# HINTS AND SOLUTIONS

## PART-C : MATHEMATICS



61. (C)

Range of $f(x) = \tan^{-1}(3x^2 + bx + c)$ is $\left[0, \frac{\pi}{2}\right)$ iff range of $g(x) = 3x^2 + bx + c$ is $[0, \infty)$, which is possible only when discriminant of the equation $g(x) = 0$ is zero.

$b^2 = 12c$

62. (D)

$$(24\sin x)^{\frac{3}{2}} = 24\cos x$$

$$\Rightarrow \sqrt{24}(\sin x)^{\frac{3}{2}} = \cos x$$

$$\Rightarrow 24\sin^3 x = \cos^2 x = 1 - \sin^2 x;\ \text{Let } \sin x = t$$

$$24t^3 + t^2 - 1 = 0$$

$$\Rightarrow (3t-1)(\underbrace{8t^2 + 3t + 1}_{D<0}) = 0$$

$$t = \frac{1}{3}$$

$$\Rightarrow \sin x = \frac{1}{3}$$

$$\Rightarrow \operatorname{cosec}^2 x = 9$$

63. (C)

We need to maximize $\dfrac{1}{|4z-1|^2} + \dfrac{1}{|3w+1|^2}$

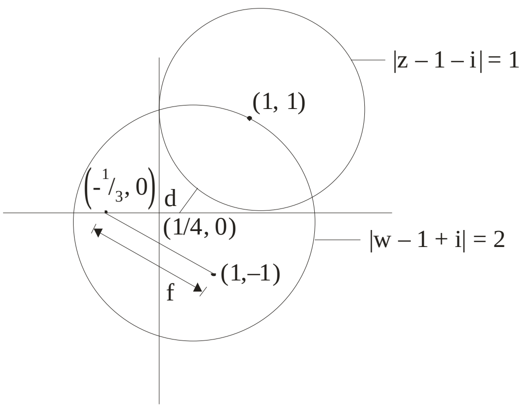
$$\Rightarrow \frac{1}{16\left|z-\frac{1}{4}\right|^2} + \frac{1}{9\left|w+\frac{1}{3}\right|^2}$$

So we need to minimize $\left|z-\frac{1}{4}\right|^2$ and $\left|w+\frac{1}{3}\right|^2$

Minimum $\left|z-\frac{1}{4}\right|^2 = \left(\sqrt{\left(1-\frac{1}{4}\right)^2 + 1^2} - 1\right)^2 = \frac{1}{16}$

Minimum $\left|w+\frac{1}{3}\right|^2 = \left(2-\sqrt{\left(1+\frac{1}{3}\right)^2+1}\right) = \left(2-\frac{5}{3}\right)^2 = \frac{1}{9}$

So answer is 2.




$$d_{min} = \left|z - \frac{1}{4}\right|$$

$$f_{min} = \left|w + \frac{1}{3}\right|$$

64. (A)

$$x-5y-y^5=0 \Rightarrow \frac{dy}{dx}=\frac{1}{5y^4-5}$$

Now x = 0 ⇒ y = 0

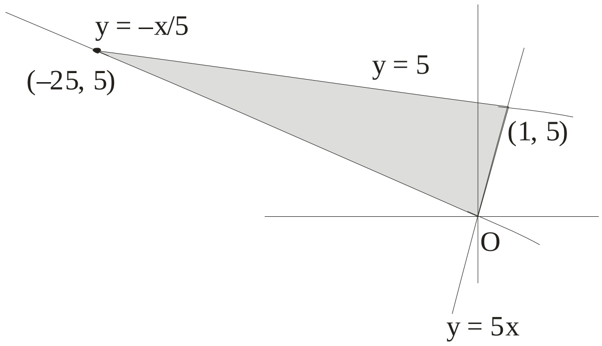
So slope of tangent at (0, 0) is $-\frac{1}{5}$

y = $\frac{1}{5}$x (Equation of tangent)

Now, equation of normal is y = 5x

Also given line is y = 5

So area of the triangle = $\frac{1}{2}(26)(5)=(13)(5)=65$




65. (A)

If a + b + c = 0 ⇒ $a^3 + b^3 + c^3 = 3abc$

So, cos A . cos B . cos C = $\lambda\left(4\left(\cos^3 A+\cos^3 B+\cos^3 C\right)-3\left(\cos A+\cos B+\cos C\right)\right)$

cos A cos B cos C = $\lambda\left(4\left(3\cos A\cos B\cos C\right)\right)$

$$\lambda=\frac{1}{12}$$

66. (B)

Conceptual

67. (A)

$p \to (q \to r) \equiv \sim p \vee (q \to r)$

$\equiv \sim p \vee (\sim q \vee r) \equiv [\sim p \vee (\sim q)] \vee r$

$\equiv \sim (p \wedge q) \vee r \equiv (p \wedge q) \to r$

68. (A)

$P(\overline{A}):P(A)=4:5 \Rightarrow P(A)=\frac{5}{9}$

$P(B):P(\overline{B}):=3:7 \Rightarrow P(B)=\frac{3}{10}$

$=P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=P(A)+P(B)-P(A)P(B)$

$=\frac{5}{9}+\frac{3}{10}-\frac{5}{9}.\frac{3}{10}=\frac{31}{45}$

69. (D)

$$\lim_{x\to\infty}\left(\frac{2+\frac{3}{x}-\frac{5}{x^2}}{3-\frac{4}{x}+\frac{1}{x^2}}\right)^{x+1}=\left(\frac{2}{3}\right)^{\infty}\to 0$$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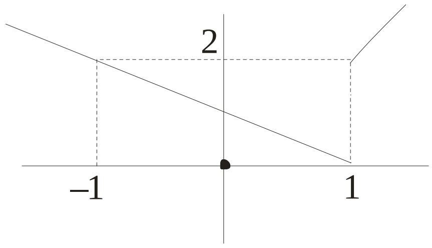
70. (A)

$$f\left(g(x)\right)=\begin{cases}1-x, & x\in(-\infty,1)\\ 1+x, & x\in[1,\infty)\end{cases}$$

$\Rightarrow a \in (1, 2)$




71. (B)

$$T_r=\frac{4r^2+2r-1}{\lfloor(2r+1)}=\frac{2r(2r+1)-1}{\lfloor 2r+1}$$

$$=\frac{1}{\lfloor 2r-1}-\frac{1}{\lfloor 2r+1}$$

$$T_1+T_2+......T_n=\left(\frac{1}{\lfloor 1}-\frac{1}{\lfloor 3}\right)+\left(\frac{1}{\lfloor 3}-\frac{1}{\lfloor 5}\right)+.........+\frac{1}{\lfloor 2n-1}-\frac{1}{\lfloor 2n+1}$$

$=1-\frac{1}{\lfloor 2n+1}$ as $n\to\infty$ $S_\infty\to 1$

Hence sum = 1

72. (B)

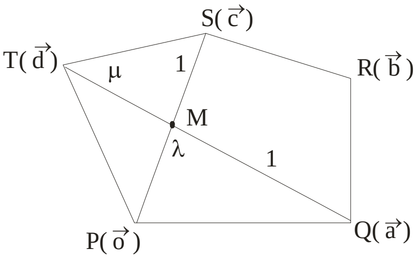
$$\left(\sin^2 x-3\sin x+2)(\sin x+1\right)=K(\sin x+1)$$

$$K=\left(\sin x-\frac{3}{2}\right)^2-\frac{1}{4}$$

$K\in[0,6);(K\neq 6 \because \sin x\neq -1$

K = {1, 2, 3, 4, 5}

73. (B)




Let position vectors of P, Q, R, S, T be $\vec{o},\vec{a},\vec{b},\vec{c},\vec{d}$

$$\left.\begin{matrix}\overrightarrow{PQ}=3\overrightarrow{SR}\Rightarrow\vec{a}=3(\vec{b}-\vec{c})\\ \overrightarrow{QR}=2\overrightarrow{PT}\Rightarrow\vec{b}-\vec{a}=2\vec{d}\end{matrix}\right\}\Rightarrow\frac{\vec{a}+3\vec{c}}{3}=\vec{a}+2\vec{d}$$

$$\Rightarrow\vec{c}=\frac{2\vec{a}+6\vec{d}}{3}$$

$$\therefore M=\frac{\mu\vec{a}+\vec{d}}{\mu+1}=\frac{\lambda\vec{c}}{\lambda+1}=\lambda\frac{\left(\frac{2\vec{a}+6\vec{d}}{3}\right)}{\lambda+1}$$

$$\frac{\mu}{\mu+1}=\frac{2\lambda}{3(\lambda+1)},\frac{1}{\mu+1}=\frac{2\lambda}{(\lambda+1)}$$

$$\frac{2\lambda\mu}{(\lambda+1)} = \frac{2\lambda}{3(\lambda+1)} \Rightarrow \lambda = \frac{1+\sqrt{10}}{3}$$

74. (A)

For n > 1, $2^n$ is a multiple of 4.

Let $2^n = 4K, K \in N$

The digit at units place in $2^4$ is 6.

$\Rightarrow$ The digit at units place in $2^{4K}$ i.e. $2^{2^n}$ is 6

The digit at units place in each of $\lfloor 5, \lfloor 6, ......, \lfloor 100$ is 0, but $\lfloor 0 + \lfloor 1 + ...... + \lfloor 4 = 1+1+2+6+24 = 34$

$\Rightarrow$ The digit at units place in $\sum_{r=0}^{100} \lfloor r = 4$

$\therefore$ The digit at units place in $\sum_{r=0}^{100} \lfloor r + 2^{2^n} = 0$

($\because$ 6 + 4 = 10)

75. (D)

$$\lim_{x\to 0} \frac{x - \left(x - \frac{x^3}{\lfloor 3} + ....\right)}{x} \sin\left(\frac{1}{x}\right)$$

$$= \lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right).\left(\frac{1}{\lfloor 3} + \text{terms of } x\right)$$

= 0

76. (C)

$f''(x) > 0 \ \forall \ x \in R$

$\Rightarrow f'(x)$ is increasing $\forall \ x \in R$

Here, for g(x) to be increasing function $g'(x) > 0$

$-f'(2-x) + f'(4+x) > 0$

$f'(4+x) > f'(2-x)$

But $f'(x)$ is increasing function

4 + x > 2 – x $\qquad \forall \ x \in R$

x > –1

$x \in (-1, \infty)$

77. (B)

$ydx - xdy + 2\sqrt{xy}dy = 0$

Divide by $y^2$

$$\frac{ydx - xdy}{y^2} + 2\sqrt{\frac{xy}{y^2}}\frac{dy}{y} = 0$$

$$\frac{d\left(\frac{x}{y}\right)}{\sqrt{\frac{x}{y}}} + 2\frac{dy}{y} = 0$$

$$2\sqrt{\frac{x}{y}} + 2\ln(y) + 2\ln c = 0$$

$$cy = e^{-\sqrt{\frac{x}{y}}}$$

78. (A)

$$f\left(\frac{\pi}{4}\right)=\int_0^{\cot\left(\frac{\pi}{4}\right)}\tan^{-1}t\ dt+\int_0^{\tan\left(\frac{\pi}{4}\right)}\cot^{-1}t\ dt=\int_0^1\tan^{-1}t\ dt+\int_0^1\cot^{-1}t\,dt$$

$$=\int_0^1\left(\tan^{-1}t+\cot^{-1}t\right)dt=\frac{\pi}{2}\int_0^1 dt=\frac{\pi}{2}$$

79. (A)

$$f(x)=\int\frac{e^{\sin x}}{x}dx$$

$$I=\int_1^4\frac{3e^{\sin x^3}}{x}dx=\int_1^4\frac{3x^2e^{\sin x^3}}{x^3}dx$$

= f(64) – f(1)

$\Rightarrow$ k = 64

80. (A)

$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\tan 8\alpha + 16\cot 16\alpha$

$$=\cot\alpha-2\cot2\alpha+2(\cot2\alpha-2\cot4\alpha)+4(\cot4\alpha-2\cot8\alpha)$$

$$8(\cot8\alpha-2\cot16\alpha)+16\cot16\alpha$$

$$=\cot\alpha-2\cot2\alpha+2\cot2\alpha-4\cot4\alpha+4\cot4\alpha-8\cot8\alpha+8\cot8\alpha-16\cot16\alpha+16\cot16\alpha=\cot\alpha$$

81. (A)

$C_1$ = centre of 1st circle ≡ (0, 0) and radius = $r_1$ = 2 units.

$C_2$ = centre of 2nd circle = (4, 3) and radius = $r_2=\sqrt{16+9+24}=7$

$$\therefore C_1C_2=\sqrt{(0-4)^2+(0-3)^2}=5$$

$$\left|r_2-r_1\right|=|7-2|=5$$

$$C_1C_2=\left|r_2-r_1\right|$$

Both circles touch each other internally.

82. (A)

Let ABCD = E

$$\therefore\left|\text{adj}\left(\text{adj}\left(\text{adj}\left(\text{adj}(ABCD)\right)\right)\right)\right|$$

$$=\left|\text{adj}\left(\text{adj}\left(\text{adj}\left(\text{adj}(E)\right)\right)\right)\right|$$

$$=|E|^{(3-1)^4}=|E|^{16}$$

$$=|ABCD|^{16}=|A|^{16}|B|^{16}|C|^{16}|D|^{16}$$

$$=|A|^{16}.\left(|A|^{3-1}\right)^{16}.\left(|A|^{(3-1)^2}\right)^{16}.\left(|A|^{(3-1)^3}\right)^{16}$$

$$=|A|^{16+32+64+128}=A^K$$

$\Rightarrow$ K = 240; which is less than 256.

83. (A)

$$25^{-\sum_{r=0}^{49}\left[x+\frac{r}{50}\right]}=25^{-[50x]}$$

$$\int_{-5}^{5}\frac{25^{-[50x]}}{25^{-50x}}dx=\int_0^{10}25^{\{50x\}}dx=500\int_0^{\frac{1}{50}}25^{\{50x\}}dx$$

$$=\frac{500}{50\ln25}\left(25^x\right)_0^1=\frac{24\times500}{50\times2\ln5}=\frac{120}{\ln5}$$

84. (D)

From the curve equation

$$3y^2\frac{dy}{dx}-3y-3x\frac{dy}{dx}=0$$

$$\left(y^2-x\right)\frac{dy}{dx}=y$$

But $y \neq 0$ otherwise $2 = 0$ therefore $y^2 = x$

$\Rightarrow y^3 - 3y^3 + 2 = 0$

So that y = 1 and x = 1

$\therefore$ m = 0 and n = 1 thus m + n = 1

85. (A)

$2\{y\} = [x] + 1$

$0 \le y < 1$

$\{y\} = y$ and $\quad 0 \le [x] + 1 < 2$

$-1 \le [x] < 1$

When $-1 < x < 0 \quad y = 0$

$0 \le x < 1 \quad y = \frac{1}{2}$

Area = $\frac{1}{2}$

86. (C)

$$f(g(x)) = g(f(x)) = x$$

$$g'(f(x)).f'(x) = 1$$

$$g''(f(x))f'(x) = \frac{d}{dx}\left(\frac{1}{f'(x)}\right)$$

$$= \frac{-f''(x)}{(f'(x))^2}$$

Put x = 1

$$g''(3) = -\frac{1}{2}$$

87. (B)

$\{x + r\} = \{x\}$ if r is integer

$x + r - [x] - r = x - [x]$

$\Rightarrow f(x) = 1000\{x\}$

$$\left[f\left(\sqrt{2}\right)\right] = [100 \times 0.414] = 414$$

88. (D)

$$\Delta = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0 \text{ for inconsistent}$$

$abc - a - b - c + 2 = 0$

$a \neq 1, b \neq 1, c \neq 1$

89. (B)

From equation

a + b + c = 13

ab + bc + ca = 54

abc = 72

$$\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$$

$$= \frac{b^2 + c^2 - a^2}{2abc} + \frac{a^2 + c^2 - b^2}{2abc} + \frac{a^2 + b^2 - c^2}{2abc}$$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$$= \frac{(a+b+c)^2 - 2(ab+bc+ca)}{2abc} = \frac{61}{144}$$

90. (A)

Centre of circle is (3, –4)

Hence distance of centre from the line

(3x + 4y – 25 = 0)

$$= \left|\frac{9-16-25}{\sqrt{9+16}}\right| = \frac{32}{5} \text{ units}$$

Shortest distance = $\frac{32}{5} - 5 = \frac{7}{5}$ units


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