



PHYSICS JEE ADVANCED PAPER – 2
SOLUTIONS

Answer key:

1.A, B, D	2.B, D	3.A, D	4.A, C	5. B,D	6.A, D	7.B,C	8.A, B, D	9. B	10.A
11.A	12.D	13.A	14. B	15.C	16.C	17.A	18.B	19.A	20.C

Section – I

This section contains eight questions. Each question has four choices, out of which one or more than one may be correct.

1. Ans. (A,B,D) For the same range, $\alpha + \beta = 90^\circ$ (or) $\beta = 90^\circ - \alpha$. Choices a), b) and d) are correct as they satisfy these relations.

2. Ans. (B) and (D)

Since the acceleration along the inclined plane ($g \sin \theta$) is less than g , the blocks take different times to reach the bottom. The speed of each block on reaching the bottom is, $v = \sqrt{2gh}$, where h is the height of the inclined plane.

3. Ans. (A) and (D)

$(P)_i = mu - mu = 0$, $(P)_f = 0$, as they come to rest.

$$\Rightarrow F_{\text{ext}} = 0 \Rightarrow W_{\text{ext}} = 0$$

$$\Rightarrow K_f - K_i = 0 - \left(\frac{1}{2} mu^2 + \frac{1}{2} mu^2 \right) = -mu^2$$

4. Ans. (A) and (C)

$F = \frac{k}{r}$, k is constant.

$$\frac{mv^2}{r} = \frac{k}{r} \Rightarrow v = \sqrt{\frac{k}{m}}, \text{ which is independent of } r.$$

$$T = \frac{2\pi r}{v} = 2\pi \sqrt{\frac{m}{k}} r \Rightarrow T \propto r$$

5. A. Using Kirchhoff's law, current through R_2 is $2.25 - 1.5 = 0.75A$. Also, since R_2 is in parallel with the 30Ω resistance, R_2 must be 60Ω since only half the current flows through it compared to the current through 30Ω resistor. Total resistance in circuit becomes, $(60+20) \Omega = 80 \Omega$
Potential drop across the battery, $E = IR = 2.25 \times 80 = 180V$

6. Ans. (A) and (D)

7. Ans. (C)

$$\beta_1 = \frac{\lambda_1 D}{d} \quad \beta_2 = \frac{\lambda_2 D}{d}$$

$$\frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2}, \text{ now } \lambda_1 = \frac{\lambda_a}{\mu} = \frac{\lambda_1}{\mu}$$

Therefore $\beta_3 = \frac{\beta_1}{\mu}$

8. Ans. (A), (B) and (D)

No. of photons released per unit time, $W/\frac{hc}{\lambda}$.

The maximum energy of emitted photo electrons is,

$$E_{\max} = hv - \phi = \frac{hc}{\lambda} - \phi = \frac{1}{\lambda}(hc - \lambda\phi), eV_s = E_{\max}$$

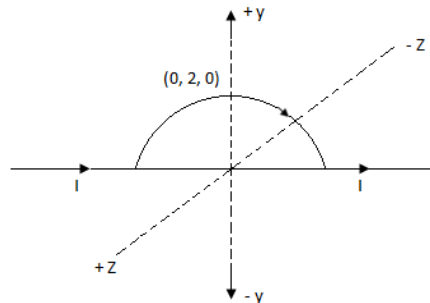
$$\text{Hence } V_s = \frac{1}{e} E_{\max} = \frac{1}{e\lambda}[hc - \lambda\phi]$$

For Photoemission to be possible, we must have, $hv \geq \phi$. Hence, $\frac{hc}{\lambda} \geq \phi$ (or) $\lambda \leq hc/\phi$

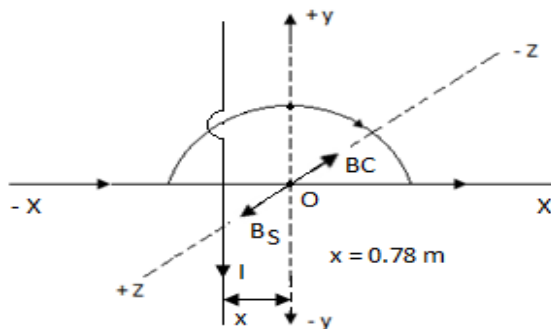
Thus, the permitted range of value of λ is $0 \leq \lambda \leq hc/\phi$. Hence the correct choices are (a), (b) and (d).

Passage - I

A current configuration is shown in figure. The radius of semicircular part is 2m. Answer the following questions on the basis of below situations.



9. Ans.



$B_c \rightarrow B_{\text{circular}}, B_s \rightarrow B_{\text{straight wire}}$

$$B_c = B_s \Rightarrow \frac{1}{2} \left[\frac{\mu_0 I}{2r} \right] = \frac{\mu_0 I}{2r}$$

$$\Rightarrow x = 0.78\text{m}$$

You can check the position of wire at $x = -0.78\text{m}$ and direction of current by right hand rule.

10. Ans. (D)

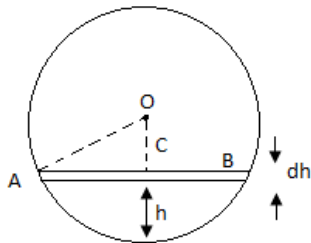
If we change the direction of current in semi circular loop the direction of B due to it will be in +Z direction, so to make net magnetic field at 'O' to be zero, the B due to the straight wire should be along -ve Z axis.

Passage – II

11. Ans. (D)

Suppose height of liquid at time t be h and it decreases dh in time dt.

From cross – section view, surface area of element of Liquid of thickness dh is, (AB) L = (2AC)L



$$(AC)^2 = R^2 - (oc)^2 = R^2 - R - h^2 + 2Rh$$

$$\Rightarrow Ac = \sqrt{2Rh - h^2}$$

$$\Rightarrow \text{Area} = 2L\sqrt{2Rh - h^2}$$

$$\Rightarrow \text{Volume of that element} = 2L(\sqrt{2Rh - h^2})dh$$

12. Ans. (B)

The rate of water coming out of orifice is equal to rate by which the level of water comes down.

Volume that goes out of orifice can also be written as, $a \times \sqrt{2gh} \times dt$

$$\text{Where, } a = \text{area of orifice} = \frac{\pi d^2}{4}$$

$$\Rightarrow 2L(\sqrt{2Rh - h^2})dh = \frac{-\pi d^2}{4} \sqrt{2gh} dt$$

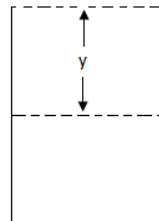
$$\text{Integrating, we get, } T = \frac{4L}{3\sqrt{2g} \frac{\pi d^2}{4}} [D^{3/2} - (D - H)^{3/2}]$$

Passage-III

In an organ pipe (may be closed or open) of 99cm length, standing wave setup, whose equation is given by longitudinal displacement,

$$S = (0.1\text{mm}) \cos \left[\frac{2\pi}{8} [y + 0.01\text{m}] \right] \cos [2\pi(400t)]$$

Where y is measured from top of Tube y is in metre, t in second and 0.01m is the end correction



13. Ans. (A)

If end correction is given in problem, that means, it is never a closed – closed Pipe.

It can be closed – Open or open – open.

Length of tube = $99 + (0.01 \times 100) = 1\text{m}$

Wavelength of standing wave = $\lambda = 0.8\text{m}$ for closed pipe it will be, $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \dots$

i.e., 0.2, 0.6, 1m. So, it should be a closed – open organ pipe with 2nd overtone.

14. Ans. Pressure maxima, $P_0 = BKS_0$

[B \rightarrow Bulk modulus,, K \rightarrow Wave number]

$S_0 \rightarrow$ displacement amplitude

$$P_0 = 5 \times 10^5 \times \frac{2\pi}{0.8} \times 0.1 \times 10^{-3} = \frac{5\pi}{4} \times 100 = (125\pi) \text{Nm}^{-2}$$

At phase difference $\frac{\pi}{2}$, pressure equation should be,

$$P_{\text{ext}} = 125\pi \sin\left[\frac{2\pi}{0.8}(y + 0.01\text{m})\right] \cos(2\pi(400t))$$

Passage – IV

15. Ans. (C)

If whole mixture becomes ice, first calorimeter will reach 0°C . So, heat given = $mC_1t_2 + mC_2t_2$. It is used by ice to increase its temperature = $mC_3(t+t_3)$

$$mC_3(t+t_3) = mC_1t_2 + mC_2t_2 \Rightarrow t = \frac{C_1t_2 + C_2t_2 - C_3t_3}{C_3}$$

If $t < 0$, that means ice is still below 0°C that means it requires more heat. So, heat supplied to water at 0°C to freeze.

$$\text{So, } \frac{C_1t_2 + C_2t_2 - C_3t_3}{C_3} < 0$$

Now, when all water freezes, then, $mc_3(t^1+t_3) = mC_1t_2 + mC_2t_2 + mL$

$$\Rightarrow t^1 = \frac{C_1t_2 + C_2t_2 + L - C_3t_3}{C_3}$$

If $t^1 < 0$, then whole mixture will freeze,

$$\text{So, } C_1t_2 + C_2t_2 + L - C_3t_3 < 0$$

16. Ans. (B)

For whole water, heat supplied by ice,

$$= mC_3t_3 + mL$$

That will be gained by calorimeter and

$$\text{water} \Rightarrow mC_1(t - t_2) + mC_2(t - t_2) = mC_3t_3 + mL$$

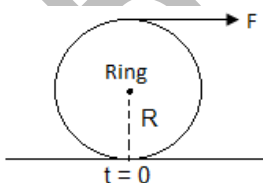
$$\Rightarrow (C_1 + C_2)t = C_3t_3 + L + C_1t_2 + C_2t_2$$

$$\Rightarrow t = \frac{C_3t_3 + L + C_1t_2 + C_2t_2}{C_1 + C_2}$$

$$\Rightarrow C_3t_3 + L + C_1t_2 + C_2t_2 > 0$$

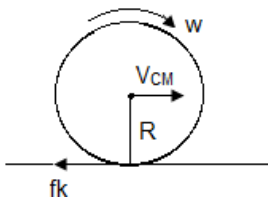
17. Ans. (A)

A.



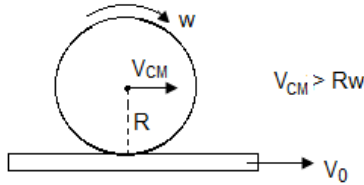
In the case of ring, there is no need of friction because a_{CM} and α provided by F will satisfy the relation $a_{\text{CM}} = R\alpha$ for pure accelerated rolling.

B.



f_k will be in backward direction to decrease V_{CM} and to increase ω until rolling body gets the pure rolling.

C.



For pure rolling, $V_{CM} = R\omega = V_0$
 So for pure rolling, (this case) $V_{CM} = V_0 + R\omega$ means $V_{CM} = R\omega$ for pure rolling in this case, As here $V_{CM} = R\omega$, it means backward slipping. So, f_k acts in forward direction.

D. In this case, f_k will be in upward direction to reduce ω and to change the direction of ω so that may be after some time pure accelerated rolling can be retained.

18. Ans. (B)

1. When non – conservative forces are acting work done in a round trip is not equal to zero.
2. In case of a pendulum, the sum of kinetic energy and potential energy is constant.
3. When there is friction, work done is negative. In projectile motion, the only force acting is gravity.
4. In uniform circular motion, there is no work done by a body.

19. Ans. (A)

1. No current because of zero charge and hence magnetic moment is zero.
 $\Rightarrow I = 0$ so $B = 0$
 At any instant E is non-zero and downward.
 $M = 0$ (magnetic moment), So, torque, $\vec{\tau} = \vec{M} \times \vec{B} = \vec{0}$
2. As, $I \neq 0 \Rightarrow M \neq 0$
 Magnetic field is always in a fixed direction so its average will not be zero.
 No electric field due to current $\Rightarrow M \neq 0$ so $\tau \neq 0$
3. We can define current $\Rightarrow M \neq 0$ so $B \neq 0$
 Moving charge will have its electric field.
 $\Rightarrow M \neq 0$ so $\tau \neq 0$

20. Ans. (C)

Thermal resistance $R = \frac{1}{KA}$
 If $K_x = 2K_y$, $R_x = \frac{R_y}{2}$ (or) $R_y = 2R_x = 2R$
 So, final

By the logic of current, for point B,

$$\frac{T_1 - 100}{R} + \frac{T_1 - T_2}{R} + \frac{T_1 - 0}{2R} = 0$$

$\Rightarrow 2$

$$2T_2 = 200 \rightarrow (1)$$

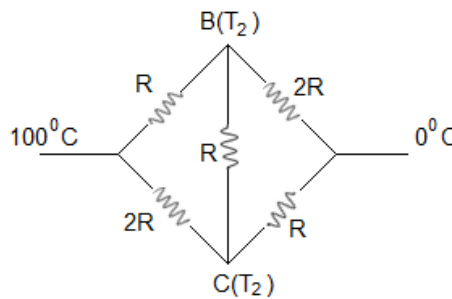
For point C, $\frac{(T_2 - 100)}{2R} + \frac{(T_2 - T_1)}{R} + \frac{(T_2 - 0)}{R} = 0$

$$\Rightarrow 5T_2 - 2T_1 = 100 \rightarrow (2)$$

Solving (1) and (2), we get,

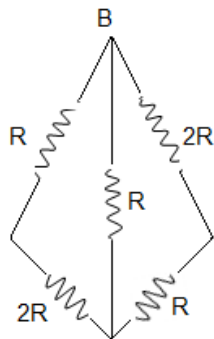
$$T_1 = \frac{400^\circ C}{7} \text{ and } T_2 = \frac{380^\circ C}{7}$$

Thermal resistance between B and D is,



$$T_1 - 200 + 2T_1 - 2T_2 + T_1 = 0 \Rightarrow 5T_1 -$$

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{3R} + \frac{1}{3R} = \frac{1}{R} + \frac{2}{3R} \Rightarrow R_{eq} = \frac{3R}{5}$$



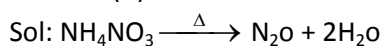
CHEMISTRY SOLUTIONS

21. A,B,C,D	22.A	23.A,C,D	24.A,B,C	25.A,C,D	26.A,C	27.B,C	28.A,D	29.D	30.B
31.D	32.A	33.C	34.B	35.A	36.A	37.C	38.B	39.B	40.A

21. Ans: (A,B,C,D)

Sol: conceptual

22. Ans. (A)



23. Ans: (A, C, D)

Sol: only aliphatic aldehydes respond.

24. Ans: (A, B, C)

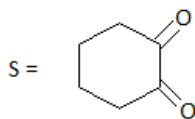
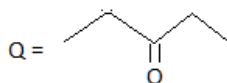
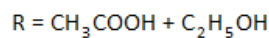
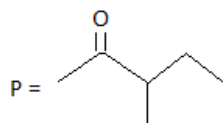
Sol: Neither 'N' or 'F' contain d- orbital's and in N_2F_4 , N-N bond is shorter than in N_2H_4 due to S-Character

25. Ans: (A, C, D)

Sol: Conceptual

26. Ans: (A, C)

Sol:



27. Ans: (B, C)

Sol: Max capacity of balloon = $\frac{8}{7} \times 480 = 548.57 \text{ ml}$

Also, $V_1 = 480 \text{ ml}$, $T_1 = 278\text{K}$, $n = 1 \text{ mol}$

$$*) \frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow \frac{480}{278} = \frac{548.57}{T_2}$$

$$T_2 = 317.71 = 44.71^\circ \text{C}$$

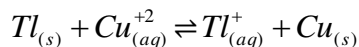
$$*) \text{ From } Pv = nRT$$

$$P \times \frac{480}{1000} = 1 \times 0.0821 \times 278$$

$$= 47.5 \text{ atm}$$

28. Ans: (A, D)

Sol: Cell reaction is



$$Q_c = \frac{[Tl^{+}]}{[Cu^{+2}]}$$

$$E = E^0 - \frac{0.0591}{n} \log Q_c \text{ at } 298 \text{ K}$$

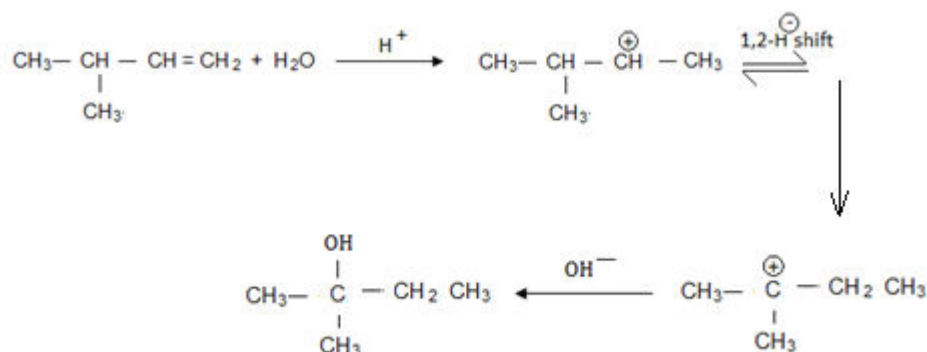
$$E = E^0 - \frac{0.059}{n} \log \left(\frac{[Tl^{+}]}{[Cu^{+2}]} \right)$$

Paragraph for question

Acid catalysed hydration of alkenes gives alcohol accordingly markownikoff's rule. Alcohols are also prepared from alkenes by oxy mercuration-de mercuration and hydro boration oxidation.

29. Ans: (D)

Sol:



30. Ans: (B)

Sol: Hydroboration – oxidation is syn addition and follows anti markownikoff's rule.

Paragraph

A solution is 0.1 M in each KCl, KBr and K_2CrO_4 to this solution solid $AgNO_3$ is gradually added. Assume there is no change in the volume of the solution given:

$$K_{sp}(AgCl) = 1.7 \times 10^{-10}$$

$$K_{sp}(AgBr) = 5.0 \times 10^{-13} \text{ and } K_{sp}(K_2CrO_4) = 1.9 \times 10^{-12}$$

31. Ans: (D)

Sol: $[Ag]^+$ to start the ppt. Of Cl^-

$$= \frac{K_{sp}(AgCl)}{[Cl^-]} = \frac{1.7 \times 10^{-10}}{0.1} = 1.7 \times 10^{-9} \text{M}$$

When Cl^- starts precipitating

$$[\text{Br}^-] = \frac{K_{sp}(\text{AgBr})}{[\text{Ag}^+]} = \frac{5 \times 10^{-13}}{1.7 \times 10^{-9}} = 3 \times 10^{-4} M$$

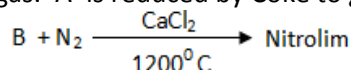
32. Ans: (A)

Sol: $[\text{Ag}^+]$ to start the ppt. Of AgBr

$$= \frac{K_{sp}(\text{AgBr})}{[\text{Br}^-]} = \frac{5 \times 10^{-13}}{0.1} = 5 \times 10^{-12} M$$

Paragraph

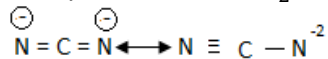
Lime stone is an ore of Ca. On decomposition gives an oxide (A) and CO_2 gas. 'A' is reduced by Coke to give 'B' and Volatile gas.



Nitrolim is a fertilizer.

33. Ans: (C)

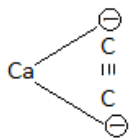
Sol: Cyanide ion = CN_2^- has



Total non bonded electrons = 8

34. Ans: (B)

Sol: 'B' is CaC_2



Paragraph

A solid crystal is composed of X, Y and Z atoms. Z atoms (radius 200 pm) form cubic closed packed structure where as X (radius 50 pm) and Y (radius 100 pm) occupy respective voids. Assume all the voids are occupied.

35. Ans: (A)

$$\text{Solution: } \frac{r_x}{r_z} = \frac{50}{200} = 0.25$$

Which is in b/w 0.225 – 0.414

\therefore 'X' atoms are in tetrahedral voids

36. Ans: (A)

Solution: For CCP, Z = 4, X = 8, Y = 4

On removing atoms along one body diagonal

The 2z atoms, 2x atoms, 1y atom is removed.

\therefore Formula $\rightarrow X_6 Y_3 Z_{3.75}$

(or) $X_8 Y_4 Z_5$

37. Ans: (C) Solution: Conceptual

38. Ans: (B) Solution: Conceptual

39. Ans: (B) Solution: Conceptual (based on Salt hydrolysis)

40. Ans: (A) Solution: Conceptual

MATHEMATICS SOLUTION

41.A,C	42.A,C,D	43. C	44.A,B	45.B,D	46.B,C,D	47.A,C,D	48.A,C,D	49. B	50. C
51. C	52. A	53. D	54. B	55. B	56. D	57. A-R, B-S, C-Q,D-P	58. A-S, B-P, C-Q,D-R	59. A-Q, B-R, C-S,D-P	60. A-R, B-P, C-Q,D-S

SECTION – 1

41. SOL. (A, C) Eliminating m from given equation, we get $3x^2 - y^2 = 48$ which is a hyperbola. Its eccentricity =

$$\sqrt{1 + \frac{48}{16}} = 2$$

42. SOL (A, C, D) Radius and center of the circle are $(1, -2)$ and $7\sqrt{2}$ respectively

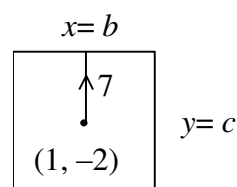
Thus diameter of the circle is $14\sqrt{2}$ and is equal to diagonal of the square, hence each side of square is 14. Since each side of the square are parallel to axes, let its equation be

$$x = a, x = b, y = c, y = d$$

also, r distance from center to each side of the circle = 7

This gives $a = -6, b = 8, c = -9, d = 5$

So vertices are $(8, 5), (8, -9), (-6, 5), (-6, -9)$



43. SOL. (C)

$$\int_{-1}^1 \frac{d}{dx} \left(\tan^{-1} \frac{1}{x} \right) dx$$

$$= \int_{-1}^1 \frac{1}{1 + \left(\frac{1}{x}\right)^2} \left(\frac{-1}{x^2} \right) dx = - \int_{-1}^1 \frac{dx}{1 + x^2} dx = -2 \int_0^1 \frac{dx}{1 + x^2} \quad (\text{even function})$$

$$= -2 \left[\tan^{-1} x \right]_0^1 = -2 \frac{\pi}{4} = -\frac{\pi}{2}$$

44. SOL. (A, B)

$2f(x)f(y) = f(x-y) + f(x+y)$ at $x = y = 0; f(0) = 1$ ($\because f(0) \neq 0$)

and at $y = 0$

$$2f(x) = f(-x) + f(x) \quad \Rightarrow f(x) = f(-x)$$

ie. $f(x)$ is an even function $f'(x) = -f'(-x)$

$$\Rightarrow f'(1) + f'(-1) = 0$$

Also $f'(x) = -f'(-x)$

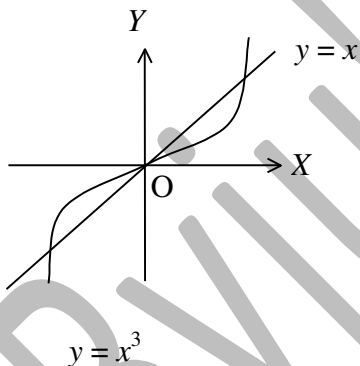
$$f'(2) + f'(-2) = 0$$

45. SOL. (B, D) Limit = $\lim_{x \rightarrow 0} \frac{e^{\frac{1}{x} \ln(1+x)} - e}{x}$

$$= \lim_{x \rightarrow 0} \frac{e^{\left(1 - \frac{x}{2} + \frac{x^2}{3} \dots\right)} - e}{x} = \lim_{x \rightarrow 0} \frac{e^{\left\{e^{\left(\frac{-x}{2} + \frac{x^2}{3} \dots\right)} - 1\right\}}}{\left(\frac{-x}{2} + \frac{x^2}{3} \dots\right)} \times \frac{\left(\frac{-x}{2} + \frac{x^2}{3} \dots\right)}{x}$$

$$= e(\ln e) \left(-\frac{1}{2}\right) = \frac{-e}{2} < -1 \quad (\because e \cong 2.7)$$

46. SOL. (B, C, D)



Clearly not differentiable at $x = 0, \pm 1$

2nd method

Solving $x^3 = x$, we have $x = 0, \pm 1$

$$\begin{aligned} \therefore \text{Max}\{x, x^2\} &= x && \text{when } x < -1 \\ &= x^3 && \text{when } -1 \leq x \leq 0 \\ &= x && \text{when } 0 < x \leq 1 \\ &= x^3 && \text{when } x > 1 \end{aligned}$$

47. SOL. (A, C, D) $f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3x \frac{1}{x} \left(x + \frac{1}{x}\right)$

$$\therefore f(x) = x^3 - 3x$$

Since $x + \frac{1}{x} \geq 2$ for $x > 0$ and $x + \frac{1}{x} \leq -2$ for $x < 0$, the domain of

$f(x)$ is $(-\infty, -2] \cup [2, \infty)$

$$f'(x) = 3(x^2 - 1) > 0 \quad \forall x \in D.$$

Thus, $f(x)$ is increasing in D , injective there and $f(x) = 3$ has a unique real root. Since f is not defined at $x = -1$, (b) does not hold.

48. SOL. (A, C, D) $\int \frac{dx}{1+y^2} = \int \frac{dx}{1+x^2}$

$$\text{or } \tan^{-1} y = \tan^{-1} x + c \quad (\text{c is correct})$$

$$\text{or } \frac{\pi}{2} - \cot^{-1} y = \tan^{-1} x + c$$

$$\text{or } \tan^{-1} x + \cot^{-1} y = \frac{\pi}{2} - c \quad (\text{d is correct})$$

$$\text{also } \tan^{-1} y - \tan^{-1} x = c$$

$$\Rightarrow \tan^{-1} \left(\frac{y-x}{1+xy} \right) = c \Rightarrow \frac{y-x}{1+xy} = \tan c = k \quad (\text{a is correct})$$

SECTION - 2

Comprehension-1

A square matrix A such that $A^\theta = A$ is called Hermitian matrix i.e. $a_{ij} = \overline{a_{ji}}$ for all values of i and j and a square matrix A such that $A^\theta = -A$ is called skew-Hermitian matrix i.e. $a_{ij} = -\overline{a_{ji}}$ for all values of i and j , where A^θ is conjugate transpose matrix.

Let $f : M \rightarrow \{1, -1\}$, M is set of all hermitian or Skew-hermitian matrixes, be a function defined as

$$f(A) = \begin{cases} 1 & A \text{ is hermitian} \\ -1 & A \text{ is skewhermitian} \end{cases}$$

49. SOL. (B)

$X = A - A^\theta$ is a skew hermitian

$$(\because X^\theta = (A - A^\theta)^\theta = A^\theta - A = -X)$$

$$f(X) = -1$$

50. SOL. (C)

$$Y^\theta = (A^n)^\theta = (A^\theta)^n = \begin{cases} A^n = Y & \text{if } A \text{ is hermitian} \\ (-A)^n = \begin{cases} Y & \text{if } A \text{ is skew hermitian and } n \text{ is even} \\ -Y & \text{if } A \text{ is skew hermitian and } n \text{ is odd} \end{cases} \end{cases}$$

Comprehension-2

For any angle A, $\sqrt{1+\sin A} = \left| \cos \frac{A}{2} + \sin \frac{A}{2} \right|$, $\sqrt{1-\sin A} = \left| \cos \frac{A}{2} - \sin \frac{A}{2} \right|$

Use this information to answer the following.

SOL.

Given $\sqrt{1+\sin A} = \left| \cos \frac{A}{2} + \sin \frac{A}{2} \right|$, $\sqrt{1-\sin A} = \left| \cos \frac{A}{2} - \sin \frac{A}{2} \right|$

51. (C)

$$\sqrt{1+\sin A} - \sqrt{1-\sin A} = -2\cos \frac{A}{2} \Rightarrow \left| \cos \frac{A}{2} + \sin \frac{A}{2} \right| - \left| \cos \frac{A}{2} - \sin \frac{A}{2} \right| = -2\cos \frac{A}{2} \Rightarrow \frac{5\pi}{2} < A < \frac{7\pi}{2}$$

52. (A)

If A = 280 then

$$\begin{aligned} \sqrt{1+\sin A} + \sqrt{1-\sin A} &= \left| \cos \frac{A}{2} + \sin \frac{A}{2} \right| + \left| \cos \frac{A}{2} - \sin \frac{A}{2} \right| \\ &= |\cos 140^\circ + \sin 140^\circ| + |\cos 140^\circ - \sin 140^\circ| \\ &= |-\sin 50^\circ + \cos 50^\circ| + |-\sin 50^\circ - \cos 50^\circ| \\ &= \sin 50^\circ - \cos 50^\circ + \sin 50^\circ + \cos 50^\circ = 2\sin 50^\circ \end{aligned}$$

Comprehension-3

Consider the function $f : \mathbb{R} \rightarrow (0, \infty)$ defined by $f(x) = 2^x + 2^{|x|}$

53. (D)

$$f(x) = 2^x + 2^{|x|} \quad \text{For } x \geq 0, f(x) = 2 \cdot 2^x \quad \text{And for } x < 0; f(x) = 2^x + 2^{-x}$$

and in this case $f'(x) = 2^x \ln 2 - 2^{-x} \ln 2 = \ln 2 \left(\frac{2^{2x} - 1}{2^x} \right) = \frac{\ln 2}{2^x} \cdot (2^x + 1)(2^x - 1) < 0$

So; $f(x)$ is decreasing in $x \in (-\infty, 0)$ and increasing in $x \in (0, \infty)$

So, $f(x)$ is many one

Also, Range of $f(x)$ is $[2, \infty)$

f is many-one into

54. Sol. (B)

$$\text{Now the required area} = \int_{-1}^0 (2^x + 2^{-x}) dx + \int_0^1 2 \cdot 2^x dx = \frac{7}{2 \ln 2} = \frac{7}{2} \cdot \log_2 e = \log_2 (e^{7/2})$$

Comprehension-4

A JEE aspirant estimates that she will be successful with an 80% chance if she studies 10 hours per day, with a 60% chance if she studies 7 hours per day and with a 40% chance if she studies 4 hours per day. She further believes that she will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7 respectively

55. SOL. (B)

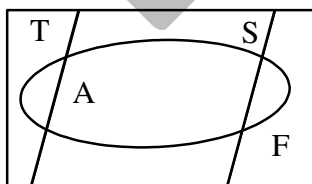
56. Sol. (D)

A : She get a success

T : She studies 10 hrs : $P(T) = 0.1$

S : She studies 7 hrs : $P(S) = 0.2$

F : She studies 4 hrs : $P(F) = 0.7$



$$P(A/T) = 0.8; P(A/S) = 0.6; P(A/F) = 0.4$$

$$\text{Now, } P(A) = P(A \cap T) + P(A \cap S) + P(A \cap F)$$

$$= P(T) \cdot P(A/T) + P(S) \cdot P(A/S) + P(F) \cdot P(A/F)$$

$$= (0.1)(0.8) + (0.2)(0.6) + (0.7)(0.4)$$

$$= 0.08 + 0.12 + 0.28 = 0.48$$

$$P(F/A) = \frac{P(F \cap A)}{P(A)} = \frac{(0.7)(0.4)}{0.48} = \frac{0.28}{0.48} = \frac{7}{12}$$

$$P(F/\bar{A}) = \frac{P(F \cap \bar{A})}{P(\bar{A})} = \frac{P(F) - P(F \cap A)}{0.52}$$

$$= \frac{(0.7) - 0.28}{0.52} = \frac{0.42}{0.52} = \frac{21}{26}$$

Section – 3

57. $A \rightarrow R, B \rightarrow S, C \rightarrow Q, D \rightarrow P$. (A) Required number of ways = 6C_2

(B) Since pots are identical, there will be 4 cases (4,0,0), (3,1,0), (2,2,0); (2,1,1) but all the

Coins are different hence selection of coins matters

For the first case No of selections = $4C_4 = 1$

For the second case No of selections = $4C_3 \times 1C_1 = 4$

For the third case No of selections = $\frac{4C_2 \times 2C_2}{2!} = 3$

For the fourth case No of selections = $\frac{4C_2 \times 2C_1 \times 1C_1}{2!} = 6$

(C) Since no box is empty and all pots are identical so the possible case is (1,1,2). But since all the Coins are different, the 2 balls can be selected in $4C_2$ ways and rest can be put in $\frac{2C_1 \times 1C_1}{2!}$

Required number of distributions = $\frac{4C_2 \times 2C_1 \times 1C_1}{2!} = 6$

(D) Since no pot is empty and all coins are identical the possible case is (1,1,2). But since all three pots are different hence, a pot (which contains 2 coins together) can be selected in $3C_1$ ways. Hence, the required number of distributions = $3C_1 \times 1 = 3$

58. $A \rightarrow S; B \rightarrow P; C \rightarrow Q; D \rightarrow R$

A. $\vec{r} = \vec{a} + t\vec{b}$
 $\vec{r} = \vec{c} + s\vec{d}$ shortest distance between the lines = $\frac{|\vec{c} - \vec{a} \cdot \vec{b} \times \vec{d}|}{|\vec{b} \times \vec{d}|}$

b. (2, -1, 2) satisfies the plane

find distance between (2, -1, 2) and (-1, -5, -10)

c. Find d.r's of A(5, 4, -1) and B(1+2k, 9k, 5k)

$$((2k-4)\vec{i} + (9k-4)\vec{j} + (5k+1)\vec{k}) \cdot (2\vec{i} + 9\vec{j} + 5\vec{k}) = 0 \text{ find } k.$$

d. projection of PQ on xy-plane = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

projection of PQ on yz-plane = $\sqrt{(y_1 - y_2)^2 + (z_1 - z_2)^2}$

projection of PQ on xz-plane = $\sqrt{(x_1 - x_2)^2 + (z_1 - z_2)^2}$

59. (A)Q(B)R(C)S(D)P

(A) \rightarrow (Q), $\arg \frac{z+1}{z-1} = \frac{\pi}{4}$ will be half the arc of the circle

(B) \rightarrow (R), $|z-2| = 4$ will be a full circle

(C) \rightarrow (S), $\arg z = \frac{\pi}{4}$ will be a line $x = y$ (such that $x > 0, y > 0$).

(D) \rightarrow (P), $x + iy = t + it^2$

$$\Rightarrow x = t, y = t^2$$

On eliminating t, we get

$$y = x^2 \text{ which is a parabola}$$

60. (A-R), (B-P), (C-Q), (D-S)

(ANS:) (A) The required event will occur if last digit in all the chosen numbers is 1, 3, 7 or 9.

$$\therefore \text{Req. probability} = \left(\frac{4}{10}\right)^n$$

(B) required probability = P (that the last digit is (2, 4, 6, 8)) = P(That the last digit is 1, 2, 3, 4, 6, 7, 8, 9) – P (that the last digit is 1, 3, 7, 9) = $\frac{8^n - 4^n}{10^n}$.

(C) $P(1, 3, 5, 7, 9) - P(1, 3, 7, 9)$

$$= \frac{5^n - 4^n}{10^n}$$

(D) required prob = $P(0, 5) - P(5)$

$$= \frac{(10^n - 8^n) - (5^n - 4^n)}{10^n}$$

$$= \frac{10^n - 8^n - 5^n + 4^n}{10^n}$$

Byju's Classes