thus, L.H.S. = R.H.S.

(c) Let A be the position of the airplane and let BC be the river. Let D be the point in BC just below the airplane.

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Thus, the width of the river = DB + DC = 250 $\frac{250}{\sqrt{3}}$ 394 m