1. The function \( f : \mathbb{N} \to \mathbb{N} \) defined by
\[ f(x) = x - 5 \left\lfloor \frac{x}{5} \right\rfloor, \]
where \( \mathbb{N} \) is the set of natural numbers and \([x]\) denotes the greatest integer less than or equal to \( x \), is:
(1) one-one and onto.
(2) one-one but not onto.
(3) onto but not one-one.
(4) neither one-one nor onto.

2. The sum of all the real values of \( x \) satisfying the equation
\[ 2(x-1)(x^2+5x-50) = 1 \]
is:
(1) 16
(2) 14
(3) −4
(4) −5

3. The equation
\[ \text{Im} \left( \frac{iz - 2}{z - i} \right) + 1 = 0, \quad z \in \mathbb{C}, \quad z \neq i \]
represents a part of a circle having radius equal to:
(1) 2
(2) 1
(3) \( \frac{3}{4} \)
(4) \( \frac{1}{2} \)
4. For two $3 \times 3$ matrices $A$ and $B$, let $A + B = 2B'$ and $3A + 2B = I_3$, where $B'$ is the transpose of $B$ and $I_3$ is $3 \times 3$ identity matrix. Then:
   (1) $5A + 10B = 2I_3$
   (2) $10A + 5B = 3I_3$
   (3) $B + 2A = I_3$
   (4) $3A + 6B = 2I_3$

5. If $x = a$, $y = b$, $z = c$ is a solution of the system of linear equations
   
   $x + 8y + 7z = 0$
   $9x + 2y + 3z = 0$
   $x + y + z = 0$

   such that the point $(a, b, c)$ lies on the plane $x + 2y + z = 6$, then $2a + b + c$ equals:
   (1) $-1$
   (2) $0$
   (3) $1$
   (4) $2$

6. The number of ways in which 5 boys and 3 girls can be seated on a round table if a particular boy $B_1$ and a particular girl $G_1$ never sit adjacent to each other, is:
   (1) $5 \times 6!$
   (2) $6 \times 6!$
   (3) $7!$
   (4) $5 \times 7!$
7. The coefficient of \(x^{-5}\) in the binomial expansion of
\[
\left(\frac{x + 1}{x^2 - x + 1} - \frac{x - 1}{x - x^2}\right)^{10}
\]
where \(x \neq 0, 1\), is:

(1) 1  
(2) 4  
(3) -4  
(4) -1  

8. If three positive numbers \(a\), \(b\) and \(c\) are in A.P. such that \(abc = 8\), then the minimum possible value of \(b\) is:

(1) 2  
(2) \(\frac{1}{3}\)  
(3) \(\frac{2}{3}\)  
(4) 4  

9. Let
\[
S_n = \frac{1}{1^3} + \frac{1 + 2}{1^3 + 2^3} + \frac{1 + 2 + 3}{1^3 + 2^3 + 3^3} + \ldots + \frac{1 + 2 + \ldots + n}{1^3 + 2^3 + \ldots + n^3}.
\]
If 100 \(S_n = n\), then \(n\) is equal to:

(1) 199  
(2) 99  
(3) 200  
(4) 19  

9. \(a, b\) and \(c\) are in an A.P. such that \(abc = 8\), then the minimum possible value of \(b\) is:

(1) 2  
(2) \(\frac{1}{3}\)  
(3) \(\frac{2}{3}\)  
(4) 4  

9. \(S_n = 1 + \frac{1 + 2}{1^3 + 2^3} + \frac{1 + 2 + 3}{1^3 + 2^3 + 3^3} + \ldots + \frac{1 + 2 + \ldots + n}{1^3 + 2^3 + \ldots + n^3}.
\]
If 100 \(S_n = n\), then \(n\) is equal to:

(1) 199  
(2) 99  
(3) 200  
(4) 19  

IX - MATHEMATICS
10. The value of $k$ for which the function

$$f(x) = \begin{cases} 
\frac{4}{5} \tan \frac{4x}{5}, & 0 < x < \frac{\pi}{2} \\
k + \frac{2}{5}, & x = \frac{\pi}{2}
\end{cases}$$

is continuous at $x = \frac{\pi}{2}$, is:

(1) $\frac{17}{20}$

(2) $\frac{2}{5}$

(3) $\frac{3}{5}$

(4) $-\frac{2}{5}$

11. If $2x = y^\frac{1}{2} + y^{-\frac{1}{2}}$ and

$$(x^2 - 1) \frac{d^2y}{dx^2} + \lambda x \frac{dy}{dx} + ky = 0,$$

then $\lambda + k$ is equal to:

(1) $-23$

(2) $-24$

(3) $26$

(4) $-26$
12. The function \( f \) defined by
\[ f(x) = x^3 - 3x^2 + 5x + 7, \]
is:
(1) increasing in \( R \).
(2) decreasing in \( R \).
(3) decreasing in \((0, \infty)\) and increasing in \((-\infty, 0)\).
(4) increasing in \((0, \infty)\) and decreasing in \((-\infty, 0)\).

13. Let \( f \) be a polynomial function such that
\[ f(3x) = f'(x) \cdot f''(x), \quad \text{for all } x \in R. \]
Then:
(1) \( f(2) + f'(2) = 28 \)
(2) \( f''(2) - f'(2) = 0 \)
(3) \( f''(2) - f(2) = 4 \)
(4) \( f(2) - f'(2) + f''(2) = 10 \)

14. If
\[ f\left(\frac{3x - 4}{3x + 4}\right) = x + 2, \quad x \neq -\frac{4}{3}, \]
and
\[ \int f(x) \, dx = A \log |1 - x| + Bx + C, \]
then the ordered pair \((A, B)\) is equal to:
(1) \( \left(\frac{8}{3}, \frac{2}{3}\right) \)
(2) \( \left(-\frac{8}{3}, \frac{2}{3}\right) \)
(3) \( \left(-\frac{8}{3}, -\frac{2}{3}\right) \)
(4) \( \left(\frac{8}{3}, -\frac{2}{3}\right) \)
15. If \[ \int_1^2 \frac{dx}{(x^2 - 2x + 4)^{\frac{3}{2}}} = \frac{k}{k+5}, \] then \( k \) is equal to:

(1) 1  
(2) 2  
(3) 3  
(4) 4

16. If \[ \lim_{n \to \infty} \frac{1^a + 2^a + \ldots + n^a}{(n+1)^{a-1} [ (n+1) + (n+2) + \ldots + (na+n)]} = \frac{1}{60} \] for some positive real number \( a \), then \( a \) is equal to:

(1) 7  
(2) 8  
(3) \( \frac{15}{2} \)  
(4) \( \frac{17}{2} \)
17. A tangent to the curve, \( y = f(x) \) at \( P(x, y) \) meets \( x \)-axis at \( A \) and \( y \)-axis at \( B \). If \( \frac{AP}{BP} = 1:3 \) and \( f(1) = 1 \), then the curve also passes through the point:

(1) \( \left( \frac{1}{3}, 24 \right) \)

(2) \( \left( \frac{1}{2}, 4 \right) \)

(3) \( \left( 2, \frac{1}{8} \right) \)

(4) \( \left( 3, \frac{1}{28} \right) \)

18. A square, of each side 2, lies above the \( x \)-axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30° with the positive direction of the \( x \)-axis, then the sum of the \( x \)-coordinates of the vertices of the square is:

(1) \( 2\sqrt{3} - 1 \)

(2) \( 2\sqrt{3} - 2 \)

(3) \( \sqrt{3} - 2 \)

(4) \( \sqrt{3} - 1 \)
19. A line drawn through the point P(4, 7) cuts the circle \(x^2 + y^2 = 9\) at the points A and B. Then \(PA \cdot PB\) is equal to:
   
   (1) 53  
   (2) 56  
   (3) 74  
   (4) 65

20. The eccentricity of an ellipse having centre at the origin, axes along the co-ordinate axes and passing through the points (4, -1) and (-2, 2) is:
   
   (1) \(\frac{1}{2}\)  
   (2) \(\frac{2}{\sqrt{5}}\)  
   (3) \(\frac{\sqrt{3}}{2}\)  
   (4) \(\frac{\sqrt{3}}{4}\)

21. If \(y = mx + c\) is the normal at a point on the parabola \(y^2 = 8x\) whose focal distance is 8 units, then |c| is equal to:
   
   (1) \(2\sqrt{3}\)  
   (2) \(8\sqrt{3}\)  
   (3) \(10\sqrt{3}\)  
   (4) \(16\sqrt{3}\)

19. यदि बिंदु P(4, 7) से खींची गई एक रेखा, जूझ \(x^2 + y^2 = 9\) को बिंदुओं A तथा B पर काटती है, तो \(PA \cdot PB\) बराबर है:
   
   (1) 53  
   (2) 56  
   (3) 74  
   (4) 65

20. एक दीर्घवृत्त जिसका केन्द्र मूल बिंदु है, अक्ष निर्देशांक \(4, -1\) और \((-2, 2)\) से होकर जाता है, उसके दर (eccentricity) है:
   
   (1) \(\frac{1}{2}\)  
   (2) \(\frac{2}{\sqrt{5}}\)  
   (3) \(\frac{\sqrt{3}}{2}\)  
   (4) \(\frac{\sqrt{3}}{4}\)

21. यदि \(y = mx + c\) परवलय \(y^2 = 8x\) के उस बिंदु पर अभिलंब है जिसकी नाभिसे दूरी 8 इकाई है, तो |c| बराबर है:
   
   (1) \(2\sqrt{3}\)  
   (2) \(8\sqrt{3}\)  
   (3) \(10\sqrt{3}\)  
   (4) \(16\sqrt{3}\)
22. If a variable plane, at a distance of 3 units from the origin, intersects the coordinate axes at A, B and C, then the locus of the centroid of \( \triangle ABC \) is:

1. \[ \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 1 \]
2. \[ \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 3 \]
3. \[ \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{9} \]
4. \[ \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 9 \]

23. If the line, \( \frac{x-3}{1} = \frac{y+2}{-1} = \frac{z+\lambda}{-2} \) lies in the plane, \( 2x-4y+3z=2 \), then the shortest distance between this line and the line, \( \frac{x-1}{12} = \frac{y}{9} = \frac{z}{4} \) is:

1. 2
2. 1
3. 0
4. 3

IX - MATHEMATICS
24. If the vector \( \vec{b} = 3 \hat{j} + 4 \hat{k} \) is written as the sum of a vector \( \vec{b}_1 \), parallel to \( \vec{a} = \hat{i} + \hat{j} \) and a vector \( \vec{b}_2 \), perpendicular to \( \vec{a} \), then \( \vec{b}_1 \times \vec{b}_2 \) is equal to:

1. \( -3\hat{i} + 3\hat{j} - 9\hat{k} \)
2. \( 6\hat{i} - 6\hat{j} + \frac{9}{2}\hat{k} \)
3. \( -6\hat{i} + 6\hat{j} - \frac{9}{2}\hat{k} \)
4. \( 3\hat{i} - 3\hat{j} + 9\hat{k} \)

25. From a group of 10 men and 5 women, four member committees are to be formed each of which must contain at least one woman. Then the probability for these committees to have more women than men, is:

1. \( \frac{21}{220} \)
2. \( \frac{3}{11} \)
3. \( \frac{1}{11} \)
4. \( \frac{2}{23} \)
26. Let E and F be two independent events. The probability that both E and F happen is \( \frac{1}{12} \) and the probability that neither E nor F happens is \( \frac{1}{2} \), then a value of \( \frac{P(E)}{P(F)} \) is:

(1) \( \frac{4}{3} \)

(2) \( \frac{3}{2} \)

(3) \( \frac{1}{3} \)

(4) \( \frac{5}{12} \)

27. The sum of 100 observations and the sum of their squares are 400 and 2475, respectively. Later on, three observations, 3, 4, and 5, were found to be incorrect. If the incorrect observations are omitted, then the variance of the remaining observations is:

(1) 8.25

(2) 8.50

(3) 8.00

(4) 9.00
28. A value of $x$ satisfying the equation 

$$\sin[cot^{-1}(1+x)] = \cos[tan^{-1}x]$$

is:

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<td>(4)</td>
<td>1</td>
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29. The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is $60^\circ$. If the area of the quadrilateral is $4\sqrt{3}$, then the perimeter of the quadrilateral is:

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<td>(3)</td>
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<td>(4)</td>
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30. Contrapositive of the statement

‘If two numbers are not equal, then their squares are not equal’, is:

(1) If the squares of two numbers are equal, then the numbers are equal.

(2) If the squares of two numbers are equal, then the numbers are not equal.

(3) If the squares of two numbers are not equal, then the numbers are not equal.

(4) If the squares of two numbers are not equal, then the numbers are equal.