JEE ADVANCED PAPER-II SOLUTIONS

PART-I PHYSICS

1.(A) $K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda d^2}$

$\frac{hc}{\lambda} = \phi_0 + \frac{h^2}{2m\lambda d^2}$

$\Rightarrow \quad -\frac{hc}{\lambda^2}\frac{d\lambda}{d\lambda_d} = -\frac{h^2\,2}{2m\lambda_d^3} \quad \Rightarrow \quad \frac{\Delta\lambda_d}{\Delta\lambda} \propto \frac{\lambda_d^3}{\lambda^2}$

2.(C) From the figure, we can write

$\vec{P} + b\vec{R} = \vec{S}$ (From ΔOPS)

and $\vec{R} = \vec{Q} - \vec{P}$ (Given)

$\vec{P} + b(\vec{Q} - \vec{P}) = \vec{S}$

$\Rightarrow \quad \vec{S} = (1-b)\vec{P} + b\vec{Q}$

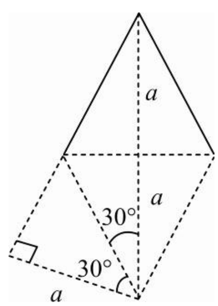
3. (D) Magnetic field due to one section of star can be calculated as follows

$B_0 = \frac{\mu_0 i}{4\pi a}[\sin 60° - \sin 30°]$

$B_0 = \frac{\mu_0 i}{4\pi a}\left(\frac{\sqrt{3}-1}{2}\right)$

Total field $B = 12B_0$

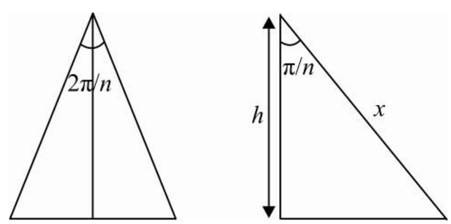
$= \frac{\mu_0 i}{2\pi a}3\left(\sqrt{3}-1\right)$



4.(D) For any n-sided polygon

$\frac{h}{x} = \cos\left(\frac{\pi}{n}\right) \quad \Rightarrow \quad x = \frac{h}{\cos\left(\frac{\pi}{n}\right)}$

$\Delta = x - h \quad \Rightarrow \quad \Delta = \frac{h}{\cos\left(\frac{\pi}{n}\right)} - h$



$\Rightarrow \quad \Delta = h\left[\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right]$

5.(B) Total mass = constant

$\rho\frac{4}{3}\pi R^3 = \text{constant}\,C$

$\therefore \quad \rho R^3 = C$

Differentiating with respect to time

$R^3\frac{d\rho}{dt} + \rho 3R^2\frac{dR}{dt} = 0$

Now $\quad R^3\left(\frac{d\rho}{dt}\right)+3R^2\rho v=0$

$\therefore \quad v=-\frac{R^3}{3R^2\rho}\frac{d\rho}{dt}=-\frac{R}{3}\frac{1}{\rho}\frac{d\rho}{dt}$

Since $\frac{1}{\rho}\frac{d\rho}{dt}$ is given to be constant

$v \propto R$

6.(C) Time taken for the stone to reach the bottom of the well.

$$t_1=\sqrt{\frac{2L}{g}}$$

time taken by the sound to reach from the bottom of the well to the observer

$$t_2=\frac{L}{V_s}$$

Total time $T=t_1+t_2$

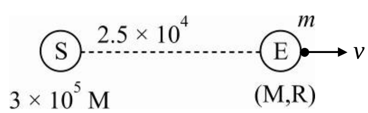
$$\Rightarrow \quad T=\sqrt{\frac{2L}{g}}+\frac{L}{V_s}$$

$$\frac{dT}{dL}=-\frac{1}{2}\sqrt{\frac{2}{g}}\times\frac{1}{\sqrt{L}}+\frac{1}{V_s}=\frac{1}{2}\times\sqrt{\frac{2}{10\times 20}}+\frac{1}{300}=\frac{1}{20}+\frac{1}{300}$$

$$\frac{0.01}{dL}=\frac{15+1}{300} \quad \Rightarrow \quad dl=0.01\times\frac{300}{16}=\frac{3}{16}$$

Percentage error, $\frac{dl}{l}\times 100=\frac{3}{16}\times\frac{1}{20}\times 100 \approx 1\%$

7.(B) For earth, $\quad Ve=\sqrt{\frac{2GM}{R}}$



For Sun + Earth,

Potential energy $=\left(-\frac{GM}{R}-\frac{3\times 10^5}{2.5\times 10^4}\frac{GM}{R}\right)m$

$$=-\frac{GMm}{R}(1+12)=-\frac{13GMm}{R}$$

$\therefore \quad \frac{1}{2}mv^2=\frac{13GMm}{R}$

$$v=\sqrt{\frac{2GM}{R}}.\sqrt{13}=\sqrt{13}\,v_e$$

$\therefore \quad v_e=40.4\ km/s \simeq 42\ km/s.$

8.(BCD) Let the currents through inductors be i_1 and i_2. Then, $V_L=\frac{L_1 di_1}{dt}=L_2\frac{di_2}{dt}$

i.e., $\quad \int_0^t d(L_1 i_1)=\int_0^t d(L_2 i_2) \quad \Rightarrow \quad L_1 i_1=L_2 i_2$

i.e. $\quad \frac{i_1}{i_2}=\frac{L_2}{L_1}$ is fixed at all times.

Hence (D) is correct

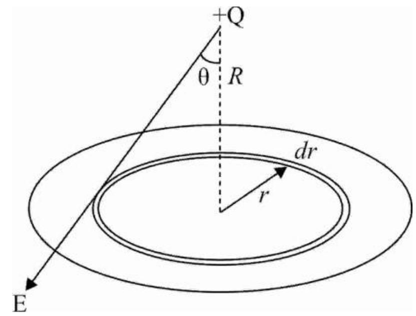
Also, since inductor acts as an open circuit at t = 0, current through all branches is zero at t = 0.

After a long time current through battery

$$i_0=\frac{V}{R} \text{ and } L_1 i_1=L_2 i_2 \text{ and } i_1+i_2=i_0$$

$$i_1 = \frac{i_0 L_2}{L_1 + L_2} \text{ and } i_2 = \frac{i_0 L_1}{L_1 + L_2}$$

9.(AD) Since the charge lies outside the sphere, net flux passing through the sphere is zero.



$$\phi_{\text{curved surface}} + \phi_{\text{disc}} = 0$$

Option (C) is incorrect

$$\phi_{\text{curved surface}} = -\phi_{\text{disc}}$$

$$\cos\theta = \frac{R}{\sqrt{R^2 + r^2}} \qquad E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(R^2 + r^2\right)}$$

$$\phi_{\text{disc}} = \int \vec{E} \cdot d\vec{A}$$

$$= \int\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(R^2 + r^2\right)} \times 2\pi r dr \times \cos\theta\right) = \frac{Q \cdot 2\pi}{4\pi\varepsilon_0}\int \frac{rdr}{R^2 + r^2} \times \frac{R}{\left(R^2 + r^2\right)^{1/2}}$$

$$= \frac{QR}{2\varepsilon_0}\int_0^R \frac{rdr}{\left(R^2 + r^2\right)^{3/2}} = \frac{QR}{2\varepsilon_0}\left[\frac{1}{2}\frac{\left(R^2 + r^2\right)^{-1/2}}{-1/2}\right]_0^R = \frac{QR}{2\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2 + R^2}}\right] = \frac{Q}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{2}}\right]$$

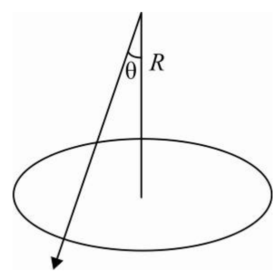
$$\Rightarrow \qquad \phi_{\text{curved surface}} = -\frac{Q}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{2}}\right]$$

Option (A) is correct

Potential at any point on the circumference of the flat surface is $\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2 + R^2}} = \frac{Q}{4\pi\varepsilon_0\left(\sqrt{2}R\right)}$

Hence it is equipotential

Option (D) is correct



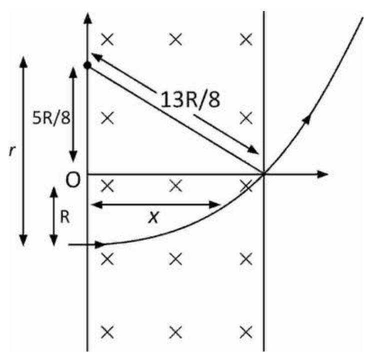
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(R/\cos\theta)^2} = \frac{\theta}{4\pi\varepsilon_0 R^2}\cos^2\theta$$

$$E_{\text{normal}} = E\cos\theta = \frac{Q}{4\pi\varepsilon_0 R^2}\cos^3\theta$$

Which is not constant

Option (B) is in correct

10.(AB) For (A) For $B = \frac{8p}{13QR}$



$$r = \frac{p}{QB} = \frac{p13QR}{Q8p} = \frac{13R}{8}$$

Clearly $x = \sqrt{\left(\frac{13r}{8}\right)^2 - \left(\frac{5R}{8}\right)^2}$

$$x = \frac{3R}{2}$$

Hence particle enters region 3 through point P_2

(r = radius of circle in which charge moves)

For (B) The particle will reenter region 1 if $r < \frac{3R}{2}$

$$\frac{p}{QB} < \frac{3R}{2} \quad \Rightarrow \quad B > \frac{2p}{3QR}$$

For (C) Distance between point p_1 and point of re-entry into region 1 = $2r$

$$y = \frac{2mv}{qB} \propto m$$

For (D) When particle re-enters region 1 through longest possible path, it will re-enter horizontally only.
Hence at farthest point from y axis, if must be vertical with magnitude of momentum being p. hence change in its linear momentum must be $\sqrt{2}p$.

11.(AC) At P_1, $\Delta x = 0$

P_2, $\Delta x = d = 3000\lambda$

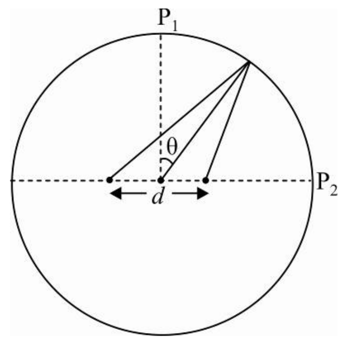
so, between P_1 and P_2 2999 bright fringes will be formed.

So, A is correct.

At P_2, there will be bright fringe.

So, B is incorrect

At P_2, there will be 3000th bright fringe



At angle θ, $\Delta x = d\sin\theta$

(assuming the circle to be of large radius)

Rate of change of path difference is $\frac{d}{d\theta}(\Delta x) = d\cos\theta$

Which decreases with increasing θ. so for small values of θ, path difference increases sharply with increases in θ resulting in closer fringes.

So, angular separation between consecutive bright sports increases as we move from P_1 to P_2.

So, D is incorrect

12.(BD) If connected across X and Y : $V_{XY} = V_X - V_Y$

$$= V_0 \sin\omega t - V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) = V_0\left[\sin\omega t - \sin\omega t\cos\frac{2\pi}{3} - \cos\omega t \sin\frac{2\pi}{3}\right]$$

$$= V_0\left[\sin\omega t + \frac{1}{2}\sin\omega t - \frac{\sqrt{3}}{2}\cos\omega t\right] = \sqrt{3}\,V_0\left[\frac{\sqrt{3}}{2}\sin\omega t - \frac{1}{2}\cos\omega t\right] = \sqrt{3}\,V_0 \sin\left(\omega t - \frac{\pi}{3}\right)$$

$$\therefore \quad V_{XY}^{rms} = \frac{\sqrt{3}V_0}{\sqrt{2}} = \sqrt{\frac{3}{2}}V_0$$

If connected across X and Z.

$$V_{XZ} = V_X - V_Z = V_0\sin\omega t - V_0 \sin\left(\omega t + \frac{4\pi}{3}\right) = \sqrt{3}\,V_0 \sin\left(\omega + \frac{\pi}{6}\right)$$

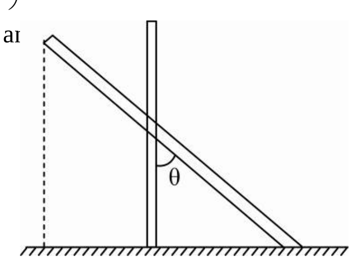
If connected across Y and Z:

$$Y_{YZ} = V_Y - V_Z = V_0\sin\left(\omega t + \frac{2\pi}{3}\right) - V_0 \sin\left(\omega t + \frac{4\pi}{3}\right) = \sqrt{3}V_0 \cos\omega t$$

$\therefore$ V_{rms} is same if the voltmeter is connected across a

13.(BCD) **(A)** $x = \frac{L}{2}\sin\theta$

$y = L\cos\theta$



Hence $\frac{x^2}{\left(\frac{L}{2}\right)^2} + \frac{y^2}{L^2} = 1$

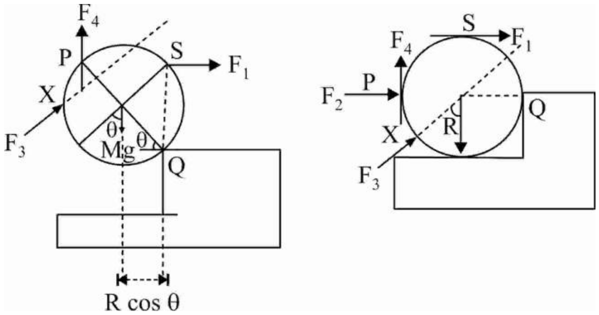
Hence NOT parabola

(B) $(\text{Torque about})_B = \left(mg\frac{L}{2}\sin\theta\right)$ Hence correct

(C) As there is no external horizontal force and zero initial velocity Hence it will move downwards

(D) Displacement of mid point $=\left(\frac{L}{2}-\frac{L}{2}\cos\theta\right)$

14.(BD)



(A) Distance of F_1 from point Q increases

$\Rightarrow$ Torque of F_1 increases (clockwise)

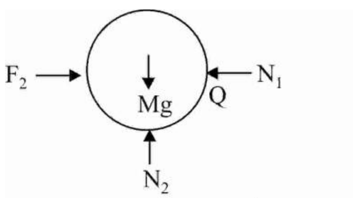
Distance of force Mg from Q decreases.

$\Rightarrow$ torque of Mg decreases (anti clockwise)

$\Rightarrow$ Net torque is anti-clockwise direction increases

$\Rightarrow$ $\tau \neq 0$ Incorrect

(B) Net torque = 0



correct

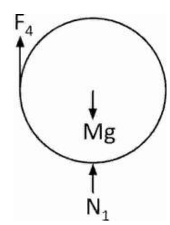
(C) As the wheel begins to climb, the distance of the force F_3 from point Q increases initially

$\Rightarrow$ Torque of F_3 through X increases clockwise.

Torque of mg decreases anticlockwise

$\Rightarrow$ Net torque increases clockwise initially Incorrect

(D) When force is applied vertically, then there will be no normal reaction at point Q and hence friction will also be zero.

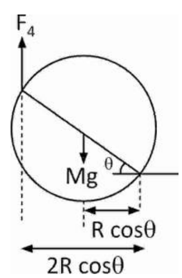


This will cause slipping of the wheel.

Since, it is given that there is no slipping at point Q, therefore we can write

$\tau = F_4(2R\cos\theta) - MgR\cos\theta$ (Clockwise)

$\tau = (2F_4R - MgR)\cos\theta$



As θ increases from 0 to $\frac{\pi}{2}$, $\cos\theta$ decreases from 1 to 0.

$\Rightarrow$ τ decreases as the wheel climbs.

Correct

15. (D) When voltage is set to $\frac{V_0}{3}$, charge supplied by battery $=\frac{CV_0}{3}$

When voltage is raised to $\frac{2V_0}{3}$, additional charge supplied $=\frac{2CV_0}{3}-\frac{CV_0}{3}=\frac{CV_0}{3}$.

When voltage is raised to V_0, additional charge supplied $= CV_0 - \frac{2CV_0}{3} = \frac{CV_0}{3}$

Total energy supplied by cell $= \frac{V_0}{3}\left(\frac{CV_0}{3}\right) + \frac{2V_0}{3}\left(\frac{CV_0}{3}\right) + V_0\left(\frac{CV_0}{3}\right)$

$$= \frac{2}{3}CV_0^2$$

Final charge on capacitor $= CV_0$

Energy stored in capacitor $= \frac{1}{2}CV_0^2$

$\therefore$ Energy dissipated across resistor $E_D = \frac{2}{3}CV_0^2 - \frac{1}{2}CV_0^2 = \frac{1}{6}CV_0^2$

16. (C) Energy supplied by cell $= V_0(CV_0)$

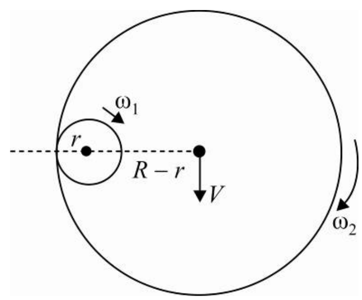
$$= CV_0^2$$

Energy stored in capacitor $(E_C) = \frac{1}{2}CV_0^2$

$\therefore$ Energy dissipated across resistor $(E_D) = CV_0^2 - \frac{1}{2}CV_0^2 = \frac{1}{2}CV_0^2$

$\therefore$ $E_C = E_D$

17.(B) The motion can be visualized as a larger ring spinning around a smaller rotating disk, without slipping. Let angular speeds of smaller and larger ring be ω_1 & ω_2. In addition to rotation about its own axis center of larger ring it is moving say with a speed *v*. Center of larger ring is moving in a circle of radius (*R* - *r*).



So, $V = (R-r)\omega_2$ ...(i)

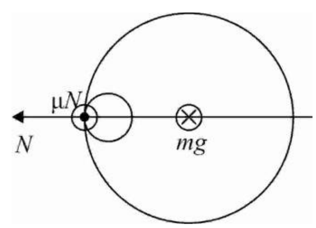
Also, no slipping at the point of contact gives

$r\omega_1 = R\omega_2 - V$ ...(ii)

(i) and (ii) give $\omega_1 = \omega_2 \longrightarrow \omega_0$

Now draw the FBD of ring

$N = ma_c s = m\omega_0^2(R-r)$ ...(iii)



Balancing forces in vertical direction for translational equilibrium,

$mg = \mu N$

$\therefore$ $mg = \mu m\omega_0^2(R-r)$ i.e. $\omega_0 = \sqrt{\frac{g}{\mu(R-r)}}$

18. **No answer matches**

Kinetic energy of ring can be calculated by two methods

(i) Basic method

$$\text{KE(Total)} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\cdot\omega^2 = \frac{1}{2}m\omega_0^2(R-r)^2 + \frac{1}{2}(mR^2)\omega_0^2$$

$$= \frac{1}{2}m\omega_0^2\left[(R-r)^2 + R^2\right]$$

(ii) Using ICOR : Centre of smaller disk is the Instantaneous Centre of Rotation for ring so,

$$\text{KE} = \frac{1}{2}I_{\text{ICOR}}\cdot\omega^2$$

$$= \frac{1}{2}\left[mR^2 + m(R-r)^2\right]\omega_0^2 = \frac{1}{2}m\omega_0^2\left[(R-r)^2 + R^2\right]$$

PART-II CHEMISTRY

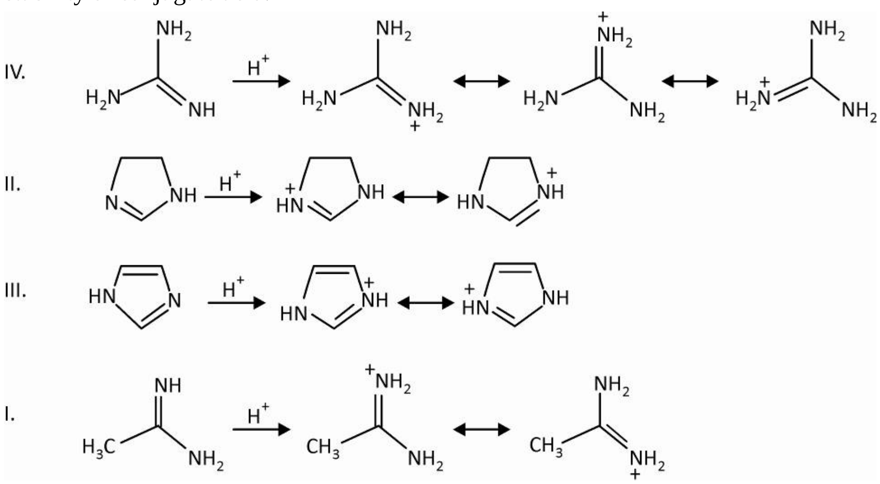
19. (A) $Zn(s) + CuSO_4(aq) \longrightarrow ZnSO_4(aq) + Cu(s)$

$$E = E^\circ - \frac{2.303RT}{2F}\log\frac{[Zn^{2+}]}{[Cu^{2+}]} = 1.1 - \frac{2.303RT}{2F}\log 10 = 1.1 - \frac{2.303RT}{2F}$$

$$\Delta G = -2FE = -2F\left(1.1 - \frac{2.303RT}{2F}\right) = -2.2F + 2.303RT = 2.303\ RT - 2.2\ F$$

20. (C) IV > I > II > III

Gaunidine is most basic due to more number of equivalent resonating structures of conjugate acid hence more stability of conjugate acid.



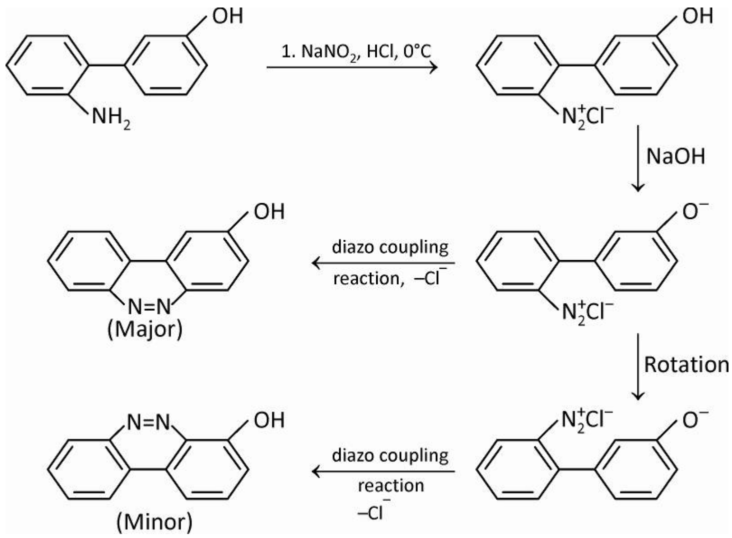
III is least basic because of electron withdrawing effect of – CH = CH– group.

21.(A) $P = \frac{\Delta G}{\Delta V}$

$$= \frac{2.9\,kJmol^{-1}}{2\times10^{-6}\ m^3\ mol^{-1}} = \frac{2.9\times10^3\ Jmol^{-1}}{2\times10^{-6}\ m^3mol^{-1}} = 1.45\times10^9\ Nm^{-2}$$

$$P = \frac{1.45\times10^9}{10^5}bar = 1.45\times10^4\ bar = 14500\,bar$$

22.(C)



23. (D)

Acid	oxidation state	Acid	oxidation state
H_3PO_2	+1	H_3PO_4	+5
H_3PO_3	+ 3	$H_4P_2O_6$	+4

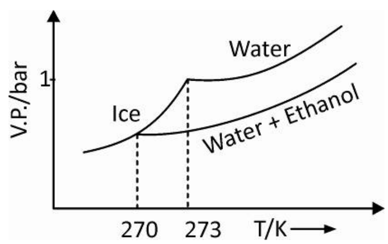
24.(C) Mole of $C_2H_5OH = \frac{34.5}{46}$

$m = \frac{34.5}{46} \times \frac{1000}{500}$

$\Delta T_f = \frac{34.5}{46} \times \frac{1000}{500} \times 2 = 3$

$T_f^\circ - T_f = 273 - T_f = 3$

$T_f = 273 - 3 = 270K$



25. (D) $4Au + 8NaCN + 2H_2O + O_2 \longrightarrow 4Na[Au(CN)_2] + 4NaOH$

$\underset{\text{conc.}}{Cu + 4HNO_3} \longrightarrow Cu(NO_3)_2 + 2NO_2 + 2H_2O$

$\underset{\text{conc}}{2Fe + 6HNO_3} \longrightarrow Fe_2O_3(s) + 6NO_2 + 3H_2O$ (Fe become passive on reaction with concentrated HNO_3)

$Zn + 2NaOH(aq) \longrightarrow Na_2ZnO_2 + H_2$

26.(AC) BeO, Al_2O_3, SnO, PbO, Cr_2O_3, SnO_2, PbO_2 are amphoteric oxides.

CrO is basic oxide , B_2O_3 is acidic oxide , NO is Neutral oxide

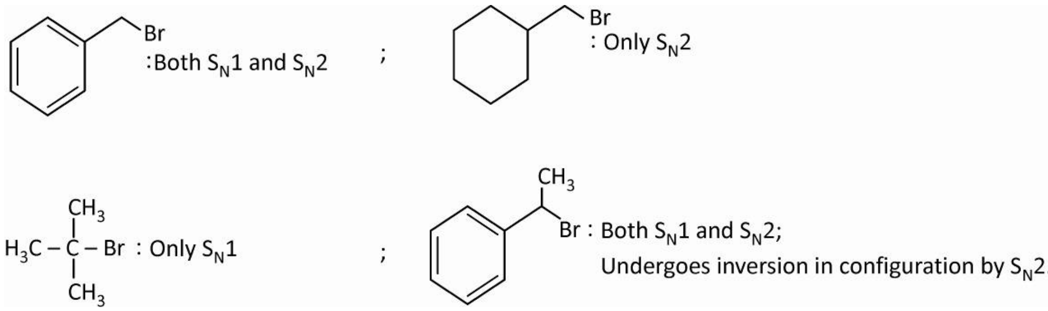
27.(BC) Cloud is not emulsion type colloid. It is an aerosol type colloid.

For adsorption $\Delta_r H < 0$ and $\Delta_r S < 0$.

Higher is critical temperature greater is extent of adsorption.

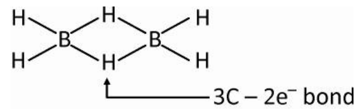
Brownian motion of colloidal particles depends on the size of the particles and viscosity of the solution.

28.(ABC)

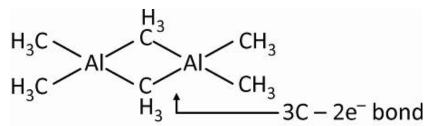


Reactivity order for nucleophilic substitution reaction is IV > III > I > II

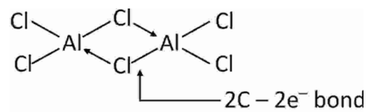
29.(ABD) Dimeric structure of BH_3 :



Dimeric structure of $Al(CH_3)_3$:



Dimeric structure of $AlCl_3$:



Lewis acidity decreases down the group.

30.(AC) $K = P \cdot Z\, e^{-Ea/RT}$ (According to Arrhenius equation)

$K = A\, e^{-Ea/RT}$

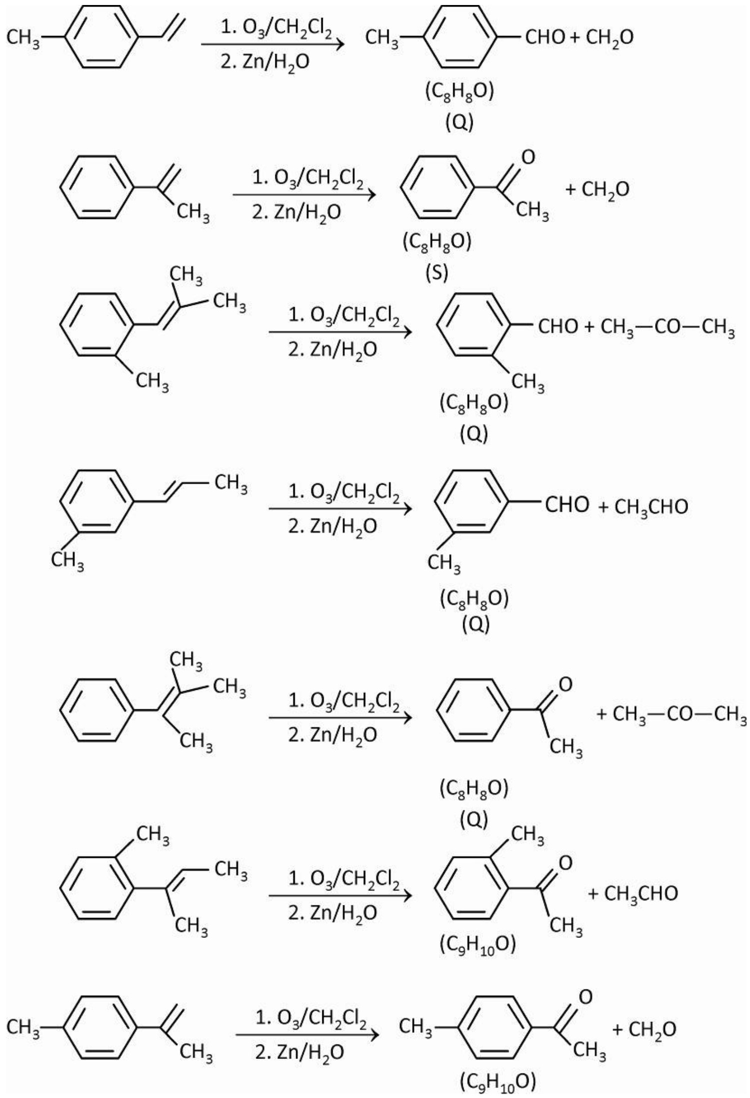
$P = \frac{A}{Z}$

According to collision theory, P value is generally less than unity but for some reactions P is greater than one and for such reactions, observed rate is greater than rate predicted from Arrhenius equation.

For a reaction with P value greater than 1, implies that the experimentally determined value of frequency factor (A) is higher than that predicted by Arrhenius equations and such reactions proceeds rapidly without the use of a catalyst.

$\therefore$ (A) and (C).

31.(AB)



32.(BD) $\Delta G^\circ = -RT\ \ell n\ K$, $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$-RT\ell n\,K = \Delta H^\circ - T\Delta S^\circ$, $\ell nK = -\frac{\Delta H^\circ}{R}\frac{1}{T} + \frac{\Delta S^\circ}{R}$

$\Delta H^\circ < 0,\ \Delta S^\circ > 0 \Rightarrow$ K decreases with increase in T ; $\Delta H^\circ > 0,\ \Delta S^\circ > 0 \Rightarrow$ K increases with increase in T

$\Delta H^\circ > 0,\ \Delta S^\circ < 0 \Rightarrow$ K decreases with increase in T ; $\Delta H^\circ < 0,\ \Delta S^\circ > 0 \Rightarrow$ K decreases with increase in T

$\Delta S_{surr} = \frac{-q_{sys}}{T_{surr}}$

ΔS_{surr} favourable means ΔS_{surr} is positive while ΔS_{surr} unfavourable means ΔS_{surr} is negative. If value of K increases with increase in temperature for endothermic reaction it means reaction shift toward forward direction because of unfavourable change in entropy of the surrounding decreases. Similarly if value of K for exothermic reaction decreases with increase in temperature it means reaction shift toward backward direction due to decrease in favourable change in entropy of surrounding.

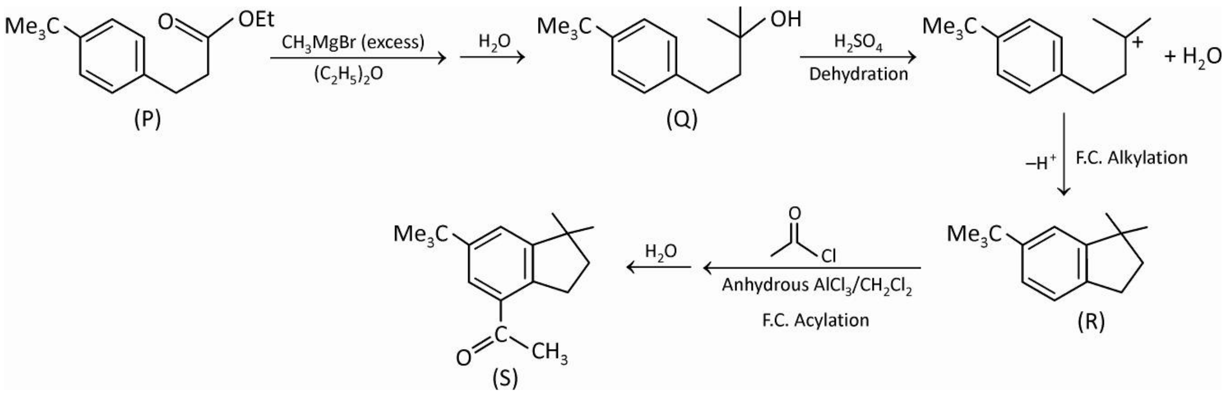
33-34. **33.(A)** **34.(A)**

$KClO_3(s) \xrightarrow{MnO_2} KCl(s) + \frac{3}{2}O_2(g)$

$P_4(s) + \underset{(W)}{O_2(g)} \xrightarrow{\Delta} \underset{(C)}{P_4O_{10}(s)}$

$12HNO_3 + P_4O_{10} \longrightarrow 6N_2O_5 + 4H_3PO_4$ or HPO_3

35-36. **35.(C)** **36.(C)**



PART-III MATHEMATICS

37.(C) $x+y+z=10$

Total number of non-negative integers satisfying this equation $= {}^{10+3-1}C_{3-1} = {}^{12}C_2$

If z is even i. e, $z = 2m;\ \ m = \{0,1,2,3,4,5\}\ \ x+y = 10-2m$

No of . Solutions $= {}^{10-2m+2-1}C_{2-1} = {}^{11-2m}C_1 = 11-2m$

Therefore number of favourable cases $= \sum_{m=0}^{5}(11-2m) = 66-30 = 36$

Probability $= \frac{36}{12_{C_2}} = \frac{36}{66} = \frac{6}{11}$

38.(A) $f''(x) > 0$

Since $f\left(\frac{1}{2}\right) = \frac{1}{2}, f(1) = 1$

Let $g(x) = f(x) - x$

$$g\left(\frac{1}{2}\right) = 0 = g(1)$$

Therefore, $g'(x) = 0\ \forall e \in (0,1) \quad \Rightarrow \quad f'(e) = 1$

$$f''(x) > 0$$

$$\int_e^1 f''(x)\,dx > \int_0^1 0\,dx$$

$$f'(1) - f'(e) > 0$$

$$f'(1) > f'(e)$$

$$\Rightarrow \quad f'(e) > 1$$

39.(C) The normal to the plane is :

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = \hat{i}(-2-12) - \hat{j}(-4+6) + \hat{k}(-12-3) = -14\hat{i} - 2\hat{j} - 15\hat{k}$$

Therefore, the equation of the plane is $(x-1)14 + (y-1)(2) + (z-1)(15) = 0$.

40.(B) Taking 1st two

$$\overline{OP}\cdot\left(\overline{OQ}-\overline{OR}\right)=\overline{OS}\cdot\left(\overline{OQ}-\overline{OR}\right)$$

$\Rightarrow \quad \left(\overline{OP}-\overline{OS}\right)\cdot\left(\overline{OQ}-\overline{OR}\right)=0 \quad \Rightarrow \quad \overline{PS}\perp\overline{QR}$

Similarly $\quad \overline{QS}\perp\overline{PR}$

$\overline{RS}\perp\overline{PQ}$

$\Rightarrow$ S is orthocentre

41.(B) $M=\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$M^T=\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

$M^TM=\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$T_r\left(M^TM\right)=\left(a_1^2+b_1^2+c_1^2\right)+\left(a_2^2+b_2^2+c_2^2\right)+\left(a_3^2+b_3^2+c_3^2\right)=5$

$5=1^2+1^2+1^2+1^2+1^2+0^2+0^2+0^2+0^2 \quad \Rightarrow \quad 5,\ 1's,\ 4,\ 0's \quad \Rightarrow \quad {}^9C_5$

$1^2+2^2+0^2+0^2+0^2+0^2+0^2+0^2+0^2 \quad \Rightarrow \quad 1\to 1,\ 1\to 2,\ 7\to 0's \quad \Rightarrow \quad {}^9C_7\times{}^2C_1$

$\Rightarrow \quad 126+\dfrac{9.8}{2}\times 2=198$

42.(D) $|N_1|={}^5C_1\times{}^4C_4$

$|N_2|={}^5C_2\times{}^4C_3$

$|N_3|={}^5C_3\times{}^4C_2$

$|N_4|={}^5C_4\times{}^4C_1$

$|N_5|={}^5C_5\times{}^4C_0$

$N_1+......+N_5={}^9C_5=\dfrac{9\cdot 8\cdot 7\cdot 6}{4\cdot 3\cdot 2\cdot 1}=126$

43.(C) $dy=\dfrac{1}{8\sqrt{x}\,\sqrt{9+\sqrt{x}}\,\sqrt{4+\sqrt{9+\sqrt{x}}}}dx, \qquad x>0$

$y=\dfrac{1}{2\sqrt{4+\sqrt{9+\sqrt{x}}}}\cdot\dfrac{1}{2\sqrt{9+\sqrt{x}}}\cdot\dfrac{1}{2\sqrt{x}}dx=dt \qquad \sqrt{4+\sqrt{9+\sqrt{x}}}=t$

$y=\sqrt{4+\sqrt{9+\sqrt{x}}}+C$

$y(0)=7 \quad \Rightarrow \quad C=0$

$y(256)=\sqrt{4+\sqrt{9+\sqrt{256}}}=3$

44.(BC) $\lim\limits_{x\to 1^-}\dfrac{1-x(1+1-x)}{1-x}\cos\left(\dfrac{1}{1-x}\right)$

$$\frac{1-2x+x^2}{(1-x)}\cos\left(\frac{1}{1-x}\right)$$

$$\lim_{x\to 1^-}\frac{(x-1)^2}{(1-x)}\cos\left(\frac{1}{1-x}\right)=0$$

$$\lim_{x\to 1^+}\frac{1-x(1+x-1)}{(x-1)}\cos\left(\frac{1}{1-x}\right) \qquad \text{does not exist}$$

45.(CD) $2\cos\beta-2\cos\alpha+\cos\alpha\cos\beta=1$

$\cos\beta-\cos\alpha=\left(\frac{1-\cos\alpha\cos\beta}{2}\right)$ Assume $x=\tan\frac{\alpha}{2}$ and $y=\tan\frac{\beta}{2}$

$\frac{1-y^2}{1+y^2}-\frac{1-x^2}{1+x^2}=\frac{(1+x^2)(1+y^2)-(1-x^2)(1-y^2)}{2(1+x^2)(1+y^2)}$ Apply C and D

$$\frac{1+x^2-y^2-x^2y^2-(1-x^2y^2-x^2+y^2)}{(1+x^2)(1+y^2)}=\frac{1+x^2+y^2+x^2y^2-(1+x^2y^2-x^2-y^2)}{2(1+x^2)(1+y^2)}$$

$$2(x^2-y^2)=\frac{2x^2+2y^2}{2}$$

$$\frac{x^2}{y^2}=3.$$

46.(BC) $\cos 2x(1)-\cos 2x(-\cos^2 x+\sin^2 x)+\sin 2x(-\cos x.\sin x-\cos x.\sin x)$

$f(x)=\cos 2x+\cos 4x$

$f'(x)=-2\sin 2x-4\sin 4x$

$-2\sin 2x[1+4\cos 2x]$

$\sin 2x=0 \qquad \cos 2x=-\frac{1}{4}$

$\left\{\frac{-\pi}{2},0,\frac{\pi}{2}\right\}.$

47.(AD) $x^3\le y\le x \qquad 0\le x\le 1$

$$\int_0^{\alpha}(x-x^3)dx=\int_{\alpha}^{1}(x-x^3)dx$$

$$\frac{\alpha^2}{2}-\frac{\alpha^4}{4}=\frac{1}{2}-\frac{1}{4}-\left(\frac{\alpha^2}{2}-\frac{\alpha^2}{4}\right)$$

$$2\alpha^4-4\alpha^2+1=0.$$

48.(CD) $k=1,2,-,98$

$k\le x\le k+1$

$$\frac{1}{x+1}\le\frac{k+1}{x(x+1)}\le\frac{1}{x}$$

$$\int_k^{k+1}\frac{1}{x+1}dx\le\int_k^{k+1}\frac{k+1}{x(x+1)}dx\le dx$$

$$\ell n\left(\frac{k+2}{k+1}\right)\le\int_k^{k+1}\frac{k+1}{x(x+1)}dx\le\ell n\left(\frac{k+1}{k}\right)$$

$$\ell n\left(\frac{3}{2}\times\frac{4}{3}\times\frac{5}{4}\times\ldots\times\frac{100}{99}\right) < I < \ell n\left(\frac{2}{1}\times\frac{3}{2}\times\frac{4}{3}\times\ldots\frac{99}{98}\right)$$

$$\ell n 50 < I < \ell n 99$$

49.(CD) $f'(x) > 2f(x)$

$f'(x) - 2f(x) > 0$

$\frac{dy}{dx} - 2y > 0$

$e^{-2x}\frac{dy}{dx} - 2e^{-2x}y > 0$

$\frac{d}{dx}\left(e^{-2x}y\right) > 0 \quad \Rightarrow \quad e^{-2x}f(x)$ is increasing

and $\quad x > 0$

$e^{-2x}f(x) > f(0) \quad \Rightarrow \quad f(x) > e^{2x} > 0 \quad \Rightarrow \quad f'(x) > 0$

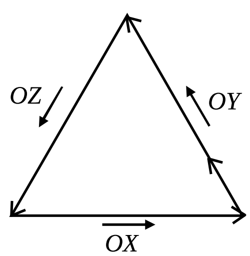
50. $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)\,dt$

$g'(x) = \sin^{-1}(\sin 2x)\cdot\cos 2x\cdot 2 - \sin^{-1}(\sin x)\cdot\cos x$

$g'\left(\frac{\pi}{2}\right) = \sin^{-1}(\sin\pi)\cdot\cos\pi\cdot 2 - \sin^{-1}\left(\sin\frac{\pi}{2}\right)\cdot\cos\frac{\pi}{2} = 0$

$g'\left(-\frac{\pi}{2}\right) = 0$

51.(D)



$|\overrightarrow{OX}\times\overrightarrow{OY}| = |\overrightarrow{OX}|\ |\overrightarrow{OY}|\sin(\pi - R)$

$\equiv 1.1\sin R$

$\equiv \sin(P + Q)$

52.(A) $\cos(P+Q) + \cos(Q+R) + \cos(R+P)$

In a ΔABC

$\cos A + \cos B + \cos C \le \frac{3}{2}$

$\therefore \quad -[\cos P + \cos Q + \cos R]$ has minimum value $-3/2$.

53.(D) **54.(A)**

$a_n - a_{n-1} = p\alpha^{n-1}(\alpha - 1) + q\beta^{n-1}(\beta - 1)$

$= p\alpha^{n-1}\left(\frac{1}{\alpha}\right) + q\beta^{n-1}\left(\frac{1}{\beta}\right)$

$$a_n - a_{n-1} = p\alpha^{n-2} + q\beta^{n-2}$$

$$a_n = a_{n-1} + a_{n-2}$$

$$a_4 = a_3 + a_2 = a_2 + a_1 + a_1 + a_0 = a_1 + a_0$$

$$= 3a_1 + 2a_0$$

$$= 3\alpha p + 3\beta q + 2p + 2q$$

$$\alpha = \frac{1-\sqrt{5}}{2}, \beta = \frac{1+\sqrt{5}}{2}$$

$$\left(\frac{3}{2} - \frac{3}{2}\sqrt{5}\right)p + \left(\frac{3}{2} + \frac{3}{2}\sqrt{5}\right)q + 2p + 2q$$

$$\frac{7p}{2} + \frac{7q}{2} = 28, \quad p = q$$

$$28 = 7p \quad \Rightarrow \quad p = 4$$