1. The velocity of water in a river is 18 km/hr near the surface. If the river is 5 m deep , find the shearing stress between the horizontal layers of water. The co-efficient of viscosity of water = 10^{-2} poise.

(1) 10^{-1} N/m^2 (2) 10^{-4} N/m^2

(3) 10^{-2} N/m^2 (4) 10^{-3} N/m^2

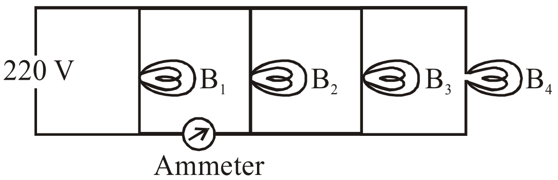
Ans. (4)

Sol. F = – nA dv/dx

$$= -(10^{-3})\left(\frac{5}{5}\right) = -\ 10^{-3}$$

now stress = F/A = 10^{-3}

2. Four bulbs B_1, B_2, B_3 and B_4 of 100 W each are connected to 220 V main as shown in the figure. Ihe reading in an ideal ammeter will be



(1) 1.35 A (2) 0.45 A

(3) 1.80 A (4) 0.90 A

Ans. (1)

Sol. Resistance of any bull

$$R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\ \Omega$$

Net resistance of the cks

$$R_{eq} = \frac{R}{4} = \frac{484}{4} = 121\ \Omega$$

$v = i\ R_{eq}$

$220 \times i \times 121$

i = 1.81 Amp. (total current supplied by the battery)

Current in each branch in $i = \frac{1.81}{4}$ amp.

= 0.45 amp.

Reading of ammeter = 3 × 0.45 = 1.3575 amp.

3. In a Young's double slit experiment, the distance between the two identical slits is 6.1 times larger than the slit width. Then the number of intensity maxima observed within the central maximum of the single slit diffraction pattern is

(1) 12 (2) 24 (3) 3 (4) 6

Ans. (1)

Sol. d = 6.1 a

width of central maxima

$$B_0 = 2\ \frac{D\lambda}{a}$$

$$n \times \frac{D\lambda}{d} = \frac{2D\lambda}{a}$$

$n = 6.1 \times 2$

$= 12$

4. A body is in simple harmonic motion with time period half second (T = 0.5 s) and amplitude one cm (A = 1 cm). Find the average velocity in the interval in which it moves from equilibrium position to half of its amplitude.

(1) 6 cm/s (2) 4 cm/s

(3) 12 cm/s (4) 16 cm/s

Ans. (3)

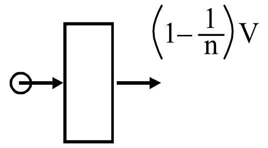
5. A bullet looses $\left(\frac{1}{n}\right)^{th}$ of its velocity passing through one plank. The number of such planks that are required to stop the bullet can be :

(1) n (2) $\frac{n^2}{2n-1}$

(3) Infinte (4) $\frac{2n^2}{n-1}$

Ans. (2)

Sol.



$$\left(1-\frac{1}{n}\right)^2 V^2 = V^2 - 2as$$

$$2as = V^2\left(1-\left(\frac{n-1}{n}\right)^2\right) = V^2\left(\frac{2n-1}{n^2}\right)$$

$$O = V^2 - 2\ ans$$

$$n = \frac{V^2}{2as} = \frac{V^2}{V^2\left(\frac{2n-1}{n^2}\right)} = \frac{n^2}{2n-1}$$

6. Long range radio transmission is possible when the radiowaves are reflected from the ionosphere. For this to happen the frequency of the radiowaves must be in the range :

(1) 150-500 kHz (2) 8-25 MHz

(3) 80-150 MHz (4) 1-3 MHz

Ans. (2)

7. A ray of light is incident from a denser to a rarer medium. The critical angle for total internal reflection is θ_{iC} and the Brewster's angle of incidence is θ_{iB}, such that $\sin\theta_{iC}/\sin\theta_{iB} = \eta = 1.28$. The relative refractive index of the two media is :

(1) 0.4 (2) 0.9

(3) 0.8 (4) 0.2

Ans. (3)

8. If denote microwaves, X rays, infrared, gamma rays, ultra-violet, radio waves and visible parts of the electromagnetic spectrum by M, X, I, G, U, R and V, the following is the arrangement in ascending order of wavelength :

(1) M, R, V, X, U, G and I

(2) R, M, I, V, U, X and G

(3) G, X, U, V, I, M and R

(4) I, M, R, U, V, X and G

Ans. (3)

Sol. The desending order of energy for following waves $E_y > E_x > E_{uv} > E_{vissible} > V_{iR} > V_{MW}$

9. A piece of wood from a recently cut tree shows 20 decays per minute. A wooden piece of same size placed in a museum (obtained from a tree cut many years back) shows 2 decays per minute. If half life of C^{14} is 5730 years, then age of the wooden piece placed in the museum is approximately :

(1) 19039 years

(2) 10439 years

(3) 39049 years

(4) 13094 years

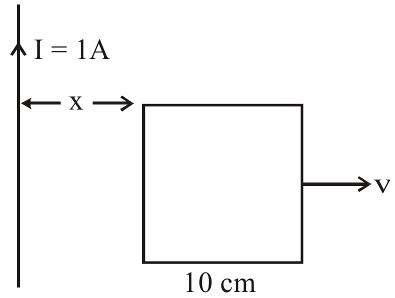
Ans. (1)

Sol. use, $A = \frac{A_0}{2^n}$, $n = \frac{t}{T} = \frac{t}{5730}$

$2 = \frac{20}{2^n}$, $\frac{t}{T} \log 2 = \log 10$

or $t = \frac{5730}{0.3010} = 19039$ year

10. A square frame of side 10 cm and a long straight wire carrying current 1 A are in the plane of the paper. Starting from close to the wire, the frame moves towards the right with a constant speed of 10 ms^{-1} (see figure). The e.m.f induced at the time the left arm of the frame is at x = 10 cm from the wire is :



(1) 1 μV (2) 0.5 μV

(3) 2 μV (4) 0.75 μV

Ans. (1)

Sol. $e = B \times v \times \ell$

$= \left(\frac{\ell wi}{2\pi x}\right) \times v \times \ell$

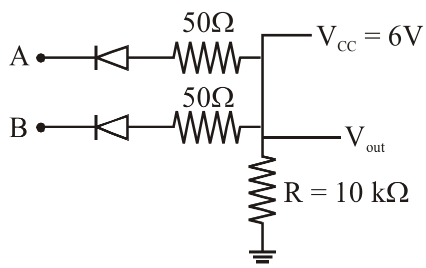
$= \frac{4\pi \times 10^{-7} \times 1 \times 10 \times 10 \times 10^{-2}}{2\pi \times 10 \times 10^{-2}}$

$= 2 \times 10^{-5} \times 10 \times 10^{-2}$

$= 2 \times 10^{-6}$ volt

$= 2\mu v$

11.



Given : A and B are input terminals.

Logic 1 = > 5 V

Logic 0 = < 1 V

Which logic gate operation, the following circuit does ?

(1) OR Gate (2) XOR Gate

(3) AND Gate (4) NOR Gate

Ans. (BONUS)

12. The gravitational field in a region is given by $\vec{g} = 5N/kg\,\hat{i} + 12N/kg\,\hat{j}$. The change in the gravitational potential energy of a particle of mass 2 kg when it is taken from the origin to a point (7m, –3 m) is :

(1) 1 J (2) $13\sqrt{58}$ J

(3) – 71 J (4) 71 J

Ans. (BONUS)

Sol. $\Delta U = -\int \vec{E}_g - \overline{dr}$

$= -\int (5\hat{i} + 12\hat{j}) \cdot (dx\,\hat{i} + dy\hat{j})$

$= -\int_0^7 5dx - \int_0^{-3} 12dy$

$= -5\,[7°] - 12\,[-3]$

$= 1$ J

Energy = 1 × 2 = 2J

13. Match **List-I** (Event) with **List-II** (Order of the time interval for happening of the event) and select the correct option from the options given below the lists.

List-I		List-II	
(a)	Rotation period of earth	(i)	10^5 s
(b)	Revolution period of earth	(ii)	10^7 s
(c)	Period of a light wave	(iii)	10^{-15} s
(d)	Period of sound wave	(iv)	10^{-3} s

(1) (a)-(i), (b)-(ii), (c)-(iv), d-(iii)

(2) (a)-(ii), (b)-(i), (c)-(iv), d-(iii)

(3) (a)-(ii), (b)-(i), (c)-(iii), d-(iv)

(4) (a)-(i), (b)-(ii), (c)-(iii), d-(iv)

Ans. (4)

14. Match **List-I** (Experiment performed) with **List-II** (Phenomena discovered/ associated) and select the correct option from the options given below the lists :

List-I		List-II	
(a)	Davisson and Germer experiment	(i)	Wave nature of electrons
(b)	Millikan's oil drop experiment	(ii)	Charge of an electron
(c)	Rutherford experiment	(iii)	Quantisation of energy levels
(d)	Franck Hertz experiment	(iv)	Existence of nucleus

(1) (a)-(i), (b)-(ii), (c)-(iii) (d)-(iv)

(2) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

(3) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

(4) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

Ans. (2)

Sol. **(a)** Davisson - germar give experimental verification for wave nature of electron.

(b) Millikna's formed experiment about change of an electron

(c) Rutherford performed gold foil experiment and found the exisitance of nucleus.

(d) Franck - Hertz gives information about quantisation of energy level.

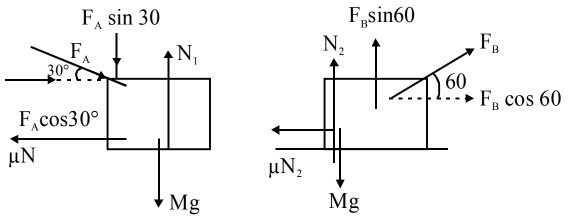
15. A heavy box is to be dragged along a rough horizontal floor. To do so, person A pushes it at an angle 30° from the horizontal and requires a minimum force F_A, while person B pulls the box at an angle 60° from the horizontal and needs minimum force F_B. If the coefficient of friction between the box and the floor is $\frac{\sqrt{3}}{5}$, the ratio $\frac{F_A}{F_B}$ is :

(1) $\sqrt{\frac{3}{2}}$ (2) $\frac{2}{\sqrt{3}}$

(3) $\frac{5}{\sqrt{3}}$ (4) $\sqrt{3}$

Ans. (2)

Sol.



16. A gas is compressed from a volume of 2 m^3 to a volume of 1 m^3 at a constant pressure of 100 N/m^2. Then it is heated at constant volume by supplying 150 J of energy. As a result, the internal energy of the gas :

(1) Increases by 250 J

(2) Decreases by 50 J

(3) Decreases by 250 J

(4) Increases by 50 J

Ans. (BONUS)

Sol. **Case 1** $F \cos 30 = (mg + F_A \sin 30)\mu$

$$\Rightarrow F_A = \frac{\mu mg}{\cos 30 - \mu \sin 30} \quad .....(i)$$

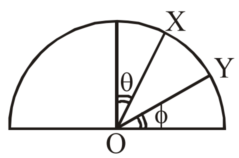
Case 2 $F_B \cos 60 = (mg - f \sin 60)\ \mu$

$$\Rightarrow \quad F_B = \frac{\mu mg}{\cos 60 + \mu \sin 60} \quad .....(ii)$$

Dividing (1) by (2) we get

$$\frac{F_A}{F_B} = \frac{2}{\sqrt{3}}$$

17. A particle is released on a vertical smooth semicircular track from point X so that OX makes angle θ from the vertical (see figure). The normal reaction of the track on the particle vanishes at point Y where OY makes angle ϕ with the horizontal. Then :

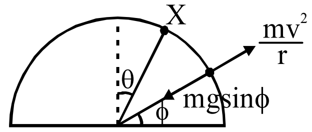


(1) $\sin\phi = \cos\theta$ (2) $\sin\phi = \frac{1}{2}\cos\theta$

(3) $\sin\phi = \frac{3}{4}\cos\theta$ (4) $\sin\phi = \frac{2}{3}\cos\theta$

Ans. (4)

Sol.



$$\frac{mv^2}{r} = mg \sin \phi \quad .....(i)$$

$$mg\ r \cos\theta = \frac{1}{2}mv^2 + rgsin\ \phi$$

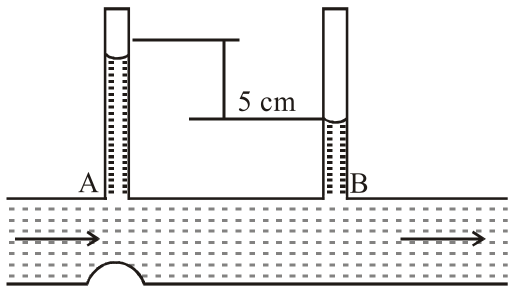
$$\frac{v^2}{rg} = 2\cos\theta - 2\sin\phi \ .....(ii)$$

$\sin\phi = 2\cos\theta - 2\sin\phi$

$3\sin\phi = 2\cos\theta$

$$\sin\phi = \frac{2}{3}\cos\phi$$

18. In the diagram shown, the difference in the two tubes of the manometer is 5 cm, the cross section of the tube at A and B is 6 mm^2 and 10 mm^2 respectively. The rate at which water flows through the tube is ($g = 10\ ms^{-2}$)



(1) 10.0 cc/s (2) 7.5 cc/s

(3) 8.0 cc/s (4) 12.5 cc/s

Ans. (2)

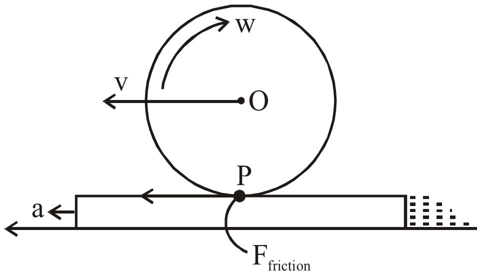
Sol. $$Q = A_1A_2\sqrt{\frac{2g(h_1 - h_2)}{(A_1^2 - A_2^2)}}$$

$$= 6 \times 10 \times 10^{-4}\sqrt{\frac{2\times 980(5)}{(-36+100)\times 10^{-2}}}$$

$$= 6 \times 10^{-3}\sqrt{\frac{9800\times 10^2}{64}}$$

$$= \frac{6\times 10^{-3}\times 1000}{8} = 7.5$$

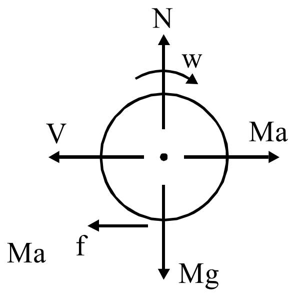
19. Consider a cylinder of mass M resting on a rough horizontal rug that is pulled out from under it with acceleration 'a' perpendicular to the axis of the cylinder. What is $F_{friction}$ at point P ? It is assumed that the cylinder does not slip.



(1) Mg (2) $\frac{Ma}{3}$ (3) Ma (4) $\frac{Ma}{2}$

Ans. (2)

Sol.



Ma = f

20. A large number of liquid drops each of radius r coalesce to from a single drop of radius R. The energy released in the process is converted into kinetic energy of the big drop so formed. The speed of the big drop is (given surface tension of liquid T, density ρ)

(1) $\sqrt{\frac{4T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$ (2) $\sqrt{\frac{T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$

(3) $\sqrt{\frac{2T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$ (4) $\sqrt{\frac{6T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$

Ans. (4)

Sol. $T\, 4\pi R^3 \left\{\frac{1}{r}-\frac{1}{R}\right\}=\frac{1}{2}\left(\frac{4}{3}\pi R^3 d\right)$

$$V = \sqrt{\frac{6T}{d}\left\{\frac{1}{r}-\frac{1}{R}\right\}}$$

21. The gap between the plates of a parallel plate capacitor of area A and distance between plates d, is filled with a dielectric whose permittivity varies linearly from $\in_1$ at one plate to $\in_2$ at the other. The capacitance of capacitor is :

(1) $\in_0(\in_2 + \in_1)A/2d$

(2) $\in_0 A/[d\ell n(\in_2/\in_1)]$

(3) $\in_0(\in_1 + \in_2)A/d$

(4) $\in_0(\in_2 - \in_1)A/[d\ell n(\in_2/\in_1)]$

Ans. (4)

Sol. $\varepsilon = \left(\frac{\varepsilon_2-\varepsilon_1}{d}\right) x + \varepsilon_1$

$$dC = \frac{\varepsilon_0 \varepsilon A}{dx}$$

$C_{eq} = \int dc$

22. A gas molecule of mass M at the surface of the Earth has kinetic energy equivalent to 0°C. If it were to go up straight without colliding with any other molecules, how high it would rise ? Assume that the height attained is much less than radius of the earth, (k_B is Boltzmann constant)

(1) $\frac{819k_B}{2Mg}$ (2) $\frac{546k_B}{3Mg}$

(3) 0 (4) $\frac{273k_B}{2Mg}$

Ans. (1)

Sol. $\frac{3}{2}KT=\frac{1}{2}mv^2$

$$v^2 = \frac{3KT}{m}$$

$v^2 = u^2 + 2as$

$$O = \frac{3KT}{m} - 2gs$$

$$S = \frac{3K}{2}\frac{(273)}{mg}$$

$$S = \frac{819K}{2mg}$$

23. The electric field in a region of space is given by, $\vec{E} = E_0\hat{i} + 2E_0\hat{j}$ where $E_0 = 100$ N/ C. The flux of this field through a circular surface of radius 0.02 m parallel to the Y-Z plane is nearly

(1) 0.02 Nm^2/C (2) 0.125 Nm^2/C

(3) 3.14 Nm^2/C (4) 0.005 Nm^2/C

Ans. (2)

Sol. $\phi = (E_0\hat{i} + 2E_0\hat{j}) \cdot (\pi R^2)\hat{i}$

$\phi = 100 \times \pi \times (0.02)^2$

$= 0.125$

24. A ball of mass 160 g is thrown up at an angle of 60° to the horizontal at a speed of 10 ms^{-1}. The angular momentum of the ball at the highest point of the trajectory with respect to the point from which the ball is thrown is nearly ($g = 10\ ms^{-2}$)

(1) 3.0 kg m^2/s

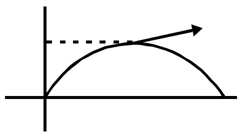
(2) 3.46 kg m^2/s

(3) 1.73 kg m^2/s

(4) 6.0 kg m^2/s

Ans. (1)

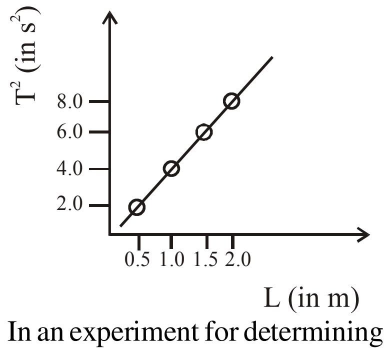
Sol. $\overline{L} = \overline{r} \times \overline{10}$



$$= \frac{160}{1000} \times 10 \times \frac{1}{2} \times \frac{10 \times 10}{2 \times 10}\left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 10 \times \frac{1}{2} \times \frac{1}{2} \times \frac{3}{4} = 3$$

25.



In an experiment for determining the gravitational acceleration g of a place with the help of a simple pendulum, the measured time period square is plotted against the string length of the pendulum in the figure.

What is the value of g at the place ?

(1) 9.91 m/s^2 (2) 10.0 m/s^2

(3) 9.87 m/s^2 (4) 9.81 m/s^2

Ans. (3)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$T^2 = 4\pi^2\frac{\ell}{g}$$

$$g = 4\pi^2\frac{\ell}{T^2}$$

$$T^2 = \frac{4\pi^2}{g}$$

$$\frac{4\pi^2}{g} = 4$$

$g = \pi^2 = 9.87$

26. An example of a perfect diamagnet is a superconductor. This implies that when a superconductor is put in a magnetic field of intensity B, the magnetic field Bs inside the superconductor will be such that :

(1) $B_s < B$ but $B_s \neq 0$ (2) $B_s = B$

(3) $B_s = 0$ (4) $B_s = -B$

Ans. (4)

Sol.

27. The total length of a sonometer wire between fixed ends is 110 cm. Two bridges are placed to divide the length of wire in ratio 6 : 3 : 2. The tension in the wire is 400 N and the mass per unit length is 0.01 kg/m. What is the minimum common frequency with which three parts can vibrate ?

(1) 166 Hz (2) 1000 Hz

(3) 1100 Hz (4) 100 Hz

Ans. (2)

Sol. $\ell_1 : \ell_2 : \ell_3 = 6 : 3 : 2$

so $\ell_1 = 60$ cm

$\ell_2 = 30$ cm

$\ell_3 = 20$ cm

60, 30, 20

$$\frac{\lambda}{2} = 10 \text{ cm}$$

$$f = \frac{200}{2} = 1000 \text{ Hz}$$

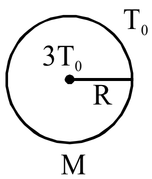
28. A black coloured solid sphere of radius R and mass M is inside a cavity with vacuum inside. The walls of the cavity are maintained at temperature T_0. The initial temperature of the sphere is $3T_0$. If the specific heat of the material of the sphere varies as αT^3 per unit mass with the temperature T of the sphere, where α is a constant, then the time taken for the sphere to cool down to temperature $2T_0$ will be
(σ is Stefan Boltzmann constant)

(1) $\frac{M\alpha}{16\pi R^2\sigma}\ell n\left(\frac{16}{3}\right)$ (2) $\frac{M\alpha}{16\pi R^2\sigma}\ell n\left(\frac{3}{2}\right)$

(3) $\frac{M\alpha}{4\pi R^2\sigma}\ell n\left(\frac{16}{3}\right)$ (4) $\frac{M\alpha}{4\pi R^2\sigma}\ell n\left(\frac{3}{2}\right)$

Ans. (1)

Sol. $\frac{d\theta}{dt} = -\frac{\sigma Ae}{ms}(\theta^4 - \theta_0^4)$



$\frac{d\theta}{dt} = -\frac{\sigma Ae}{m^{-}\alpha\theta^3}(\theta^4 - \theta_0^4)$

$\int\frac{\theta^3}{\theta^4 - \theta_0^4}d\theta = -\frac{\sigma Ae}{m^{-}\alpha}\int dt$

let $\theta^4 - \theta_0^4 = x$

$4\theta^3 d\theta = dx$

$\int\frac{dx}{4x} = -\frac{\sigma Ae}{m^2\alpha}t$

$\frac{1}{4}[\log x] = -\frac{\sigma Ae}{m^{-}\alpha}[t]_0^t$

$\frac{1}{4}\left[\log(\theta^4 - \theta_0^4)\right]_{3T_0}^{2T_0} = -\frac{\sigma Ae}{m^{-}\alpha}t$

$\frac{1}{4}\log\left(\frac{(2T_0)^4 - T_0^4}{(3T_0)^4 - T_0^4}\right) = -\frac{\sigma Ae}{m^{-}\alpha}t$

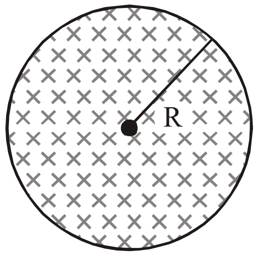
$t = -\frac{\alpha m}{4\sigma Ae}\ln\frac{T_0^4}{80T_0^4}$

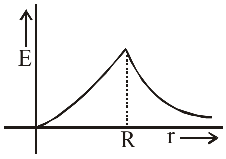
$= \frac{\alpha M}{4\sigma Ae}\ln\frac{3}{16}$

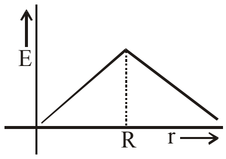
$= \frac{\alpha M}{4\sigma(4\pi R^2)e}\ln\frac{3}{16}$

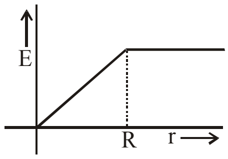
$t = t = \frac{\alpha M}{16\pi\sigma R^2}\ln\frac{16}{3}$

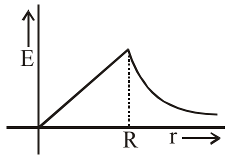
29. Figure shows a circular area of radius R where a uniform magnetic field $\vec{B}$ is going into the plane of paper and increasing in magnitude at a constant rate. In that case, which of the following graphs, drawn schematically, correctly shows the variation of the induced electric field E(r)?



(1) 

(2) 

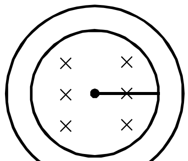
(3) 

(4) 

Ans. (4)

Sol. $B = B_0 t$

r < R



$r > R$

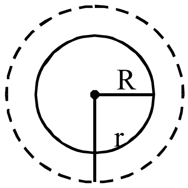
$\oint\vec{\varepsilon}\cdot de = \frac{d\phi}{dt}$

$\Rightarrow \varepsilon(2\pi r) = B\pi r^2$

$e \propto r$

$r > R$

$\varepsilon\,(2\pi r) = B\pi R^2$



$\varepsilon \propto \frac{1}{r}$

30. The diameter of the objective lens of microscope makes an angle β at the focus of the microscope. Further, the medium between the object and the lens is an oil of refractive index n. Then the resolving power of the microscope.

(1) Increases with decreasing value of n

(2) Increases with increasing value of $\frac{1}{n\sin 2\beta}$

(3) Increases with decreasing value of β

(4) Increases with increasing value of n sin 2β

Ans. (4)

PART B – CHEMISTRY

1. For an ideal Solution of two components A and B, which of the following is true ?
(1) $\Delta H_{mixing} < 0$ (zero)
(2) A – A, B – B and A – B interactions are identical
(3) A – B interaction is stronger than A – A and B – B interactions
(4) $\Delta H_{mixing} > 0$ (zero)

Ans. (2)

2. Which of these statements is not true ?
(1) In aqueous solution, the Tl^{+} ion is much more stable than Tl (III)
(2) $LiAlH_4$ is a versatile reducing agent in organic synthesis.
(3) NO^{+} is not isoelectronic with O_2
(4) B is always covalent in its compounds

Ans. (4)

3. Nickel(Z = 28) combines with a uninegative monodentate ligand to form a dia magnetic complex $[NiL_4]^{2-}$. The hybridisation involved and the number of unpaired electrons present in the complex are respectively :
(1) sp^3, zero (2) sp^3. two
(3) dsp^2 one (4) dsp^2, zero

Ans. (4)

4. For the reaction
$3A + 2B \rightarrow C + D$
the differential rate law can be written as

(1) $-\frac{1}{3}\frac{d[A]}{dt} = \frac{d[C]}{dt} = k[A]^n[B]^m$

(2) $+\frac{1}{3}\frac{d[A]}{dt} = -\frac{d[C]}{dt} = k[A]^n[B]^m$

(3) $\frac{1}{3}\frac{d[A]}{dt} = \frac{d[C]}{dt} = k[A]^n[B]^m$

(4) $-\frac{d[A]}{dt} = \frac{d[C]}{dt} = k[A]^n[B]^m$

Ans. (1)

Sol. Rate $= -\frac{1}{3}\frac{d(A)}{dt} = -\frac{1}{2}\frac{d(B)}{dt} = \frac{d(C)}{dt} = \frac{d(D)}{dt}$

Rate = K $(A)^n$ $(B)^n$

$$-\frac{1}{3}\frac{d(A)}{dt} = -\frac{1}{2}\frac{d(B)}{dt} = \frac{d(C)}{dt} = \frac{d(D)}{dt}$$
$$= K(A)^n(B)^m$$

so $-\frac{1}{3}\frac{d(A)}{dt} = \frac{d(C)}{dt} = K(A)^n(B)^m$

5. Which one of the following molecules is paramagnetic?
(1) NO (2) O_3
(3) N_2 (4) CO

Ans. (1)

6. Ionization energy of gaseous Na atoms is 495.5 $kjmol^{-1}$. The lowest possible frequency of light that ionizes a sodium atom is
($h = 6.626 \times 10^{-34}$ Js, $N_A = 6.022 \times 10^{23}$ mol^{-1})
(1) 3.15×10^{15} s^{-1} (2) 4.76×10^{14} s^{-1}
(3) 1.24×10^{15} s^{-1} (4) 7.50×10^{4} s^{-1}

Ans. (3)

Sol. $\Delta E = hv$

$$v = \frac{\Delta E}{h}$$

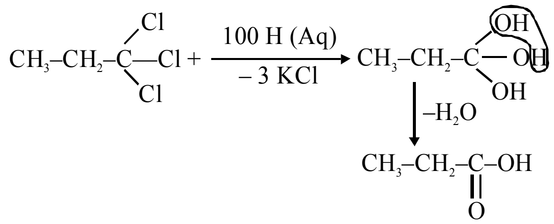
$$v = \frac{495.5 \times 10^3 \text{ Joule}}{6.023 \times 10^{23}} \times \frac{1}{6.626 \times 10^{-34}}$$

$v = 1.24 \times 10^{15}$ sec^{-1}

7. The major product formed when 1,1,1 - trichloro - propane is treated with aqueous potassium hydroxide is :
(1) 2 - Propanol (2) Propionic acid
(3) Propyne (4) 1 - Propanol

Ans. (2)

Sol.



8. The final product formed when Methyl amine is treated with $NaNO_2$ and HCI is :
(1) Methylcyanide (2) Methylalcohol
(3) Nitromethane (4) Diazomethane

Ans. (2)

Sol. $CH_3–NH_2 + HNO_2 \longrightarrow CH_3–OH$
it is third order reaction.

9. Which one of the following is an example of thermosetting polymers?
(1) Nylon 6, 6 (2) Bakelite
(3) Buna-N (4) Neoprene

Ans. (2)

Sol. Bakelite becomes find on heating and the process is irreversible

10. Which one of the following has largest ionic radius ?

(1) Li^+ (2) F^- (3) O_2^{2-} (4) B^{3+}

Ans. (3)

11. Consider the reaction

$$H_2SO_{3(aq)} + Sn^{4+}_{(aq)} + H_2O_{(l)} \rightarrow Sn^{2+}_{(aq)} + HSO^-_{4(aq)} + 3H^+_{(aq)}$$

Which of the following statements is correct?

(1) H_2SO_3 is the reducing agent because it undergoes oxidation

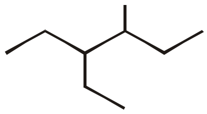
(2) H_2SO_3 is the reducing agent because it undergoes reduction

(3) Sn^{4+} is the reducing agent because it undergoes oxidation

(4) Sn^{4+} is the oxidizing agent because it undergoes oxidation

Ans. (1)

12. The correct 1UPAC name of the following compound is:



(1) 3, 4 - ethyl methylhexane

(2) 3 - ethyl - 4 - methylhexane

(3) 4 - ethyl - 3 - methylhexane

(4) 4 - methyl - 3 - ethylhexane

Ans. (2)

13. Which one of the following ores is known as Malachite

(1) Cu_2O (2) $CuFeS_2$

(3) Cu_2S (4) $Cu(OH)_2.CuCO_3$

Ans. (4)

14. For the decomposition of the compound, represented as

$$NH_2COONH_4(s) \rightleftharpoons 2NH_3(g) + CO_2(g)$$

the $K_P = 2.9 \times 10^{-5}$ atm^3.

If the reaction is started with 1 mol of the compound, the total pressure at equilibrium would be :

(1) 38.8×10^{-2} atm (2) 1.94×10^{-2} atm

(3) 5.82×10^{-2} atm (4) 7.66×10^{-2} atm

Ans. (3)

Sol. $NH_2COONH_4 \rightleftharpoons 2NH_3 + CO_2$

2P P

$Kp = (2p)^2 (P) = 4p^3 = 725 \times 10^{-6}$

$p = 1.94 \times 10^{-2}$

total pressure = 2P + P = 5.82×10^{-2} atm

15. The reason for double helical structure of DNA is the operation of :

(1) Electrostatic attractions

(2) Hydrogen bonding

(3) Dipole - Dipole interactions

(4) van der Waals forces

Ans. (2)

16. Amongst LiCl, RbCl, $BeCl_2$ and $MgCl_2$ the compounds with the greatest and the least ionic character, respectively are :

(1) RbCl and $MgCl_2$ (2) LiCl and RbCl

(3) $MgCl_2$ and $BeCl_2$ (4) RbCl and $BeCl_2$

Ans. (4)

17. Which one of the following compounds will not be soluble in sodium bicarbonate ?

(1) Benzene sulphonic acid

(2) Benzoic acid

(3) O-Nitrophenol

(4) 2, 4, 6 - Trinitrophenol

Ans. (3)

Sol. Bicarbonates are weak bases can't react with wenler acid

18. Among the following organic acids, the acid present in rancid butter is:

(1) Lactic acid (2) Acetic acid

(3) Pyruvic acid (4) Butyric acid

Ans. (4)

19. The total number of octahedral void(s) per atom present in a cubic close packed structure is :-

(1) 1 (2) 2 (3) 3 (4) 4

Ans. (1)

Sol. CCP no. of octrahedral void = $12 \times \frac{1}{4} + 1 = 4$

(edge) (centre)

per atom octrahedral void is 1.

20. The observed osmotic pressure for a 0.10 M solution of $Fe(NH_4)_2(SO_4)_2$ at 25°C is 10.8 atm. The expected and experimental (observed) values of Van't Hoff factor (i) will be respectively :

(R= 0.082 L atm k^- mol^{-1})

(1) 3 and 5.42 (2) 5 and 3.42

(3) 4 and 4.00 (4) 5 and 4.42

Ans. (4)

Sol. $\pi = CRTi$

$$i = \frac{\pi}{CRT} = \frac{10.8}{0.1 \times 0.0821 \times 298}$$

i = 4.42 (observed)

21. Sulphur dioxide and oxygen were allowed to diffuse through a porous partition. 20 dm^3 of SO_2 diffuses through the porous partition in 60 seconds. The volume of O_2 in dm^3 which diffuses under the similar condition in 30 seconds will be (atomic mass of sulphur = 32u)

(1) 28.2 (2) 14.1

(3) 7.09 (4) 10.0

Ans. (2)

Sol. $\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$

$$\frac{20/60}{V/30} = \sqrt{\frac{32}{64}}$$

$$\frac{10}{V} = \sqrt{\frac{1}{2}} \quad \Rightarrow \quad V = 10\sqrt{2} = 14.1$$

22. An octahedral complex with molecular composition $M.5NH_3.Cl.SO_4$ has two isomers, A and B. The solution of A gives a white precipitate with $AgNO_3$ solution and the solution of B gives white precipitate with $BaCl_2$ solution. The type of isomerism exhibited by the complex is :

(1) Geometrical isomerism

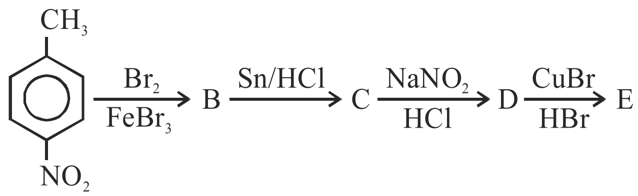
(2) Coordinate isomerism

(3) Ionisation isomerism

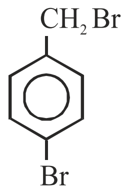
(4) Linkage isomerism

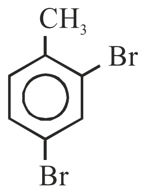
Ans. (3)

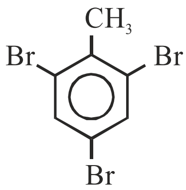
23. In a set of reactions p-nitrotoluene yielded a product E

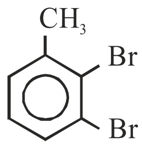


The product E would be :-

(1) 

(2) 

(3) 

(4) 

Ans. (2)

24. How many electrons are involved in the following redox reaction ?

$Cr_2O_7^{2-} + Fe^{2+} + C_2O_4^{2-} \rightarrow$

$Cr^{3+} + Fe^{3+} + CO_2$ (Unbalanced)

(1) 3 (2) 4

(3) 5 (4) 6

Ans. (4)

25. Zirconium phosphate $[Zr_3(PO_4)_4]$ dissociates into three zirconium cations of charge +4 and four phosphate anions of charge –3. If molar solubility of zirconium phosphate is denoted by S and its solubility product by K_{sp} then which of the following relationship between S and K_{sp} is correct ?

(1) $S = \{K_{sp}/144\}^{1/7}$

(2) $S = \{K_{sp}/(6912)^{1/7}\}$

(3) $S = (K_{sp}/6912)^{1/7}$

(4) $S = \{K_{sp}/6912\}^7$

Ans. (3)

Sol. $Zr_3(PO_4)_4 \longrightarrow 3Zr^{+3} + 4\ PO_4^{-3}$

$\qquad\qquad\qquad\qquad 3s \qquad 4s$

$Ksp = (3s)^3\ (4s)^4$

$$s = \left(\frac{ksp}{6912}\right)^{1/7}$$

26. Choose the correct statement with respect to the vapour pressure of a liquid among the following :-

(1) Increases linearly with increasing temperature

(2) Decreases non-linearly with increasing temperature

(3) Decreases linearly with increasing temperature

(4) Increases non-linearly with increasing temperature

Ans. (4)

27. Amongst the following, identify the species with an atom in +6 oxidation state :

(1) $[MnO_4]^-$

(2) $[Cr(CN)_6]^{3-}$

(3) Cr_2O_3

(4) CrO_2Cl_2

Ans. (4)

28. Example of a three-dimensional silicate is :

(1) Beryls

(2) Zeolites

(3) Feldspars

(4) Ultramarines

Ans. (2, 3, 4)

29. Williamson synthesis of ether is an example of

(1) Nucleophilic addition

(2) Electrophilic substitution

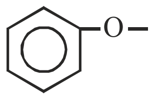
(3) Nucleophilic substitution

(4) Electrophilic addition

Ans. (3)

Sol. Nucleophilic substitution

$$R\overset{\ominus}{O}\ \overset{\oplus}{Na}\ (\text{or } K^{\oplus}) + R' - X \longrightarrow RO{-}R'$$

30. Which one of the following substituents at *para*-position is most effective in stabilizing the phenoxide  ion?

(1) $-\ CH_3$ (2) $-\ CH_2OH$

(3) $-\ OCH_3$ (4) $-\ COCH_3$

Ans. (4)

Sol. $-\overset{O}{\overset{\|}{C}}-CH_3$ is is an electron withdrawing group which stabilises the anion.

PART C – MATHEMATICS

1. The circumcentre of a triangle lies at the origin and its centroid is the mid point of the line segment joining the points $(a^2 + 1, a^2 + 1)$ and $(2a, -2a)$, $a \neq 0$. Then for any a, the orthocentre of this triangle lies on the line :

(1) $y - (a^2 + 1)x = 0$

(2) $y + x = 0$

(3) $(a - 1)^2x - (a + 1)^2 y = 0$

(4) $y - 2ax = 0$

Ans. (3)

Sol. $H = (x, y) = \left(\frac{3}{2}(a+1)^2, \frac{3}{2}(a-1)^2\right)$

$$\frac{x}{y} = \frac{(a+1)^2}{(a-1)^2}$$

$(a - 1)^2x = (a + 1)^2y$

2. Let $f(n) = \left[\frac{1}{3} + \frac{3n}{100}\right]n$, where [n] denotes the greatest integer less than or equal to n. Then $\sum_{n=1}^{56} f(n)$ is equal to :-

(1) 56 (2) 1399 (3) 689 (4) 1287

Ans. (2)

Sol. $\sum_{n=1}^{56} f(x) = \left[\frac{1}{3} + \frac{3\times1}{100}\right]\times1 + \ldots\ldots + \left[\frac{1}{3} + \frac{3\times22}{100}\right]\times22$

$+ \left[\frac{1}{3} + \frac{3\times23}{100}\right]\times23$ …….. + ………

$\left[\frac{1}{3} + \frac{3\times55}{100}\right]\times55 + \left[\frac{1}{3} + \frac{3\times56}{100}\right]\times56$

$= 0 + \ldots\ldots\ldots + 0 + 23 + 24 + \ldots\ldots\ldots + 55 + 2 \times 56$

$= \frac{55(56)}{2} - \frac{22(23)}{2} + 112$

$= 11 (5 \times 28 - 23) + 112$

$= 11 \times 117 + 112$

$= 1287 + 112$

$= 1399$

3. If $\vec{x} = 3\hat{i} - 6\hat{j} - \hat{k}$, $\vec{y} = \hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{z} = 3\hat{i} - 4\hat{j} - 12\hat{k}$ then the magnitude of the projection of $\vec{x} \times \vec{y}$ on $\vec{z}$ is :-

(1) 13 (2) 14 (3) 15 (4) 12

Ans. (2)

Sol. $\left|\frac{(\bar{x}\times\bar{y})\cdot\bar{z}}{|\bar{z}|}\right| = \frac{\begin{vmatrix} 3 & -6 & -1 \\ 1 & 4 & -3 \\ 3 & -4 & -12 \end{vmatrix}}{\sqrt{9+16+144}} = 14$

4. The equation of the circle described on the chord $3x + y + 5 = 0$ of the circle $x^2 + y^2 = 16$ as diameter is :-

(1) $x^2 + y^2 + 3x + y - 2 = 0$

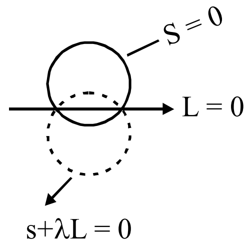
(2) $x^2 + y^2 + 3x + y - 22 = 0$

(3) $x^2 + y^2 + 3x + y + 1 = 0$

(4) $x^2 + y^2 + 3x + y - 11 = 0$

Ans. (4)

Sol.



5. The principal value of $\tan^{-1}\left(\cot\frac{43\pi}{4}\right)$ is:-

(1) $-\frac{3\pi}{4}$ (2) $\frac{\pi}{4}$ (3) $-\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

Ans. (3)

Sol. $\tan^{-1} [\cot (11\pi - \pi/4)]$

$= \tan^{-1} [-\cot \pi/4]$

$= -\tan^{-1} (\cot \pi/4)$

$= -\tan^{-1} (1)$

$= -\pi/4$

6. Let A and E be any two events with positive probabilities :

Statement - 1 : $P(E/A) \geq P(A/E)P(E)$

Statement - 2 : $P(A/E) \geq P(A\cap E)$.

(1) Both the statements are false

(2) Both the statements are true

(3) Statement - 1 is false, Statement - 2 is true.

(4) Statement -1 is true, Statement - 2 is false

Ans. (2)

Sol. L.H.S.

St- I $P\left(\frac{E}{A}\right) = \frac{P(E \cap A)}{P(A)}$

R.H.S. $P\left(\frac{A}{E}\right)P(E)$

$\frac{P(A \cap E)}{(PE)} \cdot P(E)$

$\because O \le P(A) \le 1$

So $\frac{P(E \cap A)}{P(A)} \ge P(A \cap E)$

St. - I is true

St. - 2 $P\left(\frac{A}{E}\right) = \frac{P(A \cap E)}{P(E)}$

So $\frac{P(A \cap E)}{P(E)} \ge P(A \cap E)$ $\because 0 \le P(E) \le 1$

st - 2 is true

7. If a line L is perpendicular to the line $5x - y = 1$, and the area of the triangle formed by the line L and the coordinate axes is 5, then the distance of line L from the line $x + 5y = 0$ is :-

(1) $\frac{7}{\sqrt{5}}$ (2) $\frac{5}{\sqrt{13}}$ (3) $\frac{7}{\sqrt{13}}$ (4) $\frac{5}{\sqrt{7}}$

Ans. (2)

Sol. Equation = of line L is

$x + 5y + c = 0$

$\frac{c^2}{2|1 \times 5|} = 5$

$\Rightarrow c = \pm 5\sqrt{2}$

$\therefore$ Eq of line L is

$x + 5y \pm 5\sqrt{2} = 0$

Its distance from $x + 5y = 0$ is $\frac{5}{\sqrt{13}}$

8. Let A and B be any two 3×3 matrices. If A is symmetric and B is skewsymmetric, then the matrix $AB - BA$ is :

(1) neither symmetric nor skewsymmetric

(2) skewsymmetric

(3) symmetric

(4) I or - I, where I is an identity matrix.

Ans. (3)

Sol. $A = A^T$, $B = -B^T$

Let $P = AB - BA$

$P^T = (AB - BA)^T$

$= (AB)^T - (BA)^T$

$= B^TA^T - A^TB^T$

$= -BA + AB$

$= AB - BA$

$= P$

So $AB - BA$ is symmetric

9. If the angle between the line $2(x + 1) = y = z + 4$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is $\frac{\pi}{6}$, then the value of λ is :-

(1) $\frac{45}{11}$ (2) $\frac{135}{7}$ (3) $\frac{45}{7}$ (4) $\frac{135}{11}$

Ans. (3)

Sol. $\sin\frac{\pi}{6} = \frac{1 - 1 + \sqrt{\lambda}}{\sqrt{\frac{1}{4} + 1 + 1}\sqrt{4 + 1 + \lambda}}$

$\Rightarrow \frac{1}{2} = \frac{\sqrt{\lambda}}{\sqrt{\frac{9}{4}}\sqrt{5 + \lambda}} \Rightarrow \lambda = \frac{45}{7}$

10. If $\frac{dy}{dx} + y\tan x = \sin 2x$ and $y(0) = 1$, then $y(\pi)$ is equal to :-

(1) 5 (2) -1 (3) 1 (4) -5

Ans. (4)

Sol. I.F. $= e^{\int \tan x dx} = e^{\log \sec x} = \sec x$

y.sec x = ∫ sin 2x.sec x dx + c

= ∫ 2 sinx.cos x. sec x dx + c

y sec x = – 2 cos x + c

y(0) = 1 1.1 = – 1.2 + c ⇒ c = 3

y sec x = – 2 cos x + 3

y(π) = ? y (– 1) = (–2) (–1) + 3

⇒ – y = 5 ⇒ y = – 5

11. Two women and some men participated in a chess tournament in which every participant played two games with each of the other participants. If the number of games that the men played between them- selves exceeds the number of games that the men played with the women by 66, then the number of men who participated in the tournament lies in the interval :

(1) (14, 17) (2) [8, 9]
(3) (11, 13] (4) [10, 12)

Ans. (4)

Sol. Let n men participated

$2({}^nC_2 - 2n) = 66 \Rightarrow 2\left\{\frac{n(n-1)}{2} - 2n\right\} = 66$

$\Rightarrow n^2 - 5n - 66 = 0 \quad \Rightarrow n = 11$, (not possible)

which lies in [10, 12]

12. The number of terms in an A.P. is even; the sum of the odd terms in it is 24 and that the even terms is 30. If the last term exceeds the first term by $10\frac{1}{2}$, then the number of terms in the A.P. is :

(1) 8 (2) 4 (3) 16 (4) 12

Ans. (1)

Sol. Let no. of terms = 2n

a, (a + d), (a + 2d),........ a + (2n – 1)d

sum of even terms

$\frac{n}{2}[2(a + d) + (n - 1)2d] = 30$ ......(i)

sum of odd terms

$\frac{n}{2}[2a + (n-1)2d] = 24$ ....(ii)

$a + (2n - 1) d - a = \frac{21}{2}$ ......(iii)

eq. (i)....eq. (ii)

$\frac{n}{2} \times 2d = 6 \Rightarrow nd = 6$ ......(iv)

$(2n - 1)d = \frac{21}{2}$ ......(v)

$\frac{eq(iv)}{eq(v)} = \frac{n}{2n-1} = \frac{4}{7} \Rightarrow 8n - 4 = 7n$

n = 4

so no. of terms = 8

13. If $\Delta_r = \begin{vmatrix} r & 2r-1 & 3r-2 \\ \frac{n}{2} & n-1 & a \\ \frac{1}{2}n(n-1) & (n-1)^2 & \frac{1}{2}(n-1)(3n+4) \end{vmatrix}$

then the value of $\sum_{r=1}^{n-1}\Delta_r$:-

(1) depends only on n

(2) is independent of both a and n

(3) depends only on a

(4) depends both on a and n

Ans. (2)

Sol. $\sum_{r=1}^{n-1}\Delta_r = \begin{vmatrix} \sum_{r=1}^{n-1} r & \sum_{r=1}^{n-1} 2r-1 & \sum_{r=1}^{n-1} 3r-2 \\ \frac{n}{2} & n-1 & a \\ \frac{n(n-1)}{2} & (n-1)^2 & \frac{1}{2}(n-1)(3n+4) \end{vmatrix}$

$= \begin{vmatrix} \frac{n(n-1)}{2} & (n-1)^2 & \frac{(n-1)(3n+4)}{2} \\ \frac{n}{2} & n-1 & a \\ \frac{n(n-1)}{2} & (n-1)^2 & \frac{(n-1)(3n+4)}{2} \end{vmatrix}$

R_1 and R_3 are identical so

$\sum_{r=1}^{n-1}\Delta_r = 0$ is independent of a, and n

14. If non-zero real numbers b and c are such that min f(x) > max g(x), where
$f(x) = x^2 + 2bx + 2c^2$ and
$g(x) = -x^2 - 2cx + b^2$ $(x \in R)$;

then $\left|\frac{c}{b}\right|$ lies in the interval:-

(1) $\left[\frac{1}{\sqrt{2}}, \sqrt{2}\right]$ (2) $(\sqrt{2}, \infty)$

(3) $\left[\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ (4) $\left(0, \frac{1}{2}\right)$

Ans. (2)

Sol. $\frac{4\times1\times2c^2 - 4b^2}{4} > \frac{4\times-1\times b^2 - 4c^2}{-4}$

$2c^2 - b^2 > b^2 + c^2$

$c^2 > 2b^2$

$\frac{c^2}{b^2} > 2$

$\left|\frac{c}{b}\right| > \sqrt{2}$

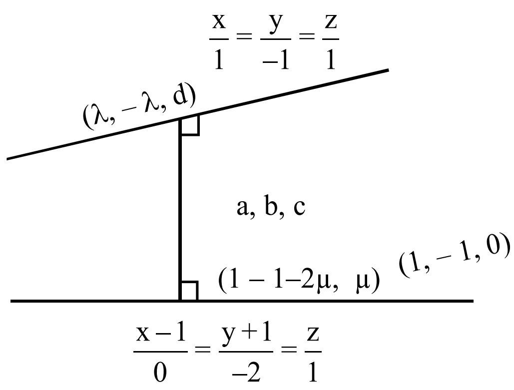
15. Equation of the line of the shortest distance between the lines $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ and $\frac{x-1}{0} = \frac{y+1}{-2} = \frac{z}{1}$ is :-

(1) $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{1}$ (2) $\frac{x}{-2} = \frac{y}{1} = \frac{z}{2}$

(3) $\frac{x}{1} = \frac{y}{-1} = \frac{z}{-2}$ (4) $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{-2}$

Ans. (4)

Sol. $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 0 & -2 & 1 \end{vmatrix}$



$\frac{1-\lambda}{1} = \frac{-1-2\mu+\lambda}{-1} = \frac{\mu-\lambda}{-2}$

$-2 = 3\lambda + \mu$

$-1 + \lambda = -1 - 2\mu + \lambda$

$\mu = 0$ $\lambda = -\frac{2}{3}$

16. The coefficient of x^{1012} in the expansion of $(1 + x^n + x^{253})^{10}$, (where $n \le 22$ is any positive integer), is:-

(1) $^{253}C_4$ (2) 4n (3) 1 (4) $^{10}C_4$

Ans. (4)

Sol. let x^{1012} occurs in general terms

$\frac{\lfloor 10}{\lfloor r_1 \lfloor r_2 \lfloor r_3} 1^{r_1}(x^n)^{r_2}(x^{253})^{y_3}$ $0 \le r_1, r_2\ r_3 \le 10$

when $r_1 + r_2 + r_3 = 10$
$nr_2 + 253r_3 = 1012$

only one case possible

$r_1 = 6, r_2 = 0, r_3 = 4$

so coeff = $\frac{\lfloor 10}{\lfloor 6 \lfloor 0 \lfloor 4} = {}^{10}C_4$

17. The contrapositive of the statement "if I am not feeling well, then 1 will go to the doctor" is :

(1) If I will go to the doctor, then I am not feeling well.
(2) If I will not go to the doctor, then I am feeling well
(3) If I will go to the doctor, then I am feeling well
(4) If 1 am feeling well, then I will not go to the doctor

Ans. (2)

Sol. Let p: I m not feeling well

q: I will go to doctor

Given statement :- $p \rightarrow q$

its contrapositive $\sim q \rightarrow \sim p$

$\because$ If I will not go to the doctor then i am feeling well.

18. The function $f(x) = |\sin 4x| + |\cos 2x|$, is a periodic function with period :-

(1) 2π (2) $\frac{\pi}{2}$ (3) π (4) $\frac{\pi}{4}$

Ans. (2)

Sol. $f(x) = |\sin 4x| + |\cos 2x|$

$\downarrow$ $\frac{\pi}{4}$ $\downarrow$ $\frac{\pi}{2}$

$\frac{\pi}{2}$

$f(x + \pi/4) = |\sin 4x| + |\cos 2(\pi/4 + x)| \neq f(x)$

Period $= \pi/2$

19. Let $f: R \rightarrow R$ be a function such that $|f(x)| \leq x^2$, for all $x \in R$. Then, at $x = 0$, f is:

(1) Neither continuous nor differentiable

(2) differentiable but not continuous

(3) continuous as well as differentiable

(4) continuous but not differentiable

Ans. (3)

Sol. $|f(x)| \leq x^2$

$|f(0)| \leq 0$

$f(0) = 0$

$$\lim_{x\to 0}|f(x)| \leq \lim_{x\to 0} x^2$$

$$\leq 0$$

$$= 0$$

Conti at x = 0

$$\text{LHD} = \lim_{h\to 0}\frac{f(-h)-f(0)}{-h}$$

$$= \lim_{h\to 0}\frac{h^2-0}{-h} = 0$$

$$\text{RHD} = \lim_{h\to 0}\frac{f(h)-f(0)}{h}$$

$$= \lim_{h\to 0}\frac{h^2-0}{h} = 0$$

L.H.D. = R.H.D.

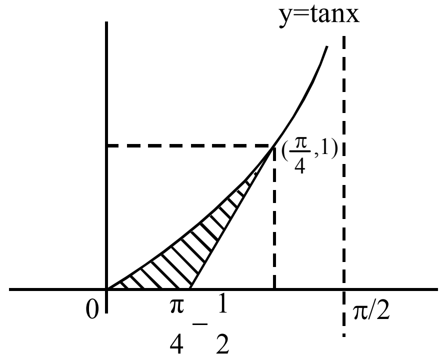
20. The area of the region above the x-axis bounded by the curve $y = \tan x$, $0 \leq x \leq \frac{\pi}{2}$ and the tangent to the curve at $x = \frac{\pi}{4}$ is:-

(1) $\frac{1}{2}(1+\log 2)$ (2) $\frac{1}{2}\left(\log 2+\frac{1}{2}\right)$

(3) $\frac{1}{2}(1-\log 2)$ (4) $\frac{1}{2}\left(\log 2-\frac{1}{2}\right)$

Ans. (4)

Sol.



$$\frac{dy}{dx} = \sec^2 x$$

$$\left.\frac{dy}{dx}\right|_{x=\pi/4} = 2$$

tangent at point $\left(\frac{\pi}{4},1\right)$

$y - 1 = 2(x - \pi/4)$

on x-axis

$0 - 1 = 2(x - \pi/4) \Rightarrow \frac{\pi}{4} - \frac{1}{2} = x$

$$\text{Area} = \int_0^{\pi/4}\tan x\,dx - \frac{1}{2}\frac{1}{2}1$$

$$= (\log\sec x)_0^{\pi/4} - \frac{1}{4}$$

$$= \log\sqrt{2} - \frac{1}{4}$$

$$= \frac{1}{2}\left[\log 2 - \frac{1}{2}\right]$$

21. If m is a non-zero number and $\int \frac{x^{5m-1}+2x^{4m-1}}{(x^{2m}+x^m+1)^3}dx = f(x)+c$, then f(x) is:-

(1) $\frac{x^{5m}}{2m(x^{2m}+x^m+1)^2}$ (2) $\frac{x^{4m}}{2m(x^{2m}+x^m+1)^2}$

(3) $\frac{2m(x^{5m}+x^{4m})}{(x^{2m}+x^m+1)^2}$ (4) $\frac{(x^{5m}-x^{4m})}{2m(x^{2m}+x^m+1)^2}$

Ans. (2)

Sol. $\int \frac{x^{5m-1}+2x^{4m-1}}{(x^{2m}+x^m+1)^3}\ dx$

$$\int \frac{x^{5m-1}+2x^{4m-1}}{x^{6m}\left(1+\frac{1}{x^m}+\frac{1}{x^{2m}}\right)^3} = \int \frac{x^{-m-1}+2x^{-2m-1}}{\left(1+\frac{1}{x^m}+\frac{1}{x^{2m}}\right)^3}\ dx$$

put $1+\frac{1}{x^m}+\frac{1}{x^{2m}} = t$

$= -\frac{1}{m}\int \frac{dt}{t^3} = -\frac{1}{m}\cdot\frac{t^{-3+1}}{-3+1}+C$

$= -\frac{1}{m}\left(\frac{t^{-2}}{-2}\right)+C$

$= \frac{1}{2m\left(1+\frac{1}{x^m}+\frac{1}{x^{2m}}\right)^2}+C$

$= \frac{x^{4m}}{2m(x^{2m}+x^m+1)^2}+C$

22. Let $\overline{x}$, M and σ^2 be respectively the mean, mode and variance of n observations $x_1, x_2, ..., x_n$ and $d_i = -x_i - a$, i = 1, 2, ..., n, where a is any number.

Statement I : Variance of $d_1, d_2, ..., d_n$ is σ^2.

Statement II : Mean and mode of $d_1, d_2, ..., d_n$ are $-\overline{x} - a$ and $-M - a$, respectively

(1) Statement I and statement II are both true

(2) Statement I and statement II are both false

(3) Statement I is true and Statement II is false

(4) Statement I is false and Statement II is true

Ans. (1)

Sol. Mean & mode depends upon change in origin and scale so mean & mode of –x; – a is –x – a and – M – a

but veriance never depends upon change in origin & it is always positive so veriance if – x – a is same i.e. σ^2

23. Let function F be defined as

$F(x) = \int_1^x \frac{e^t}{t}dt,\ x > 0$ then the value of the integral $\int_1^x \frac{e^t}{t+a}dt$, where a > 0, is:-

(1) e^a [F(x + a) – F(1 + a)]

(2) e^{-a} [F(x + a) – F (1 + a)]

(3) e^{-a} [F(x + a) – F (a)]

(4) e^a [F(x) – F (1 + a)]

Ans. (2)

Sol. $F(x) = \int_1^x \frac{e^t}{t}\ dt \Rightarrow F'(x) = \frac{e^x}{x}\cdot 1 - 0$

$F'(x) = \frac{e^x}{x}$

$\int_1^x \frac{e^t}{t+a}\ dt$ $\quad t + a = p$, $dt = dp$

$\int_{1+a}^{x+a} \frac{e^{p-a}}{p}\ dp \Rightarrow e^{-a}\int_{1+a}^{x+a}\frac{e^p}{p}\ dp$

$= e^{-a}\int_{1+a}^{x+a}\frac{e^t}{t}\ dt = e^{-a}\int_{1+a}^{x+a}F'(t)dt$

$= e^{-a}\left[F(t)\right]_{1+a}^{x+a}$

$= e^{-a}$ [F(x+a) – F(1 + a)]

24. If the function $f(x) = \begin{cases} \frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}, & x \neq \pi \\ k, & x = \pi \end{cases}$

is continuous at x = π, then k equals:-

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$

(3) 2 (4) 0

Ans. (1)

Sol. $x = \pi - h$

$$k = \lim_{h \to 0} \frac{\sqrt{2 + \cos(\pi - h)} - 1}{h^2}$$

$$k = \frac{\sqrt{2 - \cos h} - 1}{h^2}$$

$$= -\left[\frac{1 - \sqrt{2 - \cos h}}{h^2}\right]\left[\frac{1 + \sqrt{2 - \cos h}}{1 + \sqrt{2 - \cos h}}\right]$$

$$= -\left[\frac{1 - 2 + \cos h}{h^2[1 + \sqrt{2 - \cos h}]}\right]$$

$$= \frac{1 - \cos h}{h^2[1 + \sqrt{2 - \cos h}]}$$

$$= \frac{2\sin^2 h/2}{4(h^2/4)[1 + \sqrt{2 - \cos h}]}$$

$$= \frac{1}{2} \times \frac{1}{1 + \sqrt{2 - 1}} = \frac{1}{4}$$

25. For all complex numbers z of the form $1 + i\alpha$, $\alpha \in R$, If $z^2 = x + iy$, then:-

(1) $y^2 - 4x + 2 = 0$ (2) $y^2 - 4x + 4 = 0$

(3) $y^2 + 4x - 4 = 0$ (4) $y^2 + 4x + 2 = 0$

Ans. (3)

Sol. $(1 + i\alpha)^2 = x + iy$

$1 - \alpha^2 + 2i\alpha = x + iy$

so $x = 1 - \alpha^2$, $y = 2\alpha$

putting $\alpha = y/2$

$x = 1 - \left(\frac{y}{2}\right)^2 \Rightarrow y^2 + 4x - 4 = 0$

26. If the volume of a spherical ball is increasing at the rate of 4π cc/sec, then the rate of increase of its radius (in cm/sec), when the volume is 288π cc, is :-

(1) $\frac{1}{24}$ (2) $\frac{1}{6}$ (3) $\frac{1}{9}$ (4) $\frac{1}{36}$

Ans. (4)

Sol. $\frac{dv}{dt} = 4\pi$: $\frac{dr}{dt} = ?$ $v = 288\pi$

$288\pi = \frac{4}{3}\pi(r^3)$

$2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = \frac{4}{3}r^3$

$r = 6$

$v = \frac{4}{3}\pi r^3 \Rightarrow \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$4\pi = 4\pi(6)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{36}$

27. The equation $\sqrt{3x^2 + x + 5} = x - 3$, where x is real, has:-

(1) no solution

(2) exactly two solutions

(3) exactly one solution

(4) exactly four solutions

Ans. (1)

Sol. $3x^2 + x + 5 = x^2 - 6x + 9$

$2x^2 + 7x - 4 = 0$

$x = -4, 1/2$

$\times \quad \times$

No solution [$\because$ both values are not satisfied]

28. The tangent at an extremity (in the first quadrant) of latus rectum of the hyperbola $\frac{x^2}{4} - \frac{y^2}{5} = 1$, meets x -axis and y-axis at A and B respectively. Then $(OA)^2 - (OB)^2$, where O is the origin, equals:-

(1) $\frac{16}{9}$ (2) $-\frac{20}{9}$ (3) $-\frac{4}{3}$ (4) 4

Ans. (2)

Sol. Eq of tangent

$3x - 2y = 4$

$A = \left(\frac{4}{3}, 0\right)$; $B = (0, 2)$

$OA^2 - OB^2 = \frac{16}{9} - 4 = -\frac{20}{9}$

29. A chord is drawn through the focus of the parabola $y^2 = 6x$ such that its distance from the vertex of this parabola is $\frac{\sqrt{5}}{2}$, then its slope can be :-

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{\sqrt{5}}{2}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{2}{\sqrt{5}}$

Ans. (2)

Sol. Eq of chord

$y - 0 = m\,(x - 3/2)$

$$\left|\frac{-\frac{3}{2}m}{\sqrt{m^2+1}}\right| = \frac{\sqrt{5}}{2} \quad \Rightarrow m = \frac{\sqrt{5}}{2}$$

30. Let $f : R \to R$ be defined by $f(x) = \frac{|x|-1}{|x|+1}$ then f is:-

(1) one-one but not onto

(2) neither one-one nor onto

(3) both one-one and onto

(4) onto but not one-one

Ans. (2)

Sol. $f(x) = \frac{|x|-1}{|x|+1} = \begin{cases} \frac{x-1}{x+1} & x \geq 0 \\ \frac{-(x+1)}{-x+1} & x < 0 \end{cases}$