JEE MAINS Sample Paper 2018 (Answer key & Solutions)

Part - A (PHYSICS)

1. b	2. b	3. a	4. b	5. c	6. c	7. d	8. c	9. a	10. a
11. c	12. d	13. c	14. d	15. d	16. a	17. d	18. c	19. c	20. c
21. b	22. b	23. a	24. d	25. c	26. b	27. c	28. b	29. b	30. d

PART – B (CHEMISTRY)

31. b	32. a	33. b	34. a	35. d	36. a	37. c	38. b	39. c	40. d
41. d	42. c	43. d	44. a	45. a	46. a	47. b	48. a	49. d	50. c
51. d	52. d	53. c	54. a	55. a	56. d	57. b	58. d	59. a	60. b

PART – C (Mathematics)

61. b	62. c	63. b	64. c	65. b	66. a	67. d	68. a	69. c	70. a
71. b	72. d	73. d	74. a	75. b	76. a	77. c	78. c	79. c	80. a
81. c	82. d	83. d	84. b	85. a	86. b	87. a	88. d	89. c	90. c

1. Intensity at a distance x from point source is given by,

$I = \frac{P}{4\pi x^2}$

Consider an elemental shell of radius 'x' and thickness dx, in this region, the energy contained is,

$dE = \frac{I}{C}(4\pi x^2 dx) = \frac{Pdx}{C}$

Let dn be the number of photons in this elemental shell,

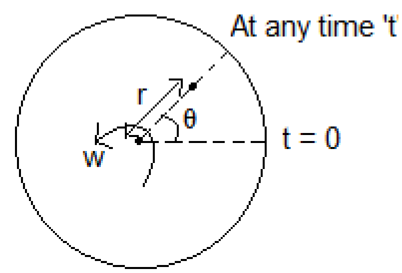
Thendn $\times \frac{hC}{\lambda} = dE = \frac{Pdx}{C}$

$\Rightarrow dn = \frac{p\lambda dx}{hc^2}$

Total no. of photons in the shell of inner radius 'r' and outer radius 2r is

$n = \int dn = \int_r^{2r} \frac{P\lambda dx}{hC^2} = \frac{P\lambda r}{hC^2}$

2. The situation is as shown in the figure. At any time 't' its location is described by two parameters –ri.e, the distance from centre and θ, the angle rotated by spoke in time 't'.



Velocity of bead at time t, $\vec{v} = u\,\hat{e}_r + r\omega\hat{e}_t$

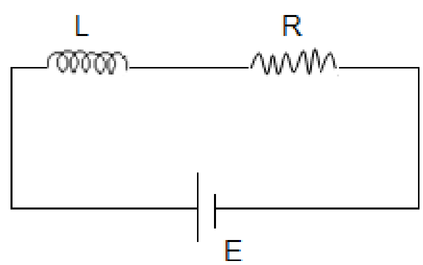
Where $\hat{e}_r$&$\hat{e}_t$ one unit vectors along radial and tangential directions

$\therefore \vec{v} = u\,\hat{e}_r + (ut)\omega\hat{e}_t$where r = ut as $\frac{dr}{dt} = u$

3. In LR growth circuit current grows to 63.2% of its maximum value, in one time constant, it means required time is

$t = T = \frac{L}{R}$

If the circuit is switched on at t = 0, then



$i(t) = \frac{E}{R}[1 - e^{-t/T}]\left[T = \frac{L}{R}\right]$

$\therefore i_{max} = \frac{E}{R}$

$\Rightarrow 3 = \frac{8}{R} \Rightarrow \quad R = \frac{8}{3}\Omega$

So, $t = T = \frac{L}{R} = \frac{6\times10^{-3}}{\frac{8}{3}} = \frac{9}{4}\text{ms}$

4. $\frac{dv}{dt} = v\frac{dv}{dx} = -Kx^2$

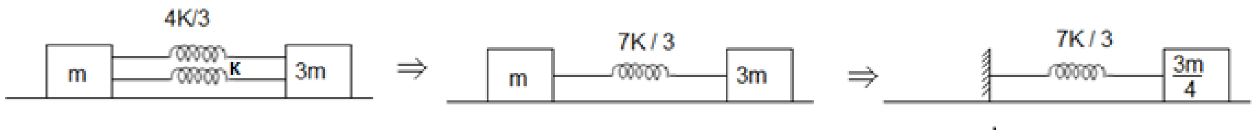
$\Rightarrow Vdv = -Kx^2 dx$

Let V_1 and V_2 be the velocities of the particle at locations x_1 and x_2 respectively. Then

$$\frac{V_2^2 - V_1^2}{2} = -K\left[\frac{x_2^3 - x_1^3}{3}\right]$$

So, loss in K.E α x^3

5. The given system can be redrawn as

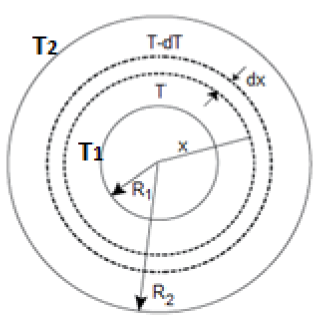


The above simplification has been done by using series and parallel combinations of springs and the reduced mass concept.

In series, $\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}$,In parallel, $K_{eq} = K_1 + K_2$. Reduced mass, μ = $\frac{m_1 m_2}{m_1 + m_2}$

Required time period, T = 2π $\sqrt{\frac{\mu}{K_{eq}}} = 3\pi\sqrt{\frac{m}{7k}}$.

6. Temperature is decreasing we are going out, let at a distance 'x' from the centre, the temperature gradient is $\frac{-dT}{dx}$



At this location, K = $a_0 Tx$

From H = –KA $\left(\frac{-dT}{dx}\right)$

$\Rightarrow$H = $(-a_0 Tx)(4\pi x^2)\left(\frac{dT}{dx}\right)$

$\Rightarrow \int_{R_1}^{R_2} \frac{Hdx}{x^3} = -\int_{T_1}^{T_2} 4\pi a_0 T\, dT$

$\Rightarrow$ H = $\frac{4\pi a_0 R_1^2 R_2^2 (T_1^2 - T_2^2)}{R_2^2 - R_2^2}$

7. In this case, we can't take reference point for potential at infinity as wire itself is of infinite dimension and hence we can't define absolute potential due to infinite

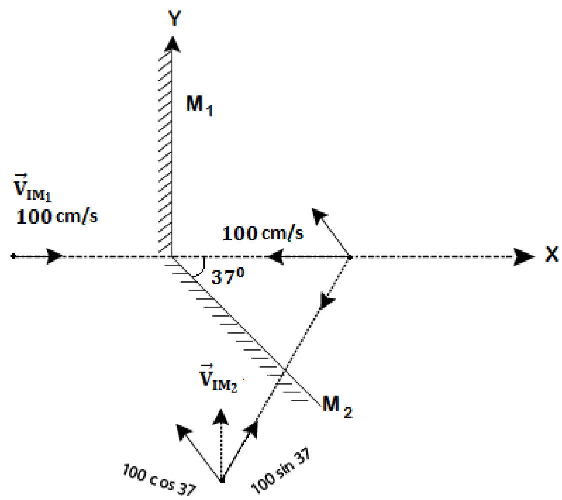
(charged) wire. Only potential difference between two points can be found, provided none of the point lies on wire.

8. $\vec{V}_{cm} = \frac{m_1\vec{V}_1+m_2\vec{V}_2}{m_1+m_2} = \frac{6\hat{\imath}+5\hat{\jmath}}{5}$

$\vec{a}_{cm} = \frac{m_1\vec{a}_1+m_2\vec{a}_2}{m_1+m_2} = \frac{6\hat{\imath}+6\hat{\jmath}}{5}$

As $\vec{V}_{cm} \parallel \vec{a}_{cm}$, so centre of mass follows a straight line path.

9. The components ofvarious velocities are as shown in the figure below:



$\vec{V}_{IM_1} = (100\,\hat{\imath})$cm/s

$\vec{V}_{IM_2}$is given by the vector sum of components of velocity of image w.r. to M_2 along the normal and perpendicular to the normal

$\vec{V}_{IM_2} = [100\sin^2 37^0\hat{\imath} + 100\sin 37^0\cos 37^0\,\hat{\jmath}] + [-100\cos^2 37^0\hat{\imath} + 100\sin 37^0\cos 37^0\,\hat{\jmath}]$

$\Rightarrow\ = (-28\hat{\imath} + 48\hat{\jmath})\text{cm/s}$

$\therefore \vec{V}_{IM_2} - \vec{V}_{IM_1} = [-128\hat{\imath} + 48\hat{\jmath}]\text{cm/s}$

10. Let a satellite of mass 'm' is revolving around the earth of mass 'M' in a circle of radius 'r'(> radius of earth), then its total mechanical energy in the orbit is given by,

T.E = P.E+K.E = $-\frac{GMm}{r} + \frac{GMm}{2r} = -\frac{GMm}{2r}$ = -(K.E)

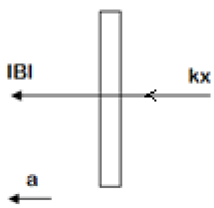
So , T.E = $-E_K$ (Given K.E=E_K)

Let 'x' is the amount of energy supplied to the satellite in its orbit so that it goes to infinity where its total energy is zero.

Appling law of conservation of energy, x +T.E = 0

So, x = -(T.E) = - ($-E_K$) = E_K .

11. Let the velocity of rod be 'V' when it has been displaced by 'x' due to motion of rod an emf, will be induced in rod given by e = BVL, due to this induced emf, charging of the capacitor takes place as a current, flows in the circuit [for very small time] as a result of this current, the rod experiences a magnetic force given by IBL.



From Newton's $\mathrm{II^{nd}}$ law,

$LIBl + Kx = ma$

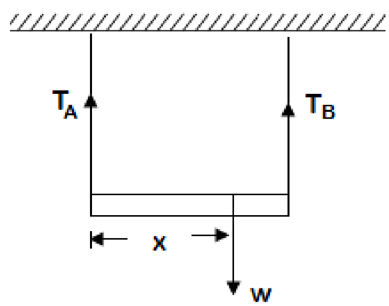
$\Rightarrow I = \frac{d}{dt}(Q) = \frac{d}{dt}(C \times BvL) = CBl \times \frac{dv}{dt}$

$\Rightarrow \quad a = \frac{Kx}{m - B^2L^2C} = \omega^2 x$

12. Distance between two adjacent nodes is $\frac{\lambda}{2} = \frac{2\pi/K}{2} = \frac{\pi}{K}$

13. $T_A + T_B = W$

$(T_A)(x) = T_B(L - x)$



Solving the above equations

$T_A = \frac{w(L-x)}{L}; T_B = \frac{Wx}{L}$

Stress in 'A' = $\frac{T_A}{A_A}$ Where A_A is cross section area of wire 'A'.

Stress in 'B' = $\frac{T_B}{A_B}$ where A_B is cross section area of wire 'B'

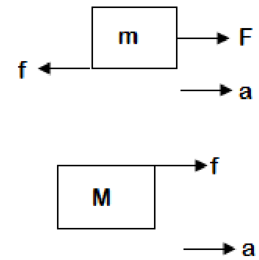
It is given $A_A = \frac{A_B}{2}, \frac{T_A}{A_A} = \frac{T_B}{A_B} \Rightarrow x = \frac{2L}{3}$

14. Electric potential of the common centre, is

$V = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2}$

$\Rightarrow \quad v = \frac{\sigma}{\varepsilon_0} \times r_1 + \frac{\sigma}{\varepsilon_0} \times r_2 = \frac{\sigma}{\varepsilon_0}[r_1 + r_2] \quad \begin{bmatrix} \because q_1 = (4\pi r_1^2)\sigma \\ q_2 = (4\pi r_2^2)\sigma \end{bmatrix}$

15. Let both the blocks are moving together with same acceleration a, then



$a = \frac{F}{M+m}$, $f = Ma = \frac{MF}{M+m}$

For no relative motion to be there between the blocks, $f \le f_L$

i.e, $\frac{MF}{M+m} \le \mu mg \Rightarrow F \le \frac{\mu m(M+m)g}{M}$

16. $\lambda_n = \frac{h}{P_n} = \frac{h}{mv_n}$

$J_n = \frac{nh}{2\pi}$

We know $v_n \propto \frac{1}{n}$ i.e., $\lambda_n \propto n$ and $J_n \propto n$

So $\lambda_n \propto J_n$

17. The position of final image is independent of b only when the rays are incident parallel to principal axis on concave lens, which is possible only when the object is kept at focus of plano convex lens.

$\therefore \frac{1}{a} = \frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$= \left(\frac{3}{2} - 1\right)\left(\frac{1}{10} - \frac{1}{\infty}\right)$

$\Rightarrow$ a = 20 cm.

18. When screen is parallel to line joining coherent sources S_1 and S_2, then shape of fringe is hyperbolic but central bright fringe is straight line.

When screen is perpendicular to the line joining the sources S_1 and S_2, then the shape of fringe is circular.

19. T = mg

Wave speed $C = \sqrt{\frac{T}{PA}}$

From the given equation of wave,

$C = \frac{w}{K}$

$\therefore \frac{w}{K} = \sqrt{\frac{T}{PA}} \Rightarrow \frac{w^2}{K^2} = \frac{T}{pA}$

$\therefore T = \frac{pAw^2}{K^2}$ or $mg = \frac{pAw^2}{K^2} \Rightarrow m = \frac{pAw^2}{K^2 g}$

20. $\mu = \tan 60^0 = \sqrt{3}$

$\frac{\sin i}{\sin r} = \sqrt{3} \Rightarrow \sin r = \frac{\sin 45^0}{\sqrt{3}} = \frac{1}{\sqrt{6}}$

$\Rightarrow$ r = $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

21. only D_1 and D_4 are forward biased. So, I = $\frac{V}{R} = \frac{5}{50} = 0.1A$.

22. $N = N_0 e^{-\lambda t}$
$\therefore N_1 = N_0 e^{-10\lambda_0 t}$
$N_2 = N_0 e^{-\lambda_0 t}$
$\therefore \frac{N_1}{N_2} = \frac{1}{e} = e^{-9\lambda_0 t}$
$9\lambda_0 t = 1$
$\Rightarrow$ t = $\frac{1}{9}\lambda_0$.

23. ${}_{92}U^{238} \longrightarrow {}_{82}Pb^{214} + 6\ {}_{2}He^{4} + 2e^{-1}$
So 6α and 2β are possible.

24. $U_i = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$
$U_f = \frac{1}{2}KCV^2 + \frac{1}{2}(KC)\frac{V^2}{K^2} = \frac{1}{2}CV^2\left(K + \frac{1}{K}\right)$
$\therefore \frac{U_i}{U_f} = \frac{1}{\frac{1}{2}\left(\frac{K^2+1}{K}\right)} = \frac{2K}{K^2+1} = \frac{6}{10} = \frac{3}{5}$

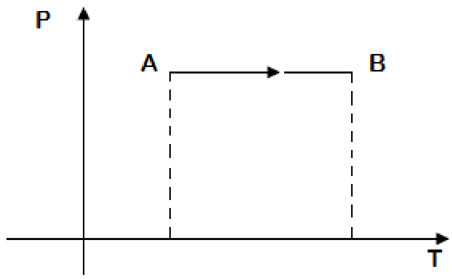
25. $R_1 = \frac{V^2}{P_1}$; $R_2 = \frac{V^2}{P_2}$
$P_{net} = \frac{V^2}{R_{net}} = \frac{V^2}{\left(\frac{R_1R_2}{R_1+R_2}\right)} = \frac{V^2(R_1+R_2)}{R_1R_2}$

$\Rightarrow P_{net} = \frac{V^2\left(\frac{V^2}{P_1}+\frac{V^2}{P_2}\right)}{\frac{V^2}{P_1}\times\frac{V^2}{P_2}} = P_1 + P_2$

26. f = $\frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{5\times80\times10^{-6}}}$
$\Rightarrow$ f = $\frac{25}{\pi}$ Hz

27. As relative velocity is doubled. $\frac{d\phi}{dt}$ is also doubled hence emf induced becomes doubled.

28.



dw = nRdT
anddw = PdV
∴PdV = nRdT = (2) (R)(200)
⇒PdV = 400 R

29. At $x = \sqrt{\frac{2E}{K}}$
Potential energy U = 0 [∵ x > 0]
Total energy is purely kinetic
$\therefore E = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2E}{m}}$

30. No friction is required for pure rolling of ring.

PART – B (CHEMISTRY)

31. b	32. a	33. b	34. a	35. d	36. a	37. c	38. b	39. c	40. d
41. d	42. c	43. d	44. a	45. a	46. a	47. b	48. a	49. d	50. c
51. d	52. d	53. c	54. a	55. a	56. d	57. b	58. d	59. a	60. b

SOLUTION:

31. Sol: (b)
$W_{salt} = 1g$ Let Wt% of H = x
$W_{Ag} = 0.5934\ g$ Let Wt% of C = 8x
Let Wt% of O = 16x

Since its dibasic acid

∴ |mole of salt = 2 moles of Ag

∴moles of Ag = $\frac{0.5934}{108}$

∴ moles of salt/Acid = $\frac{0.5934}{108} \times \frac{1}{2}$

Given amount of salt = 1g

∴ Molecular amount of salt = $\frac{1}{0.5934} \times 108 \times 2$

= 364 g/mol

now x + 8x + 16x = 364

x = 14.5g

∴moles of H $\oint 0 = 14.5$

Since 4+0 are equal hence option b.

32. Sol: (a)

Orbital angular momentum = $\sqrt{l(l+1)}\frac{h}{2\pi}$

For a d – orbital l = 2

Orbital angular momentum = $\sqrt{2(2+1)}$ h/2π

= $\frac{\sqrt{6}\,h}{2\pi}$

33. Sol: (b)

Higher the cationic charge smaller the radius

Higher the anionic charge higher the radius for isoelectronic species.

34. Sol: (a)

Conceptual of VSEPR THEORY

35. Sol: (d)

Draw the Mo diagram and this will have unpaired e^-

36. Sol: (a)

$N_1V_1 + N_2V_2 = NV$

Where $V_1 + V_2$ = 1 L

Or V_2 = $(1 - V_1)\,L$

10 × $V_1 + 4\,(1 - V_1) = 1 \times 7$

10 $V_1 + 4 - 4V_1$ = 7

$6V_1 = 3$

$$V_1 = \frac{3}{6} = 0.5\ L$$

$V_2 =$ 0.5 L

37. Sol: (c)

Vol of 1 mol of ideal gas at 273 K
and 1atm is 22.4 L
$\therefore$ Volat 373 K and 1 atm
V = $\frac{RT}{P} = \frac{0.082 \times 373}{1} = 30.6L$

38. Sol: (b)

$\Delta s\ (A \longrightarrow B) = \ \Delta s\ (A \longrightarrow C) + \ \Delta s\ (C \longrightarrow D) + \ \Delta s\ (D \longrightarrow B)$

= 50 + 30 + (– 20) = 60 eV

39. Sol: (c) For pH = 3 ; $[H_3O^+] = 10^{-3}$

pH = 4 ; $[H_3O^+] = 10^{-4}$

Moles of H_3O^+ in 100 ml of solution pH = 3 = $\frac{10^{-3}}{1000} \times 100 = 10^{-4}$

Moles of H_3O^+ in 400 ml of solution of pH = 4

= $\frac{10^{-4}}{1000} \times 400 = 4 \times 10^{-5}$

Total moles of H_3O^+ on mixing

= $10^{-4} + 4 \times 10^{-5} = 14 \times 10^{-5}$

$[H_3O^+]$ = $\frac{14 \times 10^{-5} \times 1000}{500} = 2.8 \times 10^{-4}$

pH = – log (2.8×10^{-4})

= 4 – log 2.8

40. Sol: (d)

	$H_2O(g) \rightleftharpoons$	$H_2(g) +$	$\frac{1}{2}O_2(g)$
Initial com	1	0	0

At equation $1-\propto \quad \propto \quad \propto/2$
Total no of moles at $eq^b = 1-\propto + \propto + \propto/2$
$= 1 + \propto/2$

$P(H_2O) = \frac{1-\propto}{1+\propto/2} P \qquad P(O_2) = \frac{\propto/2}{1+\propto/2} P$

$P(H_2) = \frac{\propto}{1+\propto/2} P$

$K_P = \frac{P(H_2)(Po_2)^{1/2}}{(PH_2O)} = \frac{\propto^{3/2} P^{1/2}}{(1-\propto)(2+\propto)^{1/2}}$

41. Sol: (d)
Conceptual

42. Sol: (c)
Very pure$H_2(99.9\%)$ is prepared by the action of water on salt hydrides (eqNaH)
Na H + $H_2O \longrightarrow Na\,OH + H_2$

43. Sol: (d) Conceptual

44. Sol: (a)
Maximum covalency of Boron is 4 only

45. Sol: (a)
In (b), lone pair of N is taking part in resonance
In (c), nitrogen is attached to electron with drawing group.
In (d), lone pair of nitrogen is in resonance with benzene ring
∴ All these are less nucleophilic

46. Sol: (a) Ph C ≡ $C\,CH_3 \xrightarrow[n^+,\ H_2O]{Hg^{2+}}$ Ph C(OH) = $CHCH_3$

$\rightleftharpoons$ Tautomerism

ph $C-CH_2CH_3$
$\;\;\|$
$\;\;O$

47. Sol: (b)
For each I^- ion, there are 2 tetrahedral voids since number of Ag^+ ions is equal to I^- ion thus, only 50% tetrahedral voids are occupied by Ag^+ ions.

48. Sol: (a) $K_3[Fe\ (CN)_6] \rightleftharpoons 3K^+ + [Fe(CN)_6]^{3-}$

i = 4

$\Delta\, Tf = \frac{i \times kf \times 1000 \times w_2}{W_1 \times M_2} = \frac{4 \times 1.86 \times 1000 \times 0.1}{100 \times 329}$

f.pt = 0 – 2.3 × 10^{-2}

= - 2.3 × 10^{-2} C

49. Sol: (d) $\propto = \frac{\Lambda}{\Lambda_\infty} = \frac{8}{100} = 2 \times 10^{-2}$

Dissociation constant K = $\propto^2\ C$

K = $(2 \times 10^{-2})^2 \times \frac{1}{32} = 1.25 \times 10^{-5}$

50. Sol: (c)

pH = 2, $[\mathrm{H}^+] = 10^{-2}$ pH = 1 $[\mathrm{H}^+] = 10^{-1}$

Initial rate $(rate)_0 = K\,[\mathrm{H}^+]^n$

$(rate)_1 = K\,[10^{-2}]^n$ $(rate)_2 = K\,[10^{-1}]^n$

$\frac{rate_2}{rate_1} = 100 = \left[\frac{10^{-1}}{10^{-2}}\right]^n$

100 = $10^n\ or\ \boxed{n = 2}$

51. Sol: (d)

Conceptual

52. Sol: (d)

In haematite($\mathrm{Fe_2O_3}$) oxidation number of Fe is 2x + 3 (-2) = 0 ; x = 3

Magnetite ($\mathrm{Fe_3O_4}$)is an equimolar mixture of Fe O of $\mathrm{Fe_2O_3}$

$\therefore$ oxidationnumber of iron is Fe O is 2 and in $\mathrm{Fe_2O_3}$ is 3

53. Sol: (c)

Br_2 reacts with NaI only to give I_2

2NaI + $Br_2 \longrightarrow$ 2NaBr + I_2

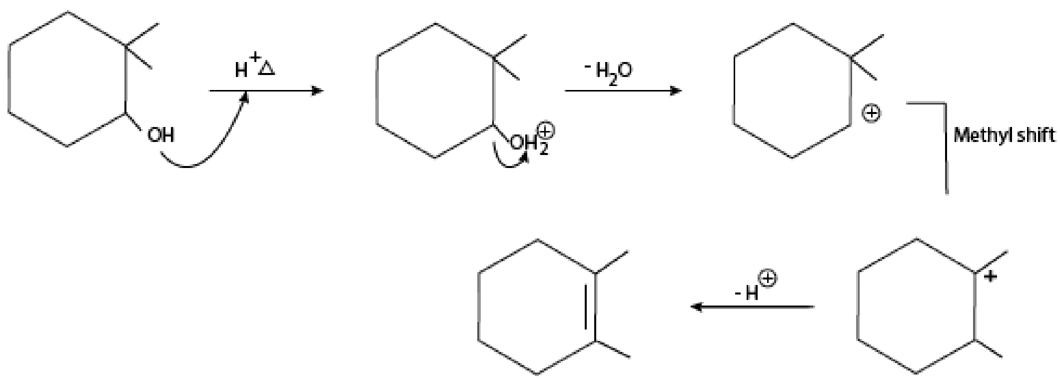
54. Sol: (a) Conceptual

$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + 4H_2O$

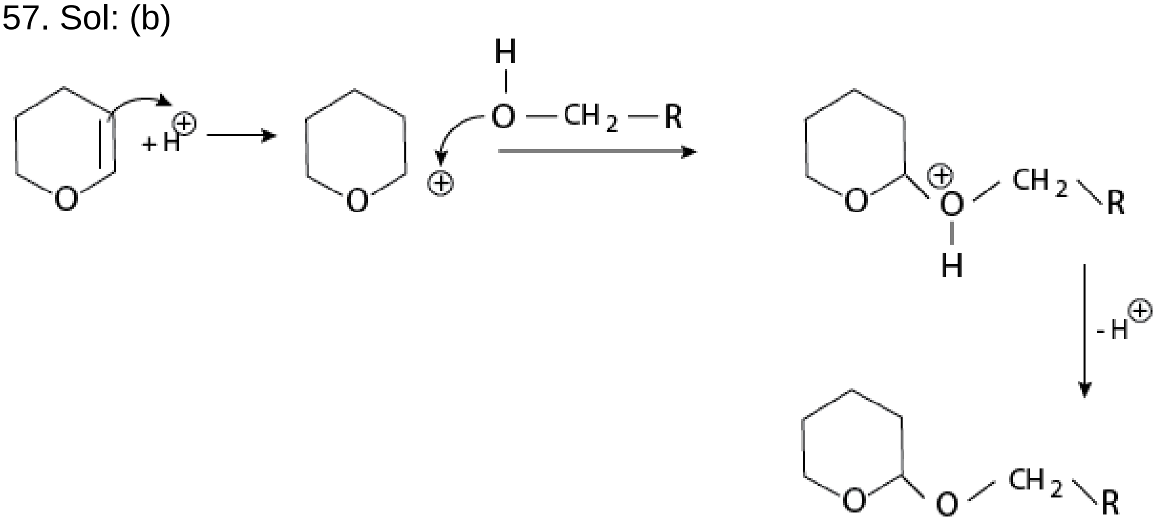
Colourless green
gas solid

55. Sol: (a) Conceptual

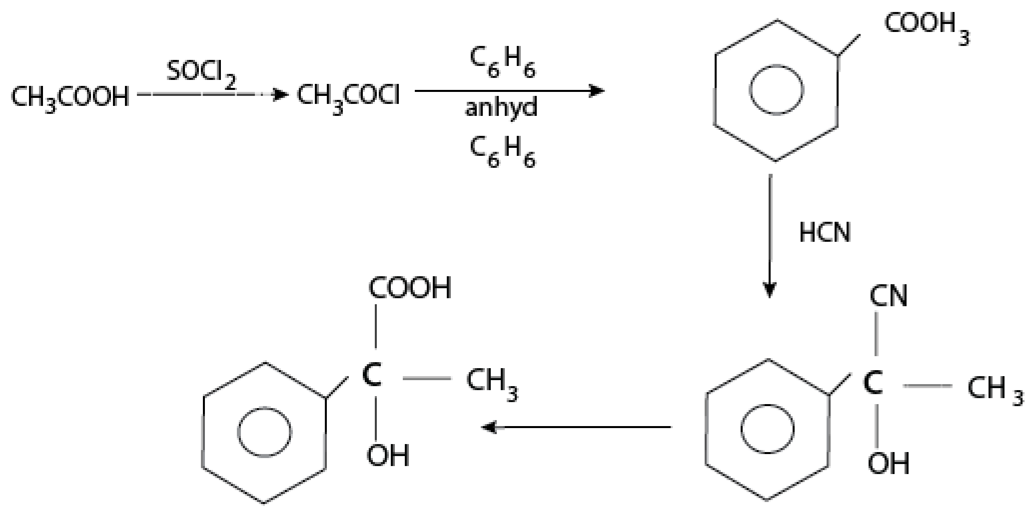
56. Sol: (d)



57. Sol: (b)



58. Sol: (d)



59. Sol: (a)

Only treatment of amide with Br_2 in aqNaOH or kOH will give amine with lesser number of carbon atoms

R $CONH_2 \xrightarrow[\Delta]{Br_2/\ KOH} RNH_2$

60. Sol: (b)

Nucleic acid is a poly nucleotide which contains of Nitrogenous base, phosphoric acid and ribose sugar.

PART – C (MATHS)

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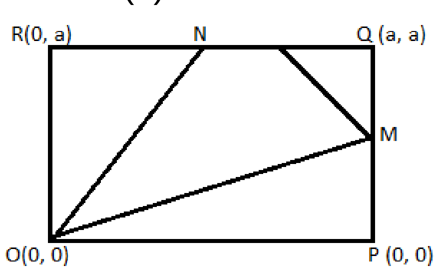
61. Sol: (b)



M = $\left(a, \frac{a}{2}\right)$, N = $\left(\frac{a}{2}, a\right)$

Area ΔOMN = $\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ a & a/2 & 1 \\ a/2 & a & 1 \end{vmatrix} = \frac{3a^2}{8}$, Area of square = a^2

$\therefore$ Ratio is $a^2 : \frac{3a^2}{8} \Rightarrow 8 : 3$

62. Sol: (c)

The line $5x - 2y + 6 = 0$ cuts y-axis at Q(0, 3). Clearly PQ is the length of the tangent drawn from Q on
the circle
$x^2 + y^2 + 6x + 6y = 2 \quad \Rightarrow PQ = \sqrt{0 + 9 + 6 \times 0 + 6 \times 3 - 2} = 5$

63. Sol: (b)

Equation of tangent of slope m is $y = mx + \frac{1}{m}$ which passes through (1, 4) $\Rightarrow m^2 - 4m + 1 = 0$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow \tan\theta = \frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{1 + m_1 m_2} \Rightarrow \tan\theta = \frac{\sqrt{16 - 4}}{2} = \sqrt{3}$

$\therefore \theta = 60°$

64. Sol: (c)

Sum of 100 items = 49 × 100 = 4900
Sum of items = 60 + 70 + 80 = 210
Sum of items replaced = 40 + 20 + 50 = 110
$\therefore$ New sum = 4900 – 110 + 210 = 5000
$\therefore$ Correct mean = $\frac{5000}{100} = 50$

65. Sol: (b)

Equation of the pair of Asymptotes is $3x^2 - y^2 + k = 0$ But passes through origin
$\Rightarrow k = 0$
$\therefore$ Asymptotes are $3x^2 - y^2 = 0$
$\therefore$ Angle α between them $\alpha = 2\text{Tan}^{-1}\left\{\frac{2\sqrt{0 + 3}}{3 - 1}\right\}$

$\therefore \alpha = \frac{2\pi}{3}$

66. Sol: (a)

$\sim p \to (q \vee r)$ is F $\Rightarrow$ P is T, $q \vee r$ is F

$\Rightarrow$ P is T, q is F, r is F

$\Rightarrow$ P is T, q is T, r is F

67. (d) $\underset{x \to 0}{Lt} \frac{(1-\cos x)(1+\cos x+\cos^2 x)}{\sin 3x \cdot \sin 5x} = \frac{3/2}{3.5} = \frac{1}{10}$
Sol:

68. (a) If $\sin^{-1}x + \sin^{-1}y = \pi/2$, then $x=\sqrt{1-y^2}$
or $x^2 + y^2 = 1$ $\Rightarrow 2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-x}{y}$

69.(c) $\frac{dy}{dx} = m = 3x^2 - 4x = 12 - 8 = 4$

AT = $\left|\frac{y_1\sqrt{1+m^2}}{m}\right| = \frac{4\cdot\sqrt{17}}{4} = \sqrt{17}$

70. (a)acosx + $\frac{1}{3}\cdot$ 3 cos3x = 0 for $x = \frac{\pi}{3}$

a $\frac{1}{2}$+ (– 1) = 0

a = 2

71. (b) Let $f(x) = ax^3 + bx^2 + cx$
f(0) = 0 = f(1) $\Rightarrow$ a + b + c = 0

72. (d)$\int e^x\left(\frac{x+2}{x+3} + \frac{1}{(x+3)^2}\right)dx = e^x\cdot\left(\frac{x+2}{x+3}\right) + C$

73. (d) f(x) = log $\left(\frac{2-\sin x}{2+\sin x}\right)$ is an odd function

$\therefore \int_{-\pi/2}^{\pi/2} f(x)dx = 0$

74. (a)Required area is $2\int_0^{\pi} \sin x\,dx = 4$ sq.units

75. (b)The given equation is the homogenous differential equation:

By using y=vx we will get the required function as $\sin^{-1}\left(\frac{y}{x}\right) = \log x$

$\Rightarrow y = x = e^{\pi/2}$

76. Sol: (a)

$\bar{a}\cdot\bar{b} > 0 \Rightarrow 2\lambda^2 - 3\lambda + 1 > 0 \Rightarrow \lambda < \frac{1}{2}$ or $\lambda > 1$ . . . .(1)

$\bar{b}\cdot i < 0$, $\bar{b}\cdot j < 0$, $\bar{b}\cdot k < 0 \Rightarrow \lambda < 0$ . . . . (2)

Form (1) and (2), $\lambda \in (-\infty, 0)$

77. Sol: (c) $\bar{U}\cdot(\bar{V} \times \bar{W}) = \bar{U}\cdot(3i - 7j - k)$

$= |\bar{U}||3i - 7j - k| \cos\theta$

$= \sqrt{59} \cos\theta$

∴Maximum value of $[\bar{U}, \bar{V}, \bar{W}] = \sqrt{59}$. (∵$\cos\theta \le 1$)

78. Sol: (c)

Equation of the plane is $3(x - 1) + 4(z - 1) = 0$

$\Rightarrow 3x + 4z - 7 = 0$

∴Dist. from the origin = $\frac{|-7|}{\sqrt{3^2+4^2}} = \frac{7}{5}$

79. Sol: (c)

Let P be the required point on AB. Let P divides AB in the ratio λ : 1

$P = \left(\frac{11\lambda-9}{\lambda+1}, \frac{4}{\lambda+1}, \frac{5-\lambda}{\lambda+1}\right)$, $OP \perp AB \Rightarrow 20\left(\frac{11\lambda-9}{\lambda+1}\right) - 4\left(\frac{4}{\lambda+1}\right) - 6\left(\frac{5-\lambda}{\lambda+1}\right) = 0$

∴$\lambda = 1 \Rightarrow P = (1, 2, 2)$

80. Sol: (a)

$\begin{vmatrix} 3 & -2 & 1 \\ 4 & -3 & 4 \\ 2 & -1 & m \end{vmatrix} = 0 \Rightarrow m = -2$

81. Sol: (c)

$n(s) = 3^5 = 243$

$n(E) = 3({}^5C_2\cdot{}^3C_2\cdot{}^3C_1) + 3({}^5C_1\cdot{}^4C_1\cdot{}^3C_3) = 150$

$P(E) = \frac{n(E)}{n(S)} = \frac{50}{81}$

82. Sol: (d)

P(W) = $\frac{1}{6}$, P(L) = $\frac{5}{6}$

P(A) = $\frac{1}{6}+\left(\frac{5}{6}\right)^{2}\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^{4}\cdot\frac{1}{6}+\ .\ .\ .\ .\ . = \frac{6}{11}$

83. Sol: (d)

Sol: $(x^2 + (x^6-1)^{1/2})^5 + (x^2 - (x^6-1)^{1/2})^5$

$= 2\,({}^5C_0\,(x^2)^5 + {}^5C_2\,(x^2)^3\,(x^6-1) + {}^5C_4\,\mathbf{x^2}(x^6-1)^2)$

Here last term is of 14 degree.

84. **Sol. (b)** $|z| = \left|z - \frac{4}{z} + \frac{4}{z}\right|$

$\leq \left|z - \frac{4}{z}\right| + \left|\frac{4}{z}\right|$

$= \left|z - \frac{4}{z}\right| + \frac{4}{|z|}$

$|z| \leq 2 + \frac{4}{|z|}$

$|z|^2 - 2|z| \leq 4$

$|z|^2 - 2|z| + 1 \leq 5$

$\Rightarrow |z| \leq \sqrt{5} + 1$

85. Ans. (a) $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$

$= (1 - \log_z y \log_y z) - \log_x y(\log_y x - \log_z x \log_y z) + \log_x z(\log_y x \log_z y - \log_z x)$

$= (1-1) - (1 - \log_x y \log_y x) + (\log_x z \log_z x - 1) = 0$

{Since $\log_x y.\log_y x = 1\}$.

86. Sol: (b)

Last digit is zero and the remaining from digits are 1,2,4,5. Number of arrangements = 4! =24

87. Sol: (a)

x - $\frac{2}{x-1} = 1 - \frac{2}{x-1} ------ (1)$

$\Longrightarrow\ x^2 - x - 2 = x - 1 - 2$

$\Rightarrow\ x^2 - 2x + 1 = 0$

$\Rightarrow\ (x-1)^2 = 0$

$\Rightarrow\ x - 1 = 0 \Rightarrow\ x = 1$

But when x = 1 (1) is not defined $\therefore$ ***No root***

88. Sol: (d)

$$\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$$
$$= \tan 9^\circ + \cot 9^\circ - (\tan 27^\circ + \cot 27^\circ)$$
$$= \frac{1}{\sin 9^\circ.\cos 9^\circ} - \frac{1}{\sin 27^\circ \cos 27^\circ}$$
$$= \frac{2[\sin 54^\circ - \sin 18^\circ]}{\sin 18^\circ.\sin 54^\circ} = 4$$

89. Sol: (c)

$\mathbf{2^{78} = 2^3.2^{75} = 8\ (2^5)^{15} = 8(1+31)^{15} = 8}\{{}^{15}C_0 + {}^{15}C_1\,31 + \ldots\ldots\ldots + {}^{15}C_{15}(\mathbf{31})^{\mathbf{15}}\}$

$\mathbf{2^{78} = 8 + \text{an integer multiple of } 31}$

$\mathbf{\frac{2^{78}}{31} = \frac{8}{31} + \text{an integer}}$

90. Sol: (c) $|\mathbf{z_1 + z_2}|^2 = |\mathbf{z_1 - z_2}|^2$

$\Rightarrow \mathbf{z_1.\bar{z}_2 + \bar{z}_1 z_2} = 0$

i.e $\mathbf{z_1.\bar{z}_2 + \overline{z_1.\bar{z}_2}} = 0$

$\Rightarrow \mathrm{Re}(\mathbf{z_1.\bar{z}_2}) = 0$

Let $z_1 = r,\ e^{i\theta_1}$, $z_2 = r_2 e^{i\theta_2}$

$\Rightarrow \mathrm{Re}\ z_1.\bar{z}_2 = r_1 r_2 e^{i(\theta_1 - \theta_2)} = 0$

$\Rightarrow \cos(\theta_1 - \theta_2) = 0$

$\Rightarrow \theta_1 - \theta_2 = \pm \frac{\pi}{2}$