JEE MAINS Sample PAPER Answer key with Solutions

PHYSICS (PART – A)

Answer Key:

1.b	2.b	3.b	4.c	5.b	6.b	7.b	8.c	9.c	10.c
11.a	12.a	13.c	14.c	15.c	16.a	17.c	18.a	19.d	20.b
21.a	22.c	23.a	24.d	25.a	26.c	27.d	28.c	29.b	30.b

SOLUTIONS:

1. **Sol: (b)**

$[m] = [p]^x [v]^y$

$[ML^{-2} T^{-1}] = [ML^{-1} T^{-2}]^x [LT^{-1}]^y$

$[ML^{-2} T^{-1}] = [Mx\, L^{-x+y} T^{-2x-y}]$

$\Rightarrow x = 1, -x + y = -2$

$y = -2 + 1$

$= -1$

$\Rightarrow x = -y$

2. **Sol: (b)**

Using the notations,

C : coin

E : Elevator

$a_{CE} = a_{CO} - a_{EO}$

$= -g - 0$

$= -9.8\ ms^{-2}$

$u_{CE} = 0\ ms^{-1}$

$s_{CE} = -2.45$ m

We have,
$s_{CE} = u_{CE}t + \frac{1}{2}a_{CE}\ t^2$

$-2.45 = 0 + \frac{1}{2}(-9.8)t$

$\Rightarrow t^2 = \frac{1}{2} \Rightarrow t = \frac{1}{\sqrt{2}}$ s

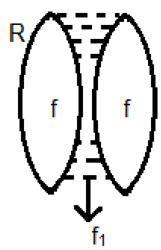
3. **Sol: (b)**

Acceleration is + ve till it becomes zero. Thus velocity is maximum at t = 11s
Area under a – t graph = change in velocity

$\frac{1}{2} \times 11 \times 10 = v - 0$

$\Rightarrow v = 55\ ms^{-1}$

4. **Sol: (c)**



$\frac{1}{f_{eq}} = \frac{1}{f} + \frac{1}{f_1} + \frac{1}{f}$

$= \frac{2}{f} + \frac{1}{f_1}$

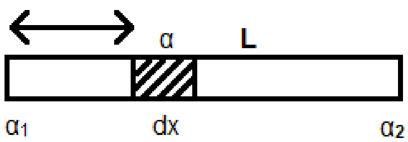
Where, $\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{+R} - \frac{1}{(-R)}\right)$

$= \frac{1}{R}$

$\therefore \frac{1}{f_1} = \frac{-2}{3f}$

$$\Rightarrow \frac{1}{f_{eq}} = \frac{2}{f} - \frac{2}{3f} \Rightarrow f_{eq} = \frac{3f}{4}$$

5. **Sol: (b)**



Consider a small length dx at which $\alpha = \alpha_1 + \frac{\alpha_2 - \alpha_1}{L}x$. Let dx^1 be the change in its length due to and increase in temperature ΔT.

$$dx^1 = dx \propto \Delta T$$
$$= \Delta T\left(\alpha_1 + \frac{\alpha_2 - \alpha_1}{L}x\right)dx$$

$$\int_0^{\Delta \ell} dx^1 = \Delta T\left[\alpha_1 \int_0^L dx + \frac{\alpha_2 - \alpha_1}{L}\int_0^L x dx\right]$$

= ∟ $\Delta T\left[\alpha_1 + \frac{\alpha_2 - \alpha_1}{2}\right]$

$$\Delta \ell = L\left[\frac{\alpha_1 + \alpha_2}{2}\right]\Delta T$$

But
$$\Delta \ell = L \propto_{eff} \Delta T$$

$$\therefore \propto_{eff} = \frac{\alpha_1 + \alpha_2}{2}$$

6. **Sol: (b)**

$$q = CE\left[1 - e^{-\frac{t}{RC}}\right]$$

$$\frac{q}{c} = E\left[1 - e^{-\frac{t}{RC}}\right]$$

$$0.75 = 1.5\left[1 - e^{-\frac{t}{1}}\right] (RC = 1)$$

$$\Rightarrow e^{-1} = \frac{1}{2} \Rightarrow e^t = 2$$

$$\Rightarrow t = \ell n 2$$

7. **Sol: (b)**

Stopping potential remains the same as it is independent of intensity of light.
Saturation current ∝ Intensity

$$i_1 \propto I_1 \propto \frac{1}{r_1^2}$$

$i_2 \propto I_2 \propto \frac{1}{r_1^2}$

$\Rightarrow \frac{i_2}{i_1} = \left(\frac{r_2}{r_1}\right)^2$

$= \left(\frac{0.2}{0.6}\right)^2$

$i_2 = \frac{1}{9} \times i_1$

$= \frac{27}{9}$

= 3mA

8. **Sol: (c)**

$y_1 = \frac{10\lambda D}{d}$

$y_2 = \frac{5\lambda_2 D}{d}$

$\Rightarrow \frac{y_1}{y_2} = \frac{2\lambda_1}{\lambda_2}$

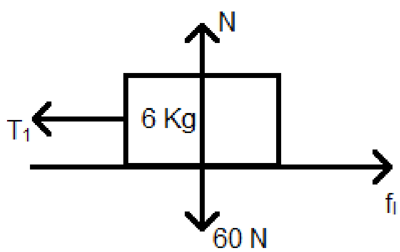
$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{y_1}{2y_2}$

9. **Sol: (c)**

From Lami's theorem

$\frac{T_1}{\sin 135^\circ} = \frac{W}{\sin 135} \Rightarrow T_1 = W$

Also,



$T_1 = f_\ell$

$= 0.5 \times 60$

$= 30N$

$\Rightarrow W = 30N$

10. **Sol: (c)**

Applying Bernoulli's equation at A and C,

$\frac{1}{2}\rho v_A^2 + \rho g h_1 + P_A = \frac{1}{2}+\rho v_C^2 + \rho g h_2 + P_C$ . . . .(1)

at B and D,

$\frac{1}{2}\rho v_B^2 + P_B = \frac{1}{2}\rho v_D^2 + P_D$ . . . .(2)

Also,

$P_B = P_A + \rho g h_1$

$P_D = P_C + \rho g h_2$

$\Rightarrow P_B - P_D = (P_A + \rho g h_1) - (P_C + \rho g h_2)$

$= \frac{1}{2}\rho(v_C^2 - v_A^2)$ from (1)

Substituting this result in (2),

$\frac{1}{2}\rho v_D^2 = \frac{1}{2}\rho v_B^2 + \frac{1}{2}\rho(v_C^2 - v_A^2)$

$v_D^2 = v_B^2 + v_C^2 - v_A^2$

$= (4)^2 + (4)^2 - (2)^2$

$v_D = \sqrt{28}$ ms^{-1}

11. **Sol: (a)**

Applying WET from A to C,

$K_A + U_A = K_C + U_C$

$0 + mg(14) = K_C + mg(7)$

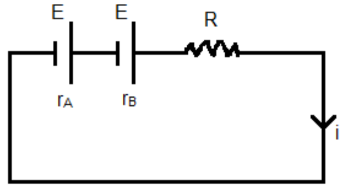
$\Rightarrow K_C = 7mg$

$= 7 \times 2 \times 10$

$= 140$ J

12. **Sol: (a)**

At steady state the capacitor behaves like infinite resistance and inductor behaves like short. Between x and y we have a balanced wheat stone network of resistance R. The circuit at steady state is



i = $\frac{1}{R + r_A + r_B}$

$V_A = 0$

But

$V_A = E - ir_A$

⇒ $E = ir_A$

E = $\frac{2Er_A}{R + r_A + r_B}$

$R + r_A + r_B = 2r_A$

$R = r_A - r_B$ $(r_A > r_B)$

13. **Sol: (c)**

R = $\frac{mv \sin 30°}{qB}$

= $\frac{mv}{2qB}$

Pitch,

x = $\frac{2\pi mv \cos 30°}{qB}$

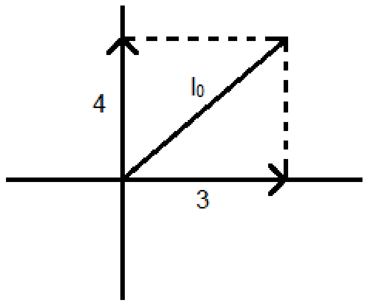
= $\frac{\sqrt{3}\,\pi mv}{q_B}$

x = $\sqrt{3}\pi(2R)$

⇒ R = $\frac{x}{2\sqrt{3}\pi}$

14. **Sol: (c)**

In the phasor diagram representation

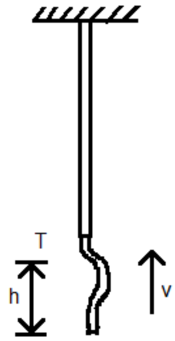


$I_0 = \sqrt{4^2 + 3^2}$

= 5

I_{nms} = $\frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

15. **Sol: (c)**



v = $\sqrt{\frac{T}{\mu}}$

T = μhg

v = $\sqrt{\frac{\mu hg}{\mu}}$

$\Rightarrow v \propto \sqrt{h}$

16. **Sol: (a)**

$v \propto \frac{1}{n}$

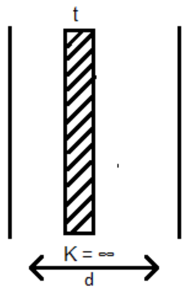
$\frac{v_1}{v_2} = \frac{n_2}{1} \Rightarrow \frac{v_1}{\frac{v_1}{3}} = n_2 \Rightarrow$ $n_2 = 3$

$r \propto n^2$

$\Rightarrow \frac{r_1}{r_2} = \left(\frac{n_1}{n_2}\right)^2$

$\frac{r}{r_2} = \left(\frac{1}{3}\right)^2 \Rightarrow$ $r_2 = 9r$

17. **Sol: (c)**



$C_f = \frac{\epsilon_0 A}{d - t + \frac{t}{K}}$

Initially,

$C = \frac{\epsilon_0 A}{d}$

$\Rightarrow C_f = \frac{cd}{d - t + \frac{t}{\infty}}$

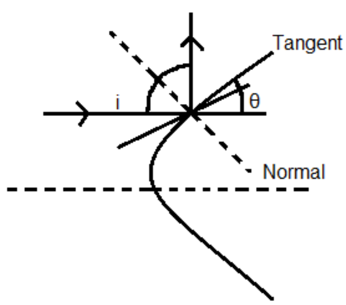
$\Rightarrow C_f = \frac{cd}{d - t}$

4.5d – 4.5t = d

4.5t = 3.5d

$t = \frac{7d}{9}$

18. **Sol: (a)**



$i = r = 45°,\ \theta = 45°$

$\tan 45° = \frac{dy}{dx}$

$1 = \frac{2}{2\sqrt{2x}}$

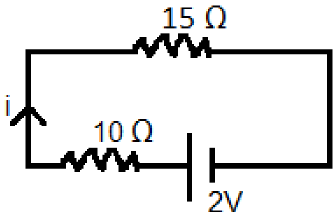
$\Rightarrow \quad x = \frac{1}{2}$

$y = \sqrt{2 \times \frac{1}{2}} = 1$

Thus (x, y) = $\left(\frac{1}{2}, 1\right)$

19. **Sol: (d)**

D$_1$ is forward biased and D$_2$ is reverse biased. Thus



i = $\frac{2}{25}$

= 0.08 = 80 mA

20. **Sol: (b)**

$I_Q = m_Q(nr)^2$
$= \mu(2\pi nr)(nr)^2$ [μ : linear density]
$= 2\pi\mu n^3r^3$

$I = mr^2$
$= \mu(2\pi r)r = 2\pi\mu r^3$

$I_Q = 4I_p$
$2\pi\mu n^3r^3 = 4\ 2\pi\mu r^3$
$\Rightarrow$ $n = (4)^{\frac{1}{3}}$

21. **Sol: (a)**

The net gravitational force on a given star provides the necessary centripetal force.

$\frac{mv^2}{r} = \frac{GMm}{r^2} + \frac{G(m)(m)}{(2r)^2}$

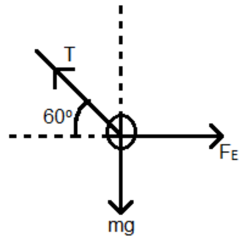
$\Rightarrow$ $v = \frac{\sqrt{G(4M+m)}}{2\sqrt{r}}$

The time period is,

$T = \frac{2\pi r}{v}$

$= \frac{4\pi r^{3/2}}{\sqrt{G(4M+m)}}$

22. **Sol: (c)**



$T\cos 60° = F_E = 9 \times 10^9 \times \frac{q^2}{(0.3)^2}$

$\Rightarrow \frac{T}{2} = q^2 \times 10^{+11}$

and

T sin30° = mg

$T \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{10} \times 10^{-3} \times 10$

$\Rightarrow \quad T = 2 \times 10^{-3}$ N

$\Rightarrow \quad q^2 \times 10^{11} = \frac{2 \times 10^{-3}}{2}$

$q = 10^{-7}$ C

23. **Sol: (a)**

The specific heat is given by

$C = C_V + \frac{P}{n}\frac{dv}{dT}$

We have,

$T = T_0 + \alpha V$

$\frac{dT}{dV} = 0 + \alpha$

$\Rightarrow \frac{dV}{dT} = \frac{1}{\alpha}$

$\frac{P}{n} = \frac{RT}{V} = \frac{R}{V}[T_0 + \alpha V]$

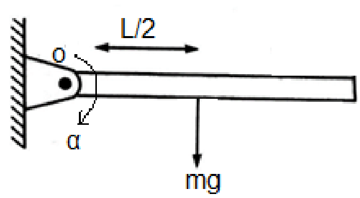
$= \frac{RT}{V} + R\alpha$

$C = C_V + \left(\frac{RT_0}{V} + R\alpha\right)\times\frac{1}{\alpha}$

$= (C_V + R) + \frac{RT_0}{\alpha V}$

$C = C_P + \frac{RT_0}{\alpha V}$

24. **Sol: (d)**



Once the string is cut,

$\tau_0 = I_0\alpha$

$mg\frac{L}{2} = \frac{mL^2}{2}\alpha$

$\Rightarrow \alpha = \frac{3g}{2L}$

25. **Sol: (a)**

$= \alpha - \beta t$

when it stops $\omega = 0$

$\Rightarrow \quad t = \frac{\alpha}{\beta}$

Also

$\frac{d\theta}{dt} = \alpha - \beta t$

$\int d\theta = \alpha\int_0^{\alpha/\beta} dt - \beta\int_0^{\alpha/\beta} t dt$

$\theta = \alpha\left(\frac{\alpha}{\beta}\right) - \frac{\beta}{2}\times\frac{\alpha^2}{\beta^2}$

$= \frac{\alpha^2}{2\beta}$

26. **Sol: (c)**

We have,

$X = \frac{S\ell_1}{\ell_2}$

where

$\ell_1 = 40 \pm 0.1$ cm = 40 ± 0.25 %

$\ell_1 = 60 \pm 0.1$ cm = $60 + 0.16$ %

$\frac{\Delta x}{x} = \frac{\Delta\ell_1}{\ell_1} + \frac{\Delta\ell_2}{\ell_2}$

= 0.25 + 0.16

= 0.41 %

$X_m = \frac{90 \times 40}{60}$

= 60 cm

$\therefore X = 60 \pm 0.41\%$

$= (60 \pm 0.25)\Omega$

27. **Sol: (d)**

From the given data

$\frac{K(q_1 + q_2)q_3}{L^2} = +2$

$\frac{K(q_2 + q_3)q_1}{L^2} = -4$

$\frac{K(q_1 + q_3)q_2}{L^2} = -18$

$q_1q_3 + q_2q_3 = \frac{2L^2}{K}$

$q_2q_1 + q_3q_1 = -\frac{4L^2}{K}$

$q_1q_2 + q_3q_2 = -\frac{18L^2}{K}$

on solving them simultaneously,

$q_1q_2 = -\frac{12L^2}{K}$

$q_2q_3 = -\frac{6L^2}{K}$

$q_1q_3 = -\frac{8L^2}{K}$

$\Rightarrow \frac{q_1}{q_2} = -\frac{4}{3}$

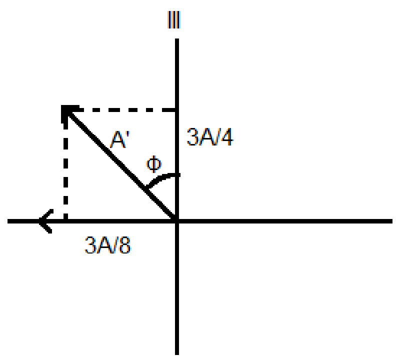
$\frac{q_1}{q_3} = -2$

$\frac{q_2}{q_3} = -\frac{3}{2}$

$\Rightarrow q_1 : q_2 : q_3 = 1 : -\frac{3}{4} : \frac{1}{2}$

$= 4 : -3 : 2$

28. **Sol: (c)**



$A' = \sqrt{\frac{9A^2}{16} + \frac{9A^2}{64}}$

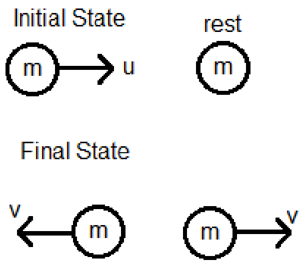
$= \sqrt{\frac{45A^2}{64}}$

$= \frac{3\sqrt{5}\,A}{8}$

$\tan\phi = \frac{\frac{3A}{8}}{\frac{3A}{4}} = \frac{1}{2}$

$\Rightarrow \phi = \tan^{-1}\left(\frac{1}{2}\right)$

29. **Sol: (b)**



<u>COM</u>
$mu = -mv + Mv$

$\Rightarrow \quad v = \frac{mu}{M - m}$ . . . .(1)

<u>NLC</u>
$v_2 - v_1 = e(U_1 - U_2)$
$v - (-v) = (1)\ (v - 0)$
$2v = u$ . . . .(2)

(2) in (1)

$\frac{u}{2} = \frac{mu}{M - m}$

$\Rightarrow \quad 3m = M$

$\frac{m}{M} = \frac{1}{3}$

30. **Sol: (b)**
From Einstein's equation of the photoelectric effect
$hv = hv_0 + ev_0$

$\Rightarrow \quad V_0 = \frac{h}{e}\gamma - \frac{h}{e}\gamma_0$

Slope = $\frac{h}{e} \Rightarrow$ h = (slope)

PART – B (CHEMISTRY)

<u>KEY:</u>

31. c	**32. d**	**33. d**	**34. b**	**35. b**	**36. b**	**37. a**	**38. d**	**39. d**	**40. c**
41. c	**42. b**	**43. d**	**44. a**	**45. a**	**46. a**	**47. d**	**48. b**	**49. a**	**50. c**
51. c	**52. b**	**53. b**	**54. a**	**55. b**	**56. b**	**57. a**	**58. c**	**59. c**	**60. d**

<u>Solutions:</u>

31. Ans: (c)

Solution:

de-Broglie's wave length is given by

$\lambda = \frac{h}{mV}$; $V = \frac{h}{m\lambda}$

$KE = \frac{1}{2}mV^2 = \frac{1}{2}m\left(\frac{h}{m\lambda}\right)^2$

$KE \; \alpha \; \frac{1}{m}$

Hence, $E_1 > E_3 > E_2$

32. Ans: (d)

Solution: Radiodecay is a first order reaction and hence the half life remains constant. Thus 48g of the sample will reduce to 3g after 4 half lives.

33. Ans: (d)

Solution: Reactions (a), (b) and (c) involve a proton transfer and hence they are Bronsted acid – base reactions. Reaction (d) has no proton transfer but the electron pairs of ammonia is transferred to copper ion and hence it is a Lewis acid-base reaction.

34. Ans: (b)

Solution: For Adiabatic process

W = $\Delta E - p\Delta V$

$$nC_v\Delta T = -P\Delta V$$

$$1 \times \frac{R}{r-1} \times (T_j - T_i) = -P(V_2 - V_1)$$

$$T_j = T - \frac{r-1}{R}P(V_2 - V_1)$$

$$T_j = T - \frac{\left(\frac{S}{3} - 1\right) \times 1 \times (2-1)}{0.0821}\left(r = \frac{5}{3} \; for \; mono \; atomic \; gas\right)$$

$$T_j = T - \frac{2}{3 \times 0.0821}$$

35. Ans: (b)

Aldehydes with Grignard reagent gives secondary alcohol.

36. Ans: (b)

Solution: Conceptual

37. Ans: (a)

Solution: Normality = 5.6 x volume strength

$$N = \frac{N_1 + N_2 + N_3}{3} = \frac{\frac{20+25+50}{5.6}}{3} = 5.65N$$

38. Ans: (d) The racemic mixture formed will be resolvable.

39. Ans: (d)

Solution: since compound 'X' donot give any ppt with H_2S, it cannot be $ZnCl_2$ or $SnCl_2$.

The salt should contain chloride radical as it gives dense white fumes with NH4OH. The salt solution gives precipitate with NH_4OH which is soluble in excess of NaOH. Hence it is not ferric chloride. The salt should be $AlCl_3$.

40. Ans: (c)

Solution:

$$2d\,\mathrm{Sin}\theta = n\lambda$$

$$d = \frac{n\lambda}{2\mathrm{Sin}\theta} = \frac{1\times 2.29}{2\times 0.4561} = 2.51\text{angstroms}$$

41. Ans: (c)

Solution:

$$n_1 = \frac{20}{180} = 0.111\ ;\ n_2 = \frac{70}{18} = 3.89$$

$$\frac{P_0 - P_S}{P_0} = \frac{n_1}{n_1 + n_2}$$

$$\frac{25.21 - P_S}{25.21} = \frac{0.111}{0.111 + 3.89} = 0.0277$$

$$25.21 - P_S = 0.0277 \times 25.21 = 0.699$$

$$P_S = 25.21 - 0.699 = 24.5\text{torr}$$

42. Ans: (b)

Solution: More the stability of carbocation, easier is the dehydration.

43. Ans: (d)

    Solution: Conceptual

44. Ans: (a)

    Solution: NaOH reacts with $AgNO_3$ to form AgOH which is unstable and converts into a brown ppt of Ag_2O.

45. Ans: (a)

    Solution: conceptual

46. Ans: (a)

    Solution: the gas with a higher value of critical temperature will have stronger vanderwaal's forces and hence easily adsorbed on charcoal.

47. Ans: (d)

    Solution: Using salt bridge we can eliminate the liquid junction potential

48. Ans: (b)

    Solution: Conceptual

49. Ans: (a)

    Solution:

$$k = \frac{0.693}{T_{1/2}} = \frac{0.693}{2\times10^{-9}} = 3.46\times10^{8}\,s^{-1}$$

50. Ans: (c)

    Solution: It is pinacol – pinacolone rearrangement and methyl group migrates in preference to benzene ring. If benzene ring migrates it results in a 4-membered ring which is not stable.

51. Ans: (c)

    Solution:

$$Ag\,|\,Ag^{+}(1M)\,||\,Cu^{2+}(1M)\,|\,Cu \quad E_{cell} = -0.46V$$
$$E^{0}_{Cu^{2+}/Cu} - E^{0}_{Ag^{+}/Ag} = -0.46V$$
$$Zn\,|\,Zn^{2+}(1M)\,||\,Cu^{2+}(1M)\,|\,Cu \quad E_{cell} = +1.10V$$
$$E^{0}_{Cu^{2+}/Cu} - E^{0}_{Zn^{2+}/Zn} = 1.10V$$
$$Zn\,|\,Zn^{2+}(1M)\,||\,Ag^{+}(1M)\,|\,Ag$$
$$E^{0}_{Ag^{+}/Ag} - E^{0}_{Zn^{2+}/Zn} = \left(E^{0}_{Cu^{2+}/Cu} - E^{0}_{Zn^{2+}/Zn}\right) - \left(E^{0}_{Cu^{2+}/Cu} - E^{0}_{Ag^{+}/Ag}\right) = 1.1 - (-0.46) = 1.56V$$

52. Ans: (b)

Solution: $5Cl_2 + I_2 + 6H_2O \longrightarrow 2HIO_3 + 10HCl$

53. Ans: (b)

Solution:

Milliequivalents of acid = Milliequivalents of Na_2CO_3 + Milliequivalents of NaOH

$$50 \times N = (25 \times 0.5) + (20 \times 0.5)$$
$$N = 0.45N$$

54. Ans: (a)

Solution: Lower the gold number, more effective is its protective action.

55. Ans: (b)

Solution: Radioactivity is independent of temperature.

56. Ans: (b)

Solution: conceptual

57. Ans: (a)

Solution:

	$NH_4HS \rightleftharpoons$	NH_3	$+ H_2S$
Initial	–	0.5	0
Equilibrum	–	$0.5 + p$	p

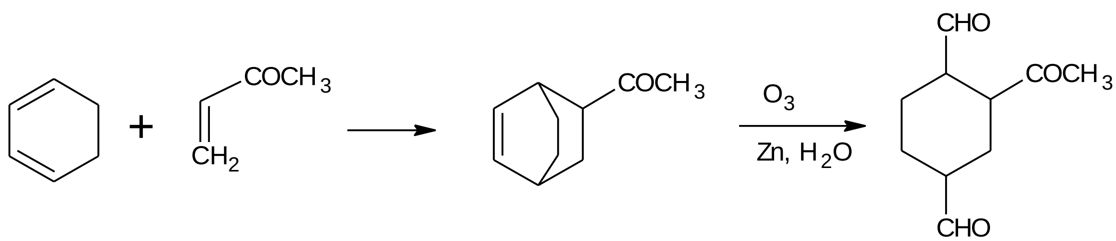
$$0.5 + p + p = 0.84$$
$$p = 0.17$$
$$K_p = p_{NH_3} \times p_{H_2S} = 0.67 \times 0.17 = 0.114 atm^2$$

58. Ans: (c)

Solution: From the graph we can make out that $P^0_A < P^0_B$, the component 'B' is more volatile than 'A'. Hence the mole fraction of B is higher than A in vapour phase.

59. Ans: (c)

Solution:



60. Ans: (d)

Solution:

Weight of CO_2 = 1.0g

Weight of oxygen in oxide = Weight of oxygen in Carbon dioxide = $\frac{32}{44}\times 1 = 0.727g$

Mass of metal oxide = 3.7g, Mass of oxygen = 0.72g, so mass of metal = 3.7-0.72 = 2.973g.

Eq. Mass of metal = $\frac{2.973}{0.727}\times 8 = 32.7$

App. Atomic mass = $\frac{6.4}{\text{Sp.heat}} = \frac{6.4}{0.095} = 67.36$

$n = \frac{\text{App.Atomicmass}}{\text{Eq.mass}} = \frac{67.36}{32.7} \approx 2$

Exact atomic mass $= 32.7 \times 2 = 65.4$

PART – C (MATHEMATICS)

KEY:

61.d	**62.c**	**63.a**	**64.a**	**65.b**	**66.c**	**67.b**	**68.a**	**69.b**	**70.c**
71.c	**72.c**	**73.a**	**74.d**	**75.b**	**76.a**	**77.a**	**78.c**	**79.b**	**80.b**

81.c	82.c	83.b	84.d	85.d	86.b	87.a	88.b	89.d	90.b

SOLUTION:

61. As z and I z are mutually perpendicular and z+iz is resultant of z &iz
$\therefore$ Triangle in argand plane is OAC where OA = z, AC = iz

$$\therefore Area = \frac{1}{2}|z||iz| = 16$$
$$\Rightarrow |z|^2 = 32 \Rightarrow |z| = 4\sqrt{2}$$

62. Let $z = x + iy$
Given cure is $x^2 + y^2 - \frac{3}{\sqrt{2}} \times + \frac{3}{\sqrt{2}} y - \sqrt{2} = 0$
Let distance from centre$\left(\frac{3}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}\right)$ to origin is d = $\frac{3}{2}$
Then maximum distance from origin = d+ r
Minimum distance from origin = d – r
$\therefore r_1 + r_2 = 2d$= 3

63. $I = \int_{-a}^{a} \frac{a-x}{\sqrt{a^2-x^2}} dx$

$$= \int_{-a}^{a} \frac{a}{\sqrt{a^2 - x^2}} dx - \int_{-a}^{a} \frac{x}{\sqrt{a^2 - x^2}} dx$$
$$= 2a \int_{0}^{a} \frac{1}{\sqrt{a^2 - x^2}} dx - 0 \left[\because \frac{x}{\sqrt{a^2 - x^2}} \text{ is odd function}\right]$$
$$= 2a \sin^{-1} \frac{x}{a}\Big|_0^a = 2a[\sin^{-1}(1) - \sin^{-1}(0)] = \pi a$$

64. $f'(x) = \frac{\frac{1}{2} - \sin^2 x}{f(x)} \Rightarrow 2f(x).f'(x) = 1 - 2\sin^2 x$

$$\Rightarrow \frac{d}{dx}[f(x)]^2 = \cos 2x$$
$$\Rightarrow f(x)^2 = \int \cos 2x \, dx = \frac{\sin 2x}{2} + c$$
$$\therefore f(x) = \pm\sqrt{\frac{\sin 2x}{2} + c} \Rightarrow period = \pi$$

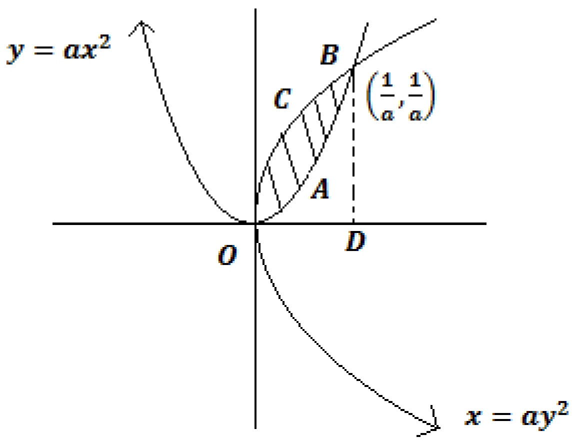
65. Here D > 0; $\alpha < 2 < B$
D = $(p+1)^2 - 4(p^2 + p - 8) > 0$

$\Rightarrow 3p^2 + 2p - 33 < 0$
$\Rightarrow (3p+11)(p-3) < 0 \Rightarrow p \in \left(-\frac{11}{3}, 3\right)$ -------- (1)
$\Rightarrow f(2) < 0$

Also f(2) = 4 –2 (p+1)+p^2 + p –8 < 0
$\Rightarrow (p-3)(p+2)$<0 $\Rightarrow p \in (-2,3)$--------(2)

Common solution of 1 and 2 is (-2,3).

66.



Required area = OABCO
$= \int_0^{1/a}\left(\sqrt{\frac{x}{a}} - ax^2\right)dx = 1 \Rightarrow a^2 = \frac{1}{3} \Rightarrow a = \frac{1}{\sqrt{3}}$

67. $X_w + X_R + X_B = 10$
$\therefore Required\ ways = 3 + 10 - 1_{C_{10}} = 12_{C_2} = \frac{12.11}{2} = 66$

68. S = $\sum_{r=0}^{n}(-1)^r\,(3+5r).n_{C_r}$
$$= 3\left(\sum_{r=0}^{n}(-1)^r.n_{C_r}\right) + 5\left(\sum_{r=1}^{n}(-1)^r.n.n-1_{C_{r-1}}\right)$$
= $3(1-1)^n + 5n(1-1)^{n-1} = 0$

69. (abcd)$^{10}\left[\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right]^{10}$

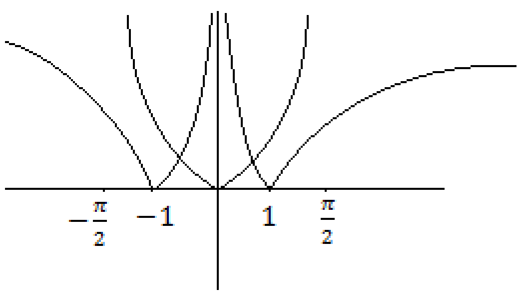
$\therefore Required\ co-efficent = a^{-2}b^{-6}c^{-1}d^{-1}\ in\ \left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)^{10}$

$= \frac{10!}{2!6!1!1!} = 2520$ *(Number of ways to form the group of 6, 2, 1 and 1 from 10 objects)*

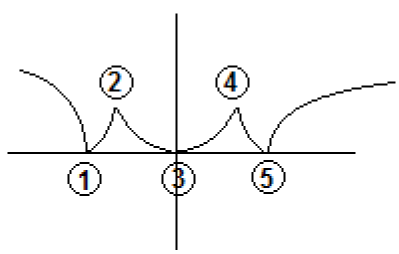
70. $x^m . y^n = a^{m+n}$

$$\Rightarrow m\log x + n\log y = (m+n)\log a$$
$$\Rightarrow \frac{m}{x}+\frac{n}{y}.\frac{dy}{dx} = 0$$
$$\Rightarrow \frac{dy}{dx} = \frac{-my}{nx}$$
$$\therefore sub-tangent = \left|\frac{y}{m}\right| = \frac{nx}{m} = \frac{n}{m}.x$$
$$\Rightarrow propotional\ to 'x'$$

71.



$$\therefore Required\ curve\ is$$



$$\therefore 5\ non-differentiable\ points\ are\ there$$

72. Clearly given points lie on the circle $x^2 + y^2 = 18$

$$\therefore Circum\ centre\ (s) = (0,0)$$

Centroid (G) = $\left(\frac{\sum x}{3}, \frac{\sum y}{3}\right)$

Orthocetre divides line joining ‘G’ and ‘S’ in the ratio 2:3 external

$$\therefore G = \left(\frac{\sum x}{3}, \frac{\sum Y}{3}\right) \quad s = (0,0)$$

2 : 3 (external)
0 = $(\sum X, \sum Y)$
= $(\sqrt{13} + \sqrt{7} - \sqrt{18}, \sqrt{5} - \sqrt{11})$

73. Equation of tangent to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ *is* $y = mx \pm \sqrt{16m^2 + 9}$

$$\Rightarrow mx - y \pm \sqrt{16m^2 + 9} = 0$$

If it is also Tangent to the circle $x^2+y^2 = 12$ then
r = d holds good.

$$\therefore \sqrt{12} = \frac{\sqrt{16M^2 + 9}}{\sqrt{M^2 + 1}}$$
$$\Rightarrow 12(m^2 + 1) = 16m^2 + 9$$
$$\Rightarrow m^2 = \frac{3}{4} \Rightarrow m = \pm \frac{\sqrt{3}}{2}$$

$$\therefore Tangent\ is\ 2.y = \sqrt{3}x \pm \sqrt{21}$$

74. $\bar{r} = (2 + \lambda)i - (1 + 2\lambda)j + (1 - 3\lambda)k$

$$= (2i - j + k) + \lambda(i - 2j - 3k)$$

$\Rightarrow this\ line\ is\ passing\ through\ (2,-1,1) and\ paralel\ to\ the\ line\ with\ d.r^1s(1,-2,-3)$

Required plane passing through (1,2,3)
∴dir's of line joining (1,2,3) and (2, -1,1) is (1,-3,-2)

$$\therefore Equation\ of\ plane\ is \begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & -3 & -2 \\ 1 & -2 & -3 \end{vmatrix} = 0$$

i.e 5x +y +z = 10

75. $\bar{r} = (2i - j + k) + \lambda(i - 2j - 3k) = \bar{a} + \lambda\bar{b}$

$$\bar{r} = (i + 2j + 3k) + \mu(3i - 2j - k) = \bar{c} + \mu\bar{d}$$

S.D = $\frac{|[\overline{a-c}\ \bar{b}\ \bar{d}]|}{|b \times d|} = \frac{|[i-3j-2k\ \ i-2j-2k\ \ 3i-2j-k]|}{|(i-2j-3k)\times(3i-2j-k)|}$

$$= \frac{\begin{vmatrix} 1 & -3 & -2 \\ 1 & -2 & -3 \\ 3 & -2 & -1 \end{vmatrix}}{\begin{vmatrix} i & j & k \\ 1 & -2 & -3 \\ 3 & -2 & -1 \end{vmatrix}} = \frac{12}{|-4(i + 2j + k)|} = \frac{3}{\sqrt{6}} = \frac{\sqrt{3}}{\sqrt{2}}$$

76. If A,B,C and D are consecutive vertices of a parallelogram
then D = A+C–B

Here A = (1,2,-1), B = (2,3,0), C = (–3,1,–2)
Since these are not consecutive vertices, fourth vertex will be either of A+B–C (or) A+C–B (or) B+C–A
i.e A+B –C = (6, 4, 1)
A+C –B = (–4, 0, –3)
B+C –A = (–2, 2, –1)

77. Let (x, y) is the point on the curve y = $\frac{e^x+e^{-x}}{2}$ nearer to origin

$$\therefore op^2 = x^2 + y^2 = x^2 + \left(\frac{e^x + e^{-x}}{2}\right)^2$$

Let f(x) = $x^2 + \frac{1}{4}(e^{2x} + e^{-2x} + 2)$

$$\Rightarrow f'(x) = 2x + \frac{1}{2}(e^{2x} - e^{-2x})$$

For maximum (or) minimum, we have f'(x) = 0 ⇒ $e^{-2x} - e^{2x} = 4x$
⇒ $x = 0$ also f''(0) > 0
∴ $at\ x = 0$ shortest distance exist i.e '1'

78. $y^2 = 4x$

$$\Rightarrow 2y\frac{dy}{dx} = 4$$
$$\Rightarrow \frac{dy}{dx} - \frac{2}{y}$$
$$\therefore M_1 = 1$$

$$x^2 + y^2 - 6x + 1 = 0$$
$$\Rightarrow 2x + 2y.\frac{dy}{dx} - 6 = 0$$
$$\Rightarrow \frac{dy}{dx} = \frac{3-x}{y}$$
$$\therefore M_2 = \left(\frac{dy}{dx}\right)_{1,2} = 1$$
$$\therefore M_1 = M_2 \Rightarrow curves\ touch\ each\ other$$

79. y = (c_1 + c_2) cos (x + c_3) –c_4 . e^{c5}. e^x
Let c_1 + c_2 = A and c_3 = B, c_4. e^{c5} = C

$$\therefore y = A\cos(X + B) - ce^x$$

3 arbitrary constants ⇒ $order = 3$
Actual equation after solving is $\frac{d^3y}{dx^3} + \frac{dy}{dx} = \frac{d^2y}{dx^2} + y$

80. n(s) = $10_{C_3}; E = \{(10,5,2)(8,2,4)(6,2,3)\}$

$$\therefore p(\in) = \frac{n(\in)}{n(s)} = \frac{3}{10_{C_3}} = \frac{1}{40}$$

81. $|-A| = (-1)^n|A| = \begin{cases} -1 \ if \ \ 'n' is \ odd \\ \ 1 \ if \ \ 'n' is even \end{cases}$

82. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$

$R_1 \to R_1 + bR_3; R_2 \to R_2 - aR_3$

$$\begin{vmatrix} 1+a^2+b^2 & 0 & -b(1+a^2+b^2) \\ 0 & 1+a^2+b^2 & a(1+a^2+b^2) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

$= \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}(1+a^2+b^2)^2$

$$R_3 \to R_3 - 2bR_1 + 2aR_2$$

$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 0 & 0 & 1+a^2+b^2 \end{vmatrix} = (1+a^2+b^2)^3$

83. Let x = sin y

$$\Rightarrow \sin^{-1}(1-siny) - 2\sin^{-1}(\sin y) = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}(1-siny) \quad = \frac{\pi}{2} + 2y$$

$$\Rightarrow 1 - \sin y = \cos 2y$$

$$\Rightarrow 1 - \cos 2y = \sin y \Rightarrow 2\sin^2 y - \sin y = 0$$

$$\Rightarrow \sin y = 0 \ (or) \sin y = \frac{1}{2}$$

$$\Rightarrow x = 0 \ (or) \ x = \frac{1}{2}$$

But x = $\frac{1}{2}$ is pseudo result

$\therefore x = 0$ is the only solution

84. $r_1 - r = 4R \sin\frac{A}{2}\left[\cos\frac{B}{2}\cos\frac{C}{2} - \sin\frac{B}{2}\sin\frac{C}{2}\right]$

$$\Rightarrow 6 = 4R\sin\frac{A}{2}\cos\left(\frac{B+C}{2}\right)$$

$$\Rightarrow 6 = 4R\sin^2\frac{A}{2} \Rightarrow \sin^2\frac{A}{2} = \frac{1}{2} \Rightarrow \angle A = \frac{\pi}{2}$$

85. $2\sin^2 x - a\sin x + (2a-8) = 0$

$$\Rightarrow \sin x = \frac{a \pm (a-8)}{4} \Rightarrow \sin x = \frac{a-4}{2}$$

$$\therefore -1 \le \frac{a-4}{2} \le 1 \Rightarrow a \in [2, 6]$$

86. $\underset{x\to a}{Lt} f(x) = \frac{3}{2}; \underset{x\to a}{Lt} g(x) = \frac{1}{2}$

$$\therefore \underset{x\to a}{Lt} f(x).g(x) = \frac{3}{4}$$

87. $\underset{x\to 0}{Lt} \frac{(9^x-1)(3^x-1)}{(1-\cos x)}.\left[\sqrt{2}+\sqrt{1+\cos x}\right]$

$= \underset{x\to 0}{Lt} \frac{\frac{a^x-1}{x}.\frac{3^x-1}{x}}{\frac{\sin^2 x}{x^2}}.(\sqrt{2}+\sqrt{1+\cos x})(1+\cos x)$

$$= \frac{\log 3^2.\log 3}{1^2}.(\sqrt{2}+\sqrt{2})(1+1)$$

$$= 8\sqrt{2}(\log 3)^2$$

88. Mean = 8 ⇒ $x + y = 14$ (assume unknown observations as x & y)
Variance = 16 ⇒ $x^2 + y^2 = 100$

$$\therefore (x+y)^2 + (x-y)^2 = 2(x^2+y^2)$$

$$\Rightarrow (x-y)^2 = 4 \Rightarrow x - y = \pm 2$$

∴on solving x + y = 14 and x – y =±2, we have
x = 6, y = 8 (or) x = 8, y = 6

89. $n(\mu) = 2000, n(A) = 1720, n(B) - 1450$

$$n(A\cup B) = n(A) + n(B) - n(A\cap B) = 3170 - n(A\cap B)$$

Since A ∪ B <$\mu, n(A\cup B) \le n(\mu)$

$$\Rightarrow 3170 - n(A\cap B) \le 2000$$

$$\Rightarrow n(A\cap B) \ge 1170$$

90. $f(x) = \frac{x-1}{x+1}$

$$\Rightarrow \frac{f(x)+1}{f(x)-1} = \frac{-2x}{2}$$

$$\Rightarrow x = \frac{f(x)+1}{1-f(x)}$$

Now $f(2x) = \frac{2x-1}{2x+1}$

$$= \frac{2\left(\frac{f(x)+1}{1-f(x)}\right)-1}{2\left(\frac{f(x)+1}{1-f(x)}\right)+1}$$

$$= \frac{3f(x)+1}{f(x)+3}$$