

JEE ADVANCED PAPER-I

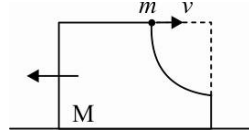
SOLUTIONS

PHYSICS

1.(AD) Net external force acting on the system along the x-axis is zero.

∴ Along the x-axis
Momentum is conserved
⇒ $mv = MV$

From conservation of energy



Loss in GPE of particle of mass $m =$ Gain in kinetic energy of both the masses.

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 \quad \Rightarrow \quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{mv}{M}\right)^2$$

$$\Rightarrow 2gh = \left(1 + \frac{m}{M}\right)v^2 \quad \Rightarrow \quad v = \sqrt{\frac{2gh}{1 + \frac{m}{M}}} \text{ hence option (A) is correct.}$$

$$V = \frac{m}{M}v = \frac{m\sqrt{2gh}}{\sqrt{M(m+M)}} \text{ along } -ve \text{ x-axis}$$

Hence option [B] is incorrect.

Since the location of center of mass does not change along the x-axis

$$m\Delta x_{m/G} + M\Delta x_{M/G} = 0 \quad \Rightarrow \quad m(R-x) - Mx = 0$$

$$x = \frac{mR}{m+M}$$

$$\Delta x_{M/G} = \frac{-mR}{m+M}$$

Hence option (D) is correct.

$$\Delta x_{m/G} = R - x = \frac{MR}{M+m}$$

$$\text{Final position of } m = -x = -\frac{mR}{m+M}$$

Hence option (C) is incorrect.

2.(AC) At $\omega \sim 0$, $X_L \sim 0$ and $X_C \rightarrow \infty$

∴ Current will be nearly zero.

At $\omega \gg 10^6$, $X_L \gg 1$ and $X_C \rightarrow 0$

Hence circuit does not behave like a capacitor.

At resonance frequency (ω_0) i.e. when $X_C = X_L$ current will be in phase with voltage and frequency is independent of R .

$$\text{Resonant frequency } \omega_0 = \frac{1}{\sqrt{LC}} = 10^6 \text{ rad/s}$$

3.(ABD/BD)

Velocity of the wave depends upon the tension in the rope and mass per unit length of the rope.

⇒ Velocity is independent of frequency and wavelength

Option (B) is correct

Since tension at the mid point is same, therefore speeds at that point will also be same. Option (A) is correct. Since the direction of motion of pulses is opposite, velocities will be equal and opposite. With this point of view A will be incorrect.

Since the velocities at position is dependent only on tension at every point ($\mu = \text{constant}$) therefore time taken for the wave the reach A from O and O from A will be same.

$$T_{OA} = T_{AO}$$

Option (D) is correct

$$v = \lambda \nu$$

Since v depends on source

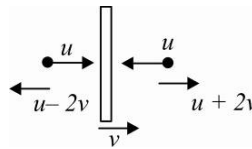
$$\Rightarrow \lambda \propto v \Rightarrow \lambda \propto \sqrt{T}$$

Tension decreases as we move from O to A, therefore λ becomes shorter. Option (C) is incorrect.

4.(ACD) Molecules hitting the forward and rear surfaces will bounce back with speeds given above. Let mass of one molecule be m_0 . Then,

$$\Delta P_0 (\text{forward}) = 2m_0(u + v)$$

$$\Delta P_0 (\text{Rear}) = 2m_0(u - v)$$



Let the rates of collision with front and rear surfaces be R_1 and R_2 respectively

$$\text{So, } R_1 \propto (u + v)$$

$$R_2 \propto (u - v)$$

Force = $\Delta P \cdot R$.

$$\text{So, } F_1 = R_1 \cdot 2m_0(u + v), \quad F_2 = R_2 \cdot 2m_0(u - v)$$

$$\text{So, } F_1 - F_2 \propto 2m_0(u + v)^2 - 2m_0(u - v)^2$$

$$\propto 2m_0[4uv]$$

$$\propto uv \text{ so, (A) is correct}$$

Clearly, the net force due to gas is proportion to v , i.e. it is variable hence acceleration of plate is variable.

Finally the plate will start moving with terminal velocity.

Hence (C) is correct.

$$\text{Resistive force} = \Delta P \cdot A \propto V$$

$$\Rightarrow \text{Hence (D) is correct.}$$

5.(A or ACD) Net power radiated = $\sigma a(T^4 - T_0^4)$

$$\text{For small temperature difference, } \Delta T = T - T_0$$

$$P(\text{Net}) = \sigma A \left[(T_0 + \Delta T)^4 - T_0^4 \right] = \sigma A T_0^4 \left[\left(1 + \frac{\Delta T}{T_0} \right)^4 - 1 \right] \approx \sigma A T_0^4 \left[\frac{4\Delta T}{T_0} \right]$$

$$\therefore P(\text{Net}) = 4\sigma A T_0^3 (T - T_0)$$

(A) From the above result, $P(\text{Net})$ decreases with A

(B) Peak shifts to shorter wave lengths for rise in temperature.

$$(C) P(\text{Radiated}) = \sigma A T^4 = \sigma A (T_0 + 10)^4 > 460 \text{ watt}$$

Hence (C) is incorrect

If $P(\text{Net})$ is taken, then (C) will be correct

(D) Energy radiated by a body is dependent only on its own temperature, not the temperature of surroundings. Hence (D) is incorrect. If $P(\text{Net})$ is taken, then from the calculation shown

above,

(D) will also be correct.

6.(ACD) Angular deviation : $\delta = i_1 + i_2 - A$

For min deviation : $i_1 = i_2$ so, $i_1 = A$... (i)

Also, $r_1 + r_2 = A$

i.e. $r_1 = \frac{A}{2}$... (ii)

$\therefore r_1 = \frac{i_1}{2}$ (A is correct)

By Snell's law, $\sin i_1 = \mu \sin r_1$

i.e. $\sin A = \mu \sin\left(\frac{A}{2}\right)$

$\Rightarrow 2 \cos\left(\frac{A}{2}\right) = \mu$

$A = 2 \cos^{-1}\left(\frac{\mu}{2}\right)$

Hence (B) is incorrect

(C) is obviously correct

For tangential emergence, $i_2 \rightarrow 90^\circ$.

So, $\mu \sin r_2 = 1$

$$\cos r_2 = \sqrt{1 - \frac{1}{\mu^2}}$$

Also, $r_1 = A - r_2$

$\sin r_1 = \sin A \cos r_2 - \cos A \sin r_2$

$= (\sin A) \frac{\sqrt{\mu^2 - 1}}{\mu} - (\cos A) \frac{1}{\mu}$

By Snell's law on 1st surface,

$\sin i_1 = \mu \sin r_1$

$\Rightarrow \sin i_1 = \sin A (\mu^2 - 1)^{1/2} - \cos A \Rightarrow i_1 = \sin^{-1} \left(\sin A \sqrt{4 \cos^2 \frac{A}{2} - 1} - \cos A \right)$

i.e. (D) is correct

7.(AB) The angle that area vector makes with \vec{B} at time t is $\theta = \omega t$.

$|\phi_1| = BA \cos \omega t \Rightarrow |\epsilon_1| = BA \omega \sin(\omega t)$

$|\phi_2| = 2BA \cos(\omega t) \Rightarrow |\epsilon_2| = 2BA \omega \sin(\omega t)$

Due to orientation of loops, the two EMFs will work against each other.

So, $|\epsilon(\text{net})| = BA \omega \sin(\omega t)$

So, (A) is correct.

We can see that ϵ_{net} is maximum when $\theta = \frac{\pi}{2}$. So, (B) is correct. Obviously, (C) is incorrect. (D) is incorrect as the EMF is proportional to difference in areas.

8.(5) $V \propto \frac{1}{n^2}$

So, $\frac{V_i}{V_f} = \left(\frac{n_f}{n_i}\right)^2 = 6.25 \quad \therefore \frac{n_f}{n_i} = 2.5 = \frac{5}{2}$

Minimum integral value of n_f is 5.

9.(8) We know that for the given case,

$\mu \sin \theta = \text{constant}$

So, $1.6 \sin(30^\circ) = (n - m\Delta n) \sin 90^\circ$

i.e. $0.8 = n - m\Delta n$ Solving, $m = 8$

10.(5) $A = A_0 e^{-\lambda t}$

Here, $\lambda = \frac{\ln 2}{8} (\text{days})^{-1}$

$t = 12 \text{hrs} = \frac{1}{2} \text{ day}$

So, $A = A_0 / \exp\left[\frac{\ln 2}{8} \cdot \frac{1}{2}\right] \cong \frac{A_0}{2\left(1 + \frac{1}{16}\right)}$

$A \cong \frac{A_0}{2} \left(\frac{16}{17}\right)$

If the volume of blood be $V (ml)$, then

$115 = A \cdot \left(\frac{2.5}{V}\right)$

$115 = \frac{A_0}{2} \left(\frac{16}{17}\right) \cdot \left(\frac{2.5}{V}\right)$

Solving $V \cong 5$ liters

11(6)



Frequency received by car $= \left[\frac{330+2}{330}\right] 492 = \frac{332}{330} \times 492 \cong 495 \text{ Hz}$

Frequency received by source $= \frac{330}{330-2} \times 495 \cong 498 \text{ Hz}$

Original frequency $f_1 = 492 \text{ Hz}$

Final frequency $f_2 = 498 \text{ Hz}$

Beat frequency $= |f_1 - f_2| = 6 \text{ Hz}$

12.(6) $\frac{4}{3} \pi R^3 = \frac{4}{3} \pi r^3 \times K$ (Conservation of volume)

R = radius of bigger drop

r = radius of smaller drop

$R^3 = r^3 \times K$... (i)

$U_i = S(4\pi R^2)$

$U_f = KS(4\pi r^2)$

$KS(4\pi r^2) - S \times 4\pi R^2 = 10^{-3}$

$4\pi S[Kr^2 - R^2] = 10^{-3}$

$K^{\frac{1}{3}} R^2 - R^2 = 10^{-2}$

$10^{\alpha/3} - 1 = 100$

$10^{\alpha/3} = 101$

$\alpha \approx 6$

13.(D) For constant velocity, acceleration of particle should be zero, Hence net force should be zero.

$qVB = qE \Rightarrow V = \frac{E}{B}$ (For $a = 0$)

Electric field and magnetic field should be perpendicular.

Option (D) : $\vec{V} = \frac{E_0}{B_0} \hat{y}$, $\vec{E} = -E_0 \hat{x}$, $\vec{B} = B_0 \hat{z}$ (for electron)

$$\vec{F}_B = q(\vec{V} \times \vec{B}) = -e \left(\frac{E_0}{B_0} \hat{y} \times B_0 \hat{z} \right) = -E_0 \hat{x}$$

$$F_E = E_0 \hat{x}$$

14.(B) $\vec{V} = 0$, $\vec{E} = -E_0 \hat{y}$, $\vec{B} = B_0 \hat{y}$ (Proton)

$F_B = 0$ only force along -y axis is acting due to electric field alone.

15.(D) $\vec{V} = 2 \frac{E_0}{B_0} \hat{x}$; $\vec{E} = E_0 \hat{z}$; $\vec{B} = B_0 \hat{z}$ (Proton)

$F_B =$ along -y axis

$F_E =$ along +z axis

So condition for helical path is satisfied

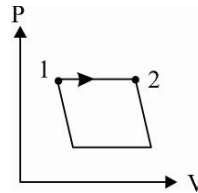
16.(C) (Please note that $W_{1 \rightarrow 2}$ is work done on the gas).

Process 1 to 2 represents isobaric process

$$W_{1 \rightarrow 2} = P(V_1 - V_2) = PV_1 - PV_2$$

$$\Delta U = \Delta Q - P\Delta V \text{ is correct.}$$

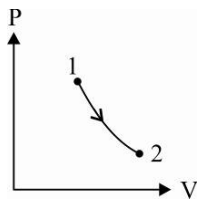
No other combination is possible.



17.(A) Laplace correction is done is correction in the determination of the speed of sound in an ideal gas, which states that the process assumed was not Isothermal but it is Adiabatic .

$$W_{adiabatic\ 1 \rightarrow 2} = \frac{1}{\gamma - 1} (P_2 V_2 - P_1 V_1)$$

Corresponding graph is



18.(B) (A) Incorrect since in Isochoric process $W = 0$

(B) Correct as $W_{isochoric} = 0$

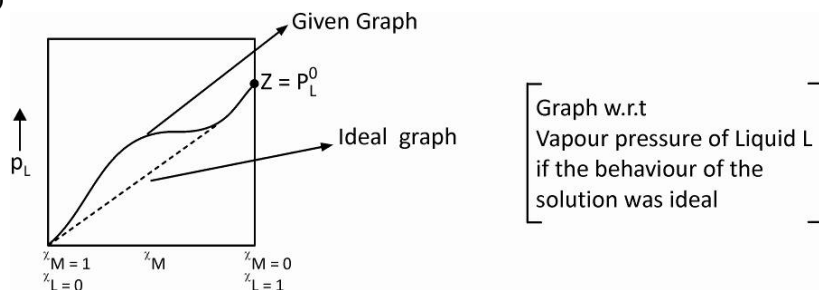
Graph is also correct.

(C) Not correct combination since $W_{1 \rightarrow 2} = -PV_2 + PV_1$ is wrong for Adiabatic process.

(D) Incorrect combination of graph and process

CHEMISTRY

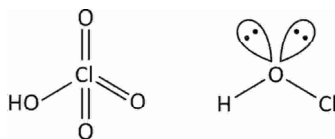
19.(BC)



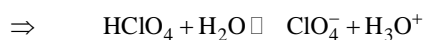
- * Z represent vapour pressure of pure liquid L and as $\chi_L \rightarrow 1$, the given graph is merging with ideal graph of p_L . Hence Option (C) is correct
- * Since, vapour pressure of liquid L (p_L) is higher than the ideal values so L-M interactions are weaker than L-L & M-M interactions. Hence Option (B) is correct choice.

20.(ABC)

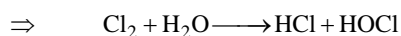
- ⇒ In oxoacids acidic strength increases with increase in number of double bonded oxygen atoms because of greater resonance stabilization of conjugate base. Hence HClO_4 is more acidic than HClO .



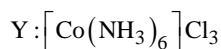
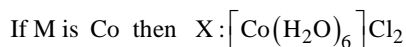
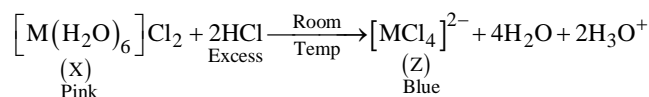
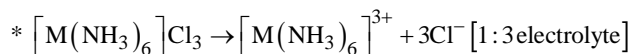
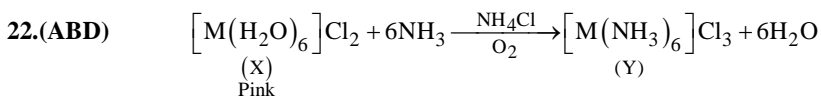
- ⇒ Central atom in both HClO_4 and HClO is sp^3 hybridized.



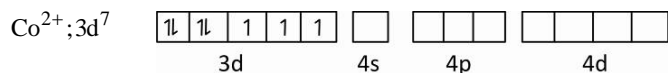
This equilibrium remain shifted in forward direction hence ClO_4^- is weaker base than H_2O . Because such equilibrium remain shifted towards weaker acid or weaker base side.



21.(AC) IUPAC name of the given compound is 1 - chloro-4-methylbenzene or 4-chlorotoluene



- ⇒ $\mu_x = 3.87$ due to presence of three unpaired electron in X



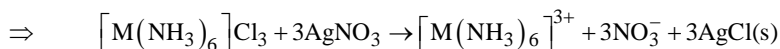
H₂O is weak field ligand hence no pairing

Hybridization state is sp³d².

⇒ μ_z = 3.87 BM due to presence of three unpaired electron in Z.



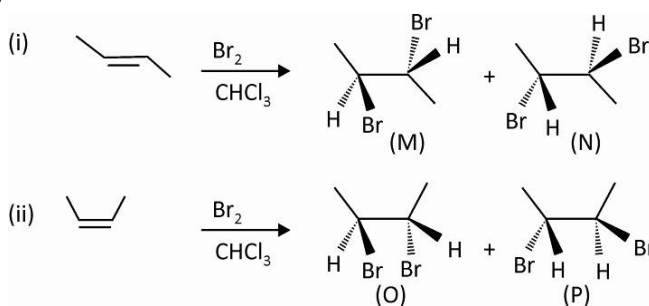
Cl⁻ is weak a field ligand and coordination number is four hence hybridization state is sp³.



(As octahedral complex changes to tetrahedral complex with H₂O ligands replaced by Cl⁻)

At 0°C, equilibrium shifts in reverse direction hence colour of the solution is pink due to [Co(H₂O)₆]²⁺

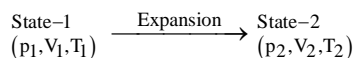
23.(CD)



* Bromination (addition of Br₂) proceeds through trans-addition in both the reactions

* (M and O) and (N and P) are two pairs of diastereomers

24.(ACD)



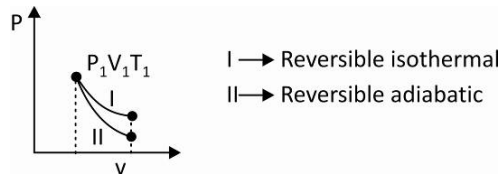
⇒ Work done in reversible compression process is smaller than the work done in irreversible compression process.

⇒ ΔU = 0 for reversible isothermal expansion.

ΔU < 0 for reversible adiabatic expansion.

⇒ For free expansion q = 0, w = 0 and ΔU = 0 hence it is simultaneously both isothermal (ΔU = 0) as well as adiabatic (q = 0).

⇒ Work done in reversible adiabatic expansion is less than the work done in reversible isothermal expansion as shown in figure.



25.(BC) Electronic configuration of F₂ molecule is σ_{1s}², σ_{1s}^{*2}, σ_{2s}², σ_{2s}^{*2}, σ_{2p_z}², π_{2p_x}², π_{2p_y}², π_{2p_x}^{*2}, π_{2p_y}^{*2}, σ_{2p_z}^{*}

⇒ Similar electronic configuration for other X₂ molecules.

⇒ π_{p_x}^{*} and π_{p_y}^{*} are highest energy occupied molecular orbital (HUMO) and σ_{p_z}^{*} is lowest energy unoccupied molecular orbital (LUMO).

⇒ Colour of X₂ molecules of group 17 elements is due to transition of electrons from π^{*} to σ^{*}.

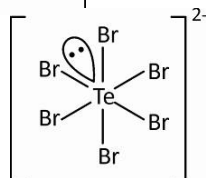
26.(6) Number of electrons around central atom (N) = $\frac{V + M \pm C}{2}$

V = Valence electrons of central atom
M = +1 for every monovalent atom
-1 for every trivalent atom
C = cationic or anionic charge

$$[\text{TeBr}_6]^{2-}: \quad N = \frac{6 + 6 + 2}{2} = 7$$

$$\text{Number of bp} = 6$$

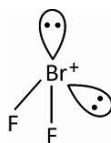
$$\text{Number of } \ell p = 1$$



$$[\text{BrF}_2]^+: \quad N = \frac{7 + 2 - 1}{2} = 4$$

$$\text{Number of bp} = 2$$

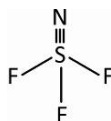
$$\text{Number of } \ell p = 2$$



$$\text{SNF}_3: \quad N = \frac{6 - 1 + 3}{2} = 4$$

$$\text{Number of bp} = 4$$

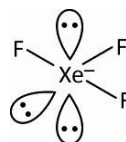
$$\text{Number of } \ell p = 0$$



$$[\text{XeF}_3]^-: \quad N = \frac{8 + 3 + 1}{2} = 6$$

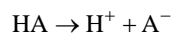
$$\text{Number of bp} = 3$$

$$\text{Number of } \ell p = 3$$



The sum of the number of lone pairs of electrons on each central atom = 1 + 2 + 0 + 3 = 6

$$27.(6) \quad \therefore \quad \Lambda_m^c = \left(\frac{1000K}{M} \right)$$



$$c\alpha = 0.0015 \times \alpha = 10^{-4}$$

$$\Rightarrow \quad \alpha = \left(\frac{1}{15} \right) \quad \therefore \quad R = \rho \left(\frac{l}{A} \right)$$

$$\Rightarrow \quad \frac{1}{\rho} = \left(\frac{1}{R} \right) \left(\frac{l}{A} \right) \quad \Rightarrow \quad K = 5 \times 10^{-7} \left(\frac{120}{1} \right)$$

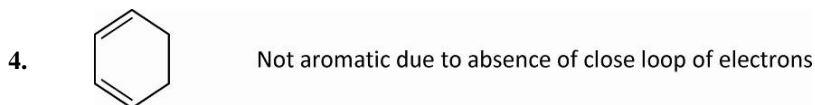
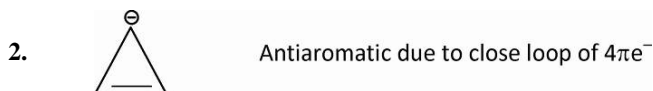
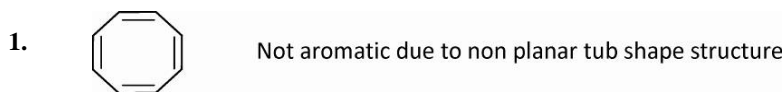
$$K = 6.0 \times 10^{-5}$$

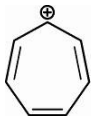


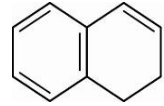
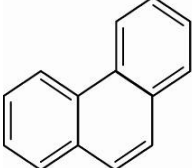
$$\text{And} \quad \Lambda_m^c = \frac{1000 \times 6.0 \times 10^{-5}}{1.5 \times 10^{-3}}$$

$$\Lambda_m^c = 40 \quad \therefore \quad \alpha = \frac{\Lambda_m^c}{\Lambda_m^0} \Rightarrow \Lambda_m^0 = 40 \times 15 = 6.0 \times 10^2 = z \times 10^2$$

$$\text{So} \quad z = 6.0$$

28.(5) Compound having a close loop of $(4n + 2)\pi$ electrons is aromatic compound.



5.  Aromatic due to close loop of $6\pi e^-$
6.  Aromatic due to close loop of $4\pi e^-$
7.  Aromatic due to close loop of $6\pi e^-$
8.  Aromatic due to close loop of $6\pi e^-$
9.  Aromatic due to close loop of $14\pi e^-$

Number of aromatic compounds is five i. e. 3, 5, 7, 8 and 9.

- 29.(6) $H_2 \rightarrow$ Diamagnetic σ_{1s}^2
 $He_2^+ \rightarrow$ Paramagnetic $\sigma_{1s}^2, \sigma_{1s}^{*1}$
 $Li_2 \rightarrow$ Diamagnetic $\sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2$
 $Be_2 \rightarrow$ Diamagnetic $\sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}$
 $B_2 \rightarrow$ Paramagnetic $\sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2p_x}^1, \pi_{2p_y}^1$
 $C_2 \rightarrow$ Diamagnetic $\sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2p_x}^2, \pi_{2p_y}^2$
 $N_2 \rightarrow$ Diamagnetic $\sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2p_x}^2, \pi_{2p_y}^2, \sigma_{2p_z}^2$
 $O_2^- \rightarrow$ Paramagnetic $\sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \sigma_{2p_z}^2, \pi_{2p_x}^2, \pi_{2p_y}^2, \pi_{2p_x}^{*2}, \pi_{2p_y}^{*1}$
 $F_2 \rightarrow$ Diamagnetic $\sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \sigma_{2p_z}^2, \pi_{2p_x}^2, \pi_{2p_y}^2, \pi_{2p_x}^{*2}, \pi_{2p_y}^{*2}$

The number of diamagnetic species is 6 i.e. $H_2, Li_2, Be_2, C_2, N_2$ and F_2 are diamagnetic species

- 30.(2) $a = 400\text{pm}$

$$d = 8 \text{ g cm}^{-3}$$

$$w = 256 \text{ g}$$

$$d = \frac{z \times M_0}{a^3 \times N_A}$$

$$M_0 = \frac{d \times a^3 \times N_A}{z} = \frac{8 \times (400 \times 10^{-10})^3 \times 6.023 \times 10^{23}}{4} = 77.09 \text{ g/mol}$$

$$\text{Number of atoms} = \frac{256}{77.09} \times 6.023 \times 10^{23} = N \times 10^{24}$$

$$2 \times 10^{24} = N \times 10^{24}$$

$$N = 2$$

31-33. The correct matching in column 1, column 2 and column 3 are

| Column 1 | Column 2 | Column 3 |
|----------|------------|-----------------|
| (I) | (i) | (Q) (R) (S) |
| (II) | (ii) | (P) (Q) (R) (S) |
| (III) | (iii) (iv) | (S) |
| (IV) | — | (Q) (S) |

31.(A) Incorrect combination is (I) (iii) (R) because in the expression of ψ_{n,ℓ,m_ℓ} for 1s-orbital, the exponential part

must have $e^{-\frac{Zr}{a_0}}$.
 \therefore [A]

32.(D) The correct combination is (I) (i) (S) as ψ_{n,ℓ,m_ℓ} in the column 2 (i) has exponential part as $e^{-\frac{Zr}{a_0}}$.

$$\frac{E_4 - E_2}{E_6 - E_2} = \frac{\frac{13.6Z^2}{16}(4-1)}{\frac{13.6Z^2}{36}(9-1)} = \frac{3 \times 13.6Z^2}{8 \times 13.6Z^2} = \frac{27}{32}$$

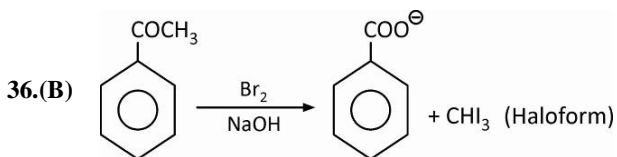
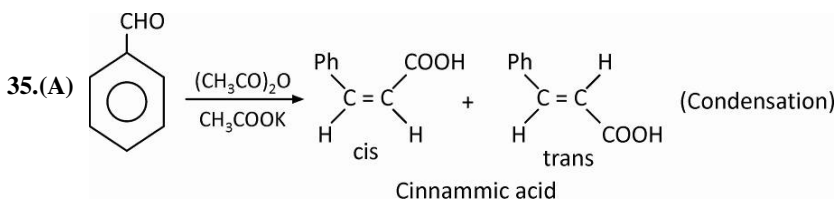
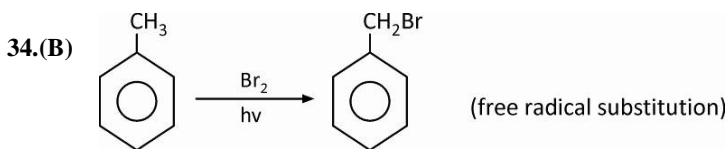
\therefore [D]

33.(C) The correct combination for any hydrogen-like species is [II] (ii) (P).

2s-orbital has one radial node and ψ_{n,ℓ,m_ℓ} v/s r plot will start from a finite value and sign changes once from

+ve to -ve.

\therefore [C]



MATHS

37.(AC) $\frac{az + b}{z + 1} - \frac{a\bar{z} + b}{\bar{z} + 1} = 2iy$

$$\frac{a\bar{z}z + az + b\bar{z} + b - (a\bar{z}\bar{z} + bz + a\bar{z} + b)}{(z + 1)(\bar{z} + 1)} = 2iy$$

$$\frac{(a-b)z - \bar{z}(a-b)}{(z + 1)(\bar{z} + 1)} = 2iy$$

$$\frac{(a-b)2iy}{(z+1)(\bar{z}+1)} = 2iy$$

$$(z+1)(\bar{z}+1) = 1$$

$$x+y^2+2x+1=1$$

$$x^2+2x+1=1-y^2$$

$$(x+1) = \pm\sqrt{1-y^2}$$

$$x+1 = \sqrt{1-y^2}$$

$$x+1 = -\sqrt{1-y^2}$$

$$x = -1 + \sqrt{1-y^2}$$

$$x = -1 - \sqrt{1-y^2}$$

38.(CD) (A) $f_1(x) = e^x - \int_0^x f(t)\sin t dt$

$$f_1(0) = 1$$

$$0 < f(x) < 1$$

$$0 < f(x)\sin x < 1$$

$$f'_1(x) = e^x - f(x)\sin x > 0$$

(B) $f(x) + \int_0^{\pi/2} f(t)\sin t dt > 0 \forall x \in (0, 1)$

(C) $g(x) = x - \int_0^{\frac{\pi}{2}-x} f(t)\cos t dt$

$$g(0) = - \int_0^{\pi/2} f(t)\cos t dt < 0$$

$$g(1) = 1 - \int_0^{\frac{\pi}{2}-1} f(t)\cos t dt > 0$$

(D) $g(x) = x^9 - f(x)$

$$g(0) = -f(0) < 0$$

$$g(1) = 1 - f(1) > 0$$

39.(AC) $P(X) = 1/3 \dots(i)$

$$P(X/Y) = 1/2 = \frac{P(X \cap Y)}{P(Y)} \dots(ii)$$

$$P(Y/X) = 2/5 = \frac{P(Y \cap X)}{P(X)} \dots(iii)$$

$$\frac{2}{5} = \frac{P(X \cap Y)}{1} \dots 3$$

$$P(X \cap Y) = \frac{2}{15}$$

$$(ii) \Rightarrow \frac{1}{2} = \frac{\frac{2}{15}}{P(Y)} = P(Y) = \frac{2 \cdot 2}{15 \cdot 1} = \frac{4}{15}$$

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{5+4-2}{15} = \frac{7}{15}$$

$$P(\bar{X}/Y) = \frac{P(\bar{X} \cap Y)}{P(Y)} = \frac{P(Y) - P(X \cap Y)}{P(Y)} = 1 - \frac{2}{15} \times \frac{15}{4} = 1/2$$

40.(ABD)

$$\begin{aligned} \lim_{x \rightarrow -1^+} &= (-1+h) \cos(\pi(-1+h) + [-1+h]) \\ &\equiv (-1+h) \cos[\pi(-1+h-1)] = (-1+h) \cos[\pi(-2+h)] \\ &\equiv (-1+h) \cos[2\pi - \pi h] \equiv (-1+h) \cos \pi h \equiv -1 \end{aligned}$$

$$\lim_{x \rightarrow 0^-} = (-h) \cos[\pi(-h-1)] = -h \cos[\pi h + \pi] \equiv 0$$

$$\lim_{x \rightarrow 0^+} = h \cos[\pi h] = 0$$

$$\lim_{x \rightarrow 2^-} \equiv (2-h) \cos[\pi(2-h+1)] \equiv (2-h) \cos[3\pi - \pi h] \equiv -(2-h) \cdot \cos \pi h \equiv -2$$

$$\lim_{x \rightarrow 2^+} = (2+h) \cos[\pi(2+h+2)] = (2+h) \cos[4\pi + \pi h] = (2+h) \cos \pi h \equiv 2$$

41.(ABD)

$$2x - y + 1 = 0$$

$$y = 2x + 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{16} = 1$$

$$c^2 = a^2 m^2 - b^2$$

$$1 = a^2 \cdot 4 - 16$$

$$a^2 = \frac{17}{4}$$

$$a = \frac{\sqrt{17}}{2}$$

42.(AB) For (A) and (B) $\det(A^2) = -1$,

and for (C)

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = -1 = \begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\text{For (C): } I^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

43.(B) $y^2 = 16x$

Equation of a chord with a given middle point (h, k) is $T = S_1$

$$ky - 8(x+h) = k^2 - 16h$$

$$ky - 8x + 8h - k^2 = 0$$

$$2x + y - p = 0$$

$$-\frac{8}{2} = \frac{k}{1} = \frac{8h - k^2}{-p} \quad \Rightarrow \quad k = -4$$

$$-\frac{4}{1} = \frac{8h - 16}{-p}$$

$$4p = 8h - 16 \quad \Rightarrow \quad p = 2h - 4$$

$$h = 3 \quad p = 2$$

$$44.(2) \quad g(x) = \left[f(t) \cos ect \right]_x^{\pi/2} = g(x) = 3 - f(x) \operatorname{cosec}(x)$$

$$\lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} \left(3 - \frac{f(x)}{\sin x} \right) = 3 - 1 = 2 = 2$$

$$45.(5) \quad x = 10!$$

$$y = {}^{10}C_9 \times {}^9C_1 \times \frac{10!}{2!}$$

$$46.(6) \quad a-d, \quad a, \quad a+d \quad a > 0$$

$$(a+d)^2 = a^2 + (a-d)^2$$

$$a^2 + d^2 + 2ad = 2a^2 - 2ad + d^2$$

$$a^2 = 4ad \quad a \neq 0$$

$$a = 4d$$

$$3d \quad 4d \quad 5d$$

$$A = \frac{1}{2} \times 3d \times 4d = 24$$

$$6d^2 = 24$$

$$d^2 = 4 \quad d = -2 \text{ Rejected}$$

$$d = 2$$

$$a = 8$$

$$6, \quad 8, \quad 10$$

47.(2) **Case I :** Touching x -axis

$$(x-1)^2 + (y+2)^2 = 4$$

$$x^2 + 2x + 1 + y^2 + 4y = 0$$

When $x = 0$

$$y^2 + 4y + 1 = 0$$

$D > 0$ (Two real root)

$$p = -1$$

Case II : Touching y -axis

$$(x+1)^2 + (y+2)^2 = 1$$

$$x^2 + y^2 + 2x + 4y + 4 = 0$$

$$y = 0$$

$$x^2 + 2x + 4 = 0$$

$D < 0$ (None real root) Rejected

Case III : $x^2 + y^2 + 2x + 4y = 0$

$$p = 0$$

$$48.(1) \quad \begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & 1 & \alpha \\ 1 & \alpha & 1 \end{vmatrix} = (\alpha^2 - 1)^2 = 0$$

$$\alpha^2 = 1$$

$$\alpha = \pm 1$$

$\alpha = 1$ (two planes are parallel) (Rejected)

$\alpha = -1$ (two planes are coincident)

49.(D) $y = x + 8$

$$y^2 = 32x, y = mx + \frac{a}{m}, \left(\frac{a}{m^2}, \frac{2a}{m} \right)$$

50.(A) $a = \sqrt{2}$

$$x^2 + y^2 = a^2, y = mx \pm a\sqrt{1+m^2}, \left(\frac{-ma^2}{c}, \frac{a^2}{m} \right)$$

$$x^2 + y^2 = 2 \text{ tangent at } (-1, 1) \text{ } -x + y = 2 \text{ or } y = x + 2$$

51.(C) $3 + \frac{a^2}{4} = a^2$

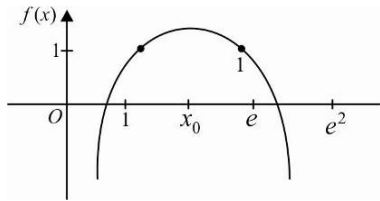
$$\Rightarrow a^2 = 4$$

$$x^2 + y^2 = 4$$

$$\sqrt{3}x + 2y = 4$$

52-54. 52.(D) 53.(A) 54.(B)

$$f(x) = x + \ln x - x \ln x \quad n \in (0, \infty)$$



$$f'(x) = 1 + \frac{1}{x} - x \cdot \frac{1}{x} - \ln x = \frac{1}{x} - \ln x = \frac{1 - x \ln x}{x} = 0$$

$$\Rightarrow \ln x = \frac{1}{x}$$

$$f(1) = f(x) = 1$$

$$f(0^+) = -\infty$$

$$f(e^2) < 0$$

$$f''(x) = -\frac{1}{x^2} - \frac{1}{x} < 0$$

