Answers & Solutions For JEE MAIN- 2014
(Code-E)

Time Durations : 3 hrs. Maximum Marks: 360

(Physics, Chemistry and Mathematics)

Important Instructions :

1. The test is of 3 hours duration.
2. The Test Booklet consists of 90 questions. The maximum marks are 360.
3. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for each correct response.
4. Candidates will be awarded marks as stated above in Instructions No. 3 for correct response of each question. ¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
5. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.
6. Use Blue/Black Ball Point Pen only for writing particulars/marking responses on Side-1 and Side-2 of the Answer Sheet. Use of pencil is strictly prohibited.
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination hall/room.
8. The CODE for this Booklet is E. Make sure that the CODE printed on Side-2 of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.

1. The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)$ mA, where the applied V is in volts and the temperature T is in degree kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA?

(1) 0.2 mA (2) 0.02 mA
(3) 0.5 mA (4) 0.05 mA

Answer (1)

Sol. $I = (e^{1000\,V/T} - 1)$mA

When I = 5 mA, $e^{1000\,V/T}$ = 6 mA

Also, $dI = (e^{1000\,V/T}) \times \dfrac{1000}{T} \cdot dV$

$= (6\text{ mA}) \times \dfrac{1000}{300} \times (0.01)$

= 0.2 mA

2. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is:

(1) $2gH = n^2u^2$ (2) $gH = (n - 2)^2u^2$
(3) $2gH = nu^2(n - 2)$ (4) $gH = (n - 2)u^2$

Answer (3)

Sol. Time taken to reach highest point is $t_1 = \dfrac{u}{g}$

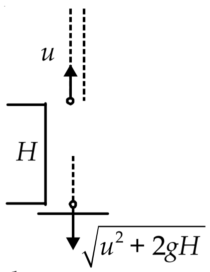
Speed on reaching ground $= \sqrt{u^2 + 2gh}$

Now, $v = u + at$

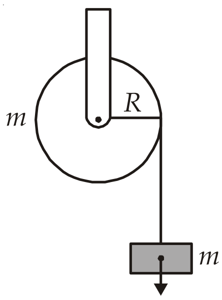
$\Rightarrow \sqrt{u^2 + 2gh} = -u + gt$

$\Rightarrow t = \dfrac{u + \sqrt{u^2 + 2gH}}{g} = \dfrac{nu}{g}$

$\Rightarrow 2gH = n(n-2)u^2$



3. A mass m is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release?



(1) $\dfrac{2g}{3}$ (2) $\dfrac{g}{2}$

(3) $\dfrac{5g}{6}$ (4) g

Answer (2)

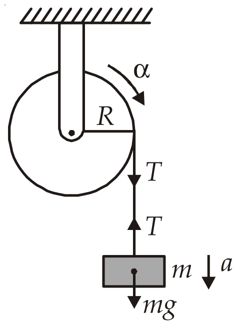
Sol. $a = R\alpha$

$mg - T = ma$

$T \times R = mR^2\alpha$

or $T = ma$

$\Rightarrow a = \dfrac{g}{2}$



4. A block of mass m is placed on a surface with a vertical cross-section given by $y = \dfrac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is

(1) $\dfrac{1}{6}$ m (2) $\dfrac{2}{3}$ m

(3) $\dfrac{1}{3}$ m (4) $\dfrac{1}{2}$ m

Answer (1)

Sol. $\tan\theta = \dfrac{dy}{dx} = \dfrac{x^2}{2}$

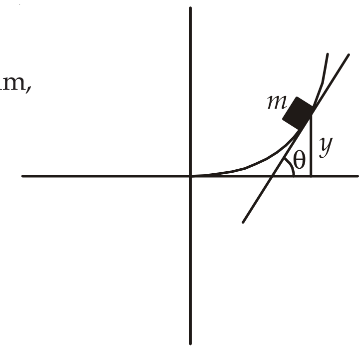
At limiting equilibrium,

$\mu = \tan\theta$

$0.5 = \dfrac{x^2}{2}$

$\Rightarrow x = \pm 1$

Now, $y = \dfrac{1}{6}$



5. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is :

(1) $aL^2 + bL^3$ (2) $\dfrac{1}{2}(aL^2 + bL^3)$

(3) $\dfrac{aL^2}{2} + \dfrac{bL^3}{3}$ (4) $\dfrac{1}{2}\left(\dfrac{aL^2}{2} + \dfrac{bL^3}{3}\right)$

Answer (3)

Sol. $\int dW = \int F \cdot dl$

$$W = \int_0^L ax\,dx + \int_0^L bx^2\,dx = \frac{aL^2}{2} + \frac{bL^3}{3}.$$

6. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension

(1) Angular momentum is conserved

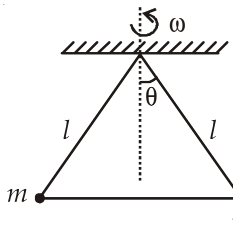
(2) Angular momentum changes in magnitude but not in direction

(3) Angular momentum changes in direction but not in magnitude

(4) Angular momentum changes both in direction and magnitude

Answer (3)

Sol. $\tau = mg \times l \sin\theta$. (Direction parallel to plane of rotation of particle)



as τ is perpendicular to $\vec{L}$, direction of L changes but magnitude remains same.

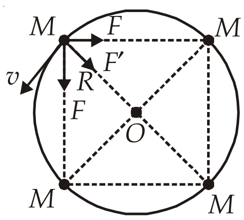
7. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(1) $\sqrt{\frac{GM}{R}}$ (2) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

(3) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (4) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

Answer (4)

Sol. $\frac{F}{\sqrt{2}} + \frac{F}{\sqrt{2}} + F' = \frac{Mv^2}{R}$



$$\frac{2\times GM^2}{\sqrt{2}(R\sqrt{2})^2} + \frac{GM^2}{4R^2} = \frac{Mv^2}{R}$$

$$\frac{GM^2}{R}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right] = Mv^2$$

$$v = \sqrt{\frac{Gm}{R}\left(\frac{\sqrt{2}+4}{4\sqrt{2}}\right)} = \frac{1}{2}\sqrt{\frac{Gm}{R}\left(1+2\sqrt{2}\right)}$$

8. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is :

(For steel Young's modulus is 2×10^{11} Nm^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1})

(1) 2.2×10^8 Pa (2) 2.2×10^9 Pa

(3) 2.2×10^7 Pa (4) 2.2×10^6 Pa

Answer (1)

Sol. As length is constant,

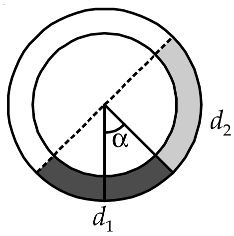
$$\text{Strain} = \frac{\Delta L}{L} = \alpha\Delta Q$$

Now pressure = stress = Y × strain

$= 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100$

$= 2.2 \times 10^8$ Pa

9. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is

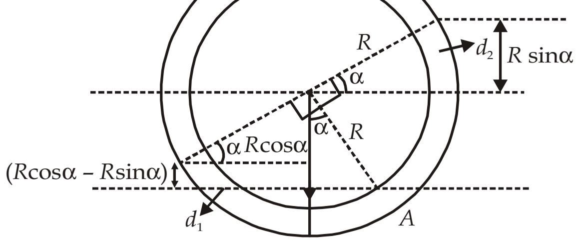


$\frac{1\ \sin\alpha}{1-\sin\alpha}$ (2) $\frac{1+\cos\alpha}{1-\cos\alpha}$

(3) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (4) $\frac{1+\sin\alpha}{1-\cos\alpha}$

Answer (3)

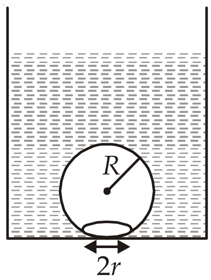
Sol. Equating pressure at A



$$(R\cos\alpha + R\sin\alpha)d_2 g = (R\cos\alpha - R\sin\alpha)d_1 g$$

$$\Rightarrow \frac{d_1}{d_2} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{1+\tan\alpha}{1-\tan\alpha}$$

10. On heating water, bubbles being formed at the bottom of the vessel detatch and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If $r << R$, and the surface tension of water is T, value of r just before bubbles detatch is (Density of water is ρ_w)



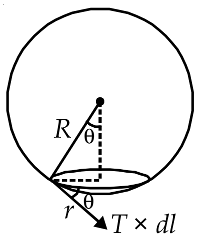
(1) $R^2\sqrt{\frac{\rho_w g}{3T}}$ (2) $R^2\sqrt{\frac{\rho_w g}{6T}}$

(3) $R^2\sqrt{\frac{\rho_w g}{T}}$ (4) $R^2\sqrt{\frac{3\rho_w g}{T}}$

Answer (No answer)

Sol. When the bubble gets detached,

Buoyant force = force due to surface tension



$$\int T \times dl \sin\theta = \frac{4}{3}\pi R^3 \rho_w g$$

$$\Rightarrow \quad T \times 2\pi r \times \frac{r}{R} = \frac{4}{3}\pi R^3 \rho_w g$$

$$\Rightarrow \quad r^2 = \frac{2R^4 \rho_w g}{3}$$

$$\Rightarrow \quad r = R^2\sqrt{\frac{2\rho_w g}{3T}}$$

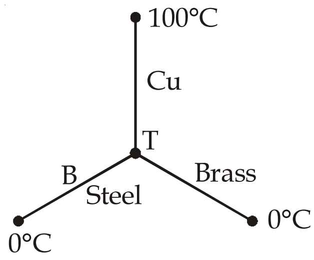
11. Three rods of copper, brass and steel are welded together to form a Y-shaped structure. Area of cross-seciton of each rod = 4 cm^2. End of copper rod is maintained at 100°C whereas ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cm respectively. The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is

(1) 1.2 cal/s (2) 2.4 cal/s

(3) 4.8 cal/s (4) 6.0 cal/s

Answer (3)

Sol.



$Q = Q_1 + Q_2$

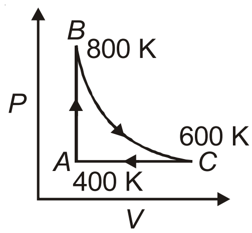
$$\frac{0.92 \times 4(100 - T)}{46} = \frac{0.26 \times 4 \times (T - 0)}{13} + \frac{0.12 \times 4 \times T}{12}$$

$\Rightarrow \quad 200 - 2T = 2T + T$

$\Rightarrow \quad T = 40°C$

$$\Rightarrow \quad Q = \frac{0.92 \times 4 \times 60}{46} = 4.8 \text{ cal/s}$$

12. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K respectively. Choose the correct statement



(1) The change in internal energy in whole cyclic process is 250R

(2) The change in internal energy in the process CA is 700R

(3) The change in internal energy in the process AB is –350R

(4) The change in internal energy in the process BC is – 500R

Answer (4)

Sol. $\Delta U = nC_V\Delta T = 1 \times \frac{5R}{2}\Delta T$

For BC, ΔT = –200 K

$\Rightarrow \quad \Delta U = -500R$

13. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now?

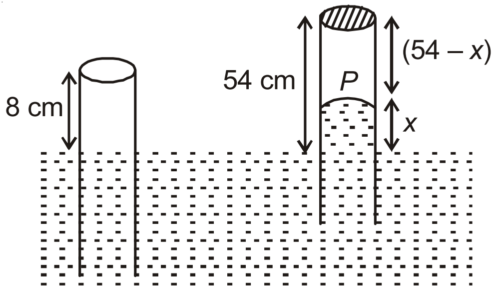
(Atmospheric pressure = 76 cm of Hg)

(1) 16 cm (2) 22 cm

(3) 38 cm (4) 6 cm

Answer (1)

Sol.



$P + x = P_0$

$P = (76 - x)$

$8 \times A \times 76 = (76 - x) \times A \times (54 - x)$

$x = 38$

Length of air column = 54 – 38 = 16 cm.

14. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels $2a$ in same direction then

(1) Amplitude of motion is $3a$

(2) Time period of oscillations is 8τ

(3) Amplitude of motion is $4a$

(4) Time period of oscillations is 6τ

Answer (4)

Sol. As it starts from rest, we have

$x = A\cos\omega t$. At $t = 0$, $x = A$

when $t = \tau$, $x = A - a$

when $t = 2\tau$, $x = A - 3a$

$\Rightarrow A - a = A\cos\omega\tau$

$A - 3a = A\cos 2\omega\tau$

As $\cos 2\omega\tau = 2\cos^2\omega\tau - 1$

$$\Rightarrow \frac{A-3a}{A} = 2\left(\frac{A-a}{A}\right)^2 - 1$$

$$\frac{A-3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$A^2 - 3aA = A^2 + 2a^2 - 4Aa$

$a^2 = 2aA$

$A = 2a$

Now, $A - a = A\cos\omega\tau$

$$\Rightarrow \cos\omega\tau = \frac{1}{2}$$

$$\frac{2\pi}{T}\tau = \frac{\pi}{3}$$

$\Rightarrow T = 6\tau$

15. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s.

(1) 12 (2) 8

(3) 6 (4) 4

Answer (3)

Sol. $f = \dfrac{(2n-1)v}{4L} \le 1250$

$$\Rightarrow \frac{(2n-1)\times 340}{0.85\times 4} \le 1250$$

$\Rightarrow 2n - 1 \le 12.5$

∴ Answer is 6.

16. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at $x = 2$ m is

(1) 120 J (2) –120 J

(3) –80 J (4) 80 J

Answer (3)

Sol. $dV = -\vec{E}\cdot\vec{dx}$

$$\int_{V_O}^{V_A} dV = -\int_0^2 30x^2\,dx$$

$$V_A - V_O = -[10x^3]_0^2 = -80 \text{ J}$$

17. A parallel plate capacitor is made of two circular plates separated by a distance 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to

(1) 6×10^{-7} C/m^2 (2) 3×10^{-7} C/m^2

(3) 3×10^4 C/m^2 (4) 6×10^4 C/m^2

Answer (1)

Sol. $E = \dfrac{\sigma}{K\varepsilon_0}$

$\sigma = K\varepsilon_0 E$

$= 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4 \approx 6 \times 10^{-7}$ C/m^2

18. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be :

(1) 8 A (2) 10 A

(3) 12 A (4) 14 A

Answer (3)

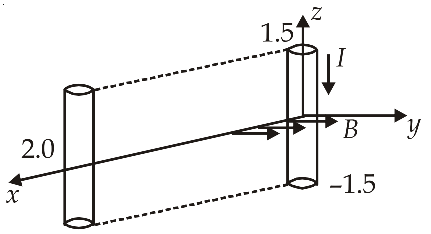
Sol. $15 \times 40 + 5 \times 100 + 5 \times 80 + 1000 = V \times I$

$600 + 500 + 400 + 1000 = 220\ I$

$$I = \frac{2500}{220} = 11.36$$

$I = 12$ A.

19. A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-\hat{a}_z$ direction (see figure). For a field $\vec{B} = 3.0 \times 10^{-4}\ e^{-0.2x}\ \hat{a}_y$ T, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = 0$ m in 5×10^{-3} s. Assume parallel motion along the x-axis



(1) 1.57 W (2) 2.97 W

(3) 14.85 W (4) 29.7 W

Answer (2)

Sol. Average Power $= \dfrac{\text{work}}{\text{time}}$

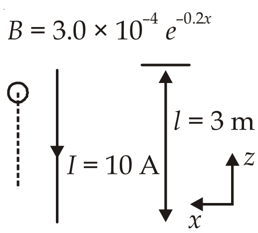
$$W = \int_0^2 F dx$$

$$= \int_0^2 3.0 \times 10^{-4} e^{-0.2x} \times 10 \times 3\ dx$$

$$= 9 \times 10^{-3} \int_0^2 e^{-0.2x}\ dx$$

$$= \frac{9 \times 10^{-3}}{0.2}\left[-e^{-0.2 \times 2} + 1\right]$$

$$= \frac{9 \times 10^{-3}}{0.2} \times \left[1 - e^{-0.4}\right]$$



$= 9 \times 10^{-3} \times (0.33)$

$= 2.97 \times 10^{-3}$ J

$$P = \frac{2.97 \times 10^{-3}}{(0.2) \times 5 \times 10^{-3}} = 2.97 \text{ W}$$

20. The coercivity of a small magnet where the ferromagnet gets demagnetized is 3×10^3 A m^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is

(1) 30 mA (2) 60 mA

(3) 3 A (4) 6 A

Answer (3)

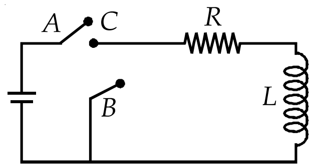
Sol. $B = \mu_0\ n\ i$

$$\frac{B}{\mu_0} = ni$$

$$3 \times 10^3 = \frac{NI}{L} = \frac{100 \times i}{10 \times 10^{-2}}$$

$I = 3$ A.

21. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to



(1) $\dfrac{e}{1-e}$ (2) 1

(3) –1 (4) $\dfrac{1-e}{e}$

Answer (3)

Sol. Applying Kirchhoff's law in closed loop, $-V_R - V_C = 0$

$\Rightarrow\ V_R / V_C = -1$

Note : The sense of voltage drop has not been defined. The answer could have been 1.

22. During the propagation of electromagnetic waves in a medium

(1) Electric energy density is double of the magnetic energy density

(2) Electric energy density is half of the magnetic energy density

(3) Electric energy density is equal to the magnetic energy density

(4) Both electric and magnetic energy densities are zero

Answer (3)

Sol. Energy is equally divided between electric and magnetic field

23. A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal lengths f_1 and f_2 respectively. The correct relation between the focal lengths is

(1) $f_1 = f_2 < f$

(2) $f_1 > f$ and f_2 becomes negative

(3) $f_2 > f$ and f_1 becomes negative

(4) f_1 and f_2 both become negative

Answer (2)

Sol. By Lens maker's formula

$$\frac{1}{f_1} = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_2} = \left(\frac{3/2}{5/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\Rightarrow f_1 = 4f$ & $f_2 = -5f$

24. A green light is incident from the water to the air - water interface at the critical angle(θ). Select the **correct** statement

(1) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal

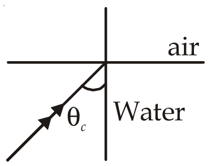
(2) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium

(3) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium

(4) The entire spectrum of visible light will come out of the water at various angles to the normal

Answer (2)

Sol. $\sin\theta_c = \frac{1}{\mu}$



For greater wavelength (i.e. lesser frequency) μ is less

So, θ_c would be more. So, they will not suffer reflection and come out at angles less then 90°.

25. Two beams, A and B, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then $\frac{I_A}{I_B}$ equals

(1) 3

(2) $\frac{3}{2}$

(3) 1

(4) $\frac{1}{3}$

Answer (4)

Sol. By law of Malus, $I = I_0\cos^2\theta$

Now, $I_{A'} = I_A\cos^2 30$

$I_{B'} = I_B\cos^2 60$

As $I_{A'} = I_{B'}$

$$\Rightarrow I_A \times \frac{3}{4} = I_B \times \frac{1}{4}$$

$$\frac{I_A}{I_B} = \frac{1}{3}$$

26. The radiation corresponding to 3→2 transition of hydrogen atoms falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to

(1) 1.8 eV

(2) 1.1 eV

(3) 0.8 eV

(4) 1.6 eV

Answer (2)

Sol. $r = \frac{mv}{qB}$

$$= \frac{\sqrt{2meV}}{eB}$$

$$= \frac{1}{B}\sqrt{\frac{2m}{e}V}$$

$$\Rightarrow V = \frac{B^2r^2e}{2m} = 0.8 \text{ V}$$

For transition between 3 to 2,

$$E = 13.6\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$= \frac{13.6 \times 5}{36} = 1.88 \text{ eV}$$

Work function $= 1.88$ eV – 0.8 eV

$= 1.08$ eV $= 1.1$ eV

27. Hydrogen ($_1H^1$), Deuterium ($_1H^2$), singly ionised Helium $(_2He^4)^+$ and doubly ionised lithium $(_3Li^6)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wave lengths of emitted radiation are λ_1, λ_2, λ_3 and λ_4 respectively then approximately which one of the following is **correct**?

(1) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

(2) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

(3) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

(4) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

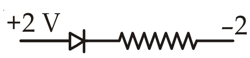
Answer (3)

Sol. $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n^2}\right]$

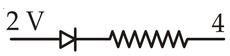
$\Rightarrow \quad \lambda \propto \frac{1}{Z^2}$ for given n_1 & n_2

$\Rightarrow \quad \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

28. The forward biased diode connection is

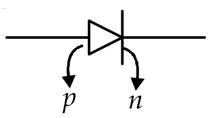
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

For forward Bias, p-side must be at higher potential than n-side.

29. Match **List-I** (Electromagnetic wave type) with **List - II** (Its association/application) and select the correct option from the choices given below the lists :

List-I		**List-II**	
(a)	Infrared waves	(i)	To treat muscular strain
(b)	Radio waves	(ii)	For broadcasting
(c)	X-rays	(iii)	To detect fracture of bones
(d)	Ultraviolet rays	(iv)	Absorbed by the ozone layer of the atmosphere

	(a)	**(b)**	**(c)**	**(d)**
(1)	(iv)	(iii)	(ii)	(i)
(2)	(i)	(ii)	(iv)	(iii)
(3)	(iii)	(ii)	(i)	(iv)
(4)	(i)	(ii)	(iii)	(iv)

Answer (4)

Sol. (a) Infrared rays are used to treat muscular strain

(b) Radiowaves are used for broadcasting

(c) X-rays are used to detect fracture of bones

(d) Ultraviolet rays are absorbed by ozone

30. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?

(1) A meter scale

(2) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm

(3) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm

(4) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm

Answer (2)

Sol. As measured value is 3.50 cm, the least count must be 0.01 cm = 0.1 mm

For vernier scale with 1 MSD = 1 mm and 9 MSD = 10 VSD,

Least count = 1 MSD – 1 VSD

= 0.1 mm

31. The correct set of four quantum numbers for the valence electrons of rubidium atom (Z = 37) is

(1) $5, 0, 0, +\frac{1}{2}$ (2) $5, 1, 0, +\frac{1}{2}$

(3) $5, 1, 1, +\frac{1}{2}$ (4) $5, 0, 1, +\frac{1}{2}$

Answer (1)

Sol. $37 \rightarrow 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 5s^1$

So last electron enters 5s orbital

Hence n = 5, l = 0, $m_l = 0$, $m_s = \pm\frac{1}{2}$

32. If Z is a compressibility factor, van der Waals equation at low pressure can be written as

(1) $Z = 1 + \frac{RT}{Pb}$ (2) $Z = 1 - \frac{a}{VRT}$

(3) $Z = 1 - \frac{Pb}{RT}$ (4) $Z = 1 + \frac{Pb}{RT}$

Answer (2)

Sol. Compressibility factor $(Z) = \frac{PV}{RT}$

(For one mole of real gas)

van der Waal equation

$$(P + \frac{a}{V^2})(V - b) = RT$$

At low pressure

$$V - b \approx V$$

$$\left(P + \frac{a}{V^2}\right)V = RT$$

$$PV + \frac{a}{V} = RT$$

$$PV = RT - \frac{a}{V}$$

$$\frac{PV}{RT} = 1 - \frac{a}{VRT}$$

So, $\boxed{Z = 1 - \frac{a}{VRT}}$

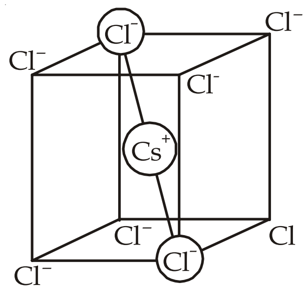
33. CsCl crystallises in body centred cubic lattice. If 'a' is its edge length then which of the following expressions is correct?

(1) $r_{Cs^+} + r_{Cl^-} = 3a$ (2) $r_{Cs^+} + r_{Cl^-} = \frac{3a}{2}$

(3) $r_{Cs^+} + r_{Cl^-} = \frac{\sqrt{3}}{2}a$ (4) $r_{Cs^+} + r_{Cl^-} = \sqrt{3}a$

Answer (3)

Sol.



$$2r_{Cl^-} + 2r_{Cs^+} = \sqrt{3}\,a$$

$$r_{Cl^-} + r_{Cs^+} = \frac{\sqrt{3}a}{2}$$

34. For the estimation of nitrogen, 1.4 g of an organic compound was digested by Kjeldahl method and the evolved ammonia was absorbed in 60 mL of $\frac{M}{10}$ sulphuric acid. The unreacted acid required 20 mL of $\frac{M}{10}$ sodium hydroxide for complete neutralization. The percentage of nitrogen in the compound is

(1) 6% (2) 10%

(3) 3% (4) 5%

Answer (2)

Sol. As per question

	Normality	Volume
H_2SO_4	$\frac{N}{5}$	60mL
NaOH	$\frac{N}{10}$	20mL

$$(n_{geq})_{H_2SO_4} = (n_{geq})_{NaOH} + (n_{geq})_{NH_3}$$

$$\frac{1}{5} \times \frac{60}{1000} = \frac{1}{10} \times \frac{20}{1000} + (n_{geq})_{NH_3}$$

$$\frac{6}{500} = \frac{1}{500} + (n_{geq})_{NH_3}$$

$$(n_{geq})_{NH_3} = \frac{5}{500} = \frac{1}{100}$$

$$(n_{mol})_N = (n_{mol})_{NH_3} = (n_{geq})_{NH_3} = \frac{1}{100}$$

$$(Mass)_N = \frac{14}{100} = 0.14\,g$$

$$\text{Percentage of "N"} = \frac{0.14}{1.4} \times 100 = 10\%$$

35. Resistance of 0.2 M solution of an electrolyte is 50 Ω. The specific conductance of the solution is 1.4 S m^{-1}. The resistance of 0.5 M solution of the same electrolyte is 280 Ω. The molar conductivity of 0.5 M solution of the electrolyte in S m^2 mol^{-1} is

(1) 5×10^{-4} (2) 5×10^{-3}

(3) 5×10^{3} (4) 5×10^{2}

Answer (1)

Sol. For 0.2 M solution

R = 50 Ω

$\sigma = 1.4 \text{ S m}^{-1} = 1.4 \times 10^{-2} \text{ S cm}^{-1}$

$$\Rightarrow \quad \rho = \frac{1}{\sigma} = \frac{1}{1.4\times10^{-2}} \ \Omega\,\text{cm}$$

Now, $R = \rho\frac{l}{a}$

$$\Rightarrow \quad \frac{l}{a} = \frac{R}{\rho} = 50\times1.4\times10^{-2}$$

For 0.5 M solution

R = 280 Ω

σ = ?

$$\frac{l}{a} = 50\times1.4\times10^{-2}$$

$$\Rightarrow \quad R = \rho\frac{l}{a}$$

$$\Rightarrow \quad \frac{1}{\rho} = \frac{1}{R}\times\frac{l}{a}$$

$$\Rightarrow \quad \sigma = \frac{1}{280}\times50\times1.4\times10^{-2}$$

$$= \frac{1}{280}\times70\times10^{-2}$$

$= 2.5 \times 10^{-3} \text{ S cm}^{-1}$

Now, $\lambda_m = \frac{\sigma\times1000}{M}$

$$= \frac{2.5\times10^{-3}\times1000}{0.5}$$

$= 5 \text{ S cm}^2 \text{ mol}^{-1}$

$= 5 \times 10^{-4} \text{ S m}^2 \text{ mol}^{-1}$

36. For complete combustion of ethanol,

$C_2H_5OH(l) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$,

the amount of heat produced as measured in bomb calorimeter, is 1364.47 kJ mol^{-1} at 25°C. Assuming ideality the enthalpy of combustion, Δ_cH, for the reaction will be

(R = 8.314 kJ mol^{-1})

(1) –1366.95 kJ mol^{-1} (2) –1361.95 kJ mol^{-1}

(3) –1460.50 kJ mol^{-1} (4) –1350.50 kJ mol^{-1}

Answer (1)

Sol. $C_2H_5OH(l) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$

Bomb calorimeter gives ΔU of the reaction

So, as per question

$\Delta U = -1364.47 \text{ kJ mol}^{-1}$

$\Delta n_g = -1$

$\Delta H = \Delta U + \Delta n_g RT$

$$= -1364.47 - \frac{1\times8.314\times298}{1000}$$

$= -1366.93 \text{ kJ mol}^{-1}$

37. The equivalent conductance of NaCl at concentration C and at infinite dilution are λ_C and λ_∞, respectively. The correct relationship between λ_C and λ_∞ is given as

(Where the constant B is positive)

(1) $\lambda_C = \lambda_\infty + (B)C$ (2) $\lambda_C = \lambda_\infty - (B)C$

(3) $\lambda_C = \lambda_\infty - (B)\sqrt{C}$ (4) $\lambda_C = \lambda_\infty + (B)\sqrt{C}$

Answer (3)

Sol. According to Debye Huckle onsager equation,

$$\lambda_C = \lambda_\infty - A\sqrt{C}$$

Here A = B

$$\therefore \quad \lambda_C = \lambda_\infty - B\sqrt{C}$$

38. Consider separate solutions of 0.500 M $C_2H_5OH(aq)$, 0.100 M $Mg_3(PO_4)_2(aq)$, 0.250 M KBr(aq) and 0.125 M $Na_3PO_4(aq)$ at 25°C. Which statement is **true** about these solutions, assuming all salts to be strong electrolytes?

(1) They all have the same osmotic pressure.

(2) 0.100 M $Mg_3(PO_4)_2(aq)$ has the highest osmotic pressure.

(3) 0.125 M $Na_3PO_4(aq)$ has the highest osmotic pressure.

(4) 0.500 M $C_2H_5OH(aq)$ has the highest osmotic pressure.

Answer (1)

Sol. $\pi = i\,CRT$

$$\pi_{C_2H_5OH} = 1\times0.500\times R\times T = 0.5\,RT$$

$$\pi_{Mg_3(PO_4)_2} = 5\times0.100\times R\times T = 0.5\,RT$$

$$\pi_{KBr} = 2\times0.250\times R\times T = 0.5\,RT$$

$$\pi_{Na_3PO_4} = 4\times0.125\times RT = 0.5\,RT$$

39. For the reaction $SO_{2(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons SO_{3(g)}$, if $K_P = K_C(RT)^x$ where the symbols have usual meaning then the value of x is (assuming ideality)

(1) -1 (2) $-\frac{1}{2}$

(3) $\frac{1}{2}$ (4) 1

Answer (2)

Sol. $SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$

$K_P = K_C(RT)^x$

$x = \Delta n_g$ = no. of gaseous moles in product – no. of gaseous moles in reactant

$= 1 - \left(1 + \frac{1}{2}\right) = 1 - \frac{3}{2} = \frac{-1}{2}$

40. For the non-stoichiometre reaction $2A + B \rightarrow C + D$, the following kinetic data were obtained in three separate experiments, all at 298 K.

Initial Concentration (A)	Initial Concentration (B)	Initial rate of formation of C (mol $L^{-1}s^{-1}$)
0.1 M	0.1 M	1.2×10^{-3}
0.1 M	0.2 M	1.2×10^{-3}
0.2 M	0.1 M	2.4×10^{-3}

The rate law for the formation of C is

(1) $\frac{dC}{dt} = k[A][B]$ (2) $\frac{dC}{dt} = k[A]^2[B]$

(3) $\frac{dC}{dt} = k[A][B]^2$ (4) $\frac{dC}{dt} = k[A]$

Answer (4)

Sol. $2A + B \longrightarrow C + D$

Rate of Reaction $= \frac{-1}{2}\frac{d[A]}{dt} = -\frac{d[B]}{dt}$

$= \frac{d[C]}{dt} = \frac{d[D]}{dt}$

Let rate of Reaction $= k[A]^x[B]^y$

Or, $\frac{d[C]}{dt} = k[A]^x[B]^y$

Now from table,

$1.2 \times 10^{-3} = k\,[0.1]^x[0.1]^y$ ...(i)

$1.2 \times 10^{-3} = k\,[0.1]^x[0.2]^y$ ...(ii)

$2.4 \times 10^{-3} = k\,[0.2]^x[0.1]^y$ ...(iii)

Dividing equation (i) by (ii)

$\Rightarrow \frac{1.2\times10^{-3}}{1.2\times10^{-3}} = \frac{k[0.1]^x[0.1]^y}{k[0.1]^x[0.2]^y}$

$\Rightarrow 1 = \left[\frac{1}{2}\right]^y$

$\Rightarrow \boxed{y = 0}$

Now Dividing equation (i) by (iii)

$\Rightarrow \frac{1.2\times10^{-3}}{2.4\times10^{-3}} = \frac{k[0.1]^x[0.1]^y}{k[0.2]^x[0.1]^y}$

$\Rightarrow \left[\frac{1}{2}\right]^1 = \left[\frac{1}{2}\right]^x$

$\Rightarrow \boxed{x = 1}$

Hence $\frac{d[C]}{dt} = k[A]^1[B]^0$.

41. Among the following oxoacids, the correct decreasing order of acid strength is

(1) $HOCl > HClO_2 > HClO_3 > HClO_4$

(2) $HClO_4 > HOCl > HClO_2 > HClO_3$

(3) $HClO_4 > HClO_3 > HClO_2 > HOCl$

(4) $HClO_2 > HClO_4 > HClO_3 > HOCl$

Answer (3)

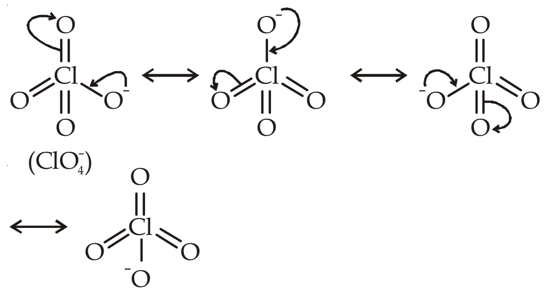
Sol. $HClO_4 \rightleftharpoons ClO_4^- + H^+$

$HClO_3 \rightleftharpoons ClO_3^- + H^+$

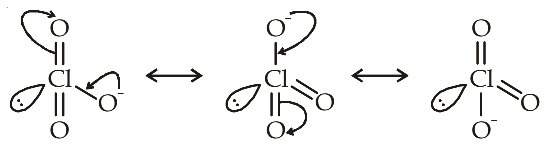
$HClO_2 \rightleftharpoons ClO_2^- + H^+$

$HOCl \rightleftharpoons ClO^- + H^+$

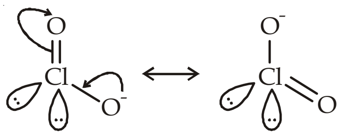
Resonance produced conjugate base.

(i) 

(ClO_4^-)

(ii) 

(ClO_3^-)

(iii) 

(ClO_2^-)

(iv) ClO^- is not resonance stabilized.

As per resonance stability order of conjugate base is

$$ClO_4^- > ClO_3^- > ClO_2^- > ClO^-$$

Hence acidic strength order is

$$HClO_4 > HClO_3 > HClO_2 > HClO$$

42. The metal that cannot be obtained by electrolysis of an aqueous solution of its salts is

(1) Ag (2) Ca

(3) Cu (4) Cr

Answer (2)

Sol. On electrolysis only in case of Ca^{2+} salt aqueous solution H_2 gas discharge at Cathode.

Case of Cr

At cathode : $Cr^{3+} + 2e^- \longrightarrow Cr$

So, Cr is deposited.

Case of Ag

At cathode : $Ag^+ + e^- \longrightarrow Ag$

So, Ag is deposited.

Case of Cu

At cathode : $Cu^{2+} + 2e^- \longrightarrow Cu$

Case of Ca^{2+}

At cathode : $H_2O + e^- \longrightarrow \frac{1}{2} H_2 + OH^-$

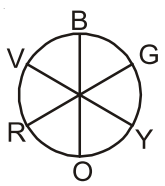
43. The octahedral complex of a metal ion M^{3+} with four monodentate ligands L_1, L_2, L_3 and L_4 absorb wavelengths in the region of red, green, yellow and blue, respectively. The increasing order of ligand strength of the four ligands is

(1) $L_4 < L_3 < L_2 < L_1$ (2) $L_1 < L_3 < L_2 < L_4$

(3) $L_3 < L_2 < L_4 < L_1$ (4) $L_1 < L_2 < L_4 < L_3$

Answer (2)

Sol.



The energy of red light is less than that of violet light.

So energy order is

Red < Yellow < Green < Blue

The complex absorbs lower energy light lower will be its strength. So order of ligand strength is

$L_1 < L_3 < L_2 < L_4$

44. Which one of the following properties is **not** shown by NO?

(1) It is diamagnetic in gaseous state

(2) It is a neutral oxide

(3) It combines with oxygen to form nitrogen dioxide

(4) It's bond order is 2.5

Answer (1)

Sol. Nitric oxide is paramagnetic in the gaseous state as it has one unpaired electron in its outermost shell. The electronic configuration of NO is

$$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\sigma_{2p_z}^2\pi_{2p_x}^2 = \pi_{2p_y}^2\pi_{2p_x}^{*1}$$

However, it dimerises at low temperature to become diamagnetic.

$$2NO \rightleftharpoons N_2O_2$$

Its bond order is 2.5 and it combines with O_2 to give nitrogen dioxide.

45. In which of the following reactions H_2O_2 acts as a reducing agent?

(a) $H_2O_2 + 2H^+ + 2e^- \rightarrow 2H_2O$

(b) $H_2O_2 - 2e^- \rightarrow O_2 + 2H^+$

(c) $H_2O_2 + 2e^- \rightarrow 2OH^-$

(d) $H_2O_2 + 2OH^- - 2e^- \rightarrow O_2 + 2H_2O$

(1) (a), (b) (2) (c), (d)

(3) (a), (c) (4) (b), (d)

Answer (4)

Sol. The reducing agent oxidises itself.

(a) $H_2O_2^{-1} + 2H^+ + 2e^- \longrightarrow 2H_2O^{-2}$

(b) $H_2O_2^{-1} - 2e \longrightarrow \overset{0}{O_2} + 2H^+$

(c) $H_2O_2^{-1} + 2e \longrightarrow 2\overset{-2}{O}H^-$

(d) $H_2O_2^{-1} + 2OH^- - 2e \longrightarrow \overset{0}{O_2} + H_2O$

Note : Powers of 'O' are oxidation number of 'O' in the compound.

46. The correct statement for the molecule, CsI_3, is

(1) It is a covalent molecule

(2) It contains Cs^+ and I_3^- ions

(3) It contains Cs^{3+} and I^- ions

(4) It contains Cs^+, I^- and lattice I_2 molecule

Answer (2)

Sol. It contains Cs^+ and I_3^- ions.

47. The ratio of masses of oxygen and nitrogen in a particular gaseous mixture is 1 : 4. The ratio of number of their molecule is

(1) 1 : 4 (2) 7 : 32

(3) 1 : 8 (4) 3 : 16

Answer (2)

Sol. Let the mass of O_2 = x

Mass of N_2 = 4x

Number of moles of $O_2 = \frac{x}{32}$

Number of moles of $N_2 = \frac{4x}{28} = \frac{x}{7}$

$\therefore$ Ratio $= \frac{x}{32} : \frac{x}{7} = 7 : 32$

48. Given below are the half-cell reactions

$Mn^{2+} + 2e^- \rightarrow Mn$; E° = – 1.18 V

$(Mn^{3+} + e^- \rightarrow Mn^{2+})$; E° = + 1.51 V

The E° for $3\,Mn^{2+} \rightarrow Mn + 2Mn^{3+}$ will be

(1) –2.69 V; the reaction will not occur

(2) –2.69 V; the reaction will occur

(3) –0.33 V; the reaction will not occur

(4) –0.33 V; the reaction will occur

Answer (1)

Sol. (1) $Mn^{2+} + 2e \rightarrow Mn$; E° = –1.18V ;

$\Delta G_1^\circ = -2F(-1.18) = 2.36F$

(2) $Mn^{3+} + e \rightarrow Mn^{2+}$; E° = +1.51 V;

$\Delta G_2^\circ = -F(1.51) = -1.51\,F$

(1) – 2 × (2)

$3Mn^{2+} \rightarrow Mn + 2Mn^{3+}$;

$\Delta G_3^\circ = \Delta G_1^\circ - 2\Delta G_2^\circ$

= [2.36 – 2(–1.51)] F

= (2.36 + 3.02) F

= 5.38 F

But $\Delta G_3^\circ = -2FE^\circ$

$\Rightarrow$ 5.38F = –2FE°

$\Rightarrow$ E° = –2.69 V

As E° value is negative reaction is non spontaneous.

49. Which series of reactions correctly represents chemical reactions related to iron and its compound?

(1) $Fe \xrightarrow{dil.H_2SO_4} FeSO_4 \xrightarrow{H_2SO_4, O_2}$
$Fe_2(SO_4)_3 \xrightarrow{heat} Fe$

(2) $Fe \xrightarrow{O_2, heat} FeO \xrightarrow{dil.H_2SO_4}$
$FeSO_4 \xrightarrow{heat} Fe$

(3) $Fe \xrightarrow{Cl_2, heat} FeCl_3 \xrightarrow{heat, air}$
$FeCl_2 \xrightarrow{Zn} Fe$

(4) $Fe \xrightarrow{O_2, heat} Fe_3O_4 \xrightarrow{CO, 600°C}$
$FeO \xrightarrow{CO, 700°C} Fe$

Answer (4)

Sol. $Fe \xrightarrow[Heat]{O_2} Fe_3O_4$

This reaction is corresponding to the combustion of Fe.

$Fe\ O \xrightarrow[600°C]{CO} FeO \xrightarrow[700°C]{CO} Fe$

These reactions correspond to the production of Fe by reduction of Fe_3O_4 in blast furnace.

50. The equation which is balanced and represents the correct product(s) is

(1) $Li_2O + 2KCl \rightarrow 2LiCl + K_2O$

(2) $[CoCl(NH_3)_5]^+ + 5H^+ \rightarrow Co^{2+} + 5NH_4^+ + Cl^-$

(3) $[Mg(H_2O)_6]^{2+} + (EDTA)^{4-} \xrightarrow{excess\ NaOH} [Mg(EDTA)]^{2+} + 6H_2O$

(4) $CuSO_4 + 4KCN \rightarrow K_2[Cu(CN)_4] + K_2SO_4$

Answer (2)

Sol. The complex

$[CoCl(NH_3)_5]^+$ decomposes under acidic medium, so

$[CoCl(NH_3)_5]^+ + 5H^+ \rightarrow Co^{2+} + 5NH_4^+ + Cl^-$.

51. In S_N2 reactions, the correct order of reactivity for the following compounds

CH_3Cl, CH_3CH_2Cl, $(CH_3)_2CHCl$ and $(CH_3)_3CCl$ is

(1) $CH_3Cl > (CH_3)_2CHCl > CH_3CH_2Cl > (CH_3)_3CCl$

(2) $CH_3Cl > CH_3CH_2Cl > (CH_3)_2CHCl > (CH_3)_3CCl$

(3) $CH_3CH_2Cl > CH_3Cl > (CH_3)_2CHCl > (CH_3)_3CCl$

(4) $(CH_3)_2CHCl > CH_3CH_2Cl > CH_3Cl > (CH_3)_3CCl$

Answer (2)

Sol. Rate of S_N2 reaction depends on steric crowding of alkyl halide. So order is

$CH_3Cl > (CH_3)CH_2 - Cl > (CH_3)_2CH - Cl > (CH_3)_3CCl$

52. On heating an aliphatic primary amine with chloroform and ethanolic potassium hydroxide, the organic compound formed is

(1) An alkanol

(2) An alkanediol

(3) An alkyl cyanide

(4) An alkyl isocyanide

Answer (4)

Sol. $R - CH_2 - NH_2 \xrightarrow[C_2H_5OH]{CHCl_3/KOH} R- CH_2 - NC$

53. The most suitable reagent for the conversion of $R - CH_2 - OH \rightarrow R - CHO$ is

(1) $KMnO_4$

(2) $K_2Cr_2O_7$

(3) CrO_3

(4) PCC (Pyridinium Chlorochromate)

Answer (4)

Sol. PCC is mild oxidising agent, it will convert

$R-CH_2-OH \longrightarrow R-CHO$

54. The major organic compound formed by the reaction of 1, 1, 1-trichloroethane with silver powder is

(1) Acetylene

(2) Ethene

(3) 2-Butyne

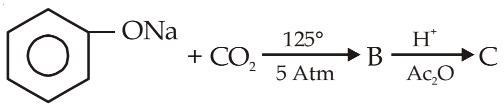
(4) 2-Butene

Answer (3)

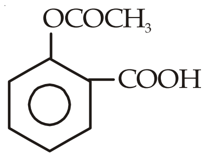
Sol. $2Cl-\overset{Cl}{\underset{Cl}{C}}-CH_3 \xrightarrow{Ag} CH_3C\equiv CCH_3 + 6AgCl$

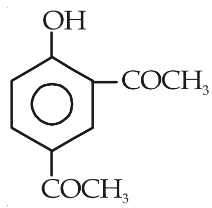
1, 1, 1-trichloroethane

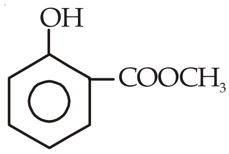
55. Sodium phenoxide when heated with CO_2 under pressure at 125°C yields a product which on acetylation produces C.

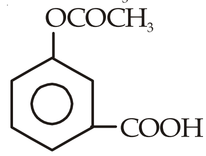


The major product C would be

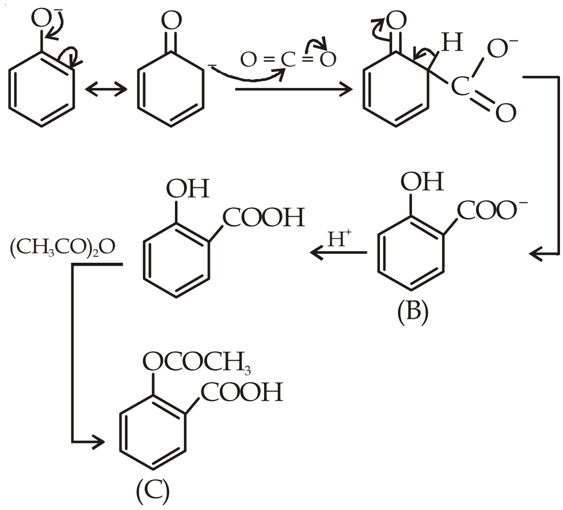
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

56. Considering the basic strength of amines in aqueous solution, which one has the smallest pK_b value?

(1) $(CH_3)_2NH$

(2) CH_3NH_2

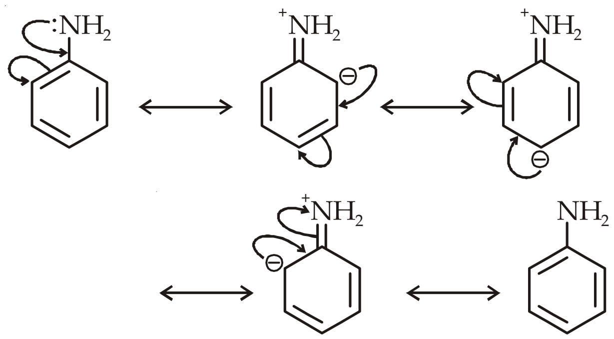
(3) $(CH_3)_3N$

(4) $C_6H_5NH_2$

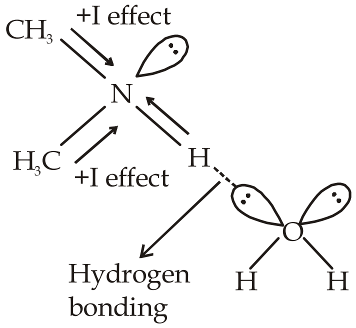
Answer (1)

Sol. Among $C_6H_5NH_2$, CH_3NH_2, $(CH_3)_2NH$,

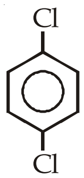
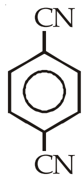
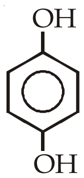
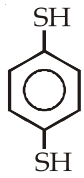
$(CH_3)_3N \cdot C_6H_5NH_2$ is least basic due to resonance.



Out of $(CH_3)_3N$, CH_3NH_2, $(CH_3)_2NH$. $(CH_3)_2NH$ is most basic due to +I effect and hydrogen bonding in H_2O.



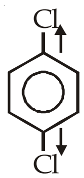
57. For which of the following molecule significant $\mu \neq 0$?

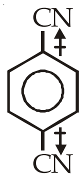
(a) 
(b) 
(c) 
(d) 

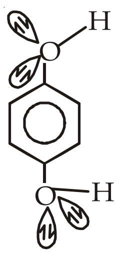
(1) Only (a)] (2) (a) and (b)

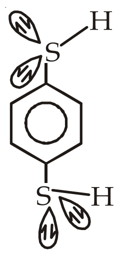
(3) Only (c) (4) (c) and (d)

Answer (4)

Sol. (a) 
$\mu = 0$

(b) 
$\mu = 0$

(c) 
$\mu \neq 0$

(d) 
$\mu \neq 0$

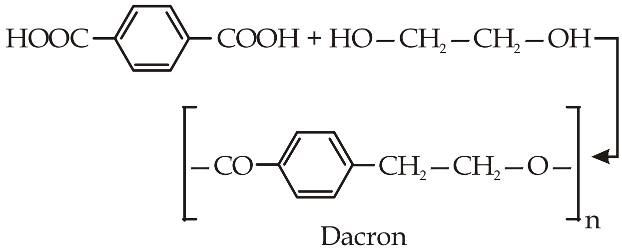
58. Which one is classified as a condensation polymer?

(1) Dacron (2) Neoprene

(3) Teflon (4) Acrylonitrile

Answer (1)

Sol. Dacron is polyester formed by condensation polymerisation of terephthalic acid and ethylene glycol.



Dacron

Acrylonitrile, Neoprene and Teflon are addition polymers of acrylonitrile, isoprene and tetrafluoro ethylene respectively.

59. Which one of the following bases is **not** present in DNA?

(1) Quinoline

(2) Adenine

(3) Cytosine

(4) Thymine

Answer (1)

Sol. DNA contains ATGC bases

A – Adenine

T – Thymine

G – Guanine

C – Cytocine

So quinoline is not present.

60. In the reaction,

$$CH_3COOH \xrightarrow{LiAlH_4} A \xrightarrow{PCl_5} B \xrightarrow{Alc.KOH} C,$$

the product C is

(1) Acetaldehyde

(2) Acetylene

(3) Ethylene

(4) Acetyl chloride

Answer (3)

Sol. Ethylene

$$CH_3COOH \xrightarrow{LiAlH_4} CH_3CH_2OH \quad 'A'$$

$$\downarrow PCl_5$$

$$CH_3CH_2Cl \quad 'B'$$

$$\downarrow Alc.\ KOH$$

$$CH_2 = CH_2 \quad 'C'$$

61. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n - 1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to

(1) X (2) Y

(3) N (4) $Y - X$

Answer (2)

Sol. $X = \{(1+3)^n - 3n - 1, n \in N\}$

$= 3^2({}^nC_2 + {}^nC_3.3 + ... + 3^{n-2}), n \in N\}$

= {Divisible by 9}

$Y = \{9(n - 1), n \in N\}$

= (All multiples of 9}

So, $X \subseteq Y$

i.e., $\boxed{X \cup Y = Y}$

62. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$

(1) Is strictly greater than $\frac{5}{2}$

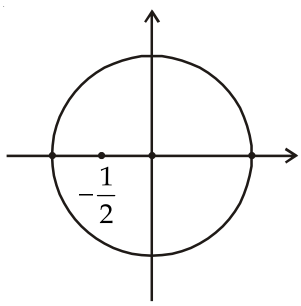
(2) Is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

(3) Is equal to $\frac{5}{2}$

(4) Lies in the interval (1, 2)

Answer (4)

Sol.



$\left|z + \frac{1}{2}\right|$

So, $\left||z| - \frac{1}{2}\right| \leq \left|z + \frac{1}{2}\right|$

$\Rightarrow \left|z + \frac{1}{2}\right| \geq \left|2 - \frac{1}{2}\right|$

$\Rightarrow \left|z_{\text{min.}} = \frac{3}{2}\right|$

63. If $a \in R$ and the equation

$-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$

(where $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval

(1) (–2, –1)

(2) $(-\infty, -2) \cup (2, \infty)$

(3) $(-1, 0) \cup (0, 1)$

(4) (1, 2)

Answer (3)

Sol. $-3(x - [x])^2 + 2[x - [x]) + a^2 = 0$

$3\{x\}^2 - 2\{x\} - a^2 = 0$

$a \neq 0,\ 3\left(\{x\}^2 - \frac{2}{3}\{x\}\right) = a^2$

$a^2 = 3\left(\{x\} - \frac{1}{3}\right)^2 - \frac{1}{3}$

$0 \leq \{x\} < 1$ and $-\frac{1}{3} \leq \{x\} - \frac{1}{3} < \frac{2}{3}$

$0 \leq 3\left(\{x\} - \frac{1}{3}\right)^2 < \frac{4}{3}$

$-\frac{1}{3} \leq 3\left(\{x\} - \frac{1}{3}\right)^2 - \frac{1}{3} < 1$

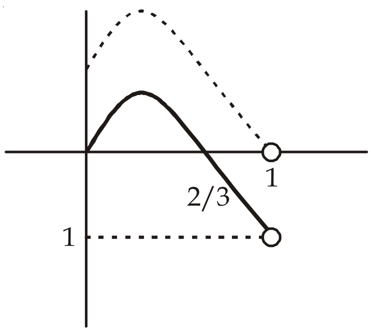
For non-integral solution

$0 < a^2 < 1$ and $a \in (-1, 0) \cup (0, 1)$

Alternative

$-3\{x\}^2 + 2\{x\} + a^2 = 0$

Now, $-3\{x\}^2 + 2\{x\}$



to have no integral roots $0 < a^2 < 1$

$\therefore\ a \in (-1, 0) \cup (0, 1)$

64. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha}+\frac{1}{\beta}=4$, then the value of $|\alpha - \beta|$ is

(1) $\frac{\sqrt{34}}{9}$ (2) $\frac{2\sqrt{13}}{9}$

(3) $\frac{\sqrt{61}}{9}$ (4) $\frac{2\sqrt{17}}{9}$

Answer (2)

Sol.

$\because$ p, q, r are in AP

$2q = p + r$ ...(i)

Also $\frac{1}{\alpha}+\frac{1}{\beta}=4$

$\Rightarrow \frac{\alpha+\beta}{\alpha\beta}=4$

$= \frac{-\frac{q}{p}}{\frac{r}{p}} = 4 \Rightarrow q = -4r$ ...(ii)

From (i)

$2(-4r) = p + r$

$p = -9r$

$q = -4r$

$r = r$

Now $|\alpha-\beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \sqrt{\left(\frac{-q}{p}\right)^2 - \frac{4r}{p}}$

$= \frac{\sqrt{q^2-4pr}}{|p|}$

$= \frac{\sqrt{16r^2+36r^2}}{|-9r|}$

$= \frac{2\sqrt{13}}{9}$

65. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$= K(1-\alpha)^2 (1-\beta)^2 (\alpha-\beta)^2$, then K is equal to

(1) 1 (2) –1

(3) αβ (4) $\frac{1}{\alpha\beta}$

Answer (1)

Sol. $$\begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & 1 \\ \alpha^2 & \beta^2 & 1 \end{vmatrix} \times \begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ 1 & 1 & 1 \end{vmatrix}$$

$= [(1-\alpha)(1-\beta)(1-\beta)]^2$

So, $\boxed{k=1}$

66. If A is an 3 × 3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals

(1) B^{-1} (2) $(B^{-1})'$

(3) $I + B$ (4) I

Answer (4)

Sol. $BB' = (A^{-1}.A')(A(A^{-1})')$

$= A^{-1}.A.A'.(A^{-1})^1$ {as $AA' = A'A$}

$= I(A^{-1}A)'$

$= I.I = I^2 = I$

67. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to

(1) $\left(14, \frac{272}{3}\right)$ (2) $\left(16, \frac{272}{3}\right)$

(3) $\left(16, \frac{251}{3}\right)$ (4) $\left(14, \frac{251}{3}\right)$

Answer (2)

Sol. $(1 + ax + bx^2)(1 - 2x)^{18}$

$(1 + ax + bx^2)[^{18}C_0 - {}^{18}C_1(2x) + {}^{18}C_2(2x)^2 - {}^{18}C_3(2x)^3 + {}^{18}C_4(2x)^4 - ....]$

Coeff. of $x^3 = -{}^{18}C_3.8 + a \times 4.{}^{18}C_2 - 2b \times 18 = 0$

$= -\frac{18\times17\times16}{6}.8 + \frac{4a+18\times17}{2} - 36b = 0$

$= -51 \times 16 \times 8 + a \times 36 \times 17 - 36b = 0$

$= -34 \times 16 + 51a - 3b = 0$

$= 51a - 3b = 34 \times 16 = 544$

$= 51a - 3b = 544$ ... (i)

Only option number (2) satisfies the equation number (i).

68. If $(10)^9 + 2(11)^1 (10)^8 + 3(11)^2 (10)^7 + ... + 10(11)^9 = k(10)^9$, then k is equal to

(1) 100 (2) 110

(3) $\frac{121}{10}$ (4) $\frac{441}{100}$

Answer (1)

Sol. $10^9 + 2\cdot(11)(10)^8 + 3(11)^2(10)^7 + ... + 10(11)^9 = k(10)^9$

$x = 10^9 + 2\cdot(11)(10)^8 + 3(11)^2(10)^7 + ... + 10(11)^9$

$\frac{11}{10}x = 11\cdot10^8 + 2\cdot(11)^2\cdot(10)^7 + ... + 9(11)^9 + 11^{10}$

$x\left(1-\frac{11}{10}\right) = 10^9 + 11(10)^8 + 11^2\times(10)^7 + ... + 11^9 - 11^{10}$

$\Rightarrow -\frac{x}{10} = 10^9\left(\frac{\left(\frac{11}{10}\right)^{10}-1}{\frac{11}{10}-1}\right) - 11^{10}$

$\Rightarrow -\frac{x}{10} = (11^{10} - 10^{10}) - 11^{10} = -10^{10}$

$\Rightarrow x = 10^{11} = k\cdot10^9$

$\Rightarrow k = 100$

69. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is

(1) $2-\sqrt{3}$ (2) $2+\sqrt{3}$

(3) $\sqrt{2}+\sqrt{3}$ (4) $3+\sqrt{2}$

Answer (2)

Sol. $a, ar, ar^2 \rightarrow$ G.P.

$a, 2ar, ar^2 \rightarrow$ A.P.

$2 \times 2ar = a + ar^2$

$4r = 1 + r^2$

$\Rightarrow r^2 - 4r + 1 = 0$

$r = \frac{4\pm\sqrt{16-4}}{2} = 2\pm\sqrt{3}$

$\boxed{r = 2+\sqrt{3}}$

$r = 2-\sqrt{3}$ is rejected

$\because$ $(r > 1)$

G.P. is increasing.

70. $\lim_{x\to0}\frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to

(1) $-\pi$ (2) π

(3) $\frac{\pi}{2}$ (4) 1

Answer (2)

Sol. $\lim_{x\to0}\frac{\sin(\pi\cos^2 x)}{x^2}$

$= \lim_{x\to0}\frac{\sin(\pi(1-\sin^2 x))}{x^2}$

$= \lim_{x\to0} \sin\frac{(\pi-\pi\sin^2 x)}{x^2}$

$= \lim_{x\to0} \sin\frac{(\pi\sin^2 x)}{x^2}$ $[\because \sin(\pi-\theta) = \sin\theta]$

$= \lim_{x\to0} \sin\frac{(\pi\sin^2 x)}{(\pi\sin^2 x)}\times\frac{\pi\sin^2 x}{x^2}$

$= \lim_{x\to0} 1\times\pi\left(\frac{\sin x}{x}\right)^2 = \pi$

71. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$, then $g'(x)$ is equal to

(1) $\frac{1}{1+\{g(x)\}^5}$ (2) $1 + \{g(x)\}^5$

(3) $1 + x^5$ (4) $5x^4$

Answer (2)

Sol. $f'(x) = \frac{1}{1+x^5} = f(g(x)) = x \rightarrow f'(g(x))\, g'(x) = 1$

$g'(x) = \frac{1}{f'(g(x))} = 1 + (g(x))^5$

72. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1)$, $g(0) = 0$ and $f(1) = 6$, then for some $c \in]0, 1[$

(1) $f'(c) = g'(c)$ (2) $f'(c) = 2g'(c)$

(3) $2f'(c) = g'(c)$ (4) $2f'(c) = 3g'(c)$

Answer (2)

Sol. Using, mean value theorem

$f'(c) = \frac{f(1)-f(0)}{1-0} = 4$

$g'(c) = \frac{g(1)-g(0)}{1-0} = 2$

so, $\boxed{f'(c) = 2g'(c)}$

73. If $x = -1$ and $x = 2$ are extreme points of $f(x)=\alpha\log|x|+\beta x^2+x$ then

(1) $\alpha=2,\ \beta=-\frac{1}{2}$ (2) $\alpha=2,\ \beta=\frac{1}{2}$

(3) $\alpha=-6,\ \beta=\frac{1}{2}$ (4) $\alpha=-6,\ \beta=-\frac{1}{2}$

Answer (1)

Sol. $f(x)=\alpha\log|x|+\beta x^2+x$

$f'(x)=\frac{\alpha}{x}+2\beta x+1=0$ at $x = -1, 2$

$-\alpha-2\beta+1=0 \Rightarrow \alpha+2\beta=1$ ...(i)

$\frac{\alpha}{2}+4\beta+1=0 \Rightarrow \alpha+8\beta=-2$ ...(ii)

$6\beta=-3 \Rightarrow \beta=-\frac{1}{2}$

$\therefore\ \alpha=2$

74. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to

(1) $(x+1)e^{x+\frac{1}{x}}+c$ (2) $-xe^{x+\frac{1}{x}}+c$

(3) $(x-1)e^{x+\frac{1}{x}}+c$ (4) $xe^{x+\frac{1}{x}}+c$

Answer (4)

Sol. $I = \int\left\{e^{\left(x+\frac{1}{x}\right)}+x\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}\right\}dx$

$= x.e^{x+\frac{1}{x}}+c$

As $\int(xf'(x)+f(x))dx=xf(x)+c$

75. The integral

$$\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx \text{ equals}$$

(1) $4\sqrt{3}-4$ (2) $4\sqrt{3}-4-\frac{\pi}{3}$

(3) $\pi - 4$ (4) $\frac{2\pi}{3}-4-4\sqrt{3}$

Answer (2)

Sol. $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$

$= \int_0^{\pi}\left|2\sin\frac{x}{2}-1\right|dx$ $\left[\begin{array}{l}\sin\frac{x}{2}=\frac{1}{2}\\ \Rightarrow\frac{x}{2}=\frac{\pi}{6}\to x=\frac{\pi}{3}\\ \frac{x}{2}=\frac{5\pi}{6}\to x=\frac{5\pi}{3}\end{array}\right]$

$= \int_0^{\pi/3}\left(1-2\sin\frac{x}{2}\right)dx+\int_{\pi/3}^{\pi}\left(2\sin\frac{x}{2}-1\right)dx$

$= \left[x+4\cos\frac{x}{2}\right]_0^{\pi/3}+\left[-4\cos\frac{x}{2}-x\right]_{\pi/3}^{\pi}$

$= \frac{\pi}{3}+4\frac{\sqrt{3}}{2}-4+\left(0-\pi+4\frac{\sqrt{3}}{2}+\frac{\pi}{3}\right)$

$= 4\sqrt{3}-4-\frac{\pi}{3}$

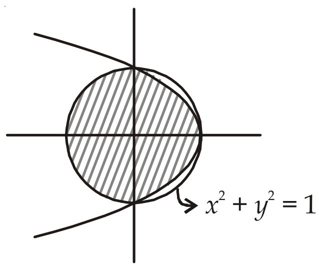
76. The area of the region described by $A = \{(x, y) : x^2 + y^2 \le 1 \text{ and } y^2 \le 1 - x\}$ is

(1) $\frac{\pi}{2}-\frac{2}{3}$ (2) $\frac{\pi}{2}+\frac{2}{3}$

(3) $\frac{\pi}{2}+\frac{4}{3}$ (4) $\frac{\pi}{2}-\frac{4}{3}$

Answer (3)

Sol.



Shaded area

$= \frac{\pi(1)^2}{2}+2\int_0^1\sqrt{(1-x)}\,dx$

$= \frac{\pi}{2}+\frac{2(1-x)^{3/2}}{3/2}(-1)\Big|_0^1$

$= \frac{\pi}{2}+\frac{4}{3}(0-(-1))$

$= \frac{\pi}{2}+\frac{4}{3}$

77. Let the population of rabbits surviving at a time t be governed by the differential equation

$\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If $p(0) = 100$, then $p(t)$ equals

(1) $600 - 500\, e^{t/2}$

(2) $400 - 300\, e^{-t/2}$

(3) $400 - 300\, e^{t/2}$

(4) $300 - 200\, e^{-t/2}$

Answer (3)

Sol. $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$

$$\int \frac{d(p(t))}{\left(\frac{1}{2}p(t) - 200\right)} = \int dt$$

$$2\log\left(\frac{p(t)}{2} - 200\right) = t + c$$

$$\frac{p(t)}{2} - 200 = e^{\frac{t}{2}}k$$

Using given condition $p(t) = 400 - 300\, e^{t/2}$

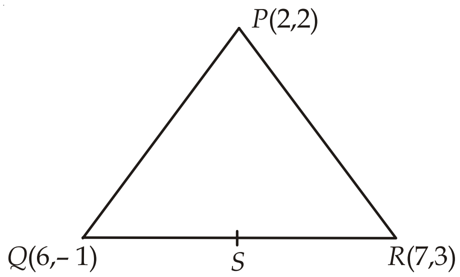
78. Let PS be the median of the triangle with vertices $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$. The equation of the line passing through $(1, -1)$ and parallel to PS is

(1) $4x + 7y + 3 = 0$ (2) $2x - 9y - 11 = 0$

(3) $4x - 7y - 11 = 0$ (4) $2x + 9y + 7 = 0$

Answer (4)

Sol.



S is mid-point of QR

So $S = \left(\frac{7+6}{2}, \frac{3-1}{2}\right)$

$= \left(\frac{13}{2}, 1\right)$

Slope of $PS = \frac{2-1}{2-\frac{13}{2}} = -\frac{2}{9}$

Equation of line $\Rightarrow y - (-1) = -\frac{2}{9}(x-1)$

$9y + 9 = -2x + 2 \Rightarrow 2x + 9y + 7 = 0$

79. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then

(1) $3bc - 2ad = 0$

(2) $3bc + 2ad = 0$

(3) $2bc - 3ad = 0$

(4) $2bc + 3ad = 0$

Answer (1)

Sol. Let $(\alpha, -\alpha)$ be the point of intersection

$\therefore \quad 4a\alpha - 2a\alpha + c = 0 \Rightarrow \alpha = -\frac{c}{2a}$

and $5b\alpha - 2b\alpha + d = 0 \Rightarrow \alpha = -\frac{d}{3b}$

$\Rightarrow 3bc = 2ad$

$\Rightarrow 3bc - 2ad = 0$

Alternative method :

The point of intersection will be

$$\frac{x}{2ad - 2bc} = \frac{-y}{4ad - 5bc} = \frac{1}{8ab - 10ab}$$

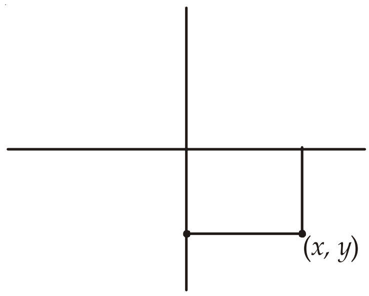
$\Rightarrow x = \frac{2(ad - bc)}{-2ab}$

$\Rightarrow y = \frac{5bc - 4ad}{-2ab}$

$\because$ Point of intersection is in fourth quadrant so x is positive and y is negative.

Also distance from axes is same

So $x = -y$ ($\because$ distance from x-axis is $-y$ as y is negative)



$$\frac{2(ad - bc)}{-2ab} = \frac{-(5bc - 4ad)}{-2ab}$$

$2ad - 2bc = -5bc + 4ad$

$\Rightarrow 3bc - 2ad = 0$ ...(i)

80. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is

(1) $(x^2 + y^2)^2 = 6x^2 + 2y^2$

(2) $(x^2 + y^2)^2 = 6x^2 - 2y^2$

(3) $(x^2 - y^2)^2 = 6x^2 + 2y^2$

(4) $(x^2 - y^2)^2 = 6x^2 - 2y^2$

Answer (1)

Sol. Here ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $a^2 = 6$, $b^2 = 2$

Now, equation of any variable tangent is

$$y = mx \pm \sqrt{a^2m^2+b^2} \quad ...(i)$$

where m is slope of the tangent

So, equation of perpendicular line drawn from centre to tangent is

$$y = \frac{-x}{m} \quad ...(ii)$$

Eliminating m, we get

$$(x^2+y^2)^2 = a^2x^2+b^2y^2$$

$$\Rightarrow \boxed{(x^2+y^2)^2 = 6x^2+2y^2}$$

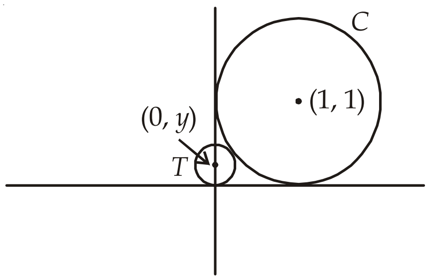
81. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) $\frac{\sqrt{3}}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

Answer (2)

Sol.



$C \equiv (x-1)^2+(y-1)^2=1$

Radius of T = $|y|$

T touches C externally

$(0 - 1)^2 + (y - 1)^2 = (1 + |y|)^2$

$\Rightarrow 1 + y^2 + 1 - 2y = 1 + y^2 + 2|y|$

If $y > 0$,

$y^2 + 2 - 2y = y^2 + 1 + 2y$

$\Rightarrow 4y = 1$

$\Rightarrow y = \frac{1}{4}$

If $y < 0$,

$y^2 + 2 - 2y = y^2 + 1 - 2y$

$\Rightarrow 1 = 2$ (Not possible)

$\therefore y = \frac{1}{4}$

82. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is

(1) $\frac{1}{8}$ (2) $\frac{2}{3}$

(3) $\frac{1}{2}$ (4) $\frac{3}{2}$

Answer (3)

Sol. $y^2 = 4x$ …(1)

$x^2 = -32y$ …(2)

m be slope of common tangent

Equation of tangent (1)

$$y = mx + \frac{1}{m} \quad ...(i)$$

Equation of tangent (2)

$$y = mx + 8m^2 \quad ...(ii)$$

(i) and (ii) are identical

$$\frac{1}{m} = 8m^2$$

$$\Rightarrow m^3 = \frac{1}{8}$$

$$\boxed{m = \frac{1}{2}}$$

Alternative method :

Let tangent to $y^2 = 4x$ be

$$y = mx + \frac{1}{m}$$

as this is also tangent to $x^2 = -32y$

Solving $x^2 + 32mx + \frac{32}{m} = 0$

Since roots are equal

$\therefore D = 0$

$$\Rightarrow (32)^2 - 4 \times \frac{32}{m} = 0$$

$$\Rightarrow m^3 = \frac{4}{32}$$

$$\Rightarrow m = \frac{1}{2}$$

83. The image of the line

$\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line

(1) $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$

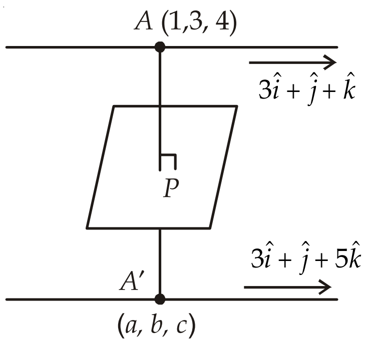
(2) $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$

(3) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

(4) $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$

Answer (3)

Sol.



$\frac{a-1}{2}=\frac{b-3}{-1}=\frac{c-4}{1}=\lambda$

$\Rightarrow$ $a = 2\lambda + 1$

$b = 3 - \lambda$

$c = 4 + \lambda$

$P \equiv \left(\lambda+1,\ 3-\frac{\lambda}{2},\ 4+\frac{\lambda}{2}\right)$

$2(\lambda+1)-\left(3-\frac{\lambda}{2}\right)+\left(4+\frac{\lambda}{2}\right)+3=0$

$2\lambda+2-3+\frac{\lambda}{2}+4+\frac{\lambda}{2}+3=0$

$3\lambda + 6 = 0 \Rightarrow \lambda = -2$

$a = -3, b = 5, c = 2$

So the equation of the required line is

$\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

84. The angle between the lines whose direction consines satisfy the equations $l + m + n = 0$ and $l^2 = m^2 + n^2$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Answer (3)

Sol. $l + m + n = 0$

$l^2 = m^2 + n^2$

Now, $(-m - n)^2 = m^2 + n^2$

$\Rightarrow$ $mn = 0$

$m = 0$ or $n = 0$

If $m = 0$

then $l = -n$

$l^2 + m^2 + n^2 = 1$

Gives

$\Rightarrow$ $n=\pm\frac{1}{\sqrt{2}}$

i.e. (l_1, m_1, n_1)

$=\left(-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$

If $n = 0$

then $l = -m$

$l^2 + m^2 + n^2 = 1$

$\Rightarrow$ $2m^2 = 1$

$\Rightarrow$ $m^2=\frac{1}{2}$

$\Rightarrow$ $m=\pm\frac{1}{\sqrt{2}}$

Let $m=\frac{1}{\sqrt{2}}$

$l=-\frac{1}{\sqrt{2}}$

$n = 0$

(l_2, m_2, n_2)

$=\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right)$

$\therefore$ $\cos\theta=\frac{1}{2}$

$\theta=\frac{\pi}{3}$

85. If $\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]=\lambda[\vec{a}\ \vec{b}\ \vec{c}]^2$ then λ is equal to

(1) 0 (2) 1

(3) 2 (4) 3

Answer (2)

Sol. L.H.S.

$=(\vec{a}\times\vec{b})\cdot[(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})]$

$=(\vec{a}\times\vec{b})\cdot[(\vec{b}\times\vec{c}\cdot\vec{a})\vec{c}-(\vec{b}\times\vec{c}\cdot\vec{c})\vec{a}]$

$=(\vec{a}\times\vec{b})\cdot[[\vec{b}\ \vec{c}\ \vec{a}]\vec{c}]$ $[\because \vec{b}\times\vec{c}.\vec{c}=0]$

$=[\vec{a}\ \vec{b}\ \vec{c}]\cdot(\vec{a}\times\vec{b}\cdot\vec{c})=[\vec{a}\ \vec{b}\ \vec{c}]^2$

$[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]=[\vec{a}\ \vec{b}\ \vec{c}]^2$

So $\lambda = 1$

86. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are

(1) Independent but not equally likely

(2) Independent and equally likely

(3) Mutually exclusive and independent

(4) Equally likely but not independent

Answer (1)

Sol. $P(\overline{A \cup B}) = \frac{1}{6} \Rightarrow P(A \cup B) = 1 - \frac{1}{6} = \frac{5}{6}$

$P(\overline{A}) = \frac{1}{4} \Rightarrow P(A) = 1 - \frac{1}{4} = \frac{3}{4}$

$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4}$

$P(B) = \frac{1}{3}$

$\because \quad P(A) \neq P(B)$ so they are not equally likely.

Also $P(A) \times P(B) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$

$= P(A \cap B)$

$\because \quad P(A \cap B) = P(A) \cdot P(B)$ so A & B are independent.

87. The variance of first 50 even natural numbers is

(1) 437 (2) $\frac{437}{4}$

(3) $\frac{833}{4}$ (4) 833

Answer (4)

Sol. Variance $= \frac{\Sigma x_i^2}{N} - (\bar{x})^2$

$\Rightarrow \quad \sigma^2 = \frac{2^2 + 4^2 + ... + 100^2}{50} - \left(\frac{2 + 4 + ... + 100}{50}\right)^2$

$= \frac{4(1^2 + 2^2 + 3^2 + ... + 50^2)}{50} - (51)^2$

$= 4\left(\frac{50 \times 51 \times 101}{50 \times 6}\right) - (51)^2$

$= 3434 - 2601$

$\Rightarrow \quad \sigma^2 = 833$

88. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals

(1) $\frac{1}{4}$ (2) $\frac{1}{12}$

(3) $\frac{1}{6}$ (4) $\frac{1}{3}$

Answer (2)

Sol. $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$

$f_4(x) - f_6(x) = \frac{1}{4}(\sin^4 x + \cos^4 x) - \frac{1}{6}(\sin^6 x + \cos^6 x)$

$= \frac{1}{4}[1 - 2\sin^2 x \cos^2 x] - \frac{1}{6}[1 - 3\sin^2 x \cos^2 x]$

$= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

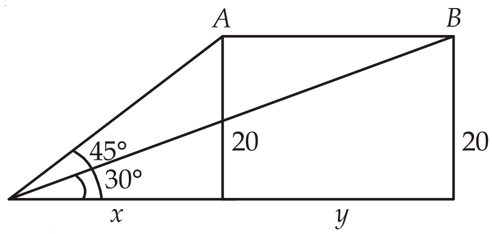
89. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is

(1) $20\sqrt{2}$ (2) $20(\sqrt{3} - 1)$

(3) $40(\sqrt{2} - 1)$ (4) $40(\sqrt{3} - \sqrt{2})$

Answer (2)

Sol.



$t = 1$ s

From figure $\tan 45° = \frac{20}{x}$

and $\tan 30° = \frac{20}{x + y}$

so, $y = 20(\sqrt{3} - 1)$

i.e., speed $= 20(\sqrt{3} - 1)$ m/s.

90. The statement $\sim(p \leftrightarrow \sim q)$ is

(1) A tautology

(2) A fallacy

(3) Equivalent to $p \leftrightarrow q$

(4) Equivalent to $\sim p \leftrightarrow q$

Answer (3)

Sol. $\sim(p \leftrightarrow \sim q)$

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$
F	F	T	F	T
F	T	F	T	F
T	F	T	T	F
T	T	F	F	T

Clearly equivalent to $p \leftrightarrow q$

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