Time Duration : 3 hrs.  

Maximum Marks : 360

Important Instructions :

1. The test is of 3 hours duration.
2. The Test Booklet consists of 90 questions. The maximum marks are 360.
3. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for each correct response.
4. Candidates will be awarded marks as stated above in Instructions No. 3 for correct response of each question. ¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
5. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.
6. Use Blue/Black Ball Point Pen only for writing particulars/marking responses on Side-1 and Side-2 of the Answer Sheet. Use of pencil is strictly prohibited.
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination room/hall.
8. The CODE for this Booklet is A. Make sure that the CODE printed on Side-2 of the Answer Sheet and also tally the serial number of the Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
1. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first?

(Assume stones do not rebound after hitting the ground and neglect air resistance, take \( g = 10 \text{ m/s}^2 \))

(The figures are schematic and not drawn to scale)

\[
\begin{align*}
(1) & \quad (y_2 - y_1) \text{ m} = 240 \\
& \quad t \to 8, 12 \quad (\text{s}) \\
(2) & \quad (y_2 - y_1) \text{ m} = 240 \\
& \quad t \to 12 \quad (\text{s}) \\
(3) & \quad (y_2 - y_1) \text{ m} = 240 \\
& \quad t \to 8, 12 \quad (\text{s}) \\
(4) & \quad (y_2 - y_1) \text{ m} = 240 \\
& \quad t \to 8, 12 \quad (\text{s})
\end{align*}
\]

Answer (3)

Sol. Till both are in air (From \( t = 0 \) to \( t = 8 \) sec)

\[
\Delta x = x_2 - x_1 = 30t
\]

\[\Rightarrow \Delta x \propto t\]

When second stone hits ground and first stone is in air \( \Delta x \) decreases.

2. The period of oscillation of a simple pendulum is \( T = 2\pi \sqrt{\frac{L}{g}} \). Measured value of \( L \) is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. The accuracy in the determination of \( g \) is

(1) 2%  \quad (2) 3%  \quad (3) 1%  \quad (4) 5%

Answer (2)

Sol. \( g = 4\pi^2 \frac{l}{T^2} \)

\[
\begin{align*}
&\Rightarrow \frac{\Delta g}{g} \times 100 = \frac{\Delta l}{l} \times 100 + 2 \frac{\Delta T}{T} \times 100 \\
&\Rightarrow \frac{\Delta l}{l} \times 100 + 2 \frac{\Delta T}{T} \times 100 \\
&= \frac{0.1}{20.0} \times 100 + 2 \times \frac{1}{90} \times 100 \\
&= \frac{100 + 200}{90} = \frac{1}{2} + \frac{20}{9} \approx 3%\]
4. A particle of mass \( m \) moving in the \( x \) direction with speed \( 2v \) is hit by another particle of mass \( 2m \) moving in the \( y \) direction with speed \( v \). If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to

(1) 44%  
(2) 50%  
(3) 56%  
(4) 62%

Answer (3)

Sol. 
\[
\begin{align*}
\text{KE loss} &= \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2 \\
-\frac{1}{2}(3m) \left( \frac{2mv\sqrt{2}}{3m} \right)^2 &= \frac{5}{3}mv^2 \\
\text{Required %} &= \frac{\frac{5}{3}mv^2}{2mv^2 + mv^2} \times 100 = 56%
\end{align*}
\]

5. Distance of the centre of mass of a solid uniform cone from its vertex is \( z_0 \). If the radius of its base is \( R \) and its height is \( h \) then \( z_0 \) is equal to

(1) \( \frac{h^2}{4R} \)  
(2) \( \frac{3h}{4} \)  
(3) \( \frac{5h}{8} \)  
(4) \( \frac{3h^2}{8R} \)

Answer (2)

Sol. 
\[
\begin{align*}
\text{dm} &= \pi r^2 \text{d}y \rho \\
y_{CM} &= \frac{\int y \text{d}m}{\int \text{d}m} = \frac{\int \pi r^2 \text{d}y \times \rho \times y}{\frac{1}{3} \pi R^2 \rho h} \\
&= \frac{3h}{4}
\end{align*}
\]

6. From a solid sphere of mass \( M \) and radius \( R \) a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is

(1) \( \frac{MR^2}{32\sqrt{2}\pi} \)  
(2) \( \frac{MR^2}{16\sqrt{2}\pi} \)  
(3) \( \frac{4MR^2}{9\sqrt{3}\pi} \)  
(4) \( \frac{4MR^2}{3\sqrt{3}\pi} \)

Answer (3)

Sol. 
\[
d = 2R = a\sqrt{3}
\]

7. From a solid sphere of mass \( M \) and radius \( R \), a spherical portion of radius \( \frac{R}{2} \) is removed, as shown in the figure. Taking gravitational potential \( V = 0 \) at \( r = \infty \), the potential at the centre of the cavity thus formed is

(\( G \) = gravitational constant)

(1) \( -\frac{GM}{2R} \)  
(2) \( -\frac{GM}{R} \)  
(3) \( -\frac{2GM}{3R} \)  
(4) \( -\frac{2GM}{R} \)
Answer (2)

\[ V = V_1 - V_2 \]
\[ V_1 = \frac{GM}{2R^3} \left[ 3R^2 - \left( \frac{R}{2} \right)^2 \right] \]
\[ V_2 = \frac{3GM}{8} \left( \frac{M}{R} \right) \]
\[ \Rightarrow V = \frac{-GM}{R} \]

8. A pendulum made of a uniform wire of cross-sectional area \( A \) has time period \( T \). When an additional mass \( M \) is added to its bob, the time period changes to \( T_M \). If the Young's modulus of the material of the wire is \( Y \) then \( \frac{1}{Y} \) is equal to \( \frac{g}{2} \) (gravitational acceleration)

\[
\begin{align*}
\left( \frac{T}{T} \right)^2 - 1 &= \frac{A}{Mg} \\
\left( \frac{T}{T} \right)^2 - 1 &= \frac{Mg}{A} \\
\left( \frac{T}{T} \right)^2 - 1 &= \frac{A}{Mg} \\
\left( \frac{T}{T} \right)^2 - 1 &= \frac{A}{Mg} \\
\left( \frac{T}{T} \right)^2 - 1 &= \frac{A}{Mg} \]
\end{align*}
\]

Answer (3)

\[ P = \frac{1}{3} \left( \frac{U}{V} \right) = \frac{kT^4}{3} \] ...(i)
\[ PV = \mu RT \] ...(ii)
\[ \frac{\mu RT}{V} = \frac{1}{3} kT^4 \]
\[ \Rightarrow V \propto T^{-3} \]
\[ \frac{R}{T} \propto \frac{1}{T} \]

10. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways:

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is

(1) \( \ln 2, 4\ln 2 \)
(2) \( \ln 2, \ln 2 \)
(3) \( \ln 2, 2\ln 2 \)
(4) \( 2\ln 2, 8\ln 2 \)

Answer (None)

\[ dS = \frac{dQ}{T} = ms \frac{dT}{T} \]

\[ \Delta S = m \int dS = m \int \frac{dT}{T} = \log_e \frac{T_2}{T_1} = \log_e \frac{473}{373} \]

11. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as \( V^q \), where \( V \) is the volume of the gas. The value of \( q \) is

\[
\begin{align*}
\gamma &= \frac{C_p}{C_v} \\
\frac{3\gamma + 5}{6} &\quad (1) \quad \frac{3\gamma - 5}{6} \quad (2) \quad \frac{\gamma + 1}{2} \quad (3) \quad \frac{\gamma - 1}{2} \quad (4)
\end{align*}
\]

Answer (3)

\[ \tau = \frac{\lambda}{v_{rms}} = \frac{1}{\sqrt{2\pi d^2 \left( \frac{N}{V} \right) \frac{3RT}{M}}} \] ...(i)
\[ \tau = \frac{V}{\sqrt{T}} \] ...(ii)
\[ TV^{-1} = k \quad \text{...(iii)} \]

\[ \Rightarrow \tau = V^2 \]

12. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement \( d \). Which one of the following represents these correctly?

(1) \begin{align*}
 KE & \quad d \\
 PE & \quad \end{align*}

(2) \begin{align*}
 KE & \quad d \\
 PE & \quad \end{align*}

(3) \begin{align*}
 KE & \quad d \\
 PE & \quad \end{align*}

(4) \begin{align*}
 KE & \quad d \\
 PE & \quad \end{align*}

Answer (2)

**Sol.**

\[ f_1 = f \left[ \frac{v}{v - v_s} \right] = f \left[ \frac{320}{320 - 20} \right] = f \times \frac{320}{300} \text{ Hz} \]

\[ f_2 = f \left[ \frac{v}{v + v_s} \right] = f \times \frac{320}{340} \text{ Hz} \]

\[ 100 \times \left( \frac{f_2}{f_1} - 1 \right) = \left( \frac{f_2}{f_1} \right) \times 100 \]

\[ = 100 \left( \frac{300}{340} - 1 \right) = 12\% \]

14. A long cylindrical shell carries positive surface charge \( \sigma \) in the upper half and negative surface charge \( -\sigma \) in the lower half. The electric field lines around the cylinder will look like figure given in

(1) \begin{align*}
 KE & \quad \end{align*}

(2) \begin{align*}
 KE & \quad \end{align*}

(3) \begin{align*}
 KE & \quad \end{align*}

(4) \begin{align*}
 KE & \quad \end{align*}

Answer (1)

**Sol.** The field line should resemble that of a dipole.

15. A uniformly charged solid sphere of radius \( R \) has potential \( V_0 \) (measured with respect to \( \infty \)) on its surface. For this sphere the equipotential surfaces with potentials \( \frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4} \) and \( \frac{V_0}{4} \) have radius \( R_1, R_2, R_3 \) and \( R_4 \) respectively. Then

(1) \( R_1 = 0 \) and \( R_2 > (R_1 - R_3) \)

(2) \( R_1 \neq 0 \) and \( (R_2 - R_1) > (R_4 - R_3) \)

(3) \( R_1 = 0 \) and \( R_2 < (R_4 - R_3) \)

(4) \( 2R < R_4 \)
Answer (3, 4)

Sol. \( V_0 = k \frac{Q}{R} \) ...(i)

\[ V_t = k \frac{Q}{2R^3}(3R^2 - r^2) \]

\[ V = \frac{3}{2} V_0 \Rightarrow R_1 = 0 \]

\[ \frac{5kQ}{4R} = kQ \left( \frac{3R^2 - r^2}{2R^3} \right) \]

\[ \Rightarrow R_2 = \frac{R}{\sqrt{2}} \]

\[ \frac{3kQ}{4R} = \frac{kQ}{R^3} \]

\[ \Rightarrow R_3 = \frac{4R}{3} \]

\[ \frac{1kQ}{4R} = \frac{kQ}{R_4} \]

\[ \Rightarrow R_4 = 4R \Rightarrow R_4 > 2R \]

16. In the given circuit, charge \( Q_2 \) on the 2 \( \mu F \) capacitor changes as \( C \) is varied from 1 \( \mu F \) to 3 \( \mu F \). \( Q_2 \) as a function of \( C \) is given properly by : (Figures are drawn schematically and are not to scale)

![Diagram](image)

Answer (2)

Sol. \( C_{aq} = \frac{3C}{3+C} \) ...(i)

Total charges \( Q = \left( \frac{3C}{3+C} \right) E \) ...(ii)

Charge upon capacitor 2 \( \mu F \),

\[ q' = \frac{2}{3} \times \frac{3CE}{3+C} = \frac{2CE}{1+\frac{3}{C}} \]

Now, \( \frac{dQ}{dC} > 0, \frac{dQ^2}{dC^2} < 0 \)

17. When 5 V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is \( 2.5 \times 10^{-4} \) ms\(^{-1} \). If the electron density in the wire is \( 8 \times 10^{28} \) m\(^{-3} \), the resistivity of the material is close to

(1) \( 1.6 \times 10^{-8} \) \( \Omega \) m  
(2) \( 1.6 \times 10^{-7} \) \( \Omega \) m  
(3) \( 1.6 \times 10^{-6} \) \( \Omega \) m  
(4) \( 1.6 \times 10^{-5} \) \( \Omega \) m

Answer (4)

Sol. \( V = IR = I \frac{1}{\rho} \)

\[ \Rightarrow \rho = \frac{VA}{I} \]

\[ \Rightarrow \rho = \frac{VA}{I \times n \times \epsilon \times v_d} \]

\[ \Rightarrow \rho = \frac{5}{0.1 \times 2.5 \times 10^{-10} \times 1.6 \times 10^{15} \times 8 \times 10^{28}} \]

\[ = 1.6 \times 10^{-5} \] \( \Omega \) m

18. In the circuit shown, the current in the 1 \( \Omega \) resistor is

![Diagram](image)

(1) 1.3 A, from \( P \) to \( Q \)  
(2) 0 A  
(3) 0.13 A, from \( Q \) to \( P \)  
(4) 0.13 A, from \( P \) to \( Q \)
Answer (3)
Sol. From KVL,
\[ 9 = 6I_1 - I_2 \quad \text{(1)} \]
\[ 6 = 4I_2 - I_1 \quad \text{(2)} \]
Solving, \[ I_1 - I_2 = -0.13 \text{A} \]

19. Two coaxial solenoids of different radii carry current \( I \) in the same direction. Let \( F_1 \) be the magnetic force on the inner solenoid due to the outer one and \( F_2 \) be the magnetic force on the outer solenoid due to the inner one. Then

(1) \( F_1 = F_2 = 0 \)
(2) \( F_1 \) is radially inwards and \( F_2 \) is radially outwards
(3) \( F_1 \) is radially inwards and \( F_2 = 0 \)
(4) \( F_1 \) is radially outwards and \( F_2 = 0 \)

Answer (1)
Sol. Net force on each of them would be zero.

20. Two long current carrying thin wires, both with current \( I \), are held by insulating threads of length \( L \) and are in equilibrium as shown in the figure, with threads making an angle \( \theta \) with the vertical. If wires have mass \( \lambda \) per unit length then the value of \( I \) is

(\( g = \) gravitational acceleration)

\[ (1) \sin \theta \sqrt{\frac{\pi \lambda g L}{\mu_0 \cos \theta}} \]
\[ (2) 2 \sin \theta \sqrt{\frac{\pi \lambda g L}{\mu_0 \cos \theta}} \]
\[ (3) \frac{2 \pi g L}{\mu_0} \tan \theta \]
\[ (4) \sqrt{\frac{\pi \lambda g L}{\mu_0 \tan \theta}} \]

21. A rectangular loop of sides 10 cm and 5 cm carrying a current \( I \) of 12 A is placed in different orientations as shown in the figures below:

If there is a uniform magnetic field of 0.3 T in the positive \( z \) direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?
(1) (a) and (b), respectively
(2) (a) and (c), respectively
(3) (b) and (d), respectively
(4) (b) and (c), respectively

Answer (3)

Stable equilibrium \( \mathbf{M} || \mathbf{B} \)

Unstable equilibrium \( \mathbf{M} || (-\mathbf{B}) \)

22. An inductor \((L = 0.03 \, \text{H})\) and a resistor \((R = 0.15 \, \text{k}\Omega)\) are connected in series to a battery of 15 V EMF in a circuit shown below. The key \(K_1\) has been kept closed for a long time. Then at \(t = 0\), \(K_1\) is opened and key \(K_2\) is closed simultaneously. At \(t = 1\, \text{ms}\), the current in the circuit will be \((e \equiv 150)\)

(1) 100 mA
(2) 67 mA
(3) 6.7 mA
(4) 0.67 mA

Answer (4)

Sol. \( I = I_0 e^{-t/\tau}, \tau = \frac{L}{R} \)

\[ I \approx \frac{15}{150} e^{-t} = 0.67 \, \text{mA} \]

23. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is

(1) 1.73 V/m
(2) 2.45 V/m
(3) 5.48 V/m
(4) 7.75 V/m

Answer (2)

Sol. \[ I = \frac{P}{4\pi r^2} = U_{av} \times c \] \( \cdots \) (1)

\[ U_{av} = \frac{1}{2} \varepsilon_0 E_0^2 \] \( \cdots \) (2)

\[ \Rightarrow \frac{P}{4\pi r^2} = \frac{1}{2} \varepsilon_0 E_0^2 \times c \]

\[ \Rightarrow E_0 = \sqrt{\frac{2P}{4\pi r^2 \varepsilon_0 c}} = 2.45 \, \text{V/m} \]

24. Monochromatic light is incident on a glass prism of angle \(A\). If the refractive index of the material of the prism is \(\mu\), a ray, incident at an angle \(\theta\), on the face \(AB\) would get transmitted through the face \(AC\) of the prism provided.

(1) \(\theta > \sin^{-1} \left[ \mu \sin A - \sin^{-1} \left( \frac{1}{\mu} \right) \right] \)
(2) \(\theta < \sin^{-1} \left[ \mu \sin A - \sin^{-1} \left( \frac{1}{\mu} \right) \right] \)
(3) \(\theta > \cos^{-1} \left[ \mu \sin A + \sin^{-1} \left( \frac{1}{\mu} \right) \right] \)
(4) \(\theta < \cos^{-1} \left[ \mu \sin A + \sin^{-1} \left( \frac{1}{\mu} \right) \right] \)

Answer (1)

Sol. \[ \sin \theta = \mu \sin r_1 \]
\[ \Rightarrow \sin r_1 = \frac{\sin \theta}{\mu} \]
\[ \Rightarrow r_1 = \sin^{-1} \left( \frac{\sin \theta}{\mu} \right) \]
\[ r_2 = A - \sin^{-1} \left( \frac{\sin \theta}{\mu} \right) \]
\[ \Rightarrow r_2 < \sin^{-1} \left( \frac{1}{\mu} \right) \]
\[ A - \sin^{-1} \left( \frac{\sin \theta}{\mu} \right) < \sin^{-1} \left( \frac{1}{\mu} \right) \]
\[ \Rightarrow A - \sin^{-1} \left( \frac{1}{\mu} \right) < \sin^{-1} \left( \frac{\sin \theta}{\mu} \right) \]
\[ \Rightarrow \sin \left( A - \sin^{-1} \left( \frac{1}{\mu} \right) \right) < \sin \theta \]
\[ \Rightarrow \mu \sin \left( A - \sin^{-1} \left( \frac{1}{\mu} \right) \right) < \sin \theta \]
\[ \Rightarrow \sin^{-1} \left( \mu \sin \left( A - \sin^{-1} \left( \frac{1}{\mu} \right) \right) \right) < \theta \]

25. On a hot summer night, the refractive index of air is smallest near the ground and increases with height form the ground. When a light beam is directed horizontally, the Huygen's principle leads us to conclude that as it travels, the light beam
(1) Becomes narrower
(2) Goes horizontally without any deflection
(3) Bends downwards
(4) Bends upwards

Answer (4)

Sol. Consider a plane wavefront travelling horizontally. As it moves, its different parts move with different speeds. So, its shape will change as shown
\[ \Rightarrow \text{Light bends upward} \]

26. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is
(1) 1 \( \mu \)m
(2) 30 \( \mu \)m
(3) 100 \( \mu \)m
(4) 300 \( \mu \)m

Answer (2)

Sol. \[ RP = \frac{1.22\lambda}{2\mu \sin \theta} = \frac{1.22 \times (500 \times 10^{-9} \text{ m})}{2 \times 1 \times \left( \frac{1}{100} \right)} \]
\[ = 3.05 \times 10^{-5} \text{ m} \]
\[ = 30 \mu \text{m} \]

27. As an electron makes a transition from an excited state to the ground state of a hydrogen-like atom/ion
(1) Its kinetic energy increases but potential energy and total energy decrease
(2) Kinetic energy, potential energy and total energy decrease
(3) Kinetic energy decreases, potential energy increases but total energy remains same
(4) Kinetic energy and total energy decrease but potential energy increases

Answer (1)

Sol. \[ \text{PE} = -27.2 \frac{z^2}{n^2} \text{ eV} \]
\[ \text{TE} = -\frac{13.6 z^2}{n^2} \text{ eV} \]
\[ \text{KE} = \frac{13.6 z^2}{n^2} \text{ eV} \]
\[ \text{KE} = \frac{13.6}{n^2} \text{ eV}, \text{ as } n \text{ decreases, KE} \uparrow \]
\[ \text{PE} = -\frac{27.2}{n^2} \text{ eV}, \text{ as } n \text{ decreases, PE} \downarrow \]
\[ \text{TE} = -\frac{13.6}{n^2} \text{ eV}, \text{ as } n \text{ decreases, TE} \downarrow \]
28. Match List-I (Fundamental Experiment) with List-II (its conclusion) and select the correct option from the choices given below the list:

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Franck-Hertz experiment</td>
<td>(i) Particle nature of light</td>
</tr>
<tr>
<td>(B) Photo-electric experiment</td>
<td>(ii) Discrete energy levels of atom</td>
</tr>
<tr>
<td>(C) Davison-Germer experiment</td>
<td>(iii) Wave nature of electron</td>
</tr>
<tr>
<td></td>
<td>(iv) Structure of atom</td>
</tr>
</tbody>
</table>

(1) (A) - (i) (B) - (iv) (C) - (iii)
(2) (A) - (ii) (B) - (iv) (C) - (iii)
(3) (A) - (ii) (B) - (i) (C) - (iii)
(4) (A) - (iv) (B) - (iii) (C) - (ii)

Answer (3)

Sol. Franck-Hertz exp.- Discrete energy level.
Photo-electric effect- Particle nature of light
Davison-Germer exp.- Diffraction of electron beam.

29. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resultant signal is/are

(1) 2 MHz only
(2) 2005 kHz and 1995 kHz
(3) 2005 kHz, 2000 kHz and 1995 kHz
(4) 2000 kHz and 1995 kHz

Answer (3)

Sol. Frequencies of resultant signal are

\[ f_c + f_m, f_c, f_c - f_m \]

(2000 + 5) kHz, 2000 kHz, (2000 - 5) kHz,
2005 kHz, 2000 kHz, 1995 kHz

30. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to \( Q_0 \) and then connected to the \( L \) and \( R \) as shown below:

If a student plots graphs of the square of maximum charge \( Q_{Max}^2 \) on the capacitor with time (\( t \)) for two different values \( L_1 \) and \( L_2 \) \((L_1 > L_2)\) of \( L \) then which of the following represents this graph correctly?

(Plots are schematic and not drawn to scale)

Answer (1)

Sol. For a damped pendulum, \( A = A_0 e^{bt/2m} \)

\[ \Rightarrow A = A_0 e^{-\left(\frac{R}{2L}\right)t} \]

(Since \( L \) plays the same role as \( m \)
31. The molecular formula of a commercial resin used for exchanging ions in water softening is C₈H₇SO₃Na (mol. wt. 206). What would be the maximum uptake of Ca²⁺ ions by the resin when expressed in mole per gram resin?

(1) \( \frac{1}{103} \)
(2) \( \frac{1}{206} \)
(3) \( \frac{2}{309} \)
(4) \( \frac{1}{412} \)

Answer (4)

Sol. \( \text{Ca}^{2+} + 2\text{C}_8\text{H}_7\text{SO}_3^-\text{Na}^+ \rightarrow \text{Ca}([\text{C}_8\text{H}_7\text{SO}_3^-])_2 + 2\text{Na}^+ \)

1 mol 2 mol

The maximum uptake = \( \frac{1}{206} \times 2 = \frac{1}{412} \) mol/g

32. Sodium metal crystallizes in a body centred cubic lattice with a unit cell edge of 4.29 Å. The radius of sodium atom is approximately

(1) 1.86 Å
(2) 3.22 Å
(3) 5.72 Å
(4) 0.93 Å

Answer (1)

Sol. Edge length of BCC is 4.29 Å.

In BCC,

\[ \text{edge length} = \frac{4}{\sqrt{3}} r \]

\[ 4.29 = \frac{4}{\sqrt{3}} r \]

\[ r = \frac{4.29 \sqrt{3}}{4} = 1.86 \, \text{Å} \]

33. Which of the following is the energy of a possible excited state of hydrogen?

(1) +13.6 eV
(2) -6.8 eV
(3) -3.4 eV
(4) +6.8 eV

Answer (3)

Sol. Energy of excited state is negative and correspond to \( n > 1 \).

\[ n = \sqrt{\frac{-13.6}{E_{\text{excited state}}}} \]

34. The intermolecular interaction that is dependent on the inverse cube of distance between the molecules is

(1) Ion-ion interaction
(2) Ion-dipole interaction
(3) London force
(4) Hydrogen bond

Answer (4)

Sol. H-bond is one of the dipole-dipole interaction and dependent on inverse cube of distance between the molecules.

35. The following reaction is performed at 298 K.

\[ 2\text{NO}(g) + \text{O}_2(g) \rightleftharpoons 2\text{NO}_2(g) \]

The standard free energy of formation of NO(g) is 86.6 kJ/mol at 298 K. What is the standard free energy of formation of NO₂(g) at 298 K?

\( K_p = 1.6 \times 10^{12} \)

(1) \( R(298) \ln(1.6 \times 10^{12}) - 86600 \)
(2) \( 86600 + R(298) \ln(1.6 \times 10^{12}) \)
(3) \( 86600 - \frac{\ln(1.6 \times 10^{12})}{R(298)} \)
(4) \( 0.5 [2 \times 86,600 - R(298) \ln 1.6 \times 10^{12}] \)

Answer (4)

Sol. \( 2\text{NO}(g) + \text{O}_2(g) \rightleftharpoons 2\text{NO}_2(g) \)

\( \Delta G^{\circ}_{\text{reaction}} = [\Delta G^{\circ}_{\text{formation}}]_{\text{product}} - [\Delta G^{\circ}_{\text{formation}}]_{\text{reactant}} \)

\( \Rightarrow -RT \ln K_p = 2 \times (\Delta G^{\circ})_{\text{NO}} - 2 (\Delta G^{\circ})_{\text{NO}_2} \)

\( \Rightarrow (\Delta G^{\circ})_{\text{NO}} = 2 (\Delta G^{\circ})_{\text{NO}_2} + RT \ln K_p \)

\( \Rightarrow (\Delta G^{\circ})_{\text{NO}} = \frac{2 \times 86600 - R(298) \ln K_p}{2} \)

\( = \frac{2 \times 86600 - R(298) \ln 1.6 \times 10^{12}}{2} \)

\( = 0.5 [2 \times 86,600 - R(298) \ln 1.6 \times 10^{12}] \)
36. The vapour pressure of acetone at 20°C is 185 torr. When 1.2 g of a non-volatile substance was dissolved in 100 g of acetone at 20°C, its vapour pressure was 183 torr. The molar mass (g mol⁻¹) of the substance is

(1) 32
(2) 64
(3) 128
(4) 488

Answer (2)

Sol. Vapour pressure of pure acetone \( P_A^o = 185 \text{ torr} \)

Vapour pressure of solution, \( P_S = 183 \text{ torr} \)

Molar mass of solvent, \( M_A = 58 \text{ g/mole} \)

as we know \( \frac{P_A^o - P_S}{P_S} = \frac{n_B}{n_A} \)

\[ \Rightarrow \frac{185 - 183}{183} = \frac{W_B \times M_A}{M_B \times W_A} \]

\[ \Rightarrow 2 \times \frac{1.2 \times 58}{183} \]

\[ \Rightarrow M_B = \frac{1.2 \times 58 \times 183}{100} \]

\[ = 63.68 \text{ g/mole} \]

37. The standard Gibbs energy change at 300 K for the reaction \( 2A \rightleftharpoons B + C \) is 2494.2 J. At a given time, the composition of the reaction mixture is

\[ [A] = \frac{1}{2}, [B] = 2 \text{ and } [C] = \frac{1}{2} \]. The reaction proceeds in the : \( [R = 8.314 \text{ J/K/mol, } e = 2.718] \)

(1) Forward direction because \( Q > K_C \)
(2) Reverse direction because \( Q > K_C \)
(3) Forward direction because \( Q < K_C \)
(4) Reverse direction because \( Q < K_C \)

Answer (1)

Sol. \( 2A \rightleftharpoons B + C, \Delta G^o = 2494.2 \text{ J} \)

As we know \( \Delta G^o = -2.303 \text{ RT log} K_C \)

\[ \Rightarrow 2494.2 = -2.303 \times 8.314 \times 300 \log K_C \]

\[ \Rightarrow -0.434 = \log K_C \]

\[ \Rightarrow K_C = \text{anti log} (-0.434) \]

\[ \Rightarrow K_C = 0.367 \]

Now \( [A] = \frac{1}{2}, [B] = 2 \text{ and } [C] = \frac{1}{2} \)

38. Two faraday of electricity is passed through a solution of CuSO₄. The mass of copper deposited at the cathode is (at. mass of Cu = 63.5 amu)

(1) 0 g
(2) 63.5 g
(3) 2 g
(4) 127 g

Answer (2)

Sol. Cu^{+2} + 2e \rightarrow Cu

So, 2 F charge deposite 1 mol of Cu. Mass deposited = 63.5 g.

39. Higher order (>3) reactions are rare due to

(1) Low probability of simultaneous collision of all the reacting species
(2) Increase in entropy and activation energy as more molecules are involved
(3) Shifting of equilibrium towards reactants due to elastic collisions
(4) Loss of active species on collision

Answer (1)

Sol. Higher order greater than 3 for reaction is rare because there is low probability of simultaneous collision of all the reacting species.

40. 3 g of activated charcoal was added to 50 mL of acetic acid solution (0.06N) in a flask. After an hour it was filtered and the strength of the filtrate was found to be 0.042 N. The amount of acetic acid adsorbed (per gram of charcoal) is

(1) 18 mg
(2) 36 mg
(3) 42 mg
(4) 54 mg

Answer (1)

Sol. Number of moles of acetic acid adsorbed

\[ = \left( 0.06 \times \frac{50}{1000} - 0.042 \times \frac{50}{1000} \right) \]

\[ = \frac{0.9}{1000} \text{ moles} \]

\[ \therefore \text{Weight of acetic acid adsorbed} = 0.9 \times 60 \text{ mg} \]

\[ = 54 \text{ mg} \]

Hence, the amount of acetic acid adsorbed per g of
charcoal = \frac{54}{3} \text{ mg} = 18 \text{ mg}

Hence, option (1) is correct.

41. The ionic radii (in Å) of N^{3-}, O^{2-} and F^{-} are respectively

(1) 1.36, 1.40 and 1.71
(2) 1.36, 1.71 and 1.40
(3) 1.71, 1.40 and 1.36
(4) 1.71, 1.36 and 1.40

Answer (3)

Sol. Radius of N^{3-}, O^{2-} and F^{-} follow order
N^{3-} > O^{2-} > F^{-}
As per inequality only option (3) is correct
that is 1.71 Å, 1.40 Å and 1.36 Å

42. In the context of the Hall-Heroult process for the extraction of Al, which of the following statement is false?

(1) CO and CO_{2} are produced in this process
(2) Al_{2}O_{3} is mixed with CaF_{2} which lowers the melting point of the mixture and brings conductivity
(3) Al^{3+} is reduced at the cathode to form Al
(4) Na_{3}AlF_{6} serves as the electrolyte

Answer (4)

Sol. In Hall-Heroult process Al_{2}O_{3} (molten) is electrolyte.

43. From the following statement regarding H_{2}O_{2}, choose the incorrect statement

(1) It can act only as an oxidizing agent
(2) It decomposes on exposure to light
(3) It has to be stored in plastic or wax lined glass bottles in dark.
(4) It has to be kept away from dust

Answer (1)

Sol. H_{2}O_{2} can be reduced or oxidised. Hence, it can act as reducing as well as oxidising agent.

44. Which one of the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy?

(1) CaSO_{4}
(2) BeSO_{4}
(3) BaSO_{4}
(4) SrSO_{4}

Answer (2)

Sol. BeSO_{4} has hydration energy greater than its lattice energy.

45. Which among the following is the most reactive?

(1) Cl_{2}
(2) Br_{2}
(3) I_{2}
(4) ICl

Answer (4)

Sol. Because of polarity and weak bond interhalogen compounds are more reactive.

46. Match the catalysts to the correct processes:

<table>
<thead>
<tr>
<th>Catalyst</th>
<th>Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>TiCl_{3}</td>
<td>(i) Wacker process</td>
</tr>
<tr>
<td>PdCl_{2}</td>
<td>(ii) Ziegler-Natta polymerization</td>
</tr>
<tr>
<td>CuCl_{2}</td>
<td>(iii) Contact process</td>
</tr>
<tr>
<td>V_{2}O_{5}</td>
<td>(iv) Deacon’s process</td>
</tr>
</tbody>
</table>

Answer (2)

Sol. TiCl_{3} - Ziegler-Natta polymerisation
PdCl_{2} - Wacker process
CuCl_{2} - Deacon’s process

47. Which one has the highest boiling point?

(1) He
(2) Ne
(3) Kr
(4) Xe

Answer (4)

Sol. Down the group strength of van der Waal’s force of attraction increases hence Xe have highest boiling point.

48. The number of geometric isomers that can exist for square planar [Pt(Cl)(py)(NH_{3})(NH_{2}OH)]^{+} is (py = pyridine)

(1) 2
(2) 3
(3) 4
(4) 6

Answer (4)
52. Which of the following compounds will exhibit geometrical isomerism?
   (1) 1 - Phenyl - 2 - butene
   (2) 3 - Phenyl - 1 - butene
   (3) 2 - Phenyl - 1 - butene
   (4) 1, 1 - Diphenyl - 1 propane

Answer (1)
Sol. For geometrical isomerism doubly bonded carbon must be bonded to two different groups which is only satisfied by 1 - Phenyl - 2 - butene.

53. Which compound would give 5-keto-2-methyl hexanal upon ozonolysis?

Answer (2)
Sol. 5-keto-2-methylhexanal is

54. The synthesis of alkyl fluorides is best accomplished by
   (1) Free radical fluorination
   (2) Sandmeyer's reaction
   (3) Finkelstein reaction
   (4) Swarts reaction

Answer (4)
Sol. Swat's reaction

\[
\text{CH}_3 - \text{Cl} + \text{AgF} \stackrel{\Delta}{\longrightarrow} \text{CH}_3\text{F} + \text{AgCl}
\]
55. In the following sequence of reactions:

\[
\text{Toluene} \xrightarrow{\text{KMnO}_4} \text{A} \xrightarrow{\text{SOCl}_2} \text{B} \xrightarrow{\text{H}_2/\text{Pd}} \text{BaSO}_4 \xrightarrow{} \text{C},
\]
the product C is

1. \( \text{C}_6\text{H}_5\text{COOH} \)
2. \( \text{C}_6\text{H}_5\text{CH}_3 \)
3. \( \text{C}_6\text{H}_5\text{CH}_2\text{OH} \)
4. \( \text{C}_6\text{H}_5\text{CHO} \)

**Answer (4)**

**Sol.**

\[
\text{CH}_3 \xrightarrow{\text{KMnO}_4} \text{COOH} \xrightarrow{\text{SOCl}_2} \text{COCl} \xrightarrow{\text{H}_2/\text{Pd}} \text{BaSO}_4 \xrightarrow{} \text{CHO}
\]

56. In the reaction

\[
\text{NH}_2 \xrightarrow{\text{NaNO}_2/\text{HCl} \ 0-5^\circ\text{C}} \xrightarrow{\text{CuCN}/\text{KCN} \ \Delta} \text{D} \xrightarrow{} \text{E} + \text{N}_2,
\]
the product E is

1. \( \text{COOH} \)
2. \( \text{H}_2\text{C} \xrightarrow{} \text{O} \xrightarrow{} \text{CH}_3 \)
3. \( \text{CN} \)
4. \( \text{CH}_3 \)

**Answer (3)**

**Sol.**

\[
\text{CH}_3 \xrightarrow{\text{NaNO}_2/\text{HCl} \ 0-5^\circ\text{C}} \text{D} \xrightarrow{\text{CuCN}/\text{KCN} \ \Delta} \text{E} + \text{N}_2
\]

57. Which polymer is used in the manufacture of paints and lacquers?

1. Bakelite
2. Glyptal
3. Polypropene
4. Poly vinyl chloride

**Answer (2)**

**Sol.** Glyptal is used in manufacture of paints and lacquers.

58. Which of the vitamins given below is water soluble?

1. Vitamin C
2. Vitamin D
3. Vitamin E
4. Vitamin K

**Answer (1)**

**Sol.** Vitamin C is water soluble vitamin.

59. Which of the following compounds is **not** an antacid?

1. Aluminium Hydroxide
2. Cimetidine
3. Phenelzine
4. Ranitidine

**Answer (3)**

**Sol.** Phenelzine is not antacid, it is anti-depressant.

60. Which of the following compounds is **not** colored yellow?

1. \( \text{Zn}_2[\text{Fe(CN)}_6] \)
2. \( \text{K}_3[\text{Co(NO}_2)_6] \)
3. \( \text{(NH}_4)_3[\text{As (Mo}_3\text{O}_{10})_4] \)
4. \( \text{BaCrO}_4 \)

**Answer (1)**

**Sol.** \( \text{(NH}_4)_3[\text{As (Mo}_3\text{O}_{10})_4] \), \( \text{BaCrO}_4 \) and \( \text{K}_3[\text{Co(NO}_2)_6] \) are yellow colored compounds but \( \text{Zn}_2[\text{Fe(CN)}_6] \) is not yellow colored compound.
61. Let $A$ and $B$ be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having at least three elements is

(1) 219       (2) 256       (3) 275       (4) 510

Answer (1)

Sol. $n(A) = 4$, $n(B) = 2$

$n(A \times B) = 8$

Required numbers = $8 \binom{3}{3} + 8 \binom{4}{4} + \ldots + 8 \binom{8}{8}$

$= 256 - 37$

$= 219$

62. A complex number $z$ is said to be unimodular if $|z| = 1$. Suppose $z_1$ and $z_2$ are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1 \overline{z_2}}$ is unimodular and $z_2$ is not unimodular. Then the point $z_1$ lies on a

(1) Straight line parallel to $x$-axis
(2) Straight line parallel to $y$-axis
(3) Circle of radius 2
(4) Circle of radius $\sqrt{2}$

Answer (3)

Sol. \[
\frac{z_1 - 2z_2}{2 - z_1 \overline{z_2}} = 1
\]

\[
\frac{(z_1 - 2z_2)(z_1 - 2\overline{z_2})}{2 - z_1 \overline{z_2} - 2\overline{z_1} z_2 + 4z_1 z_2}
\]

\[
= 4 - 2z_1 z_2 - 2\overline{z_1} z_2 + z_1 \overline{z_2} z_2
\]

\[
= 4 + z_1 \overline{z_2} z_2
\]

\[
z_1 \overline{z_1} (1 - z_2 \overline{z_2}) - 4(1 - z_2 \overline{z_2}) = 0
\]

\[
(z_1 \overline{z_1} - 4)(1 - z_2 \overline{z_2}) = 0
\]

$\Rightarrow z_1 \overline{z_1} = 4$

$|z| = 2$ i.e. $z$ lies on circle of radius 2.

63. Let $\alpha$ and $\beta$ be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to

(1) 6       (2) -6       (3) 3       (4) -3

Answer (3)

Sol. From equation,

$\alpha + \beta = 6$

$\alpha\beta = -2$

The value of $\frac{a_{10} - 2a_8}{2a_9} = \frac{\alpha^{10} + \beta^{10} + \alpha\beta(\alpha^8 + \beta^8)}{2(\alpha^9 + \beta^9)}$

$= \frac{\alpha^9(\alpha + \beta) + \beta^9(\alpha + \beta)}{2(\alpha^9 + \beta^9)}$

$= \frac{\alpha + \beta}{2} = \frac{6}{2} = 3$

64. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where $I$ is $3 \times 3$ identity matrix, then the ordered pair $(a, b)$ is equal to

(1) $(2, -1)$       (2) $(-2, 1)$
(3) $(2, 1)$       (4) $(-2, -1)$

Answer (4)

Sol. \[
\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ a & 2 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}
\]

$a + 4 + 2b = 0$

$2a + 2 + 2b = 0$

$a + 1 - b = 0$

$2a - 2b = -2$

$a + 2b = -4$

$3a = -6$

$a = -2$

$-2 + 1 - b = 0$

$b = -1$

$a = -2$

$(-2, -1)$
65. The set of all values of \( \lambda \) for which the system of linear equations
\[
2x_1 - 2x_2 + x_3 = \lambda x_1 \\
2x_1 - 3x_2 + 2x_3 = \lambda x_2 \\
-x_1 + 2x_2 = \lambda x_3
\]
has a non-trivial solution
(1) Is an empty set
(2) Is a singleton
(3) Contains two elements
(4) Contains more than two elements
Answer (3)
Sol.
\[
\begin{vmatrix}
2 - \lambda & -2 & 1 \\
2 & -\lambda - 3 & 2 \\
-1 & 2 & -\lambda
\end{vmatrix} = 0
\]
\[
(2 - \lambda)(\lambda^2 + 3\lambda - 4) + 2(-2\lambda + 2) + (4 - \lambda - 3) = 0
\]
\[
2\lambda^2 + 6\lambda - 8 - \lambda^3 - 3\lambda^2 + 4\lambda - 4\lambda + 4 - \lambda + 1 = 0
\]
\[
\Rightarrow -\lambda^3 - \lambda^2 + 5\lambda - 3 = 0
\]
\[
\Rightarrow \lambda^3 + \lambda^2 - 5\lambda + 3 = 0
\]
\[
\lambda^2(\lambda - 1) + 2\lambda(\lambda - 1) - 3(\lambda - 1) = 0
\]
\[
(\lambda - 1)(\lambda^2 + 2\lambda - 3) = 0
\]
\[
(\lambda - 1)(\lambda + 3)(\lambda - 1) = 0
\]
\[
\Rightarrow \lambda = 1, 1, -3
\]
Two elements.

66. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is
(1) 216
(2) 192
(3) 120
(4) 72
Answer (2)
Sol. 4 digit numbers
\[
\begin{array}{cccc}
3 & 5 & 6 & 7 \\
5 & 4 & 3 & 2
\end{array}
\]
5 × 4 × 3 × 2 × 1 = 120
Total number of integers = 72 + 120 = 192

67. The sum of coefficients of integral powers of \( x \) in the binomial expansion of \((1 - 2\sqrt{x})^{50}\) is
(1) \( \frac{1}{2}(3^{50} + 1) \)
(2) \( \frac{1}{2}(3^{50}) \)
(3) \( \frac{1}{2}(3^{50} - 1) \)
(4) \( \frac{1}{2}(2^{50} + 1) \)
Answer (1)
Sol. \((1 - 2\sqrt{x})^{50} = 50 C_0 - 50 C_1 (2\sqrt{x})^1 + 50 C_2 (2\sqrt{x})^2 + \ldots + 50 C_{50} (-2\sqrt{x})^{50}\)
Sum of coefficient of integral power of \( x \)
\[
= 50 C_0 2^0 + 50 C_2 2^2 + 50 C_4 2^4 + \ldots + 50 C_{50} 2^{50}
\]
We know that \((1 + 2)^{50} = 50 C_0 + 50 C_2 + \ldots + 50 C_{50} \cdot 2^{50}\)
Then,
\[
50 C_0 + 50 C_2 + \ldots + 50 C_{50} \cdot 2^{50} = \frac{3^{50} + 1}{2}
\]
68. If \( m \) is the A.M. of two distinct real numbers \( l \) and \( n \) \((l, n > 1)\) and \( G_1, G_2 \) and \( G_3 \) are three geometric means between \( l \) and \( n \), then \( G_1^4 + 2G_2^4 + G_3^4 \) equals.
(1) \( 4 l^2 m \)
(2) \( 4 l m^2 n \)
(3) \( 4 l m n^2 \)
(4) \( 4 P m^2 n^2 \)
Answer (2)
Sol. \( \frac{l + n}{2} = m \)
\( l + n = 2m \) \hspace{1cm} \ldots(1)
\[
G_1 = l \left( \frac{n}{l} \right)^{\frac{1}{4}}
\]
\[
G_2 = l \left( \frac{n}{l} \right)^{\frac{1}{2}}
\]
\[
G_3 = l \left( \frac{n}{l} \right)^{\frac{3}{4}}
\]

Now \( G_1^4 + 2G_2^4 + G_3^3 \)

\[
I^4 \frac{n}{7} + 2I^2 \left( \frac{n}{7} \right)^2 + I^4 \left( \frac{n}{7} \right)^3
\]

= \( nl^3 + 2nl^2 + n^3l \)

= \( 2nl^2 + nl(n^2 + l^2) \)

= \( 2nl^2 + nl(n + l)^2 - 2nl \)

= \( nl(n + l)^2 \)

= \( nl \cdot (2n)^2 \)

= \( 4nl^2 \)

71. If the function.

\[
g(x) = \begin{cases} 
  k\sqrt{x+1} & , \ 0 \leq x \leq 3 \\
  mx+2 & , \ 3 < x \leq 5 
\end{cases}
\]

is differentiable, the value of \( k + m \) is

(1) 2

(2) \( \frac{16}{5} \)

(3) \( \frac{10}{3} \)

(4) 4

Answer (1)

Sol. \( g(x) = \begin{cases} 
  k\sqrt{x+1} & , \ 0 \leq x \leq 3 \\
  mx+2 & , \ 3 < x \leq 5 
\end{cases} \)

R.H.D.

\[
\lim_{h \to 0} \frac{g(3+h) - g(3)}{h} = \lim_{h \to 0} \frac{m(3+h) + 2 - 2k}{h} = m
\]

and \( 3m - 2k + 2 = 0 \)

L.H.D.

\[
\lim_{h \to 0} \frac{k\sqrt{3-h} + 1 - 2k}{-h} = \frac{-k\sqrt{4-h} - 2}{h}
\]

\[
\lim_{h \to 0} -k \times \frac{4-h - 4}{h(4-h + 2)} = \frac{k}{4}
\]

From above,

\[
\frac{k}{4} = m \quad \text{and} \quad 3m - 2k + 2 = 0
\]

\[
m = \frac{2}{5} \quad \text{and} \quad k = \frac{8}{5}
\]

\[
k + m = \frac{8}{5} + \frac{2}{5} - \frac{10}{5} = 2
\]

Alternative Answer

\[
g(x) = \begin{cases} 
  k\sqrt{x+1} & , \ 0 \leq x \leq 3 \\
  mx+2 & , \ 3 < x \leq 5 
\end{cases}
\]

\( g \) is constant at \( x = 3 \)

\[
k\sqrt{4} = 3m + 2
\]

\( 2k = 3m + 2 \) \( \ldots (i) \)

Also \( \left( \frac{k}{2\sqrt{x+1}} \right)_{x=3} = m \)
\[
\frac{k}{4} = m \\
k = 4 \ m \quad \ldots (\text{ii})
\]
8 \text{ m} = 3 \text{ m} + 2 \\
m = \frac{2}{5}, \ k = \frac{8}{5} \\
m + k = \frac{2}{5} + \frac{8}{5} = 2

72. The normal to the curve, \(x^2 + 2xy - 3y^2 = 0\) at (1, 1)
(1) Does not meet the curve again
(2) Meets the curve again in the second quadrant
(3) Meets the curve again in the third quadrant
(4) Meets the curve again in the fourth quadrant

\text{Answer (4)}

\text{Sol.} \quad \text{Curve is} \quad x^2 + 2xy - 3y^2 = 0

Differentiate \(\text{w.r.t.} \ x\),

\[
\begin{bmatrix}
\frac{dy}{dx} \\
\frac{dy}{dx}
\end{bmatrix} = \begin{bmatrix}
2x + 2y \\
2x + 2y
\end{bmatrix}
\]

\[
\Rightarrow \frac{dy}{dx} = \frac{1}{1}
\]

So equation of normal at (1, 1) is

\[
y - 1 = -1 \ (x - 1)
\]

Solving it with the curve, we get

\[
x^2 + 2(x(2 - x) - 3(2 - x))^2 = 0
\]

\[
\Rightarrow -4x^2 + 16x - 12 = 0
\]

\[
\Rightarrow x^2 - 4x + 3 = 0
\]

\[
\Rightarrow x = 1, 3
\]

So points of intersections are (1, 1) & (3, -1) i.e. normal cuts the curve again in fourth quadrant.

73. Let \(f(x)\) be a polynomial of degree four having extreme values at \(x = 1\) and \(x = 2\). If \(\lim_{x \to 0} \left[1 + \frac{f(x)}{x^2}\right] = 3\), then \(f(2)\) is equal to
(1) -8
(2) -4
(3) 0
(4) 4

\text{Answer (3)}

\text{Sol.} \quad \text{Let} \quad f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4

Using \(\lim_{x \to 0} \left[1 + \frac{f(x)}{x^2}\right] = 3\),

\[
\Rightarrow \lim_{x \to 0} \frac{f(x)}{x^2} = 2
\]

\[
\Rightarrow \lim_{x \to 0} \frac{a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4}{x^2} = 2
\]

So, \(a_0 = 0, a_1 = 0, a_2 = 2\) 

\text{i.e.}, \(f(x) = 2x^2 + a_3x^3 + a_4x^4\)

Now, \(f'(x) = 4x + 3a_3x^2 + 4a_4x^3\)

\[= x[4 + 3a_3x + 4a_4x^2]\]

Given, \(f'(1) = 0\) and \(f'(2) = 0\)

\[\Rightarrow 3a_3 + 4a_4 + 4 = 0 \quad \ldots (\text{i})\]

\text{and} \(6a_3 + 16a_4 + 4 = 0 \quad \ldots (\text{ii})\)

Solving, \(a_4 = \frac{1}{2}, a_3 = -2\)

\text{i.e.,} \(f(x) = 2x^2 - 2x^3 + \frac{1}{2}x^4\)

\text{i.e.,} \(f(2) = 0\)

74. The integral \(\int \frac{dx}{x^2(x^4 + 1)^{3/4}}\) equals
(1) \(\left(\frac{x^4 + 1}{x^6}\right)^{1/2} + c\)
(2) \(\left(x^4 + 1\right)^{1/2} + c\)
(3) \(-\left(x^4 + 1\right)^{1/2} + c\)
(4) \(-\left(\frac{x^4 + 1}{x^4}\right)^{1/4} + c\)

\text{Answer (4)}

\text{Sol.} \quad I = \int \frac{dx}{x^2(x^4 + 1)^{3/4}} \quad \int \frac{dx}{x^2(x^4 + 1)^{3/4}}

Let \(1 + \frac{1}{x^4} = t \Rightarrow \frac{-4}{x^5} \ dx = dt\)

\[
\Rightarrow I = \frac{-1}{4} \frac{dt}{t^{1/4}} = \frac{-1}{4} \left(\frac{1}{t^{1/4}}\right) + c
\]

\[
= \left(-1 + \frac{1}{x^4}\right) + c
\]

So, option (4).

75. The integral \(\int \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} \ dx\) is equal to
(1) 2
(2) 4
(3) 1
(4) 6

\text{Answer (3)}

\text{Sol.} \quad I = \int \frac{\log x^2 \ dx}{\frac{1}{2} \log x^2 + \log(36 - 12x + x^2)}

I = \int \frac{\log(x - 6)^2 \ dx}{\frac{1}{2} \log x^2 + \log(6 - x)^2}
76. The area (in sq. units) of the region described by
\((x, y) : y^2 \leq 2x \text{ and } y \geq 4x - 1\) is

(1) \(\frac{7}{32}\) \hspace{1cm} (2) \(\frac{5}{64}\) \hspace{1cm} (3) \(\frac{15}{64}\) \hspace{1cm} (4) \(\frac{9}{32}\)

Answer (4)

Sol.
\[
y = 4 - \frac{y^2}{2} - 1
\]
\[
2y^2 - y - 1 = 0
\]
\[
y = \frac{1 \pm \sqrt{1 + 8}}{4} = \frac{1 \pm 3}{4}
\]
\[
y = \frac{1}{2}, \frac{-1}{2}
\]
\[
A = \int_{-1/2}^{1/2} \left(\frac{y+1}{4}\right) dy - \int_{-1/2}^{1/2} \frac{y^2}{2} dy
\]
\[
= \left[\frac{y^2}{2} + y\right]_{-1/2}^{1/2} - \frac{1}{2} \left[\frac{y^3}{3}\right]_{-1/2}^{1/2}
\]
\[
= \frac{1}{4} \left[\frac{4 + 8}{8} - 1\right] - \frac{1}{2} \left[\frac{8 + 1}{24}\right]
\]
\[
= \frac{1}{4} \left[\frac{15}{8}\right] - \frac{9}{48}
\]
\[
= \frac{15}{32} - \frac{9}{32} = \frac{6}{32} = \frac{9}{32}
\]

77. Let \(y(x)\) be the solution of the differential equation
\[
(x \log x) \frac{dy}{dx} + y = 2x \log x, \ (x \geq 1).
\]
Then \(y(e)\) is equal to

(1) \(e\) \hspace{1cm} (2) \(0\) \hspace{1cm} (3) \(2\) \hspace{1cm} (4) \(2e\)

Answer (3)*

Sol. It is best option. Theoretically question is wrong, because initial condition is not given.
\[
\frac{dy}{dx} + \frac{y}{x \log x} = 2
\]
I.F. = \(e^{-\log x} = e^{\log x} = \log x\)

Solution is \(y \log x = \int 2 \log x \, dx + c\)
\[
y \log x = 2(x \log x - x) + c
\]
\(x = 1, \ y = 0\)
Then, \(c = 2, \ y(e) = 2\)

78. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices \((0, 0), (0, 41)\) and \((41, 0)\), is

(1) 901 \hspace{1cm} (2) 861 \hspace{1cm} (3) 820 \hspace{1cm} (4) 780

Answer (4)
Answer (3)
Sol. After solving equation (i) & (ii)
\[ 2 - 3 + 4 = 0 \] ...(i)
\[ 2 - 4 + 6 = 0 \] ...(ii)
\[ x = 1 \] and \[ y = 2 \]
Slope of \( AB \times \) Slope of \( MN \) = -1
\[ 3 \quad 2 \quad 2 \quad 1 \quad 2 \]
\[ b \quad a \]
\[ \langle a, b \rangle \]
\[ (2, 3) \]
\[ (1, 2) \]
\[ A \]
\[ (2, 3) \]
\[ M(\frac{a+2}{2}, \frac{b+3}{2}) \]
\[ N(1, 2) \]
\[ (a, b) \]
\[ B \]
\[ \text{Image of } A \]
\[ y - 3) \quad (y - 1) = -(x - 2)x \]
\[ y^2 - 4y + 3 = -x^2 + 2x \]
\[ x^2 + y^2 - 2x - 4y + 3 = 0 \]
Circle of radius = \( \sqrt{2} \)

80. The number of common tangents to the circles
\[ x^2 + y^2 - 4x - 6y - 12 = 0 \] and
\[ x^2 + y^2 + 6x + 18y + 26 = 0, \] is
(1) 1 \quad (2) 2 \quad (3) 3 \quad (4) 4
Answer (3)
Sol. \[ x^2 + y^2 - 4x - 6y - 12 = 0 \]
\[ C_1(\text{center}) = (2, 3), \quad r = \sqrt{2^2 + 3^2 + 12} = 5 \]
\[ x^2 + y^2 + 6x + 18y + 26 = 0 \]
\[ C_2(\text{center}) (-3, -9), \quad r = \sqrt{9^2 + 81 - 26} \]
\[ = \sqrt{64} = 8 \]
\[ C_1C_2 = 13, \quad C_1C_2 = r_1 + r_2 \]
Number of common tangent is 3.

81. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse \[ \frac{x^2}{9} + \frac{y^2}{5} = 1, \] is
(1) \( \frac{27}{4} \) \quad (2) 18 \quad (3) \( \frac{27}{2} \) \quad (4) 27
Answer (4)
Sol. Ellipse is \[ \frac{x^2}{9} + \frac{y^2}{5} = 1 \]
i.e., \( a^2 = 9, \quad b^2 = 5 \)
So, \( e = \frac{2}{3} \)
As, required area = \[ \frac{2a^2 \cdot e}{2} = \frac{2 \times 9}{(2/3)} = 27 \]

82. Let \( O \) be the vertex and \( Q \) be any point on the parabola, \( x^2 = 8y \). If the point \( P \) divides the line segment \( OQ \) internally in the ratio 1 : 3, then the locus of \( P \) is
(1) \( x^2 = y \) \quad (2) \( y^2 = x \)
(3) \( y^2 = 2x \) \quad (4) \( x^2 = 2y \)
Answer (4)
Sol. \( x^2 = 8y \)
Let \( Q \) be \( (4t, 2t^2) \)
\[ P = \left( t, \frac{t^2}{2} \right) \]
\[ h = t, \quad k = \frac{t^2}{2} \]
\[ 2k = h^2 \]
\[ \text{Locus of } (h, k) \] is \( x^2 = 2y \).

83. The distance of the point \( (1, 0, 2) \) from the point of intersection of the line \[ \frac{x - 2}{3} = \frac{y + 1}{4} = \frac{z - 2}{12} \] and the plane \( x - y + z = 16 \), is
(1) \( 2\sqrt{14} \) \quad (2) 8 \quad (3) \( 3\sqrt{21} \) \quad (4) 13
Answer (4)
Sol. \[ \frac{x - 2}{3} = \frac{y + 1}{4} = \frac{z - 2}{12} = \lambda \]
\[ P(3\lambda + 2, \quad 4\lambda - 1, \quad 12\lambda + 2) \]
Lies on plane \( x - y + z = 16 \)
Then,
\[ 3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 = 16 \]
\[ 11\lambda + 5 = 16 \]
\[ \lambda = 1 \]
\[ P(5, 3, 14) \]
Distance = \[ \sqrt{16 + 9 + 144} = \sqrt{169} = 13 \]

84. The equation of the plane containing the line \( 2x - 5y + z = 3; \quad x + y + 4z = 5 \), and parallel to the plane, \( x + 3y + 6z = 1 \), is
(1) \( 2x + 6y + 12z = 13 \) \quad (2) \( x + 3y + 6z = -7 \)
(3) \( x + 3y + 6z = 7 \) \quad (4) \( 2x + 6y + 12z = -13 \)
Answer (3)

Sol. Required plane is

\[(2x - 5y + z - 3) + \lambda(x + y + 4z - 5) = 0\]

It is parallel to \(x + 3y + 6z = 1\)

\[\therefore \frac{2 + \lambda}{1} = \frac{-5 + \lambda}{3} = \frac{1 + 4\lambda}{6}\]

Solving \(\lambda = -\frac{11}{2}\)

\[\therefore \text{Required plane is}\]

\[(2x - 5y + z - 3) - \frac{11}{2} (x + y + 4z - 5) = 0\]

\[\therefore x + 3y + 6z - 7 = 0\]

85. Let \(\vec{a}, \vec{b}\) and \(\vec{c}\) be three non-zero vectors such that no two of them are collinear and \((\vec{a} \times \vec{b}) \cdot \vec{c} = \frac{1}{3} |\vec{b}| |\vec{c}| \vec{a}\). If \(\theta\) is the angle between vectors \(\vec{b}\) and \(\vec{c}\), then a value of \(\sin \theta\) is

\[
\begin{align*}
(1) & \quad \frac{2\sqrt{2}}{3} \\
(2) & \quad -\frac{\sqrt{2}}{3} \\
(3) & \quad \frac{2}{3} \\
(4) & \quad -\frac{2\sqrt{3}}{3}
\end{align*}
\]

Answer (1)

Sol. \((\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a} = \frac{1}{3} |\vec{b}| |\vec{c}| \vec{a}\)

\[\therefore - (\vec{b} \cdot \vec{c}) = \frac{1}{3} |\vec{b}| |\vec{c}| \vec{a}\]

\[\therefore \cos \theta = \frac{-1}{3}\]

\[\therefore \sin \theta = \frac{2\sqrt{3}}{3}\]

86. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one the boxes contains exactly 3 balls is

\[
\begin{align*}
(1) & \quad \frac{55}{3} \left(\frac{2}{3}\right)^{11} \\
(2) & \quad 55 \left(\frac{2}{3}\right)^{10} \\
(3) & \quad 220 \left(\frac{1}{3}\right)^{12} \\
(4) & \quad 22 \left(\frac{1}{3}\right)^{11}
\end{align*}
\]

Answer (1)*

Sol. Question is wrong but the best suitable option is (1).

Required probability = \(\binom{12}{3} \frac{2^9}{3^{12}} = \frac{55}{3} \left(\frac{2}{3}\right)^{11}\)

87. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is

\[
\begin{align*}
(1) & \quad 16.8 \\
(2) & \quad 16.0 \\
(3) & \quad 15.8 \\
(4) & \quad 14.0
\end{align*}
\]

Answer (4)

Sol. \(\text{Mean} = 16\)

Sum = 16 × 16 = 256

New sum = 256 - 16 + 3 + 4 + 5 = 252

Mean = \(\frac{252}{18} = 14\)

88. If the angles of elevation of the top of a tower from three collinear points \(A, B\) and \(C\), on a line leading to the foot of the tower, are 30º, 45º and 60º respectively, then the ratio, \(AB : BC\), is

\[
\begin{align*}
(1) & \quad \sqrt{3} : 1 \\
(2) & \quad \sqrt{3} : \sqrt{2} \\
(3) & \quad 1 : \sqrt{3} \\
(4) & \quad 2 : 3
\end{align*}
\]

Answer (1)

Sol. \(AO = h \cot 30^\circ\)

\[= h\sqrt{3}\]

\(BO = h\)

\(CO = h\sqrt{5}\)

\[\therefore AB : BC = AO - BO\]

\[= h\sqrt{3} - h\]

\[= h - \frac{h}{\sqrt{3}}\]

\[= \sqrt{3}\]
89. Let \( \tan^{-1} y = \tan^{-1} x + \tan^{-1} \left( \frac{2x}{1-x^2} \right) \)

where \( |x| < \frac{1}{\sqrt{3}} \). Then a value of \( y \) is

\[
\begin{align*}
(1) & \quad \frac{3x - x^3}{1 - 3x^2} \\
(2) & \quad \frac{3x + x^3}{1 - 3x^2} \\
(3) & \quad \frac{3x - x^3}{1 + 3x^2} \\
(4) & \quad \frac{3x + x^3}{1 + 3x^2}
\end{align*}
\]

Answer (1)

Sol. \( \tan^{-1} y = \tan^{-1} x + \tan^{-1} \left( \frac{2x}{1-x^2} \right) \)

\[3\tan^{-1} x = \tan^{-1} \left( \frac{3x - x^3}{1 - 3x^2} \right)\]

90. The negation of \( \sim s \vee (\sim r \wedge s) \) is equivalent to

(1) \( s \wedge \sim r \)

(2) \( s \wedge (r \wedge \sim s) \)

(3) \( s \vee (r \vee \sim s) \)

(4) \( s \wedge r \)

Answer (4)

Sol. \( \sim(\sim s \vee (\sim r \wedge s)) \)

\[
= s \wedge (r \vee \sim s)
\]

\[
= (s \wedge r) \vee (s \wedge \sim s)
\]

\[= s \wedge r\]