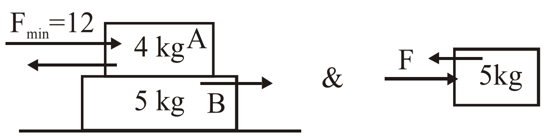
1. A block A of mass 4 kg is placed on another block B of mass 5 kg, and the block B rests on a smooth horizontal table. If the minimum force that can be applied on A so that both the blocks move together is 12 N, the maximum force that can be applied on B for the blocks to move together will be :

(1) 48 N (2) 27 N (3) 30 N (4) 25 N

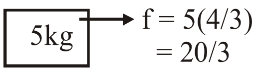
Ans. (Bonus)

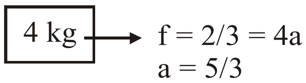
Sol.



$12 = 9a$

$a = 4/3$





$F - f = 5 \times \frac{4}{3}$

$F - \frac{20}{3} = \frac{20}{3}$

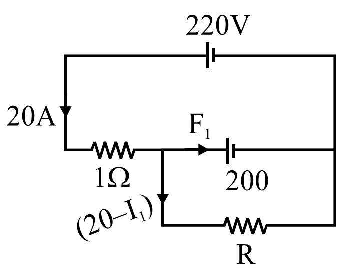
$f = \frac{40}{3}$

2. A d.c. main supply of e.m.f. 220 V is connected across a storage battery of e.m.f. 200 V through a resistance of 1Ω. The battery terminals are connected to an external resistance 'R'. The minimum value of 'R', so that a current passes through the battery to charge it is :

(1) Zero (2) 11 Ω (3) 9 Ω (4) 7 Ω

Ans. (2)

Sol.



$(20 - I_1)\, R = 200$

$$R = \frac{200}{(20 - I_1)}$$

$R \longrightarrow$ Minimum

when $20 - I_1 \longrightarrow$ maximum

& I_1 cannot be zero

so $R \simeq 11\Omega$

3. A transverse wave is represented by :

$$y = \frac{10}{\pi}\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$$

For what value of the wavelength the wave velocity is twice the maximum particle velocity?

(1) 40 cm (2) 10 cm

(3) 60 cm (4) 20 cm

Ans. **(1)**

Sol. $V = 2(V_p)_{max}$

$\because V = f\lambda$

$f\lambda = 2\omega A$

$\lambda = 4\pi A$

$= 4\pi \times \frac{10}{\pi}$

$= 40$ cm

4. The equation of state for a gas is given by $PV = nRT + \alpha V$, where n is the number of moles and α is a positive constant. The initial temperature and pressure of one mole of the gas contained in a cylinder are T_0 and P_0 respectively. The work done by the gas when its temperature doubles isobarically will be :

(1) $\frac{P_0T_0R}{P_0 - \alpha}$ (2) $\frac{P_0T_0R}{P_0 + \alpha}$

(3) $P_0\, T_0\, R$ (4) $P_0\, T_0\, R\, \ell n\, 2$

Ans. (1)

Sol. $P_0V_0 = nRT_0$

$P_0V = nRT$

$T_f = 2T_0$

$W = \int PdV$

$= \int\left(\frac{nRT}{V} + \alpha\right) dv$

$PV = nRT + \alpha V$

$$\int pdV = \int_{T_0}^{2T_0} nRdT + \int_{V_i}^{V_f} \alpha dV$$

$= nRT_0 + \alpha\, V_i$

$$= nRT_0 + \alpha\left(\frac{nRT_0}{P_0}\right)$$

$$= nRT_0\left(1+\frac{\alpha}{P_0}\right)$$

$PV = nRT + \alpha V$

$\int pdV = \int nRdT + \int \alpha dV$

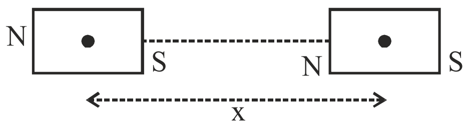
$$W = nRT_0 + \alpha\left[\frac{nRT_0}{P_0-\alpha}\right]$$

$$W = nRT_0\left[1+\frac{\alpha}{p_0-\alpha}\right]$$

$$= nR_0T_0\left[\frac{P_0}{P_0-\alpha}\right]$$

$$= \frac{nRT_0\ P_0}{P_0-\alpha}$$

5. The mid points of two small magnetic dipoles of length d in end-on positions, are separated by a distance x, (x >> d). The force between them is proportional to x^{-n} where n is :

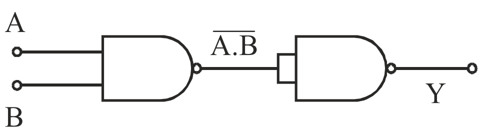


(1) 2 (2) 1 (3) 4 (4) 3

Ans. (3)

Sol. $F \propto \frac{1}{r^4}$

6. Identify the gate and match A, B, Y in bracket to check.



(1) XOR (A=0, B = 0. Y = 0)
(2) AND (A=l, B = l, Y= l)
(3) NOT (A=l, B = l, Y=l)
(4) OR (A = l, B=l, Y = 0)

Ans. (2)

7. When the rms voltages V_L, V_C and V_R are measured respectively across the inductor L, the capacitor C and the resistor R in a series LCR circuit connected to an AC source, it is found that the ratio$V_L : V_C : V_R = 1 : 2 : 3$. If the rms voltage of the AC source is 100 V, then V_R is close to:

(1) 50 V (2) 100 V (3) 70 V (4) 90 V

Ans. (4)

Sol. $I = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2+(X_L-X_C)^2}} = \frac{100}{\sqrt{9x^2+x^2}} = \frac{100}{\sqrt{10x^2}}$

Since $V_L : V_C : V_R = 1 : 2 : 3$

$X_L = X_C : X_R = 1 : 2 : 3$

$= x : 2x : 3x$

now $V_R = I\ (3x)$

$$= \frac{100}{\sqrt{10x^2}}\cdot 3x$$

≈ 94.87 V

8. A capillary tube is immersed vertically in water and the height of the water column is x. When this arrangement is taken into a mine of depth d, the height of the water column is y. If R is the radius of earth, the ratio $\frac{x}{y}$ is :

(1) $\left(1-\frac{2d}{R}\right)$ (2) $\left(1-\frac{d}{R}\right)$

(3) $\left(\frac{R-d}{R+d}\right)$ (4) $\left(\frac{R+d}{R-d}\right)$

Ans. (2)

Sol. height talances additional presence hence

$\rho g_s x = \rho\ g_{depth}\ y$

$g_s\ x = g_s(1 - d/R)y$

$$\frac{x}{y} = 1-\frac{d}{R}$$

9. In materials like aluminium and copper, the correct order of magnitude of various elastic modulii is :

(1) Bulk modulii < shear modulii < Young's modulii.

(2) Young's modulii < shear modulii < bulk modulii.

(3) Bulk modulii < Young's modulii < shear modulii.

(4) Shear modulii < Young's modulii < bulk modulii.

Ans. (4)

10. Three capacitances, each of 3 μF, are provided. These cannot be combined to provide the resultant capacitance of :

(1) 4.5 μF (2) 1 μF (3) 2μF (4) 6μF

Ans. (4)

Sol. When all in series

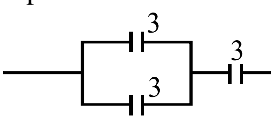
$$\frac{1}{C_{eq}} = \frac{3}{3}$$

C_{eq} = 1μF (2 not possible)

when 3 is parallel

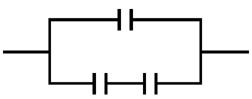
C_{eq} = 9μF

2 parallel 1 series



$C_{eq} = \frac{6\times3}{9}$ = 2μF (3 option not possible)

2 series 1 parallel



$$\frac{3\times3}{6}+3 = 4.5\ \mu F$$

(1 option not possible)

Hence answer is (4)

11. The amplitude of a simple pendulum, oscillating in air with a small spherical bob, decreases from 10 cm to 8 cm in 40 seconds. Assuming that Stokes law is valid, and ratio of the coefficient of viscosity of air to that of carbon dioxide is 1.3, the time in which amplitude of this pendulum will reduce from 10 cm to 5 cm in carbondioxide will be close to (ℓn 5 = 1.601, ℓn 2 = 0.693).

(1) 161 s (2) 208 s (3) 231 s (4) 142 s

Ans. (1)

Sol. $8 = 10e^{-\lambda \times 40}$

$$5 = 10e^{-\frac{\lambda t}{1.3}}$$

$$\ln\frac{4}{5} = -\lambda\times40$$

$2 \times 0.693 - 1.601 = -\lambda \times 40$

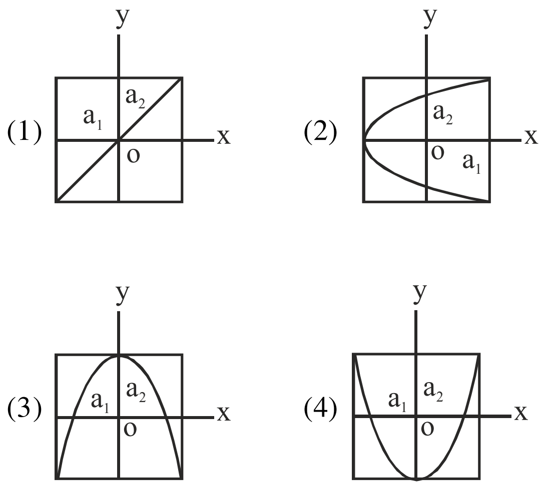
$\lambda = 0.005375$

$$\ln\frac{1}{2} = -\frac{\lambda t}{1.3}$$

$$-0.693 = -\frac{0.005375}{1.3}t$$

t = 167.6

12. A particle which is simultaneously subjected to two perpendicular simple harmonic motions represented by ; $x=a_1 \cos \omega t$ and $y = a_2 \cos 2\omega t$ traces a curve given by :



Ans. (4)

Sol. $y = a_2 [2 \cos^2\omega t - 1]$

$$= a_2\left[2\cdot\frac{x^2}{a_1^2}-1\right]$$

$$y = \frac{2a_2}{a_1^2}x^2 - a_2$$

at x = 0, y is negative and this is a equation parabola. Hence answer is 4.

13. Match **List I** (Wavelength range of electromagnetic spectrum) with **List II**. (Method of production of these waves) and select the **correct** option from the options given below the lists.

List I		List II	
(a)	700 nm to 1 mm	(i)	Vibration of atoms and molecules.
(b)	1 nm to 400 nm	(ii)	Inner shell electrons in atoms moving from one energy level to a lower level.
(c)	$< 10^{-3}$ nm	(iii)	Radioactive decay of the nucleus.
(d)	1 mm to 0.1 m	(iv)	Magnetron valve.

	(a)	(b)	(c)	(d)
(1)	(iii)	(iv)	(i)	(ii)
(2)	(i)	(ii)	(iii)	(iv)
(3)	(ii)	(iii)	(iv)	(i)
(4)	(iv)	(iii)	(ii)	(i)

Ans. (2)

Sol. 10^{19}

$E = \frac{hC}{\lambda} = hV \qquad \lambda = \frac{C}{V} = \frac{10^8}{10^{19}}$

$= 10^{-11}$ m $\qquad = 10^{-2}$ nm

Magnetron valve generate microwaves.

14. The position of a projectile launched from the origin at t = 0 is given by $\vec{r} = (40\hat{i} + 50\hat{j})$ m at t = 2s. If the projectile was launched at an angle θ from the horizontal, then θ is (take g =10 ms^{-2}).

(1) $\tan^{-1}\frac{4}{5}$ (2) $\tan^{-1}\frac{3}{2}$

(3) $\tan^{-1}\frac{2}{3}$ (4) $\tan^{-1}\frac{7}{4}$

Ans. (4)

Sol. $2u_x = 40 \Rightarrow 4x = 20$

$50 = 24y - \frac{1}{2} \times 10 \times 2^2 \Rightarrow 4y = 35$

$\tan\theta = \frac{u_y}{u_x} = \frac{35}{20} = \frac{7}{4}$

$\theta = \tan^{-1}\left(\frac{7}{4}\right)$

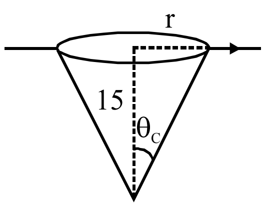
15. A diver looking up through the water sees the outside world contained in a circular horizon. The refractive index of water is $\frac{4}{3}$, and the diver's eyes are 15 cm below the surface of water. Then the radius of the circle is :

(1) $15 \times 3\sqrt{7}$ cm (2) $\frac{15 \times 3}{\sqrt{7}}$ cm

(3) $\frac{15 \times \sqrt{7}}{3}$ cm (4) $15 \times 3 \times \sqrt{5}$ cm

Ans. (2)

Sol. $\sin\theta_C = \frac{1}{4/3} = \frac{3}{4}$



$\tan\theta_C = \frac{3}{\sqrt{7}} = \frac{r}{h}$

$r = \frac{3}{\sqrt{7}} \times 15$

16. An experiment is performed to obtain the value of acceleration due to gravity g by using a simple pendulum of length L. In this experiment time for 100 oscillations is measured by using a watch of 1 second least count and the value is 90.0 seconds. The length L is measured by using a meter scale of least count 1 mm and the value is 20.0 cm. The error in the determination of g would be :

(1) 1.7% (2) 2.7% (3) 4.4% (4)2.27%

Ans. (2)

Sol. $T^2 = \frac{4\pi^2\ell}{g}$

$g = 4\pi^2\frac{\ell}{T^2}$

$\frac{\Delta g}{g} \times 100 = \left(\frac{\Delta\ell}{\ell} \times 100\right) + 2\left(\frac{\Delta T}{T} \times 100\right)$

$= \left(\frac{0.1}{20} \times 100\right) + 2\left(\frac{0.01}{.9} \times 100\right)$

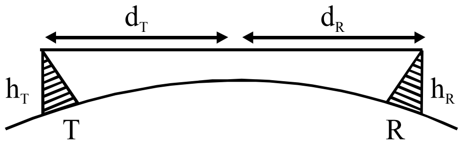
$= 0.5 + 2 \times \frac{10}{9} = 0.5 + 2.2 = 2.7\%$

17. A transmitting antenna at the top of a tower has a height 32 m and the height of the receiving antenna is 50 m. What is the maximum distance between them for satisfactory communication in line of sight (LOS) mode ?

(1) 55.4 km (2) 54.5 km
(3) 455 km (4) 45.5 km

Ans. (4)

Sol.



$d_T = \sqrt{2Rh_T} = \sqrt{2 \times 6400 \times 10^3 \times 32}$

$= 202 \times 10^2$ m $= 20.20$ km

$d_R = \sqrt{2Rh_R} = \sqrt{2 \times 6400 \times 10^3 \times 50}$

$= 25.3$ km

$\therefore$ $d = d_T + d_R = 20.2 + 25.3 = 45.5$ km

18. The magnitude of the average electric field normally present in the atmosphere just above the surface of the Earth is about 150 N/C, directed inward towards the center of the Earth. This gives the total net surface charge carried by the Earth to be :

[Given $\in_0 = 8.85 \times 10^{-12}$ $C^2/N\text{–}m^2$,
$R_E = 6.37 \times 10^6$ m]

(1) + 680 kC (2) – 680 kC
(3) – 670 kC (4) + 670 kC

Ans. (3)

Sol. $E = \frac{1}{4\pi\varepsilon_0}\frac{\theta}{R^2} = \frac{\sigma}{\varepsilon_0} \Rightarrow \sigma = \varepsilon_0 E$

$= 8.85 \times 10^{-12} \times 150$

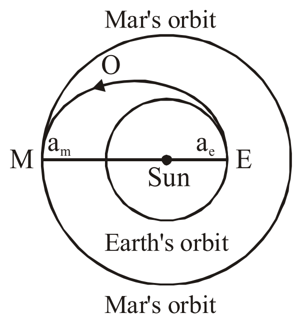
$Q = \varepsilon_0 E \times 4\pi R^2$

$= 6.76 \times 10^5 \times 10^{-12} \times$
10^{+12}

$= 680$ kC

for inward will be negative.

19. India's Mangalyan was sent to the Mars by launching it into a transfer orbit EOM around the sun. It leaves the earth at E and meets Mars at M. If the semi-major axis of Earth's orbit is $a_e = 1.5 \times 10^{11}$m, that of Mar's orbit $a_m = 2.28 \times 10^{11}$m, taken Kepler's laws give the estimate of time for Mangalyan to reach Mars from Earth to be close to :



(1) 500 days (2) 320 days
(3) 260 days (4) 220 days

Ans. (3)

Sol. $r = \frac{1.5 + 2.28}{2} = 1.89$

$\frac{T_m}{T_e} = \left(\frac{1.89}{1.5}\right)^{3/2}$

$t_m = \frac{T_m}{2} = \left(\frac{1.89}{1.5}\right)^{3/2}$

$= \frac{365}{2} \times 1.41 = 257.3$ day

20. Water of volume 2 L in a closed container is heated with a coil of 1 kW. While water is heated, the container loses energy at a rate of 160 J/s. In how much time will the temperature of water rise from 27°C to 77°C ? (Specific heat of water is 4.2 kJ/kg and that of the container is negligible).

(1) 14 min (2) 8 min 20 s
(3) 7 min (4) 6 min 2s

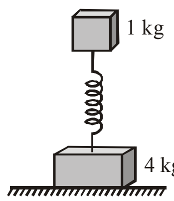
Ans. (2)

Sol. 1000 – 160 = 840 J/s

$t\text{-}840 = 2 \times 4.2 \times 10^3 \times 50$

$t = \frac{500}{60} = 8$ min 20 s.

21. Two bodies of masses 1 kg and 4 kg are connected to a vertical spring, as shown in the figure. The smaller mass executes simple harmonic motion of angular frequency 25 rad/s, and amplitude 1.6 cm while the bigger mass remains stationary on the ground. The maximum force exerted by the system on the floor is (take $g = 10$ ms^{-2}).



(1) 20 N (2) 10 N (3) 40 N (4) 60 N

Ans. (4)

Sol. $T - mg = Mw^2A$

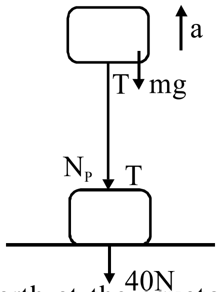
$= 1 \times 625 \times \frac{1.6}{100}$

$= 10$ N

$T = 20$ N

$N = T + 40$

$= 60$ N



22. The magnetic field of earth at the equator is approximately 4×10^{-5} T. The radius of earth is 6.4×10^6 m. Then the dipole moment of the earth will be nearly of the order of:

(1) 10^{20} A m^2 (2) 10^{23} A m^2

(3) 10^{10} A m^2 (4) 10^{16} A m^2

Ans. (2)

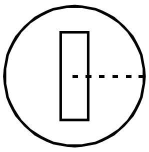
Sol. $B = 4 \times 10^{-5}$T

$B = \frac{\mu_0}{4\pi} \times \frac{M}{r^3} = 10^{-7} \times \frac{M}{(6.4 \times 10^6)^3} = 4 \times 10^{-5}$

$M = \frac{4 \times 10^{-5} \times 10^{18} \times 6.4^3}{10^{-7}}$

$= 1.048 \times 10^{3+18+7-5}$

$= 10^{23}$



23. The focal lengths of objective lens and eye lens of a Gallelian Telescope are respectively 30 cm and 3.0 cm. Telescope produces virtual, erect image of an object situated far away from it at least distance of distinct vision from the eye lens. In this condition, the Magnifying Power of the Gallelian Telescope should be :

(1) + 8.8 (2) – 11.2 (3) + 11.2 (4) – 8.8

Ans. (2)

Sol. $f_0 = 30$ cm $f_e = 3$cm

$M = \frac{f_0}{f_c}\left(1 - \frac{f_C}{D}\right)$

$= \frac{30}{3}\left(1 - \frac{3}{25}\right)$

$= \frac{22 \times 30}{3 \times 25} = \frac{44}{5} = +8.8$

24. Modern vacuum pumps can evacuate a vessel down to a pressure of 4.0×10^{-15} atm. at room temperature (300 K). Taking R= 8.3 JK^{-1} mole^{-1}, 1 atm=10^5 Pa and $N_{Avogadro} = 6 \times 10^{23}$ mole^{-1}, the mean distance between molecules of gas in an evacuated vessel will be of the order of :

(1) 0.2 nm (2) 0.2 cm

(3) 0.2 mm (4) 0.2μm

Ans. (3)

Sol. $\lambda = \frac{kT}{\sqrt{2}\pi d^2 P}$

$= \frac{1.38 \times 10^{-23} \times 300}{\sqrt{2}\pi \times 10^{-20} \times 4 \times 10^{-10}}$

$= \frac{1.38 \times 3}{\sqrt{2} \times 4\pi} \times 10^{-9}$

$= 0.2$ nm

25. An n-p-n transistor has three leads A, B and C. Connecting B and C by moist fingers, A to the positive lead of an ammeter, and C to the negative lead of the ammeter, one finds large deflection. Then, A, B and C refer respectively to :

(1) Emitter, base and collector

(2) Base, emitter and collector

(3) Base, collector and emitter

(4) Collector, emitter and base.

Ans. (1)

26. Water is flowing at a speed of 1.5 ms^{-1} through a horizontal tube of cross-sectional area 10^{-2} m^2 and you are trying to stop the flow by your palm. Assuming that the water stops immediately after hitting the palm, the minimum force that you must exert should be (density of water = 10^3 kgm^{-3}).

(1) 33.7N (2) 15N (3) 22.5N (4) 45N

Ans. (3)

Sol. $F = v\frac{dm}{dt}$

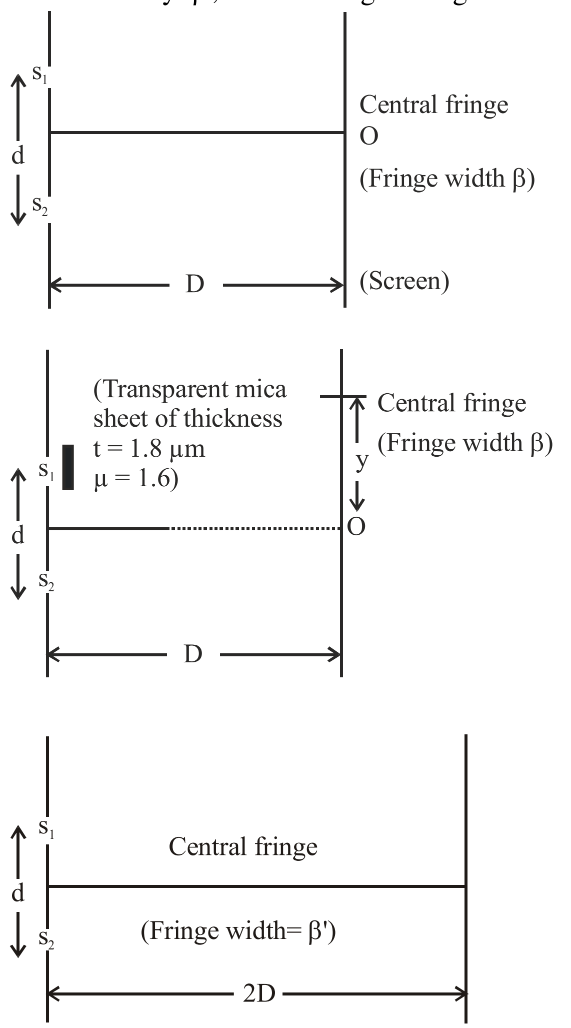
$= v\, A\rho v$

$= v^2 A\rho$

$= (1.5)^2 \times 10^{-2} \times 10^3$

$= 2.25 \times 10 = 22.5$ N

27. Using monochromatic light of wavelength λ, an experimentalist sets up the Young's double slit experiment in three ways as shown. If she observes that y=β', the wavelength of light used is



(1) 580 nm (2) 560 nm
(3) 520 nm (4) 540 nm

Ans. (4)

Sol. $B' = y$

$(\mu - 1)t = d\sin\theta$

$= d\theta = \frac{dy}{D}$

$$y = \frac{D(\mu-1)}{d}t$$

$$\frac{(2D)\lambda}{d} = \frac{D(\mu-1)t}{d}$$

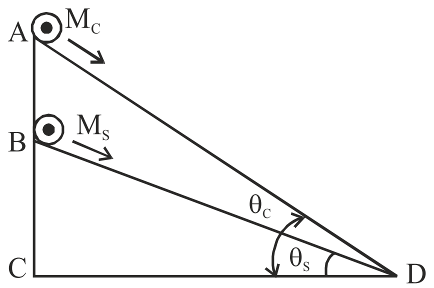
$$\lambda = \frac{(\mu-1)t}{2} = \frac{(1.6-1)\times 1.8\mu m}{2}$$

$= 0.6 \times 0.9\ \mu m$

$= .54\ \mu m$

$= 540$ nm

28. A cylinder of mass M_c and sphere of mass M_s are placed at points A and B of two inclines, respectively. (See Figure). If they roll on the incline without slipping such that their accelerations are the same, then ratio $\frac{\sin\theta_C}{\sin\theta_S}$ is:



(1) $\frac{15}{14}$ (2) $\frac{8}{7}$ (3) $\sqrt{\frac{15}{14}}$ (4) $\sqrt{\frac{8}{7}}$

Ans. (1)

Sol. $$\frac{g\sin\theta_c}{1+\frac{1}{2}} = \frac{g\sin\theta_s}{1+\frac{2}{5}}$$

$$\frac{\sin\theta_c}{\sin\theta_s} = \frac{3/2}{7/5} = \frac{15}{14}$$

29. For which of the following particles will it be most difficult to experimentally verify the de-Broglie relationship?

(1) a dust particle (2) an electron
(3) a proton (4) an α-particle

Ans. (1)

30. If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{++} is :

(1) 13.6 eV (2) 30.6 eV
(3) 122.4 eV (4) 3.4 eV

Ans. **(2)**

Sol. B.E. = 3.4 × 9 = 3.6 eV

PART B – CHEMISTRY

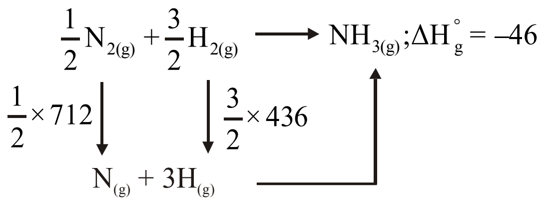
1. The standard enthalpy of formation of NH_3 is –46.0 kJ/mol. If the enthalpy of formation of H_2 from its atoms is –436 kJ/mol and that of N_2 is –712 kJ/mol, the average bond enthalpy of N—H bond in NH_3 is :

(1) + 352 kJ/mol (2) – 964 kJ/mol

(3) – 1102 kJ/mol (4) + 1056 kJ/mol

Ans. (1)

Sol. Given :



Average bond enthalpy of N–H bond = + 352 kJ mol.

2. The correct statement about the magnetic properties of $[Fe(CN)_6]^{3-}$ and $[FeF_6]^{3-}$ is : (Z = 26).

(1) $[Fe(CN)_6]^{3-}$ is paramagnetic, $[FeF_6]^{3-}$ is diamagnetic.

(2) both are diamagnetic.

(3) $[Fe(CN)_6]^{3-}$ is diamagnetic, $[FeF_6]^{3-}$ is paramagnetic.

(4) both are paramagnetic

Ans. **(4)**

Sol. In $[FeF_6]^{3-}$, 5 unpaird electron present is

ln $[Fe(CN)_6]^{3-}$ 1 unpaired electron present.

3. Allyl phenyl ether can be prepared by heating:

(1) $CH_2 = CH - CH_2 - Br + C_6H_5ONa$

(2) $C_6H_5 - CH = CH - Br + CH_3 - ONa$

(3) $C_6H_5Br + CH_2 = CH - CH_2 - ONa$

(4) $CH_2 = CH - Br + C_6H_5 - CH_2 - ONa$

Ans. **(1)**

Sol. $C_6H_5\overset{\ominus\oplus}{O}Na + CH_2{=}CH{-}CH_2{-}Br \longrightarrow C_6H_5O{-}CH_2{-}CH{=}CH_2$

Allyl phenyl ether

4. In a nucleophilic substitution reaction :

$R - Br + Cl^- \xrightarrow{DMF} R - Cl + Br^-$,

which one of the following undergoes complete inversion of configuration?

(1) $C_6H_5CCH_3C_6H_5Br$

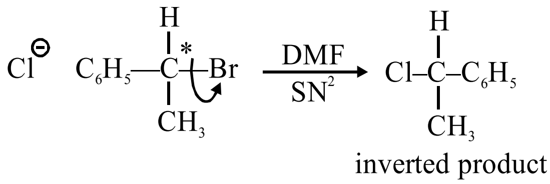
(2) $C_6H_5CHCH_3Br$

(3) $C_6H_5CHC_6H_5Br$

(4) $C_6H_5CH_2Br$

Ans. **(4)**

Sol.



5. The number and type of bonds in C_2^{2-} ion in CaC_2 are:

(1) Two σ bonds and one π – bond

(2) Two σ bonds and two π – bonds

(3) One σ bond and two π – bonds

(4) One σ bond and one π – bond

Ans. **(3)**

Sol. $Ca^{+2}[C\equiv C]^{-2}$

6. In the following sets of reactants which two sets best exhibit the amphoteric character of $Al_2O_3.\ xH_2O$?

Set-1 : $Al_2O_3.xH_2O(s)$ and OH^- (aq)

Set-2 : $Al_2O_3.xH_2O(s)$ and H_2O (ℓ)

Set-3 : $Al_2O_3.xH_2O(s)$ and H^+ (aq)

Set-4 : $Al_2O_3.xH_2O(s)$ and NH_3 (aq)

(1) 1 and 2 (2) 2 and 4

(3) 1 and 3 (4) 3 and 4

Ans. (3)

Sol. ln set 1 : $Al(OH)_4^-$ is formed

ln set 2 : Al^{+3} & H_2O is formed

7. Dissolving 120 g of a compound of (mol. wt. 60) in 1000 g of water gave a solution of density 1.12 g/mL. The molarity of the solution is:

(1) 2.00 M (2) 2.50 M

(3) 4.00 M (4) 1.00 M

Ans. **(1)**

Sol. Molarity of solution = $\dfrac{\left(\dfrac{120}{60}\right)}{\dfrac{1120}{1.12}\times\dfrac{1}{1000}}$

= 2 M

8. The standard electrode potentials $\left(E^0_{M^+/M}\right)$ of four metals A, B, C and D are –1.2 V, 0.6 V, 0.85 V and – 0.76 V, respectively. The sequence of deposition of metals on applying potential is :

(1) D, A, B, C (2) C, B, D, A
(3) B, D, C, A (4) A, C, B, D

Ans. (2)

Sol. Higher the value of reduction potential more wil be the ease of deposition.

9. Which is the major product formed when acetone is heated with iodine and potassium hydroxide ?

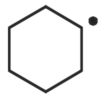
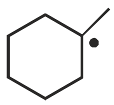
(1) Iodoacetone (2) Acetic acid
(3) Iodoform (4) Acetophenone

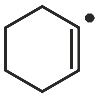
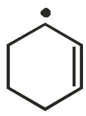
Ans. (3)

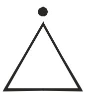
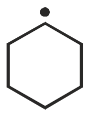
Sol. $CH_3-\overset{\overset{O}{\|}}{C}-CH_3 \xrightarrow[\text{Iodoform reaction}]{I_2 + KOH} \underset{\text{Iodoform}}{CHI_3} + CH_3-\overset{\overset{O}{\|}}{C}-\overset{\ominus}{O}\overset{\oplus}{K}$

10. In which of the following pairs A is more stable than B ?

A B

(1)  , 

(2)  , 

(3)  , 

(4) $Ph_3C^{\bullet}$, $(CH_3)_3C^{\bullet}$

Ans. (4)

Sol. $Ph_3C^{\circ} > (CH_3)C^{\circ}$
due to resonance

11. In the hydroboration - oxidation reaction of propene with diborane, H_2O_2 and NaOH, the organic compound formed is :

(1) $CH_3CH_2CH_2OH$ (2) $(CH_3)_3COH$
(3) $CH_3CHOHCH_3$ (4) CH_3CH_2OH

Ans. (1)

Sol. $CH_3-CH=CH_2 \xrightarrow[H_2O_2/\overline{O}H]{B_2H_6/THF} CH_3-CH_2-\underset{\underset{OH}{|}}{CH_2}$

12. The form of iron obtained from blast furnace is :

(1) Steel (2) Wrought Iron
(3) Cast Iron (4) Pig iron

Ans. (4)

Sol. Iron obtained in blast furnance is known as pig iron.

13. Which one of the following reactions will not result in the formation of carbon-carbon bond?

(1) Cannizzaro reaction
(2) Friedel Craft's acylation
(3) Reimer-Tieman reaction
(4) Wurtz reaction

Ans. (1)

Sol. In cannizaro reaction carbon-carbon bond not formed

$2H-\overset{\overset{O}{\|}}{C}-H \xrightarrow{\text{conc. KOH}} H-\overset{\overset{O}{\|}}{C}-\overset{\ominus}{O}\overset{\oplus}{K} + CH_2-OH$

14. The half-life period of a first order reaction is 15 minutes. The amount of substance left after one hour will be :

(1) $\frac{1}{4}$ of the original amount

(2) $\frac{1}{16}$ of the original amount

(3) $\frac{1}{32}$ of the original amount

(4) $\frac{1}{8}$ of the original amount

Ans. (2)

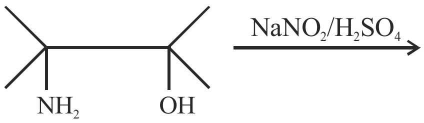
Sol. Since : $t_{1/2}$ = 15 min.

$\therefore$ No. of half lives = $\frac{60}{15} = 4$

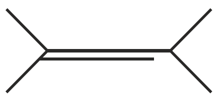
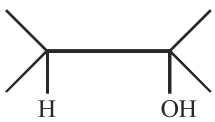
$\therefore$ Amount of substance left after one hour

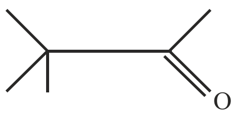
$$= \frac{A_0}{(2)^n} = \frac{A_0}{(2)^4} = \frac{A_0}{16}$$

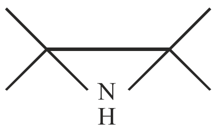
15. The major product of the reaction.



is:

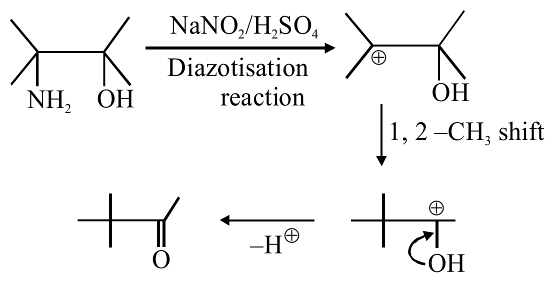
(1)  (2) 

(3) 

(4) 

Ans. (3)

Sol.



16. Vander Wall's equation for a gas is stated as,

$$p = \frac{nRT}{V-nb} - a\left(\frac{n}{V}\right)^2$$

This equation reduces to the perfect gas equation, $p = \frac{nRT}{V}$ when,

(1) both temperature and pressure are very low
(2) both temperature and pressure are very high
(3) temperature is sufficiently high and pressure is low
(4) temperature is sufficiently low and pressure is high.

Ans. **(3)**

17. The temperature at which oxygen molecules have the same root mean square speed as helium atoms have at 300 K is :
(Atomic masses : He = 4 u, O = 16 u)

(1) 1200 K (2) 600 K
(3) 300 K (4) 2400 K

Ans. **(4)**

Sol. $(U_{rms})_{o_2} = (U_{rms})_{He}$

$\Rightarrow \frac{3RTo_2}{32} = \frac{3R\ T_{He}}{4}$

$\Rightarrow T_{o_2} = 8 \times 300 = 2400$ K

18. For the compounds
CH_3Cl, CH_3Br, CH_3I and CH_3F,
the correct order of increasing C-halogen bond length is :

(1) $CH_3F < CH_3Br < CH_3Cl < CH_3I$
(2) $CH_3F < CH_3Cl < CH_3Br < CH_3I$
(3) $CH_3Cl < CH_3Br < CH_3F < CH_3I$
(4) $CH_3F < CH_3I < CH_3Br < CH_3Cl$

Ans. (2)

Sol. As the radius of halogen increases C—halogen bond length incread.

19. Chloro compound of Vanadium has only spin magnetic moment of 1.73 BM. This Vanadium chloride has the formula :
(at. no. of V = 23)

(1) VCl_4 (2) VCl_3 (3) VCl_2 (4) VCl_5

Ans. (1)

Sol. If the magnetic moment is 1.73 BM then the the number of unpaired e^- V^{4+} having our unpaired electron

20. A current of 10.0 A flows for 2.00 h through an electrolytic cell containing a molten salt of metal X. This results in the decomposition of 0.250 mol of metal X at the cathode. The oxidation state of X in the molten salt is :
(F = 96,500 C)

(1) 2 + (2) 3 + (3) 1 + (4) 4 +

Ans. (2)

Sol. $W = \frac{E}{96500} \times It$

$\Rightarrow$ No. of moles $= \frac{It}{96500 \times (n - factor)}$

$\Rightarrow 0.25 = \frac{10 \times 2 \times 60 \times 60}{96500 \times n - factor}$

$\Rightarrow$ n – factor $= \frac{720 \times 4}{965} = 3$

$\therefore$ Oxidation state of molten salt is +3

21. Which of the following has unpaired electron(s) ?

(1) O_2^- (2) N_2^{2+} (3) O_2^{2-} (4) N_2

Ans. (1)

Sol. O_2^- has one unpaired electron is π^* MO.

22. The gas evolved on heating CaF_2 and SiO_2 with concentrated H_2SO_4, on hydrolysis gives a white gelatinous precipitate. The precipitate is:

(1) silica gel
(2) silicic acid
(3) hydrofluosilicic acid
(4) calciumfluorosilicate

Ans. (3)

Sol. $CaF_2 + H_2SO_4 \longrightarrow H_2F_2 + Ca(HSO_4)_2$
$SiO_2 + 2H_2F_2 \longrightarrow SiF_4 + 2H_2O$
$SiF_4 + H_2O \longrightarrow H_2[SiF_6]$

23. The amount of oxygen in 3.6 moles of water is:

(1) 28.8 g (2) 18.4 g (3) 115.2g (4) 57.6g

Ans. (4)

Sol. 3.6 moles of H_2O = 3.6 moles of O
= 3.6 × 16 gm of oxygen
= 57.6 gm

24. Which of the following is **not** formed when H_2S reacts with acidic $K_2Cr_2O_7$ solution ?

(1) K_2SO_4 (2) $Cr_2(SO_4)_3$
(3) S (4) $CrSO_4$

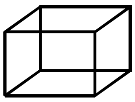
Ans. (4)

Sol. $K_2Cr_2O_7 + H_2S \rightarrow Cr_2(SO_4)_3 + S + K_2SO_4 + H_2O$

25. In a face centered cubic lattice atoms A are at the corner points and atoms B at the face centered points. If atom B is missing from one of the face centered points, the formula of the ionic compound is :

(1) AB_2 (2) A_2B_3 (3) A_5B_2 (4) A_2B_5

Ans. (4)

Sol. 

$A = 8 \times \frac{1}{8} = 1$

$B = 6 \times \frac{1}{2} - 1 \times \frac{1}{2} = \frac{5}{2}$

A : B

$1 : \frac{5}{2} \quad \Rightarrow \quad 2 : 5$

26. An octahedral complex of Co^{3+} is diamagnetic. The hybridisation involved in the formation of the complex is :

(1) d^2sp^3 (2) dsp^3d (3) dsp^2 (4) sp^3d^2

Ans. (1)

Sol. Co^{+3} is diamagnetic & having d^6 by configuration under SFL.

27. At a certain temperature, only 50% HI is dissociated into H_2 and I_2 at equilibrium. The equilibrium constant is :

(1) 3.0 (2) 0.5 (3) 0.25 (4) 1.0

Ans. (3)

Sol. $$\underset{1-\alpha}{2HI} \rightleftharpoons \underset{\frac{\alpha}{2}}{H_2} + \underset{\frac{\alpha}{2}}{I_2}$$

$$K_{eq} = \frac{\left(\frac{\alpha}{2}\right)^2}{(1-\alpha)^2} = \frac{\alpha^2}{4(1-\alpha)^2}$$

$$K_{eq} = \frac{\left(\frac{1}{2}\right)^2}{4(1/2)^2} = \frac{1}{4}$$

28. Which one of the following class of compounds is obtained by polymerization of acetylene ?

(1) Poly-ene (2) Poly-yne
(3) Poly-amide (4) Poly-ester

Ans. (2)

Sol. $$\underset{\text{yne}}{nHC{\equiv}CH} \xrightarrow{\text{Polymerisation}} \underset{\text{poly-yne}}{\left(CH{=}CH\right)_n}$$

29. Structure of some important polymers are given. Which one represents Buna-S ?

(1) $(-CH_2-\overset{\overset{CH_3}{|}}{C}=CH-CH_2-)_n$

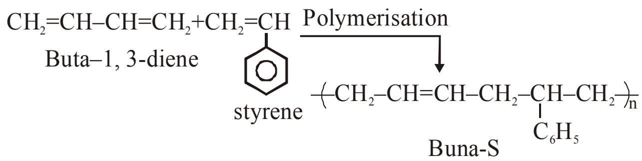
(2) $(-CH_2-CH=CH-CH_2-\underset{\underset{C_6H_5}{|}}{CH}-CH_2-)_n$

(3) $(-CH_2-\overset{\overset{Cl}{|}}{C}=CH-CH_2-)_n$

(4) $(-CH_2-CH=CH-CH_2-\underset{\underset{CN}{|}}{CH}-CH_2-)_n$

Ans. (2)

Sol.



30. The energy of an electron in first Bohr orbit of H-atom is – 13.6 eV. The energy value of electron in the excited state of Li^{2+} is :

(1) –30.6 eV (2) –27.2 eV
(3) 27.2 eV (4) 30.6 eV

Ans. (1)

Sol. Energy of e^- in the excited state of Li^{+2}

$$E = -13.6 \frac{Z^2}{n^2}$$

$$= -13.6 \times \frac{(3)^2}{(2)^2} \text{ eV}$$

$$= -\frac{9}{4} \times 13.6 \text{ eV}$$

$$= -30.6 \text{ eN}$$

PART C – MATHEMATICS

1. In a set of 2n distinct observations, each of the observation below the median of all the observations is increased by 5 and each of the remaining observations is decreased by 3. Then the mean of the new set of observations :

(1) increases by 1 (2) decreases by 2

(3) increases by 2 (4) decreases by 1

Ans. (1)

Sol. $\frac{t_1+t_2+t_3....t_n+t_{n+1}+....+t_{2n}}{2n}=M$

$$\frac{t_1+5+t_2+5+....t_n+5+t_{n+1}-3+t_{n+2}-3+....+t_{2n}-3}{2n}$$

$$\frac{t_1+t_2+....t_{n-1}+5(n)+t_n+t_{n-1}+.....+t_{2n}-3(n)}{2n}$$

$$\frac{t_1+t_2+t_3+.....t_{2n}}{2n}+\frac{5n-3n}{2n}=M+1$$

2. The number of values of α in $[0, 2\pi]$ for which $2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha = 2$, is:

(1) 6 (2) 1 (3) 4 (4) 3

Ans. (4)

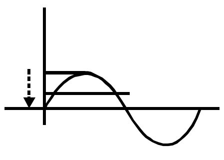
Sol. $2\sin^3\alpha-2 = 7\sin^2\alpha - 7\sin\alpha$

$2(\sin\alpha - 1)(\sin^2\alpha + 1 + \sin\alpha) = 7\sin\alpha(\sin\alpha - 1)$

$\Rightarrow \sin\alpha = 1$ or

$2\sin^2\alpha + 2 + 2\sin\alpha = 7\sin\alpha\ (\sin\alpha - t)\ (\sin\alpha - 2)$

$\Rightarrow \sin\alpha = 1$ or $\sin\alpha = \frac{1}{2}$ $\quad\because \sin\alpha \neq -2$



$\Rightarrow$ 3 solutions

3. Let P be the relation defined on the set of all real numbers such that

$P=\{(a, b) : \sec^2 a - \tan^2 b = 1\}$. Then P is :

(1) reflexive and transitive but not symmetric.

(2) reflexive and symmetric but not transitive

(3) symmetric and transitive but not reflexive

(4) an equivalence relation

Ans. (4)

Sol. for reflexive : $\sec^2a - \tan^2a = 1$ an identity forall $x\in R \Rightarrow$ reflexive

for symmetric : $\sec^2a - \tan^2b = 1$ ...(i) to prove $\sec^2b - \tan^2a = 1$

$\sec^2b - \tan^2 a = 1 + \tan^2b - (\sec^2 a - 1) = 1 + \tan^2 b + 1 - \sec^2a = \sec^2a - \tan^2b = 1 \Rightarrow$ symmetric

[$\because$ from (1)]

for transitive :

$\sec^2a - \tan^2b = 1$ ...... (ii)

$\sec^2b - \tan^2c = 1$ ......(iii)

to prove : $\sec^2 a - \tan^2c = 1$

proof L.H.S.

$1 + \tan^2b + 1 - \sec^2b$ from (ii) & (iii)

$= \sec^2 b - \tan^2b$ identity

$= 1$

$\Rightarrow$ P is reflexive, symmetric and transitive.

4. $\int\frac{\sin^8 x-\cos^8 x}{(1-2\sin^2 x\cos^2 x)}dx$ is equal to:

(1) $-\frac{1}{2}\sin 2x+c$ (2) $-\sin^2x + c$

(3) $\frac{1}{2}\sin 2x+c$ (4) $-\frac{1}{2}\sin x+c$

Ans. (1)

Sol. $I=\int\frac{(\sin^4 x+\cos^4 x)(\sin^2 x+\cos^2 x)(\sin^2 x-\cos^2 x)}{\{(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x\}}$

$=\int\frac{(\sin^4 x+\cos^4 x)(\sin^2 x-\cos^2 x)}{(\sin^4 x+\cos^4 x)}$

$= \int -\cos 2x$

$= -\frac{\sin 2x}{2}+2$

5. The integral $\int_0^{\frac{1}{2}}\frac{\ell n(1+2x)}{1+4x^2}dx$, equals:

(1) $\frac{\pi}{32}\ell n\, 2$ (2) $\frac{\pi}{8}\ell n\, 2$

(3) $\frac{\pi}{16}\ell n\, 2$ (4) $\frac{\pi}{4}\ell n\, 2$

Ans. (3)

Sol. $\int_0^{1/2} \frac{\ell n(1+2x)}{1+(2x)^2} dx$ Put $2x = \tan\theta$

$dx = \frac{1}{2}\sec^2\theta \; d\theta$

at $x = 0$, $\theta = 0$, at $x = \frac{1}{2}$, $\theta = \frac{\pi}{4}$

$$I = \int_0^{\pi/4} \frac{\log(1+\tan\theta)}{1+\tan^2\theta}, \frac{1}{2} \sec^2\theta \; d\theta$$

$$I = \frac{1}{2}\int_0^{\pi/4} \log(1+\tan\theta)\, d\theta,\ \frac{1}{2}\ I_1$$

$$I_1 = \int_0^{\pi/4} \log[1+\tan\left(\frac{\pi}{4}-\theta\right) \quad \text{using property}$$

$$= \int_0^{\pi/4} \log\left[\frac{2}{1+\tan\theta}\right] = \int_0^{\pi/4} \log 2 d\theta - \int_0^{\pi/4} \log(1+\tan\theta)\, d\theta$$

$$I_1 = \frac{\pi}{4} \log 2 - I_1$$

$$I_1 = \frac{\pi}{8} \ln 2$$

$$\Rightarrow I = \frac{\pi}{16}\ln 2$$

6. If $f(x)$ is continuous and $f(9/2) = 2/9$, then $\lim_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right)$ is equal to:

(1) 9/2 (2) 0 (3) 2/9 (4) 8/9

Ans. (3)

Sol. $f\left(\frac{2\sin^2\frac{3x}{2}}{\frac{4}{9}\cdot\frac{3x}{2}\cdot\frac{3x}{2}}\right) = f\left(\frac{9}{2}\right) = \frac{2}{9}$

7. The number of terms in the expansion of $(1 + x)^{101} (1 + x^2 - x)^{100}$ in powers of x is:

(1) 202 (2) 302 (3) 301 (4) 101

Ans. (1)

Sol. $(1 + x)(1 + x)^{100}(1 + x^2 - x)^{100} = (1 + x)(1 + x^3)^{100}$

$$= \underbrace{1(1+x^3)^{100}}_{\text{101 terms}} + \underbrace{x(1+x^3)^{100}}_{\text{101 terms}}$$

and no term is of same exponent of x

$\Rightarrow$ 202 terms

8. Equation of the plane which passes through the point of intersection of lines $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2}$ and $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and has the largest distance from the origin is:

(1) $5x + 4y + 3z = 57$ (2) $7x + 2y + 4z = 54$

(3) $4x + 3y + 5z = 50$ (4) $3x + 4y + 5z = 49$

Ans. (3)

Sol. $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2} = \alpha$

$\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \beta$

Solve the above equation to find the point of intersection i.e. (4, 3, 5)

e.g. of plane with dr's l, m, n as distance from origin is d is $\ell x + my + nz = d$

dr's of (4, 3, 5) joined with origin

$\left(\frac{4}{\sqrt{50}}, \frac{3}{\sqrt{50}}, \frac{5}{\sqrt{50}}\right)$

$\therefore$ eq of plane

$\frac{4}{\sqrt{50}}x + \frac{3}{\sqrt{50}}y + \frac{5}{\sqrt{50}}z = \sqrt{50}$

$4x + 3y + 5z = 50$

9. A line in the 3-dimensional space makes an angle $\theta \left(0 < \theta \le \frac{\pi}{2}\right)$ with both the x and y axes. Then the set of all values of θ is the interval :

(1) $\left(0, \frac{\pi}{4}\right]$ (2) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

(3) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (4) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$

Ans. (3)

Sol. for min, if the line lies on x y plane it makes angle of 45°

for max. If line at z-axis it makes an angle of 90°

$\Rightarrow \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

10. If $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ are the roots of the equation, $ax^2 + bx + 1 = 0$ $(a \neq 0,\ a,\ b \in R)$, then the equation, $x(x + b^3) + (a^3 - 3abx) = 0$ has roots:

(1) $\alpha^{-\frac{3}{2}}$ and $\beta^{-\frac{3}{2}}$ (2) $\alpha^{3/2}$ and $\beta^{3/2}$

(3) $\alpha\beta^{1/2}$ and $\alpha^{1/2}\beta$ (4) $\sqrt{\alpha\beta}$ and $\alpha\beta$

Ans. (2)

Sol. $\frac{1}{\sqrt{\alpha}}+\frac{1}{\sqrt{\beta}}=-\frac{b}{a}$ also $\frac{1}{\sqrt{\alpha\beta}}=\frac{1}{a}\Rightarrow \sqrt{\alpha}+\sqrt{\beta}=-b$

now $x(x + b^3) + a^3 - 3abx$

$= x^2 + (b^3 - 3ab)x + a^3 = x^2 + b(b^2 - 3a)x + a^3$

$= x^2 -\left(\sqrt{\alpha}+\sqrt{\beta}\right)\left\{\alpha+\beta+2\sqrt{\alpha\beta}-3\sqrt{\alpha\beta}\right\}x+\alpha\beta\sqrt{\alpha\beta}$

$= x^2 - \left(\alpha\sqrt{\alpha}+\beta\sqrt{\beta}\right)+\alpha\beta\sqrt{\alpha\beta}$

$\Rightarrow$ roots are $\alpha\sqrt{\alpha}$ and $\beta\sqrt{\beta}$

11. Given an A.P. whose terms are all positive integers. The sum of its first nine terms is greater than 200 and less than 220. If the second term in it is 12, then its 4th term is :

(1) 20 (2) 16 (3) 8 (4) 24

Ans. (1)

Sol. $(12 - d) + 12 + (12 + d) + (12 + 2d) + ... 12 + 7d$

$= 12 \times 9 + 27d = 108 + 27d$

now according to question

$200 < 108 + 27d < 220$

$92 < 27d < 112$

$\frac{92}{27}<d<\frac{112}{27}$ $\Rightarrow$ d = 4 only integer

$\Rightarrow$ 4th term $= 12 + 2d = 12 + 8 = 20$

12. Let w(Im w $\neq$ 0) be a complex number. Then the set of all complex numbers z satisfying the equation $w - \bar{w}z = k(1 - z)$, for some real number k, is :

(1) $\{z : z = \bar{z}\}$ (2) $\{z : |z| = 1, z \neq 1\}$

(3) $\{z : |z| = 1\}$ (4) $\{z : z \neq 1\}$

Ans. (2)

Sol. $w - \bar{w}z = k - kz$

$kz - \bar{w}z = k - w$

$z = \frac{k-w}{k-\bar{w}}$ ....(i)

$\bar{z} = \frac{k-\bar{w}}{k-w}$ ....(ii)

(i) $\times$ (ii)

$z\bar{z}=1$

$|z| = 1$

but $z \neq 1$

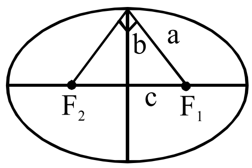
13. If OB is the semi-minor axis of an ellipse, F_1 and F_2 are its foci and the angle between F_1B and F_2B is a right angle, then the square of the eccentricity of the ellipse is :

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$

(3) $\frac{1}{4}$ (4) $\frac{1}{2\sqrt{2}}$

Ans. (2)

Sol. $a^2 = b^2 + c^2$ ....(i)



given $a^2 + a^2 = (2c)^2$

$2a^2 = 4c^2$

$a^2 = 2c^2$ ....(ii)

$c = ae$

$\frac{c}{a} = e$

$\frac{c^2}{a^2} = e^2$

$\frac{c^2}{2c^2}=e^2 \Rightarrow e^2 = \frac{1}{2}$

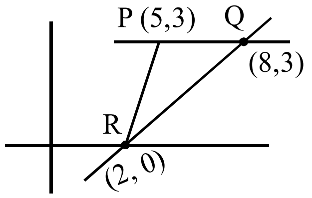
14. Given three points P, Q, R with P(5, 3) and R lies on the x-axis. If equation of RQ is $x - 2y = 2$ and PQ is parallel to the x-axis, then the centroid of ΔPQR lies on the line:

(1) $x - 2y + 1 = 0$ (2) $5x - 2y = 0$

(3) $2x + y - 9 = 0$ (4) $2x - 5y = 0$

Ans. (4)

Sol.



equation of RQ ≡ x – 2y = 2

$\Rightarrow$ R (2, 0)

equation of PQ = y = 3

point of intersection of PQ and RQ

x – 2(3) = 2

x = 8

$\Rightarrow$ R (8, 3)

Centroid $\left(\frac{2+8+5}{3}, \frac{0+3+3}{3}\right)$

≡ (5, 2) as is simplified by 2x – 5y = 0

15. The sum of the digits in the unit's place of all the 4-digit numbers formed by using the number 3, 4, 5 and 6, without repetition, is:

(1) 432 (2) 36 (3) 18 (4) 108

Ans. (4)

Sol. $(6 + 5 + 4 + 3)\ \underline{|3}$

$= 18 \times 6$

$= 108$

16. If $\operatorname{cosec}\theta = \frac{p+q}{p-q}$ ($p \neq q \neq 0$), then

$\left|\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right|$ is equal to:

(1) pq (2) $\sqrt{pq}$ (3) $\sqrt{\frac{q}{p}}$ (4) $\sqrt{\frac{p}{q}}$

Ans. (3)

Sol. $\left|\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right| = \left|\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}\right|$

$$= \frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}} \times \frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}$$

$$= \frac{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos\theta} = \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1-\frac{p-q}{p+q}}{\sqrt{1-\left(\frac{p-q}{p+q}\right)^2}} = \frac{\sqrt{q}}{\sqrt{p}}$$

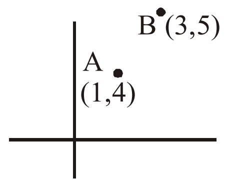
17. If the point (1, 4) lies inside the circle $x^2 + y^2 - 6x - 10y + p = 0$ and the circle does not touch or intersect the coordinate axes, then the set of all possible values of p is the interval:

(1) (25 , 29) (2) (25 , 39)

(3) (9, 25) (4) (0, 25)

Ans. (1)

Sol.



$AB = \sqrt{2^2+1} = \sqrt{5}$

according to question

$\sqrt{5} < \sqrt{3^2+5^2-p} < q$

$5 < 34 - p < q$

$-29 < -P < -25$

$29 > p > 25$

18. If $y = e^{nx}$, then $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right)$ is equal to:

(1) 1 (2) $-ne^{-nx}$ (3) ne^{-nx} (4) $n e^{nx}$

Ans. (2)

Sol. $y = e^{nx}$ | $\frac{1}{n}\log y = x$

$\frac{dy}{dx} = ne^{nx}$ | $\frac{1}{n}\left(\frac{1}{y}\right) = \frac{dx}{dy}$

$\frac{d^2y}{dx^2} = n^2e^{nx}$ ....(i) | $-\frac{1}{ny^2} = \frac{d^2x}{dy^2}$......(ii)

(i) × (ii) $= n^2e^{nx} \cdot \frac{1}{ny^2} = \frac{n^2y}{ny^2} = \frac{n}{y} = \frac{n}{e^{nx}}$

19. If $f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$, $x \in R$, then the equation $f(x) = 0$ has:

(1) One solution

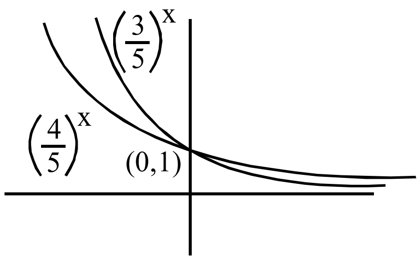
(2) no solution

(3) more than two solutions

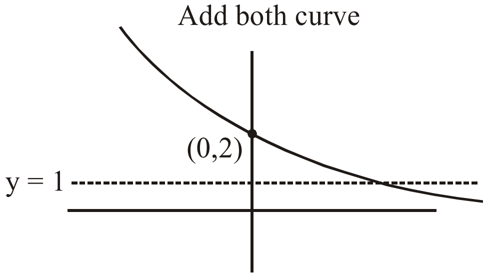
(4) two solutions

Ans. (1)

Sol.



Add both curve



20. If A and B are two events such that $P(A \cup B) = P(A \cap B)$, then the **incorrect** statement amongst the following statements is:

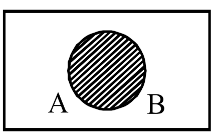
(1) P(A) + P(B) = 1 (2) $P(A' \cap B) = 0$

(3) $P(A \cap B') = 0$

(4) A and B are equally likely

Ans. (4)

Sol.



$\Rightarrow A = B$

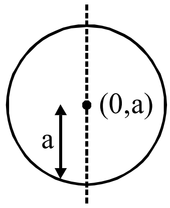
21. If the differential equation representing the family of all circles touching x-axis at the origin is $(x^2 - y^2)\frac{dy}{dx} = g(x)\,y$, then g(x) equals:

(1) $2x^2$ (2) 2x

(3) $\frac{1}{2}x^2$ (4) $\frac{1}{2}x$

Ans. (2)

Sol. $x^2 + (y - a)^2 = a^2$



$x^2 + y^2 - 2ay = 0$ ......(i)

diff. w.r.t. x

$2x + 2y\frac{dy}{dx} - 2a\frac{dy}{dx} = 0$

$a = \frac{x + y.y'}{y'}$ ......(ii)

put (ii) in (i)

$x^2 + y^2 - 2y\left(\frac{x + y.y'}{y'}\right) = 0$

$(x^2 - y^2)y' = 2xy$ ....(iii)

compare (iii) with $(x^2 - y^2)\frac{dy}{dx} = g(x).y$

gives g(x) = 2x

22. The contrapositive of the statement "I go to school if it does not rain" is:

(1) If it rains, I go to school.

(2) If it rains, I do not go to school.

(3) If I go to school, it rains.

(4) If I do not go to school, it rains.

Ans. (4)

Sol. Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

23. Let a and b be any two numbers satisfying $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{4}$. Then, the foot of perpendicular from the origin on the variable line, $\frac{x}{a} + \frac{y}{b} = 1$, lies on:

(1) a circle of radius = 2

(2) a circle of radius = $\sqrt{2}$

(3) a hyperbola with each semi-axis = $\sqrt{2}$

(4) a hyperbola with each semi-axis = 2

Ans. (1)

Sol. Equation of $\perp$

$$\frac{h-0}{\frac{1}{a}}=\frac{k-0}{\frac{1}{b}}=\frac{1}{\frac{1}{a^2}+\frac{1}{b^2}}$$

$\Rightarrow$ $a=\frac{4}{h}$ ......(i) and $b=\frac{4}{k}$ .... (ii)

to find locus of (h, k) put

(i) and (ii) in $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{4}$

i.e. $\frac{h^2}{16}+\frac{k^2}{16}=\frac{1}{4}$

$\Rightarrow$ locus is $\frac{x^2}{16}+\frac{y^2}{16}=\frac{1}{4}$

$\Rightarrow x^2 + y^2 = 4$

24. If $|\vec{a}|=2$, $|\vec{b}|=3$ and $|2\vec{a}-\vec{b}|=5$, then $|2\vec{a}+\vec{b}|$ equals:

(1) 1 (2) 17
(3) 5 (4) 7

Ans. (3)

Sol. $|2\vec{a}-\vec{b}|^2=25$

$4|\vec{a}|+|\vec{b}|^2-4\cdot\vec{a}\cdot\vec{b}=25$

$16 + 9 - 4\cdot\vec{a}\cdot\vec{b} = 25$

$4 \times \vec{a}\cdot\vec{b} = 0$ ....(i)

now

$|2\vec{a}+\vec{b}|=k$

$(2\vec{a}+\vec{b})(2\vec{a}+\vec{b})=k^2$

$4|\vec{a}|^2+|\vec{b}|^2+4\vec{a}.\vec{b}=k^2$

$\sqrt{16+9+0} = k$

$5 = k$

25. If the sum

$$\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+.....+ \text{ upto 20 terms}$$

is equal to $\frac{k}{21}$, then k is equal to:

(1) 240 (2) 120 (3) 180 (4) 60

Ans. (2)

Sol. $t_n = \dfrac{2n+1}{\frac{n(n+1)(2n+1)}{6}}=\dfrac{6}{n(n+1)}=6\left(\frac{1}{n}-\frac{1}{n+1}\right)$

$S_n = 6\left\{\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}+......-\frac{1}{21}\right\}=6\left\{\frac{1}{1}-\frac{1}{21}\right\}$

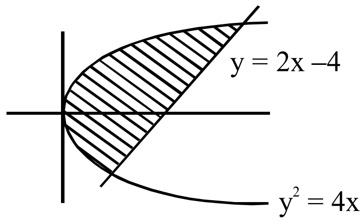
$= 6\left(\frac{20}{21}\right)=\frac{120}{21}$ $\Rightarrow$ k = 120

26. Let A = {(x , y) : $y^2 \le 4x$, $y - 2x \ge -4$}. The area (in square units) of the region A is :

(1) 11 (2) 9 (3) 8 (4) 10

Ans. (2)

Sol.



solve for y; $y^2 = 4x$ as $y - 2x = -4$
gives y = – 2,4

$\Rightarrow$ Area = $\int_{-2}^{4}\left(\frac{y+4}{2}-\frac{y^2}{4}\right)dy=\left[\frac{y^2}{4}+2y-\frac{y^3}{12}\right]_{-2}^{4}$

= 9

27. If equations $ax^2 + bx + c = 0$, (a, b, c $\in$ R, a $\neq$ 0) and $2x^2 + 3x + 4 = 0$ have a common root, then a : b : c equals :

(1) 2 : 3 : 4 (2) 3 : 2 : 1
(3) 1 : 2 : 3 (4) 4 : 3 : 2

Ans. (1)

Sol. $2x^2 + 3x + 4 = 0$ as $D \le 0$

$\Rightarrow$ both roots are imaginary $\Rightarrow$ both roots are common

$\Rightarrow$ $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}$

28. If a, b, c are non-zero real numbers and if the system of equations

(a – 1) x = y + z,
(b – 1) y = z + x,
(c – 1) z = x + y,

has a non-trivial solution, then ab + bc + ca equals :

(1) 1 (2) a+b+c (3) abc (4) – 1

Ans. (3)

Sol. for non-trivial solution D = 0

$$\begin{vmatrix} 1-a & 1 & 1 \\ 1 & 1-b & 1 \\ 1 & 1 & 1-c \end{vmatrix} = 0$$

$R_1 \rightarrow R_1 - R_3$ $\begin{vmatrix} -a & 0 & c \\ 1 & 1-b & 1 \\ 1 & 1 & 1-c \end{vmatrix}$

$\Rightarrow$ a {(1–b)(1–c)–1} + c{1–(1–b)}=0

$\Rightarrow$ ab + ac + bc – abc = 0

$\Rightarrow$ ab + ac + bc = abc

29. If the Rolle's theorem holds for the function $f(x) = 2x^3 + ax^2 + bx$ in the interval [– 1, 1] for the point $c = \frac{1}{2}$, then the value of 2a + b is:

(1) 2 (2) – 2 (3) – 1 (4) 1

Ans. (3)

Sol. f(–1) = – 2 + a – b, f(1) = 2 + a + b

f(–1) = f(1) $\Rightarrow$ –2 + a – b = 2 + a + b

–2 = b

$f'(x) = 6x^2 + 2ax + b$

$$f'\left(\frac{1}{2}\right) = 6\cdot\frac{1}{4} + 2\cdot a\cdot\frac{1}{2} + b = 0$$

$$\Rightarrow \frac{3}{2} + a + b = 0 \quad (\because b = -2)$$

$$\Rightarrow a = \frac{1}{2} \qquad \because 2a + b = -1$$

30. If B is a 3 × 3 matrix such that $B^2 = 0$, then det. $[(I + B)^{50} - 50B]$ is equal to:

(1) 1 (2) 2

(3) 3 (4) 50

Ans. (1)

Sol. $$[(1+B)^{50} - 50\,B] = 1 + 50B + \frac{50.49}{\underline{|2}}B^2 + \ldots\ldots\ldots - 50B$$

$= 1 + B^2$ {-----------} = 1 + 0 {-----} = 1