PART-C : MATHEMATICS

61. If the range of the function $f(x) = \tan^{-1}(3x^2 + bx + d)$ is $\left[0, \frac{\pi}{2}\right)$ then

(A) $b^2 = 3c$ (B) $4c = b^2$ (C) $b^2 = 12c$ (D) $b^2 = c$

62. Let $x \in \left(0, \frac{\pi}{2}\right)$ and $\log_{24\sin x}(24\cos x) = \frac{3}{2}$, then the value of $\operatorname{cosec}^2 x$ is equal to

(A) 3 (B) 6 (C) 8 (D) 9

63. If $|z - 1 - i| = 1$ and $|w - 1 + i| = 2$ where, $w, z \in C$ then maximum value of $\frac{1}{|4z-1|^2} + \frac{1}{|3w+1|^2}$ is equal to

(A) 1 (B) 3 (C) 2 (D) 4

64. Let the relation between x and y be defined implicitly by the equation $x + 5y - y^5 = 0$. Find the area of the triangle formed by tangent and normal at $x = 0$ such that $|y|$ is minimum and line $y = 5$.

(A) 65 sq. units (B) 70 sq. units (C) 75 sq. units (D) 85 sq. units

65. If three angles A, B, C are such that $\cos A + \cos B + \cos C = 0$ and if $\cos A \cos B \cos C = \lambda(\cos 3A + \cos 3B + \cos 3C)$, then λ is equal to:

(A) $\frac{1}{12}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

66. Let R be a relation defined by $R = \{(a,b) \mid a \geq b;\ a, b \in R\}$, then R is

(A) only reflexive (B) both reflexive and transitive
(C) symmetric, transitive but not reflexive (D) neither transitive nor reflexive but symmetric

67. If p, q, r be three statement, then $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$ is a

(A) Tautology (B) Fallacy
(C) Neither tautology nor fallacy (D) None of these

68. The odds against an event is 4:5 and the odds in favour of another event is 3:7. If both the events are independent, then the probability that at least one of the event will happen is

(A) $\frac{31}{45}$ (B) $\frac{77}{90}$ (C) $\frac{1}{6}$ (D) $\frac{5}{6}$

69. $\lim_{x\to\infty}\left(\frac{2x^2+3x-5}{3x^2-4x+1}\right)^{x+1}$ is equal to

(A) $\frac{2}{3}$ (B) 1 (C) $e^{\frac{2}{3}}$ (D) 0

70. If $f(x)=|x-2|, g(x)=\begin{cases}3-x, & x<1\\ x+3, & x\geq 1\end{cases}$, then the set of values of a, such that the equation $f(g(x))=a$ has exactly one negative solution is

(A) $a \in (1, 2)$ (B) $a \in (0, 3)$ (C) $a \in (-1, 1)$ (D) $a \in \phi$

71. The sum of infinite series $\frac{5}{\underline{|3}}+\frac{19}{\underline{|5}}+\frac{41}{\underline{|7}}+\frac{71}{\underline{|9}}+\ldots\ldots$ is

(A) $\frac{3}{2}$ (B) 1 (C) $\frac{9}{2}$ (D) 2

72. If $\sin^3 x-2\sin^2 x-(K+1)\sin x+2-K=0; \sin x\neq -1$ posses a solution for finite integral values of K only, then the number of positive integral value of K are equal to

(A) 4 (B) 5 (C) 6 (D) 7

73. Let PQRST be a pentagon in which the sides PQ and RS are parallel and sides TP and QR are parallel. If PQ : RS is 3 : 1 and TP : QR is 1 : 2 and diagonals PS and QT meet at M, then PM : MS equals

(A) 3 : 1 (B) $1+\sqrt{10}:3$ (C) 2 : 1 (D) 1 : 2

74. For integer n > 1, the digit at units place in the number $\sum_{r=0}^{100} \underline{|r} + 2^{2^n}$ is

(A) 0 (B) 1 (C) 2 (D) 4

75. $\lim_{x\to 0}\left(\frac{x-\sin x}{x}\right)\sin\left(\frac{1}{x}\right)$ is

(A) Non-existent (B) 1 (C) –1 (D) 0

76. Let $f''(x)>0\ \forall\ x\in R$ and $g(x)=f(2-x)+f(4+x)$. Then g(x) is increasing in

(A) $(-\infty, -1)$ (B) $(-\infty, 0)$ (C) $(-1, \infty)$ (D) $(1, \infty)$

77. The solution of differential equation $y\,dx+\left(2\sqrt{xy}-x\right)dy=0$ is

(A) $cy=e^{\sqrt{x/y}}$ (B) $cy=e^{-\sqrt{x/y}}$ (C) $cy=e^{x/y}$ (D) $cy=e^{\sqrt{2x/y}}$

78. If f(x) = $\int_0^{\cot x}\tan^{-1}(t)dt+\int_0^{\tan x}\cot^{-1}t\,dt$, if $0<x<\frac{\pi}{2}$, then $f\left(\frac{\pi}{4}\right)$ is equal to

(A) $\frac{\pi}{2}$ (B) $-\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$

79. If $\frac{d}{dx}f(x)=\frac{e^{\sin x}}{x}$, $x>0$ and $\int_1^4 \frac{3e^{\sin x^3}}{x}dx=f(k)-f(1)$ then one possible value of k is

(A) 64 (B) 32 (C) 16 (D) 8

80. The value of $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\tan 8\alpha + 16\cot 16\alpha$ is

(A) $\cot\alpha$ (B) $\cos\alpha$ (C) $\cot 2\alpha$ (D) $\tan 2\alpha$

81. The number of common tangents for circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 8x - 6y - 24 = 0$ is

(A) 1 (B) 2 (C) 3 (D) 4

82. Let A be a matrix of order 3×3 and matrices B, C, D are related such that $B = \text{adj}(A)$, $C = \text{adj}(\text{adj } A)$, $D = \text{adj}\left(\text{adj}\left(\text{adj}(A)\right)\right)$. If $\left|\text{adj}\left(\text{adj}\left(\text{adj}\left(\text{adj}(ABCD)\right)\right)\right)\right|$ is A^k then k

(A) is less than 256 (B) has 21 divisors (C) greater than 256 (D) is an odd number

83. If $\displaystyle\int_{-5}^{5} \frac{25^{-\sum_{r=0}^{49}\left[x+\frac{r}{50}\right]}}{5^{-100x}}dx$ is equal to ([.] denotes greatest integer function)

(A) $\dfrac{120}{\ln 5}$ (B) $\dfrac{240}{\ln 5}$ (C) $\dfrac{60}{\ln 5}$ (D) $\dfrac{250}{\ln 5}$

84. Let C be the curve $y^3 - 3xy + 2 = 0$. Let m be the number of points on C at which tangents are horizontal and n be the number of point on C at which tangents is vertical then 'm + n' is equal to

(A) 4 (B) 3 (C) 2 (D) 1

85. The area under the curve $2\{y\} = [x] + 1$, $0 \le y < 1$ (where {.} and [.] are the fractional part and greatest integer functions respectively) and the x axis is (in square units)

(A) $\dfrac{1}{2}$ (B) 1 (C) 0 (D) $\dfrac{3}{2}$

86. If $f(1) = 3, f'(1) = 2$ and $f''(1) = 4$ and let $f^{-1}(x) = g(x)$, then $g''(3)$ is equal to

(A) –2 (B) 2 (C) $-\dfrac{1}{2}$ (D) $\dfrac{1}{4}$

87. $f(x) = \{x\} + \{x + 1\} + \{x + 2\} + \ldots\ldots + \{x + 999\}$ then $\left[f\left(\sqrt{2}\right)\right]$ (where {.} denotes fractional part of x and [.] denotes greatest integer of x) is equal to

(A) 999×500 (B) 414 (C) 4140 (D) 510101

88. If system of equations $ax + y + z = a$, $x + by + z = b$ and $x + y + cz = c$ is inconsistent, then which of the following is correct?

(A) $abc - a - b - c + 2 = 0$ (B) $abc - a - b - c + 3 = 0$
(C) $abc - a - b - c + 3 = 0$, $a = 1$ (D) $abc - a - b - c + 2 = 0$, $a \ne 1$, $b \ne 1$, $c \ne 1$

89. If the sides a, b, c of a triangle ABC are the roots of the equation $x^3 - 13x^2 + 54x - 72 = 0$, then the value of $\dfrac{\cos A}{a} + \dfrac{\cos B}{b} + \dfrac{\cos C}{c}$ is equal to

(A) $\dfrac{59}{144}$ (B) $\dfrac{61}{144}$ (C) $\dfrac{61}{72}$ (D) $\dfrac{32}{5}$

90. The shortest distance from the line $3x + 4y = 25$ to the circle $x^2 + y^2 - 6x + 8y = 0$ is equal to (in units)

(A) $\dfrac{7}{5}$ (B) $\dfrac{9}{5}$ (C) $\dfrac{12}{5}$ (D) $\dfrac{32}{5}$